The Welfare Effects of Oligopolistic Third-Degree Price Discrimination when Own and Cross Price Elasticities Are Constants

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This study examines the welfare effects of oligopolistic third-degree price discrimination with constant own and cross price elasticities of demand under product differentiation. We verify the robustness of Adachi and Matsushima's (2014) finding on social welfare under linear demand, that price discrimination is more likely to improve social welfare for a higher value of cross price elasticity in a "strong" market (where the discriminatory price is higher than the uniform price). In contrast to Aguirre and Cowan's (2015) results for a monopoly, social welfare can be higher with price discrimination even if the relative share of the strong market under uniform pricing or the own elasticity difference between the two markets is sufficiently small. Consumer surplus can also be higher with price discrimination if the cross price elasticities are sufficiently low. This suggests that Adachi and Matsushima's (2014) result on consumer surplus (price discrimination never improves social welfare) hinges on the assumption of linearity.

Keywords: Third-Degree Price Discriminations; Oligopoly; Elasticities.

Ⅰ. Introduction

This study examines oligopolistic third-degree price discrimination when the demand in each discriminatory submarket has constant own and cross price elasticities to examine an important question since Pigou (1920) and Robinson (1933): under what conditions does third-degree price discrimination raise social welfare? In many applied studies examining the welfare effects of third-degree price discrimination, researchers typically assume that the demand in each discriminatory submarket belongs to the same functional family: linear demand is often assumed.[1)] In other theoretical studies, researchers consider nonlinear demand in a nonrestrictive manner. For example, Aguirre, Cowan, and Vickers (2010) consider the curvatures of submarket demand to synthesize the existing studies[2)] of output and welfare effects of *monopolistic* third-degree price discrimination. Notably, Proposition 2 in Aguirre, Cowan, and Vickers (2010) states that if the inverse demand is more convex a weak market than in a strong market, then price discrimination raises social welfare. However, no comparable characterization has yet come to light in the case of an oligopoly with nonlinear demand.[3)] Thus, this study aims to take one step forward toward understanding *oligopolistic* third-degree price discrimination by considering one of the most "popular" classes of nonlinear demand, namely, "log-linear" (Davis and Garcés (2009, p.447)) demand with constant own and cross price elasticities. We also review the robustness of Adachi and Matsushima's (2014) study of oligopolistic third-degree price discrimination with linear demand.

To understand Adachi and Matsushima's (2014) necessary and sufficient condition for oligopolistic third-degree price discrimination to improve social welfare, consider the case of two geographical markets, one a hot resort area, and the other a city area. Two beverage companies compete by selling their own (one) product (such as cola) in both markets. Resale by nonfirm agents for arbitrage is assumed impossible. The cost of production and sales is assumed common for both markets (for example, there are no differences in transportation costs between the markets). Although the firms are symmetric (i.e., homogeneous), consumers are not indifferent (except for the marginal ones) between the two products (given that the firms' prices are the same). In fact, horizontal product differentiation makes some consumers prefer one firm's product to the other's.[4)]

Adachi and Matsushima (2014) find that price discrimination raises social welfare if and only if the degree of substitution between two products is *sufficiently higher* in the "strong" market (where the discriminatory price is higher than the uniform price) than in the "weak" market (where it is lower).[5)] If consumers care less about firm brand in the hot resort area (because when people are thirstier it is natural for them to be less concerned about brands), then price discrimination may improve social welfare. One might think that substitution in the weak market, not the strong market, should be sufficiently higher, because fiercer competition (due to higher level of substitution) in the weak market increases the aggregate output more. However, weakening the *misallocation effect* caused by price discrimination is more important than this *output effect*: if substitution is sufficiently high in the strong market, the price increase caused by price discrimination will be small, resulting in a less number of consumers in the strong market giving up consumption after price discrimination is introduced. This benefits social welfare because, on average, consumers in the strong market have higher willingness to pay than those in the weak market *both* under uniform pricing and under price discrimination.

Note that the argument so far does not rely on the linearity of demand. We thus conjecture that this result and the intuition of Adachi and Matsushima (2014) on social welfare (see their Proposition 1) are robust to nonlinear demand. In Adachi and Matsushima's (2014) formulation, the degree of substitution in each market is characterized by one constant parameter (because of linearity) that is separable from other variables and parameters. This parameter, with normalization, coincides with cross price elasticity in equilibrium, though not elasticity *per se*. In particular, normalization needs to take into account the slope of linear demand, which in itself is less relevant to marginal conditions. Furthermore, the linearity restriction forces own price elasticity to be always one in equilibrium. Does the intuition above survive under nonlinear demand? By investigating constant own and cross price elasticities of demand, we provide a positive answer to this conjecture. In contrast, Adachi and Matsushima's (2014) result on aggregate consumer surplus (see their Proposition 2) may depend on the linearity of demand. Their Proposition 2 shows that price discrimination *always* lowers aggregate consumer surplus. This is true even if it raises social welfare. This result implies that firms "squeeze" all the surplus generated by welfare-improving price discrimination as described above. Is this still true if market demand is nonlinear?

In this paper, we extend Aguirre and Cowan's (2015) analysis of monopolistic third-degree price discrimination with constant elasticity demand[6)] to the case of differentiated oligopoly.[7)] As in Adachi and Matsushima (2014), we express the degree of product differentiation in a submarket by one parameter; however, in contrast to Adachi and Matsushima (2014), the parameter in the present paper is the cross price elasticity itself. Aguirre and Cowan (2015) show that price discrimination can raise social welfare if the output share of the strong market under uniform pricing (α in their and our notation) is sufficiently large and the (own) elasticity difference between the two markets is also sufficiently large (θ in their and our notation, which is the elasticity difference between the two markets). Aguirre and Cowan (2015) also show that price discrimination can raise consumer surplus under stricter conditions. Note the similarity between Aguirre and Cowan's (2015) sufficient condition and Adachi and Matsushima's (2014) necessary and sufficient condition. First, a fiercer level of competition in the strong market would raise the production in that market. Second, Proposition 1 in Adachi and Matsushima (2014) implies that the elasticity difference must be sufficiently high. In essence, the substitution parameter plays an important role in affecting the equilibrium elasticity in each submarket. Thus, in equilibrium, oligopolistic firms can be considered monopolistic if the strategic interactions are already incorporated: the intuition for welfare improvement in case of an oligopoly is similar to that in case of a monopoly once strategic interactions are taken into account. We also confirm that consumer surplus can be higher with price discrimination if the cross price elasticities are sufficiently low. This suggests that Adachi and Matsushima's (2014) result on consumer surplus hinges on the assumption of linearity.[8)]

One might think that greater competition in the strong market where the price rises after price discrimination is introduced is beneficial for a positive change in social welfare because greater competition would suppress an sharp increase in the price in the strong market, yielding a small decrease in output, and thus a less welfare loss. One might also expect that greater competition in the weak market where the price is lowered by price discrimination is beneficial for a welfare improvement, yielding a greater increase in the output in the weak market. As Holmes (1989) suggests, an increase in aggregate output is necessary for a welfare improvement by price discrimination, it seems natural that greater competition in *both* markets is necessary for a positive change in social welfare. However, it turns out that, in symmetric equilibrium, while the former reasoning is correct, the latter is not correct: somewhat counter-intuitively, *less* competition in the weak market is beneficial for a greater increase in the output in the weak market, and thus a welfare improvement. This is because for the weak market as a whole to be price sensitive greatly, consumers in the weak market should, in the face of an increase in one firm's price, find it difficult to switch to its rival's product. If so, they are more likely to exit from the weak market, stopping purchase of any products in the market. Because the weak market is actually where the price lowers after price discrimination, less and less substitutability implies more and more consumers are brought on to the market in response to a drop in the price, caring less about which brand to purchase.

The rest of the paper is organized as follows. The next section presents the model. We provide our welfare analysis in Section 3. Both theoretical and numerical results are presented. Section 4 concludes the paper.

