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Abstract

When we design an allocation mechanism, we should take into account not only the resource constraints but also the incentive compatibility constraints. The revelation principle asserts that the incentive compatibility constraints can be replaced with the truth-telling constraint, i.e. each player should not be worse off by telling the truth. However, the standard revelation principle implicitly assumes that the social planner or designer himself implements the designed mechanism. This implicit assumption becomes easily inadequate in reality. In many cases, the planner or designer need to delegate the implementation to multiple implementors, who follow the designer's instructions based on decentralized information. In such a case, the revelation principle should be changed accordingly. The informationally decentralized economy requires not only that each individual should tell the truth consistently to each social implementor (multiple truth-telling constraint: MTTC), but also that he should visit each class of social implementors once and only once (no repeated visit constraint: NRVC). The traditional revelation principle turns out to be a special case when there is only one social implementor. This paper further explores the implications of the generalized revelation principle in a pure exchange economy. It turns out that the linearity of value function, i.e. the existence of price, is required to ensure NRVC and that money becomes essential to ensure MTTC through cash-in-advance constraints. While the optimal allocation mechanism in the informationally decentralized economy looks similar to the first-best solution of the Arrow-Debreu economy, it is inferior even to the second-best solution. Hence it is referred as the third-best solution. Some numeric examples compare the optimal allocations under three different implemental conditions.

JEL Classification: C73, D50, D86, P51

Keywords: generalized revelation principle, decentralized implementors, multiple truth-telling constraint, no repeated visit constraint, cash-in-advance constraint, linear pricing function, third-best solution

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I. Introduction

The mechanism design theory considers an institution as a set of rules of a game and seeks for the best rules of game so that the strategic equilibrium may result in the most desirable outcome. The mechanism here simply means the set of "rules", which makes a standard game when combined with types of players, then generates the equilibrium outcome, like a machine producing output (e.g. allocation) from input (e.g. player types). In choosing rules, the designer should make sure that the outcome is incentive compatible in the sense that each player should be induced to choose the equilibrium strategy out of his own interest. For example, when we design a mechanism in which two brothers divide a cake evenly, the elder brother should be induced to cut the cake evenly to his own interest. The revelation principle, developed by Gibbard(1973), Green and Laffont(1977), Dasgupta, Hammond and Maskin(1979), Myerson(1979), Harris and Townsend(1981), Laffont and Maskin(1982), asserts that the incentive compatibility constraint can be replaced with the truth-telling constraint. To be specific, the equilibrium outcome of any mechanism can be replicated by a direct mechanism where the strategy space is the set of individual types and where the optimal strategy is to reveal the true type of each player. This principle facilitates the design process greatly, since it enables us to concentrate on the well-defined space of rules.

The standard revelation principle, however, implicitly assumes that the social planner or designer himself implements the designed mechanism. In many cases, in reality, the planner needs to delegate the implementation of mechanism to social implementors. For example, in a constitutional contract, the social planner is a symbolic representation of the unanimous public, who literally vanishes after designing the constitution. Even if the social planner survives, the implementing the designed mechanism, which involves physical transfers of goods to many citizens, may be beyond the capability of any single person. In such cases, the social planner should delegate the implementation to multiple implementors, probably multiple classes of implementors according to their specific functions, each obeying the designer's instructions independently. The primary goal of this paper is to generalize the revelation principle in a decentralized economy, where the designed mechanism should be implemented by informationally decentralized, multiple implementors. The secondary goal is to examine the implications of the generalized revelation principle on the optimal allocation mechanism of a pure exchange economy.

The primary result of the paper is that the revelation principle in an informationally decentralized economy requires two constraints: the multiple truth-telling constraints and the single visit constraints. The former requires that each individual should tell the truth consistently to each social implementer he visits, while the latter requires that each individual should visit each class of social implementers once and only once. Obviously, the standard revelation principle is a special case where there exist one and only one class of social implementers. Hence, the incentive compatibility constraint in the informationally decentralized economy is much stronger than that of the standard revelation principle in the informationally centralized economy.

