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主　論　文　の　要　旨

論文題目　Optimal Control of Continuous and Discrete Time Systems via Generating Functions （母関数を用いた連続時間及び離散時間システムの最適制御）

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論　文　内　容　の　要　旨

Optimal control which deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved, is one of the dynamic optimization techniques popularly used in robotics, computer science, and operations research. There are two major tools for studying optimal control problems, one is the minimum principle, and the other the dynamic programming. After decades of the development, there have had many other methods for studying optimally controlled systems, among which the recently proposed generating function method which exhibits theoretical insights in solving optimal control problems and practical implication for real world applications attracts increasing attention of the researchers. In theory, this method thoroughly exploit optimal control problems' geometric structures, by utilizing Hamiltonian systems' characteristics, e.g. canonical transformation, symmetry, symplecticity, and so on. In practical computation, the method moves a large amount of computational effort to the off-line part such that it is substantially useful in on-line solutions repetitive generation for different state boundary conditions.

So far, the generating function method has been studied in existing literature to solve a small number of problems that there still has a large space for this thesis to develop the related theory and extend the method for solving other typical problems in continuous and discrete time cases, including extending the generating function method to solve continuous-time state constrained problems, developing the double generating functions method for discrete-time LQ optimal control with numerical stability analysis of the optimal generators, and solving the discrete-time nonlinear optimal control problems via generating functions.

Chapter 2 introduces preliminaries of the Hamiltonian system and the generating functions. For the continuous and discrete time optimal control problems, we give necessary and sufficient (only for continuous-time case) conditions for optimality, derive Hamilton-Jacobi equations and generating functions, provide optimal solutions (only for continuous-time case), present relations between the generating function and the value function, and exhibit the LQ cases.

Chapter 3 extends the generating function method to the Hard Constraint Problem with inequality path constraints, i.e. the classical path and terminal state constrained optimal control problem. First, we formulate and design the constrained problem and the penalized Hard Constraint Problem respectively, show their convex properties, and further exhibit the convergence of the minimum cost function value and optimal solutions between these two problems under a mild condition. Second, due to the technique of Taylor series expansion, the partial differential Hamilton-Jacobi equation is reduced to ordinary differential equations for the generating function coefficients. We give the recursive condition to eliminate the coupling relations between the coefficients with lower and higher indices in these ordinary differential equations so that they can be solved recursively. This guarantees the penalized Hard Constraint Problem can be successfully solved by the generating function method. Based on this, we summarize how to design penalties which is suitable for the generating function method, and gives an algorithm presents how to generate optimal solutions repetitively for different boundary conditions.

Chapter 4 develops the discrete analogue of double generating function method. To clearly present the fundamental feature of this method and to make it convenient for further extension to the nonlinear problems, this thesis investigates the classical discrete-time LQ optimal control problem. First, we derive the left discrete Hamiltonian, Hamilton's equations, and the Hamilton-Jacobi equation for the LQ optimal control problem which is a counterpart to the right ones in the references according to discrete mechanics. Second, we choose appropriate Hamilton-Jacobi equation, left or right, to solve for the forward type II, III, and backward type III generating functions. Then by selecting any two different generating functions from the four single ones, we can construct six double generating functions which give six generators for optimal solutions, respectively. These discrete generators maintain the advantage of on-line efficient computation for different boundary conditions, which is presented by a followed algorithm. Besides, since each generator contains inverse terms, we deeply perform the numerical stability analysis to conclude that the terms in the generators constructed by double generating functions with opposite time directions are invertible under some mild conditions, while the terms with the same time directions will become singular when the time goes infinity which may cause instability in numerical computations.

Chapter 5 develops the generating function method for the general discrete-time nonlinear optimal control problems. First, we give the analytically optimal solutions, which is expressed as the state feedforward-control in terms of the generating functions. Then in the numerical implementations, we systematically perform three steps to solve the Hamilton-Jacobi equation for the generating functions. In detail, we expand all the nonlinear functions in the Hamilton-Jacobi equation as Taylor series about zeros in tensor notations such that they can clearly present the detailed structure of the Hamilton-Jacobi equation later during the reduction. Based on this, we again employ the Taylor series technique to successfully replace one variable by the other two in the Hamilton-Jacobi equation to rewrite it by the addressed theorem in the thesis. Due to this step, we achieve our objective that the Hamilton-Jacobi equation is reduced to the difference equations for the generating function coefficients, and they can be solved recursively with respect to the order of the Taylor series. The developed numerical framework can give the optimal solutions in terms of the pre-computed generating function coefficients and boundary conditions, such that we can divide the whole computation into two parts, the off-line part calculates the coefficients in advance, and the on-line part efficiently generates optimal solutions for different boundary conditions. From this viewpoint, it is useful for the on-demand optimal solutions generation for different boundary conditions.

Chapter 6 presents a brief conclusion of the research carried out in this thesis. This is followed by summarizing remarks and suggestions for the future research.

The generating function method exhibits theoretical insights in solving optimal control problems and practical implication for real world applications. After one decade of the development, there still has significant potentiality in its further research. Future work includes the study of convergence region of the Taylor series to Hamilton-Jacobi equation and the extension of the generating function method to solve stochastic optimal control problems.