Ⅱ. The Model

Consider $J(\geq 2)$ oligopolistic firms producing (horizontally) differentiated products and competing in price to sell their products (directly) to consumers. Each firm produces and sells only one type of product under its brand. The whole market can be segmented into independent $M(\geq 2)$ separate submarkets (hereafter called markets in case no ambiguity arises) based on identifiable signals (e.g., geography, age, and gender). If a firm implements uniform pricing, the price that consumers pay is uniform across all markets, but if the firm discriminates in prices across markets, consumers may have to pay different unit prices for the firm's product depending on the market they belong to. We assume that the product of a price discriminating firm cannot be resold across markets.

In this paper, we consider symmetric firms, as in Holmes (1989), and hence assume a common marginal cost for all firms. In addition, we assume that the marginal cost is constant, $c>0$. For further simplicity, we mainly consider the case of two firms (A and B) and two markets in the following analysis. Specifically, we use indices $i, j \in \{A, B\}$ for firms and index $m \in \{s, w\}$ for markets (s denotes (a set of) strong markets and w weak markets; these will be clear below in Subsection Ⅱ.2). Note, however, that our main results can be extended to the case of $J(\geq 3)$ firms and $M(\geq 3)$ markets as long as the firms are symmetric and the following s and w are considered (arbitrarily) to represent all markets.

The demand function of firm i in market m is given by $q_m^i(p_m^i, p_m^j) = a_m(p_m^i)^{-\varepsilon_m}(p_m^j)^{\sigma_m}$, where $a_2>0$ is the measure of market size, $\varepsilon_m>1$ the constant *own* price elasticity (note that $(\partial q_m^i/\partial p_m^i)(p_m^i/q_m^i) = -\varepsilon_m)$[9] and σ_m, which is assumed to be less than ε_m (the next paragraph explains the reason for this restriction), is the constant *cross* price elasticity (note that $(\partial q_m^i/\partial p_m^j)(p_m^j/q_m^i) = \sigma_m)$ that captures the *degree of product differentiation* (note that our demand form is equivalent to the following familiar form of log-linear demand: $\ln q_m^i = \ln a_m - \varepsilon_m \ln p_m^i + \sigma_m \ln p_m^j$). The assumption of identical firms requires that ε_m and σ_m are common for all firms.

Note here that ε_m *per se* indicates the total percentage of customers who will leave the firm if it raises its price by 1 percent, and *not* how many of them will switch to the other firm. Because the other firm gains σ_m percent of them as new customers, $(\varepsilon_m - \sigma_m)$ percent of the customers leave the market (i.e., they purchase no products), and σ_m percent of them switch to the other firm (i.e., they purchase the other firm's product) as a response to the 1 percent increase in the firm's price. Naturally, $(\varepsilon_m - \sigma_m)$ should be positive (thus, $\sigma_m < \varepsilon_m$ as described above).[10]

If $\sigma_m = 0$, the products of the two firms are *independent* in each market; that is, one firm's demand is not affected by the other firm's price. In other words, marginal consumers are not better off by switching to the other firm and thus leave the market only if the price goes up. In this case, the two firms behave as identical monopolists in each market: the demand function is identical (with rescaling) to the one in Aguirre and Cowan's (2015) analysis of monopoly. If $\sigma_m > 0$, the products are *substitutes*. As σ_m approaches ε_m, the competition in market m becomes fiercer. In the extreme case of $\sigma_m \approx \varepsilon_m$, all marginal consumers switch to the rival since the two products are homogeneous and (almost) perfect substitutes.

In the present study, we consider the case of substitutes only (i.e., σ_m is positive): we assume no *complementarity*} as opposed to Adachi and Matsushima's (2014) analysis with linear demand. This is because consumer surplus, as defined in the next section, assumes that consumers are segmented into three groups: (i) those who purchase product A, (ii) those who purchase product B, and (iii) those who purchase nothing. If complementarity between two products is allowed, we need to consider another type of consumer: those who purchase both products. Thus, we simply assume $\sigma_m > 0$ for $m \in \{s, w\}$ throughout the paper. In addition, we, following Aguirre and Cowan (2015), assume that the elasticity in market w is greater than that in market s: $\varepsilon_w = \varepsilon_s + \theta$ where $\theta > 0$ is the *own price elasticity difference* (more precisely, condition $\varepsilon_w > \varepsilon_s$ makes market w the weak market).

We consider two regimes, uniform pricing $(r=U)$ and price discrimination $(r=D)$. Under uniform pricing, firms set a common unit price for all separate markets, and under price discrimination, they set a different price in each market.[11] Furthermore, we impose symmetry on demand $q_m^j(p', p'') = q_m^{-j}(p'', p')$ to focus on a symmetric equilibrium where all firms set the same price in one market. With little abuse of notation, let $q_m(p) = q_m^A(p, p)$.

1. Price Discrimination

First, suppose that firm i sets a discriminatory price p_m^i in each market m. The profit function of firm i from market m can be given by $\pi_m^i(p_m^i, p_m^j) = (p_m^i - c)a_m(p_m^i)^{-\varepsilon_m}(p_m^j)^{\sigma_m}$. From the first-order condition of p_m^i, we can obtain the discriminatory price in symmetric equilibrium from

$$\frac{\partial \pi_m^i}{\partial p_m^i} = a_m(p_m^i)^{-\varepsilon_m}(p_m^j)^{\sigma_m}\left[1 - \frac{p_m^i - c}{p_m^i}\varepsilon_m\right] = 0$$

$$\Rightarrow p_m^* = \frac{\varepsilon_m}{\varepsilon_m - 1}c,$$

where superscript $*$ denotes the equilibrium under price discrimination.[12] Here, the discriminatory price in the symmetric equilibrium is (a bit surprisingly) *independent of cross price elasticity*, σ_m, and *coincides with the monopolistic discriminatory price* in Aguirre and Cowan (2015). In other words, p_m^* consists of a *dominant strategy*.[13] To explain why this is so, we write the first-order condition in general as

$$q_m^i(p_m^i, p_m^j) + (p_m^i - c)\frac{\partial q_m^i}{\partial p_m^i}(p_m^i, p_m^j) = 0$$

$$\Leftrightarrow \frac{p_m^i - c}{p_m^i} = -\frac{q_m^i / p_m^i}{\partial q_m^i / \partial p_m^i},$$

which, known as the Lerner condition, essentially implies that the competing firms' problem can be seen as a *monopolist's* problem under residual demand given the other firms' prices. The right-hand side gives the inverse of the firm's *own* price elasticity. It is in general a function of p_m^i and p_m^j. In our demand specification, however, firm i's own elasticity is independent of p_m^j (moreover, it is a constant, ε_m, which is also independent of p_m^i. This also makes firm i's optimal price independent of its belief about p_m^j.

Thus, firm i's output in market m, the aggregate output in market m, and the aggregate output in the industry are

$$q_m^i = a_m\left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)},$$

$$Q_m^* = 2a_m\left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)},$$

$$Q^* = 2\sum_{m=s,w} a_m\left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)},$$

respectively. The equilibrium profit of firm i from market m is

$$\pi_m^* \equiv [p_m^* - c]q_m^* = a_m c^{1-(\varepsilon_m - \sigma_m)}\frac{(\varepsilon_m)^{-(\varepsilon_m - \sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m - \sigma_m)}},$$

and, thus, the total profit of firm i can be written as[14]

$$\Pi^* \equiv \sum_{m=s,w}\pi_m^* = a_m\sum_{m=s,w} a_m c^{1-(\varepsilon_m - \sigma_m)}\frac{(\varepsilon_m)^{-(\varepsilon_m - \sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m - \sigma_m)}}.$$

2. Uniform Pricing

Next, we consider the case of uniform pricing. The profit function of firm i is given by $\pi_s^i(p^i, p^j) + \pi_w^i(p^i, p^j) = (p^i - c)\Sigma_{m=s,w}\, q_m^i(p^i, p^j) = (p^i - c)\Sigma_{m=s,w}\, a_m(p^i)^{-\varepsilon_m}(p^j)^{\sigma_m}$. By the first-order condition of p^i, we have[15]

$$\frac{\partial(\pi_s^i + \pi_w^i)}{\partial p^i} = \sum_{m=s,w}[a_m(p^i)^{-\varepsilon_m}(p^j)^{\sigma_m} + (p^i - c)a_m(-\varepsilon_m)(p^i)^{-\varepsilon_m - 1}(p^j)^{\sigma_m}] = 0.$$

Note that the first-order condition is reformulated as the Lerner condition under uniform pricing:

$$\frac{p^i - c}{p^i} = \frac{1}{\dfrac{\Sigma_{m=s,w}\,\varepsilon_m q_m^i(p^i, p^j)}{\Sigma_{m=s,w}\, q_m^i(p^i, p^j)}},$$

where the right-hand side is *no longer independent* of p^j. Here, the elasticity firm i takes into account is the average sum of own elasticities over markets *weighted by the outputs*. The symmetric equilibrium, where $p^i = p^j$, satisfies

$$\frac{p^0 - c}{p^0} = \frac{1}{\varepsilon(p^0)}, \tag{1}$$

where

$$\varepsilon_m(p^0) = \frac{\Sigma_{m=s,w}\,\varepsilon_m a_m(p^0)^{-(\varepsilon_m - \sigma_m)}}{\Sigma_{m=s,w}\, a_m(p^0)^{-(\varepsilon_m - \sigma_m)}},$$

and superscript 0 denotes the equilibrium outcome under uniform pricing.[16] Therefore, firm i's output in market m, the aggregate output in market m, and the aggregate output in the industry are $q_m^0 = a_m q_m(p^0, p^0) = a_m (p^0)^{-(\varepsilon_m - \sigma_m)}$, $Q_m^0 = 2a_m (p^0)^{-(\varepsilon_m - \sigma_m)}$, and $Q^0 = 2\sum_{m=s,w} a_m (p^0)^{-(\varepsilon_m - \sigma_m)}$, respectively. The differences between the quantities under price discrimination and uniform pricing are given by

$$\Delta q_m^* \equiv q_m^* - q_m^0 = a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right],$$

$$\Delta Q_m = Q_m^* - Q_m^0 = 2a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right],$$

$$\Delta Q = 2 \sum_{m=s,w} a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right]$$

respectively.

Now, following Formby, Layson, and Smith (1983), Galera and Zaratiegui (2006), Aguirre (2006), and Aguirre and Cowan (2015), we normalize the optimal uniform price to be one, $p^0 = 1$. In this case, $q_m^0 = a_m$, $Q_m^0 = 2a_m$, and $Q^0 = 2(a_s - a_w)$. The averaged elasticity is also simplified as $\varepsilon(1) = (\varepsilon_s a_s + \varepsilon_w a_w)/(a_s + a_w)$. Given Equation (1) and $p^0 = 1$, the marginal cost satisfies: $c = [a_s(\varepsilon_s - 1) + a_w(\varepsilon_w - 1)]/(a_s \varepsilon_s + a_w \varepsilon_w)$.

Each firm's equilibrium aggregate output under uniform pricing is given by $a_s + a_w$. Let the share of the strong market under uniform pricing be $\alpha \equiv a_s/(a_s + a_w)$. Analogously, the share of the weak market is defined by $1 - \alpha = a_w/(a_s + a_w)$. As a further normalization, we also assume that $a_s + a_w = 1$. Thus, under this normalization, a_m denotes the relative share of market m under uniform pricing. The above equation for c can be written as $c = [\varepsilon_s + (1-\alpha)\theta - 1]/[\varepsilon_s + (1-\alpha)\theta]$, which is less than $p^0 = 1$.[17]

In the literature of third-degree market price discrimination, a market is called *strong* if the discriminatory price is higher than the uniform price, and *weak* if the opposite is true. Now,

$$p_s^* = \frac{\varepsilon_s}{\varepsilon_s - 1} \cdot \frac{\varepsilon_s + (1-\alpha)\theta - 1}{\varepsilon_s + (1-\alpha)\theta} = \frac{\varepsilon_s^2 + (1-\alpha)\theta\varepsilon_s - \varepsilon_s}{\varepsilon_s^2 + (1-\alpha)\theta\varepsilon_s - \varepsilon_s - (1-\alpha)\theta} > 1 = p^0 (\text{because } \theta > 0)$$

$$> \frac{\varepsilon_s \varepsilon_w + (1-\alpha)\theta\varepsilon_w - \varepsilon_s - \theta}{\varepsilon_s \varepsilon_w + (1-\alpha)\theta\varepsilon_w - \varepsilon_s - \theta + \alpha\theta} = p_w^* > c.$$

Thus, market s and market w are verified to satisfy the definition. Note here that as $\theta \to 0$, $p_s^* \to 1$ and $p_w^* \to 1$.[18]

Finally, the equilibrium profit of firm i from market m is $\pi_m^0 \equiv [p_m^0 - c] q_m^0 = (1-c) a_m$, and, thus, the total profit of firm i can be written as $\Pi^0 \equiv \sum_{m=s,w} \pi_m^0 = (1-c)(a_s + a_w) = 1 - c$. Thus, the profit change between the two regimes is

$$\Delta \Pi = \Pi^* - \Pi^0 = \sum_{m=s,w} \frac{a_m c^{1-(\varepsilon_m - \sigma_m)} (\varepsilon_m)^{-(\varepsilon_m - \sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m - \sigma_m)}} - (1-c).$$

Ⅲ. Welfare Analysis

With regard to a monopoly with constant-elasticity submarket demand, Aguirre and Cowan (2015) show that price discrimination raises social welfare if the share of the strong market under uniform pricing (α)[19] as well as the elasticity difference between markets (θ) are sufficiently high. If either parameter is still higher, price discrimination raises consumer surplus as well. More precisely, Aguirre and Cowan (2015) show that if $\Delta SW \geq 0$, then $\theta\alpha > 1$. Intuitively, θ needs to be sufficiently large for a nonnegative welfare change. If θ is large, it means that price elasticity in the weak market is sufficiently large relative to that in the strong market. In other words, the strong market is sufficiently price inelastic relative to the weak market. Similar to the argument in Adachi and Matsushima's (2014) analysis of oligopoly, this helps to weaken the distortion in the strong market (i.e., the output decrease in the strong market is kept small relative to the output increase in the weak market). In this section, we argue that *it is not necessary that $\theta\alpha > 1$ for price discrimination to raise social welfare in case of an oligopoly.* As expected, the two cross elasticity parameters σ_s and σ_w play an important role.

1. Theoretical Results

First, note that under symmetric equilibrium, the equilibrium demand for firm i in market m is given by $q_m^i(p_m, p_m) = a_m(p_m)^{-\varepsilon_m}(p_m)^{\sigma_m} = a_m(p_m)^{-(\varepsilon_m-\sigma_m)}$, where p_m is the discriminatory or uniform price. This provides the inverse demand function in symmetric equilibrium for firm i: $p_m(q_m) = a_m^{1/(\varepsilon_m-\sigma_m)} q_m^{-1/(\varepsilon_m-\sigma_m)}$. Then, let SW_m^r represent the social welfare in market $m = s, w$ under regime $r = U, D$. Because of the symmetry, we define SW_m^r by[20]

$$SW_m^r \equiv 2\int_0^{q_m^r}\left(a_m^{\frac{1}{\varepsilon m-\sigma_m}} q^{-\frac{1}{\varepsilon m-\sigma_m}} - c\right)dq.$$

Therefore, the per-firm change in social welfare can be written as

$$\begin{aligned}
\frac{\Delta SW}{2} &= a_s^{\frac{1}{\varepsilon_s-\sigma_s}}\frac{\varepsilon_s-\sigma_s}{\varepsilon_s-\sigma_s-1}[q_s^*]^{\frac{\varepsilon_s-\sigma_s-1}{\varepsilon_s-\sigma_s}} - cq_s^* \\
&\quad + a_w^{\frac{1}{\varepsilon_w-\sigma_w}}\frac{\varepsilon_w-\sigma_w}{\varepsilon_w-\sigma_w-1}[q_w^*]^{\frac{\varepsilon_w-\sigma_w-1}{\varepsilon_w-\sigma_w}} - cq_w^* \\
&\quad - a_s^{\frac{1}{\varepsilon_s-\sigma_s}}\frac{\varepsilon_s-\sigma_s}{\varepsilon_s-\sigma_s-1}[q_s^0]^{\frac{\varepsilon_s-\sigma_s-1}{\varepsilon_s-\sigma_s}} + cq_s^0 \\
&\quad - a_w^{\frac{1}{\varepsilon_w-\sigma_w}}\frac{\varepsilon_w-\sigma_w}{\varepsilon_w-\sigma_w-1}[q_w^0]^{\frac{\varepsilon_w-\sigma_w-1}{\varepsilon_w-\sigma_w}} + cq_w^0 \\
&= \frac{\alpha}{\varepsilon_s-\sigma_s-1}\left[\left(\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s-1}\frac{2\varepsilon_s-\sigma_s-1}{\varepsilon_s}\right] \\
&\quad + \frac{1-\alpha}{\varepsilon_s+\theta-\sigma_w-1}\left[\left(\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_s-1}\frac{2\varepsilon_s-2\theta-\sigma_w-1}{\varepsilon_s+0}\right] \\
&\quad - \frac{1}{\varepsilon_s+(1-\alpha)\theta} - \frac{\alpha}{\varepsilon_s-\sigma_s-1} - \frac{1-\alpha}{\varepsilon_s+\theta-\sigma_w-1}.
\end{aligned}$$

Now, the following proposition is immediate.

Proposition 1. *Price discrimination lowers social welfare if* $\varepsilon_s - \sigma_s < 1$ *and* $\varepsilon_w - \sigma_w < 1$.

Thus, if the percentage of the customers who leave that market in response to a 1 percent change of the firm's price is less than one in both markets, *a change in social welfare by price discrimination is necessarily negative* ($\Delta SW/2 < 0$). In other words, for price discrimination to raise social welfare, it must be the case that $\varepsilon_s - \sigma_s \geq 1$ or $\varepsilon_w - \sigma_w \geq 1$.

Similarly, the per-firm changes in consumer surplus, profit, and output are

$$\begin{aligned}
\frac{\Delta CS}{2} &= \frac{\alpha}{\varepsilon_s-\sigma_s-1}\left(\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s-1} \\
&\quad + \frac{1-\alpha}{\varepsilon_s+\theta-\sigma_w-1}\left(\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_w-1} \\
&\quad + \frac{(\varepsilon_s-\sigma_s-1)+\alpha(\theta+\sigma_s-\sigma_w)}{(\sigma_s-\varepsilon_s+1)(\theta-\sigma_w+\varepsilon_s-1)},
\end{aligned}$$

$$\begin{aligned}
\frac{\Delta\Pi}{2} &= \frac{\Delta SW}{2} - \frac{\Delta CS}{2} \\
&= \frac{\alpha}{\varepsilon_s}\left(\frac{\varepsilon_s-1)}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s-1} \\
&\quad + \frac{1-\alpha}{\varepsilon_s+\theta}\left(\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_w-1} \\
&\quad - \frac{1}{\varepsilon_s+(1-\alpha)\theta},
\end{aligned}$$

and

$$\begin{aligned}
\frac{\Delta Q}{2} &= \alpha\left(\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s} \\
&\quad + (1-\alpha)\left(\frac{\varepsilon_s+\theta-1)}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_w} - 1,
\end{aligned}$$

respectively. Then, the following proposition is obtained.

Proposition 2. *Suppose that* $\varepsilon_s - \sigma_s < 1$ *and* $\varepsilon_w - \sigma_w < 1$. *If competition in the weak market is sufficiently high that* $\sigma_w - \varepsilon_s > \theta$, *and additionally* $\alpha > -(\varepsilon_s - \sigma_s - 1)/(\theta + \sigma_s - \sigma_w)$ *holds, then price discrimination also lowers consumer surplus.*

Proof. As $\varepsilon_s - \sigma_s < 1$ and $\varepsilon_w - \sigma_w < 1$ hold, the first two terms are always negative:

$$\begin{aligned}
\frac{\Delta CS}{2} &= \frac{\alpha}{\underbrace{\varepsilon_s-\sigma_s-1}_{(-)}}\left(\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s-1} \\
&\quad + \frac{1-\alpha}{\underbrace{\varepsilon_s+\theta-\sigma_w-1}_{(-)}}\left(\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_w-1} \\
&\quad + \frac{(\varepsilon_s-\sigma_s-1)+\alpha(\theta+\sigma_s-\sigma_w)}{\underbrace{(\sigma_s-\varepsilon_s+1}_{(+)}\underbrace{(\theta-\sigma_w+\varepsilon_s-1)}_{(-)}},
\end{aligned}$$

Thus, if the third term is always negative, the claim is established. Since the denominator is negative, if $\sigma_w-\sigma_s<\theta$ and $\alpha>-(\varepsilon_s-\sigma_s-1)/(\sigma_s+\theta-\sigma_w)$ hold, the numerator become positive, yielding the desired result. □

As a function of α, ΔSW and ΔCS can be either concave, convex, or concave-convex, depending on the other parameters (the discussion is available upon request).[21] Lastly, the following proposition with regard to ΔQ is established.

Proposition 3. *Suppose that $\varepsilon_s-\sigma_s<1$ and $\varepsilon_w-\sigma_w<1$. If the ratio of consumers who stop purchasing from either firm, relative to all leaving consumers as a response to price in－crease is greater in the strong market (i.e., $(\varepsilon_s-\sigma_s)/\varepsilon_s>(\varepsilon_w-\sigma_w)/\varepsilon_w$), then price discrimi－nation lowers total output.*

Proof. Following Aguirre (2006) and Galera and Zaratiegui (2006, p.607), we use the Bernoulli inequality which states that if $x>-1$, $x\neq 0$ and $0<a<1$, then $(1+x)^a<1+ax$. First, note that

$$\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}=1+\frac{1-(1-\alpha)\theta}{\varepsilon_s[\varepsilon_s+(1-\alpha)\theta-1]}>0$$

and

$$\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}=1+\frac{\alpha\theta}{(\varepsilon_s+\theta)[\varepsilon_s+(1-\alpha)\theta-1]}>0$$

Thus, by the Bernoulli inequality,

$$\alpha\left(\frac{\varepsilon_s-1}{\varepsilon_s}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s-\sigma_s}<\alpha\left[1+(\varepsilon_s-\sigma_s)\left(\frac{-(1-\alpha)\theta}{\varepsilon_s[\varepsilon_s+(1-\alpha)\theta-1]}\right)\right]$$

and

$$(1-\alpha)\left(\frac{\varepsilon_s+\theta-1}{\varepsilon_s+\theta}\frac{\varepsilon_s+(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\right)^{\varepsilon_s+\theta-\sigma_w}<(1-\alpha)\left[1+(\varepsilon_s-\sigma_w)\frac{\alpha\theta}{\varepsilon_s[\varepsilon_s+(1-\alpha)\theta-1]}\right]$$

hold. Now, it is immediate to see that

$$\frac{\Delta Q}{2}<\alpha\left[1+(\varepsilon_s-\sigma_s)\left(\frac{-(1-\alpha)\theta}{\varepsilon_s[\varepsilon_s+(1-\alpha)\theta-1]}\right)\right]$$

$$+(1-\alpha)\left[1+(\varepsilon_w-\sigma_w)\frac{\alpha\theta}{\varepsilon_w[\varepsilon_s+(1-\alpha)\theta-1]}\right]-1$$

$$=\frac{\alpha(1-\alpha)\theta}{\varepsilon_s+(1-\alpha)\theta-1}\left[\frac{\varepsilon_w-\sigma_w}{\varepsilon_w}\frac{\varepsilon_s-\sigma_s}{\varepsilon_s}\right],$$

which establishes the claim. □

The following corollary, which provides a sufficient condition for price discrimination to raise total output, is another application of the Bernoulli inequality which also states that if $x>-1$, $x\neq 0$ and $a>1$, then $(1+x)^a>1+ax$.

Corollary 4. *Suppose that $\varepsilon_s-\sigma_s>1$ and $\varepsilon_w-\sigma_w>1$. If the ratio of consumers who stop purchasing from either firm, relative to all leaving consumers as a response to price in－crease is greater in the weak market (i.e., $(\varepsilon_w-\sigma_w)/\varepsilon_w>1(\varepsilon_s-\sigma_s)/\varepsilon_s$), then price discrimi－nation raises total output.*

2. An Intuitive Argument on How Cross Elasticities ($\boldsymbol{\sigma_s}$ and $\boldsymbol{\sigma_w}$) Are Related to Social Welfare

It seems difficult to obtain a further analytical characterization of $\Delta Q/2$, let alone $\Delta CS/2$ or $\Delta SW/2$. In particular, it would be difficult to consider the effects of a change in σ_m with all the parameters fixed, as Adachi and Matsushima (2014) do to derive their Proposition 1.[22] We thus conduct numerical analyses of changes in social welfare, consumer surplus, and output from uniform pricing to price discrimination below. Before doing so, we provide an intuitive argument on when ΔSW is likely to be positive.

First, note that the elasticity that matters to the determination of the equilibrium price is the firm's own elasticity, ε_m. Thus, the restriction, $\varepsilon_s<\varepsilon_w$, solely determines which market is strong or weak. In symmetric equilibrium, the "virtual" market demand in the log form in market m is $\log q_m=\log a_m-(\varepsilon_m-\sigma_m)\log p_m$. Thus, the slope of the inverse demand is $-1/(\varepsilon_m-\sigma_m)$ and the intercept is $\log a_m/(\varepsilon_m-\sigma_m)$. This situation is described in Figure 1:

trapezoid *ABCD* corresponds to a welfare gain in the strong market, and trapezoid *EFGH* to a welfare loss in the weak market. Note here that one cannot directly compare area *ABCD* with *EFGH* to determine whether the corresponding welfare gain or the welfare loss is greater because the ordering of these two areas may not be preserved after logarithmic transformation. The idea here is that the welfare gain is greater than the welfare loss if and only if

$$\log\left[\frac{[(p_w^*-c)+(p^0-c)]}{2}\Delta q_w^*\right] > \log\left[\frac{[(p_s^*-c)+(p^0-c)]}{2}|\Delta q_s^*|\right]$$

$$\Leftrightarrow \log\left[\frac{\Delta q_w^*}{\Delta q_s^*}\right] > \log\left[\frac{p_s^*+p^0-2c}{p_w^*+p^0-2c}\right], \qquad (2)$$

provided that the chipped areas in both trapezoids due to the demand convexity are approximately equal.

Recall that under our normalization, we can deal with $p^0(=1)$ as a constant that is independent of parameter changes. The marginal cost is strictly lower than p^0, and it is given by $c=c(\varepsilon_s,\varepsilon_w,\alpha)\equiv 1-1/(\alpha\varepsilon_s+(1-\alpha)\varepsilon_w)$. Recall also that $p_m^*=\varepsilon_m\cdot c(\varepsilon_s,\varepsilon_w,\alpha)/(\varepsilon_m-1)$, which implies that the price change in market m, p_m^*, is *independent of the cross price elasticity*, σ_m. Thus, the upper and the lower bases in the strong market, *AB* and *DC* in Figure 1 remain the same if σ_s changes. The same argument also holds for *E*F and *HG* in the weak market. In Inequality (2) above, the right hand side is independent of σ_m. However, σ_m *does* affect the output change because it is given by

$$\Delta q_s^*=\alpha\cdot\left[\left(\frac{\varepsilon_s}{\varepsilon_s-1}c(\varepsilon_s,\varepsilon_w,\alpha)\right)^{\sigma_s-\varepsilon_s}-1\right]<0$$

for the strong market, and

$$\Delta q_w^*=(1-\alpha)\cdot\left[\left(\frac{\varepsilon_w}{\varepsilon_w-1}c(\varepsilon_s,\varepsilon_w,\alpha)\right)^{\sigma_w-\varepsilon_w}-1\right]>0$$

for the weak market. Thus, as brand substitution becomes stronger in the weak market (i.e., σ_w becomes larger), Δq_w^* becomes *smaller* because $\varepsilon_w\cdot c(\varepsilon_s,\varepsilon_w,\alpha)/(\varepsilon_w-1)$ is less than one. In Figure 1, this parameter change is expressed by a steeper slope in the weak market. On the other hand, as brand substitution becomes stronger in the strong market (i.e., σ_s becomes larger), then the slope in the strong market becomes steeper, and $|\Delta q_s^*|$ becomes smaller.

Recall again that from each firm's perspective, what matters is how many consumers it loses by raising its (discriminatory) price in market m, which is measured by ε_m. In other words, it does not care about the cross price elasticity σ_m *per se* because it is not interested in whether a leaving consumer switches to its rival or stops purchasing either product. However, in symmetric equilibrium, the cross price elasticity matters to the industry-wide elasticity, $\varepsilon_m-\sigma_m$, and the industry-wide elasticity is greater (a greater number of consumers

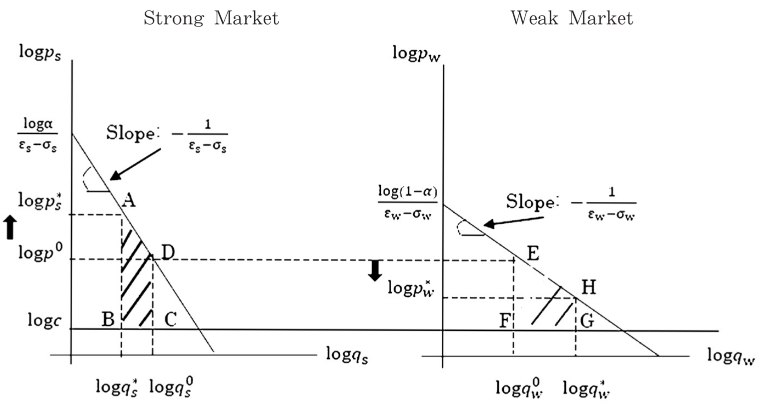

Figure 1: Log Welfare Changes in the Strong and the Weak Markets

leave the market for an increase in symmetric equilibrium price) if firms are in *less* rivalry. Intuitively, for a greater increase in the output in the weak market, the industry as a whole should stand in a shaky position so that consumers in the weak market respond to a price change greatly. As a reaction to one firm's price increase, more and more consumers leave the weak market if they are less satisfied with the rival's product. This happens if the two products are *less substitutable in the weak market*. This is why *less* competition in the weak market is beneficial for a welfare improvement in symmetric equilibrium: *less* competition in the weak market forces the weak market as a whole to be more price responsive (see also Weyl and Fabinger (2013, p.565) for a related discussion). However, in the strong market, greater competition, as expected, is beneficial for a welfare improvement in symmetric equilibrium because it reduces the welfare loss in the strong market (i.e., the line segment BC is smaller). This is an intuitive explanation for why a large value of σ_s and/or a *small* value of σ_w are likely to cause a positive change in social welfare.

3. Numerical Analyses

Now, we conduct numerical analyses in the rest of this section. With a little abuse of notation, we consider these per-firm measures below and denote them as ΔSW, ΔCS, $\Delta\Pi$, and ΔQ. More specifically, we investigate how the cross price elasticities σ_s and σ_w affect welfare changes from uniform pricing to price discrimination. In particular, our results below are in accordance with Adachi and Matsushima (2014): the greater is σ_s and less σ_w (i.e., competition is fiercer in the strong market than in the weak market), the more likely is a positive change in social welfare (i.e., $\Delta SW > 0$).

First, we consider the case of $\theta\alpha \leq 1$, where price discrimination never improves social welfare in the case of monopoly with constant elasticity (see Aguirre and Cowan (2015)). The following examples show that *under oligopoly, social welfare can be higher with price discrimination even if $\theta\alpha \leq 1$*. Table 1 shows the numerical values for Figures 2 and 3.

	ε_s	θ	α
Case 1	2	0.9	$\{0.05, 0.95\}$

Table 1: Parameter Values (for $\theta\alpha \leq 1$).

Now, Figure 2 shows that social welfare under price discrimination is higher in the lower right area below the boundary (for $\alpha = 0.95$ and 0.05). This example shows that, in contrast to the case of monopoly analyzed by Aguirre and Cowan (2015), the difference in elasticity, θ, does not always have to be large (to satisfy $\theta > 1/\alpha$) for price discrimination to benefit social welfare under oligopoly. In line with Adachi and Matsushima (2014), *cross price elasticity in the strong market, σ_s, must be sufficiently high*. Note also that the relative share of the strong market under uniform pricing, α, can be relatively small.

However, we confirm that a change in consumer surplus is negative for all (σ_s, σ_w) if α is equal to either 0.95 or 0.05 Figures 3 and 4 depict the areas of $\Delta\Pi > 0$ and $\Delta Q > 0$ for $\alpha =$ 0.95, 0.05, respectively. It seems that it is necessary that $\Delta Q > 0$ for $\Delta SW > 0$ in general.

Now, we consider the numerical values in Table 2. Figure 5 gives the corresponding figure.

	ε_s	θ	α
Case 2	2	$\{0.9, 0.3\}$	0.95

Table 2: Parameter Values (for $\theta\alpha \leq 1$).

From these numerical examples given above, we conjecture the following results on condition $\Delta SW > 0$ holds in general: price discrimination is more likely to improve social welfare ($\Delta SW > 0$) as:

1. the own price elasticity difference (θ) becomes *larger*,

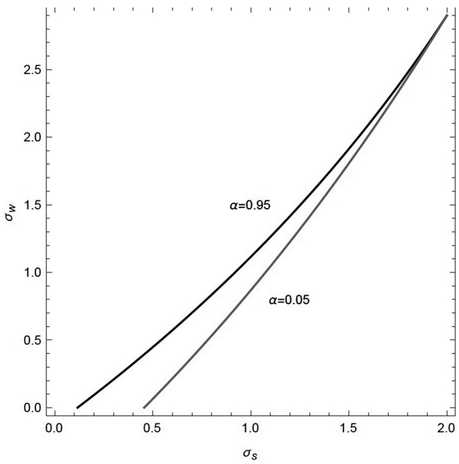

Figure 2: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$).

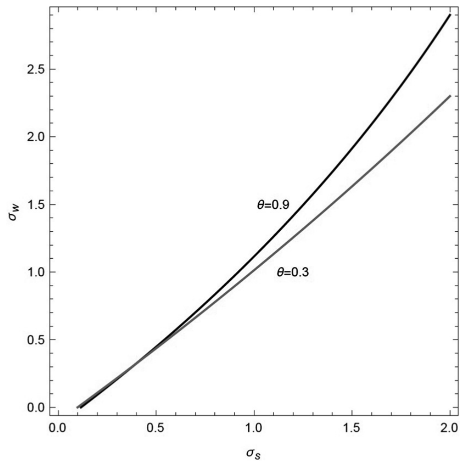

Figure 5: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 2$ and $\alpha = 0.95$.)

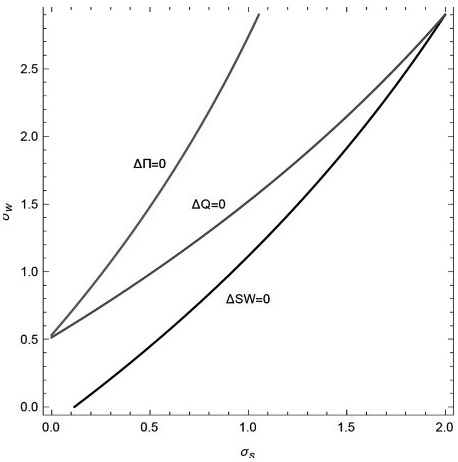

Figure 3: Area for $\Delta SW > 0$ $\Delta\Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$ and $\alpha = 0.95$.)

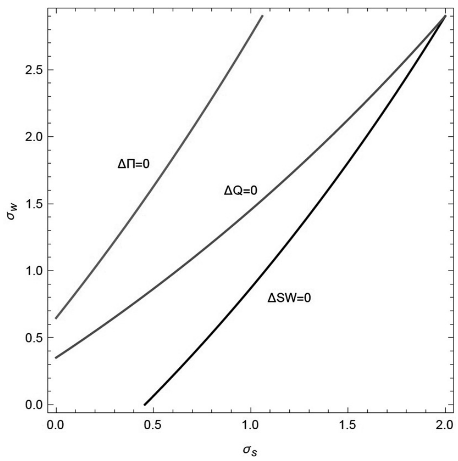

Figure 4: Areas for $\Delta SW > 0$ $\Delta\Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$ and $\alpha = 0.05$.)

2. the cross price elasticity in the strong market (σ_s) becomes *larger*, or
3. the cross price elasticity in the weak market (σ_w) becomes *smaller*.[23)]

In particular, the second and third points correspond to Adachi and Matsushima's (2014) necessary and sufficient condition in their Proposition 1, which roughly states (see the notations of the present paper) that there exists a threshold for σ_s, and that for a larger σ_s, the threshold price discrimination raises social welfare. This is an important point that should be further investigated to establish results similar to Adachi and Matsushima (2014).

Next, we analyze the case of $\theta\alpha > 1$ to see whether price discrimination can raise consumer surplus. First, the numerical values in Table 3 consider different values of own price elasticity in the strong market, ε_s. In Figure 6, the area where price discrimination raises social welfare is right below the boundary for each ε_s. Now, in contrast to the case of $\theta\alpha \leq 1$ above, *price discrimination can raise consumer surplus*, as shown in Figure 7: a change in consumer surplus is positive in the area left below the boundary for each ε_s. It appears that *both* σ_s and σ_w must be sufficiently*small*} for price discrimination to raise consumer surplus. It is conjectured that if $\Delta CS > 0$, then $\Delta SW > 0$ (and $\Delta\Pi$ is necessarily positive). Note that for a positive change in social welfare, σ_w should be kept relatively small if σ_s is sufficiently large. This would probably be the reason for the region of (σ_s, σ_w), where price discrimination raises social welfare, to be included in the region where price discrimination raises social welfare.

	ε_s	θ	α
Case 3	{1.05, 1.2, 1.35}	3	0.95

Table 3: Parameter Values (for $\theta\alpha > 1$)

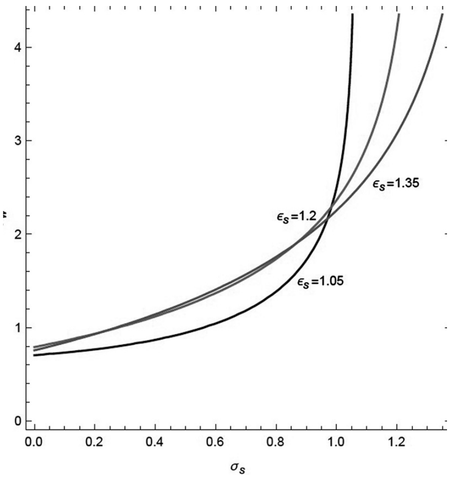

Figure 6: Area for $\Delta SW > 0$ (in the case of $\theta = 3$ and $\alpha = 0.95$.)

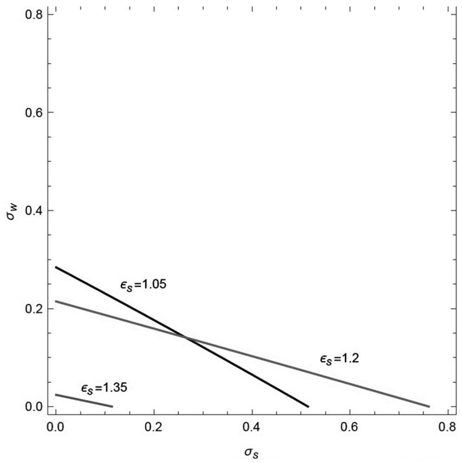

Figure 7: Areas for $\Delta CS > 0$ (in the case of $\theta = 3$ and $\alpha = 0.05$.)

Finally, by fixing the value of ε_s, we consider the effects of different values of θ and α separately. First, Table 4 gives the values for the differences in own price elasticities, θ. Interestingly, Figure 8 shows the boundaries (corresponding to $\Delta SW = 0$) quite homothetic to changes in θ. Figure 9 shows that the region of (σ_s, σ_w) where consumer surplus is higher under price discrimination is larger with $\theta = 4$ than with $\theta = 3$ (price discrimination never raises consumer surplus when $\theta = 2$). As Figure 10 (where $\Delta\Pi > 0$ for any (σ_s, σ_w)) shows in the case most favorable for a positive $\Delta SW(\theta = 4)$, consumer surplus is higher under price discrimination for a wide range of σ_s as long as σ_w is kept *small* (i.e., *less* competition in the weak market). This would probably be because own price elasticity in the weak market $\varepsilon_w = \varepsilon_s + \theta$ is already sufficiently large, suggesting that competition may work *against* consumer surplus (due to lack of coordination) if the industry as a whole faces a sufficiently elastic demand in the weak market.

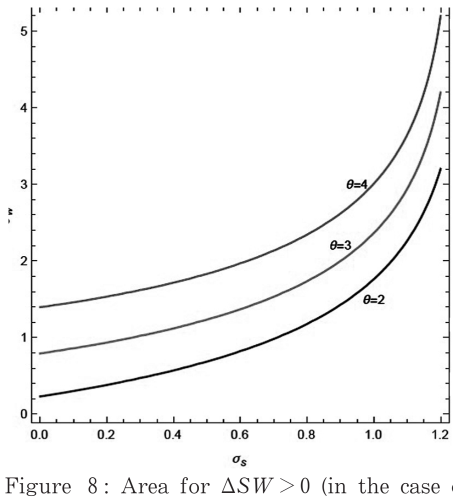

Figure 8: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 1.2$ and $\alpha = 0.95$.)

	ε_s	θ	α
Case 4	1.2	{2, 3, 4}	0.95

Table 4: Parameter Values (for $\theta\alpha > 1$).

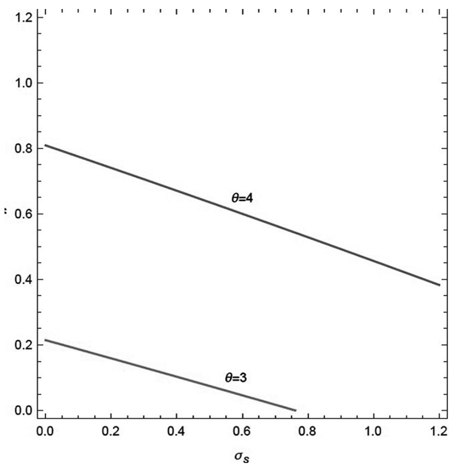

Figure 9: Area for $\Delta CS > 0$ (in the case of $\varepsilon_s = 1.2$ and $\alpha = 0.9$).

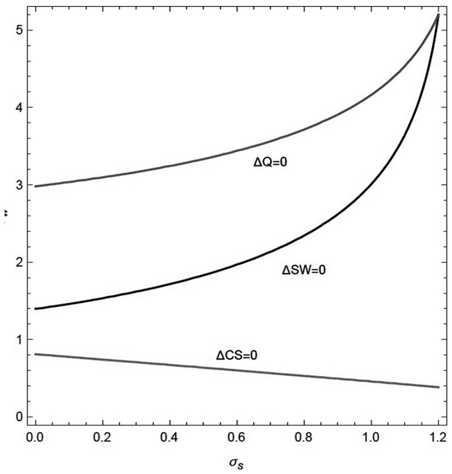

Figure 10: Area for $\Delta SW > 0$ $\Delta \Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 1.2$ and $\theta = 4$ and $\alpha = 0.95$.)

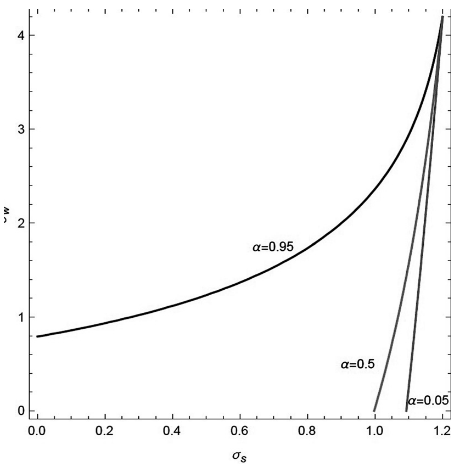

Figure 11: Areas for $\Delta SW > 0$ (in the case of $\varepsilon_s = 1.2$ and $\theta = 3$.)

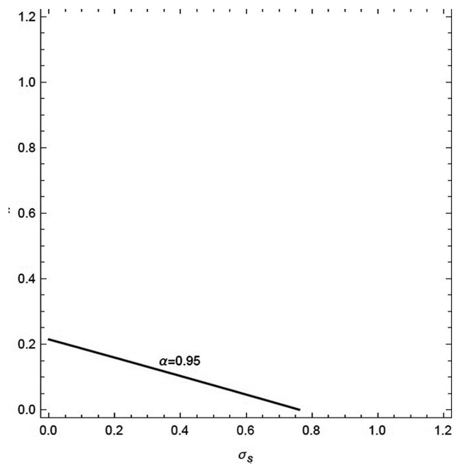

Figure 12: Area for $\Delta C > 0$ (in the case of $\varepsilon_s = 1.2$ and $\theta = 3$).

To sum up, we consider the numerical values in Table 5 to see the effects of different shares of the strong market, α. Figure 11 depicts the areas where price discrimination raises social welfare. As expected, a higher α is favorable for 10 positive change in social welfare. However, Figure 12 shows that a change in consumer surplus is negative for any (σ_s, σ_w) if $\alpha = 0.05$ or 0.5. In particular, the case of $\alpha = 0.5$ satisfies $\theta\alpha > 1$. Thus, we predict that a higher value of own elasticity difference (θ) is more important than a higher share of the strong market (α) for price discrimination to improve consumer surplus.

Ⅳ. Concluding Remarks

In this study, we investigate the welfare consequences of oligopolistic third-degree price discrimination with constant own and cross price elasticities of demand. We find that the key parameter for price discrimination to improve social welfare and even consumer surplus is *cross price elasticity*. If this parameter in the strong market is sufficiently large (i.e., competition in the strong market is fierce) with the corresponding parameter in the weak market kept sufficiently small, then price discrimination is more likely to be preferable. In comparison to the case of monopoly analyzed by

Aguirre and Cowan (2015), price discrimination can improve social welfare even with the parameter values with which it does not under monopoly. This result is consistent with Adachi and Matsushima (2014) on social welfare with linear demand. In addition, our result that consumer surplus can be higher with price discrimination shows that Adachi and Matsushima's (2014) result on consumer surplus (price discrimination never improves social welfare) hinges on the assumption of linearity.

For future research, an important issue is to explore the conditions for price discrimination to improve social welfare and consumer surplus with general nonlinear demands. As in Adachi and Ebina (2014a,b) in the context of cost pass-through (see, e.g., Bulow and Klemperer (2012) and Weyl and Fabinger (2013)) in vertical relationships, one would be able to conduct welfare analysis with exponential demand, logistic demand, and type I extreme demand.[24)]

Acknowledgments

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Notes

1) This is also true for studies that consider (nonprice) interactions within and across separate markets (such as consumption externalities). See, e.g., Layson (1998), Adachi (2002, 2004, 2005), and Bertoletti (2004).

2) See, e.g., Schmalensee (1981), Varian (1985), Layson (1988), Nahata, Ostaszewski, and Sahoo (1990), Schwartz (1990), and Cowan (2007, 2012, 2016). Armstrong (2006), Stole (2007), and Liu and Serfes (2010) are excellent surveys on third-degree price discrimination.

3) For earlier studies of oligopolistic third-degree price discrimination with price competition, see, e.g., Holmes (1989), Corts (1998), and Dastidar (2006). Galera and Zaratiegui (2006) study third-degree price discrimination with quantity competition.

4) The source of horizontal product differentiation may vary. In an extreme case, two similar products may be recognized as quite different by consumers because of advertising by manufacturers. To quote Tremblay and Tremblay (2005, p.173), "[b]ecause their colas taste very much alike, Coke and Pepsi use advertising to segment the market by creating images that appeal to different consumers. Coke pursues an image of traditional family values, while Pepsi presents a more youthful and rebellious image. This strategy benefits both firms by strengthening brand loyalty and reducing price competition."

5) The use of "strong" and "weak" markets is traditional since Robinson (1933). A more precise statement for its necessity and sufficiency is that, given the other parameter values, the substitution parameter in the strong market exceeds the threshold (see Adachi and Matsushima (2014), Proposition 1).

6) Aguirre and Cowan (2015) explain the reasons why the well-known results on third-degree price discrimination and welfare (e.g., Schmalensee (1981), Varian (1985), Aguirre, Cowan, and Vickers (2010), and Cowan (2012)) are less applicable to the case of constant elasticity demand. Aguirre and Cowan (2015) also emphasize that Aguirre, Cowan, and Vickers' (2010) sufficient condition for Proposition 2 does not hold if the submarket demand belongs to the class of *constant elasticity demand* because they are all convex with respect to own price.

7) One thing to keep in mind in an analysis of third-degree price discrimination with linear demand (as in Adachi and Matsushima (2014)) is

to guarantee conditions to keep all submarkets open under uniform pricing (the issue of "market opening"). However, under constant elasticity demand, all submarkets are necessarily open under uniform pricing because there are no intercepts.

8) Aguirre (2011) studies the case of a multimarket firm competing with a local firm in one market while it is a monopolist in another market. As in Adachi and Matsushima (2014), Aguirre (2011) assumes the linearity of demand and shows that price discrimination can improve social welfare in this setting as well.

9) Note that ε_m must be greater than one for the discriminatory price derived below to be positive and finite.

10) Holmes (1989) shows that with any symmetric price under oligopoly p, a firm's price elasticity is equal to the sum of the industry-demand and cross price elasticity. The industry demand in market m is given by $2q_m^i$ (p_m, p_m), and the industry-demand elasticity is $-(p_m/2q_m^i)(2dq_m^i/dp_m) = \varepsilon_m - \sigma_m$.

11) Note that all markets are served under both regimes: market opening is not an issue with constant elasticity demand because a positive number of consumers demand the product even if the price is tremendously high.

12) As pointed out by Nahata, Ostaszewski, and Sahoo (1990), the firm's profit function is not concave under constant elasticity demands. However, the second-order condition is satisfied at the equilibrium price because $\partial^2\pi_m^i/\partial(p_m^i)^2 < 0 \Leftrightarrow -2p_m^i + (\varepsilon_m+1)(p_m^i - c) < 0 \Leftrightarrow -p_m^i < (\varepsilon_m+1)c/(\varepsilon_m - 1)$, which implies that the first-order condition attains the unique solution.

13) As a consequence, under price discrimination, the firms' strategies are neither strategic substitutes nor strategic complements, while under uniform pricing, they can be strategic substitutes or strategic complements, depending on the parameter values (the details are available upon request).

14) Note that $\log \pi_m^* = \log(p_m^* - c) + \log q_m^* = \log a_m + [1-(\varepsilon_m - \sigma_m)][\log c - \log(\varepsilon_m - 1)] - (\varepsilon_m - \sigma_m)\log \varepsilon_m$, and thus $\partial \log \pi_m^*/\partial\sigma_m = \log[c/(\varepsilon_m - 1)] + \log \varepsilon_m > 0$. This implies that as σ_m increases, the firms as a whole, in symmetric equilibrium, become similar to monopoly. This is because $(\varepsilon_m - \sigma_m)$ becomes close to zero; almost no one leaves the market in response the price increase.

15) This profit function is not necessarily quasi-concave, and so it may have several peaks. Similar arguments for optimal price with uniform pricing $(p^i)^0$ also hold as in Aguirre and Cowan (2015). Thus, we make almost the same assumptions, and $(p^i)^0$ satisfying the first-order condition has maxima

$$\frac{\partial^2(\pi_s^i + \pi_w^i)}{\partial(p^i)^2} = 2\left[\frac{\partial q_m^A}{\partial p^A}\right] + (p^i - c)\left[\frac{\partial^2 q_m^A}{\partial(p^A)^2}\right]$$
$$= \sum_{m=s,w} \varepsilon_m a_m (p^i)^{-\varepsilon_m - 1}(p^j)^{\sigma_m}\left[-(1-\varepsilon_m) - \frac{\varepsilon_m + 1}{p^i}c\right] < 0.$$

16) Note that while p_m^* is independent of σ_m (thus, written as $p_m^*(\varepsilon_m)$), p^0 is not (thus, written as $p^0(\varepsilon_s, \varepsilon_w, \sigma_s, \sigma_w)$).

17) This simplification comes at cost: because c is no longer a free parameter, it is not possible to study the pass-through effects of an independent change in the marginal cost on the final prices (i.e., dp^0/dc under uniform pricing and dp_m^*/dc under price discrimination), though c is independent of σ_s and σ_w.

18) Thus, p_s^* and p_w^* cannot be equal for any parameter values. See Footnote 22 below for why this makes an analytical difference between Adachi and Matsushima (2014) and this paper.

19) Because of normalization, the share of the strong market under uniform pricing, which is clearly an endogenous variable, can be expressed by one parameter, α.

20) By using this welfare measure, we essentially focus on the equivalence variation when a change in consumer surplus is considered as in Figure 1 below (see, e.g., Hendel and Nevo (2013) for an analysis that uses the equivalence variation under the log-linear demands). In this sense, this definition of welfare proposed here seems natural and has a sound basis. Note that, as Adachi and Ebina (2016) argue, this welfare measure cannot be directly generated from the representative consumer's utility approach, which is another way to establish a welfare measure. If symmetry is relaxed, social welfare is not simply calculated as the area between the inverse demand and the marginal cost; one would instead consider, as Jorge and Pires (2013, p.673) do, derive expenditure function to calculate the compensating variation..

21) Note that if $\sigma_s = 0$ and $\sigma_w = 0$ are plugged in, $\Delta SW/2$ and $\Delta CS/2$ above coincide with ΔW and ΔCS, respectively, in Aguirre and Cowan (2015) (see their Equations (1) and (4)).

22) More precisely, Adachi and Matsushima (2014) find the pairs of γ_s and γ_w such that $p_s^* = p_w^*$ and define such γ_s by $\bar{\gamma}_s(\gamma_w)$, that is, $\Delta SW = 0$ at $\gamma_s = \bar{\gamma}_s(\gamma_w)$ Then, Adachi and Matsushima (2014) show $\partial\Delta SW/\partial\gamma_s < 0$, which implies that $\Delta SW > 0$

for γ_s which is close to $\bar{\gamma}_s(\gamma_w)$. Unfortunately, as mentioned in Footnote 18 above, p_s^* and p_w^* cannot be set equal in this paper's specification.

23) For α, however, we conjecture that there is no such a monotonic relationship (the discussion is available upon request).

24) By focusing on incidence properties, Fabinger and Weyl (2016) characterize the demand and supply system allowing closed-form solutions and yet flexibility to reflect the reality.

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