The implications of the generalized revelation principle are rather surprising. The single visit constraints preclude unilateral transfers so that everything should be transferred on *quid pro con* basis. Especially money becomes essential as a limited communication device, in the sense that the cash-in-advance constraints [Clower(1967), Lucas(1980)] should be binding and that money becomes individually valuable. Moreover, it implies no arbitrage condition and hence linear pricing functions. In short, the monetary market mechanism becomes optimal. However, the market mechanism is not the same as the idealistic economy construed by Arrow(1951) and Debreu(1959). It is known that the allocation with the truth-telling constraints of the standard revelation principle is less efficient than the first-best solution of the idealistic economy with only the resource constraints. Hence it is called the second-best solution. However, the constraints of the generalized revelation principle in the decentralized economy is much stronger than those of the standard revelation principle in the informationally centralized economy, the optimal allocation becomes inferior even to the second-best solution. Hence it may be called the third-best solution.

The remainder of the paper is organized as follows. In Section 2, we will examine the standard revelation principle. Since the mechanism design theory seeks for the best results out of the equilibrium behavior of the game formed by the designed rules, it depends on the equilibrium concepts of games. Three concepts are widely used: (i) dominant equilibrium, (ii) Nash equilibrium, (iii) Bayesian Nash equilibrium. We will show how the revelation principle holds in each of them and discuss the relative robustness of each.

In Section 3, we will construct a pure exchange economy of infinitely many citizens, who are subject to idiosyncratic endowment and preference shocks, which are private information. We will analyze how the public agree unanimously to a social welfare function, and hence to the optimal mechanism, in the initial situation *before* any shocks are realized. The optimal mechanism depends not only on the physical properties of the economy but also on the implemental conditions such as the honesty of citizen and the availability of loyal implementors. We classify the alleviative environments into three cases: (i) the "honest economy", where both citizens and implementors are honest, (ii) the "informationally centralized economy", where the citizens are dishonest, but the social implementors are honest(loyal), (iii) the "informationally decentralized economy", where both citizens and social implementors are dishonest(disloyal).

Section 4 analyzes the honest and the informationally centralized economy. In the honest economy, only the resource constrains matter, and hence the first-best solution can be achieved. Especially, the optimal allocation satisfies the socialist idealism, "from each according to his ability, to each according to his need."

Section 5 analyzes the informationally centralized economy. The social planner and implementor should design and implement a mechanism under the additional truth-telling conditions, from the standard revelation principle, in addition to the resource constraint. With the additional constraint, the allocation inevitably becomes inferior to the first-best solution, which is referred to the second-best solution.

Section 6, which is the core of this paper, analyzes the generalized revelation principle in the informationally decentralized economy. Each social implementor is instructed to make certain actions (including physical transfers of goods) according to visitor's responses. With uncoordinated implementors, each citizen is free to visit social implementors as many time as need be according to his chosen schedule. Moreover he may make inconsistent signals to social implementors he visits. We analyze how the revelation principle can be generalized in such an economy. It turns out that the revelation principle is given by the two conditions: (i) each citizen should tell the truth consistently to each social implementors he visits(MTTC), and (ii) each citizen should visit each class of social implementors once and only once(NRVC). The standard revelation principle may be considered as a special case, where the visit to the social implementor is automatically constrained to be once.

The remainder of the section is devoted to examine the implications of the generalized revelation principle on the optimal allocation mechanism in the informationally decentralized economy. It turns out that the allocation mechanism should use money as a limited communication devise among social implementors, by checking inconsistent announcements by imposing the cash-in advance constraints; and that the value function should be linear in which goods and money are exchanged proportionally. We should note that the additional constraints are binding, unlike the Arrow-Debreu economy, and hence the resultant optimal allocation is inferior to the second-best solution. And hence it seems to be proper to name it the third-best solution.

It appears surprising that the allocation in the informationally decentralized economy is inferior not only to the first-best solution in the idealistic economy but also to the second-best solution in the informationally centralized economy, while the third-best solution uses money and prices like the first-best solution.

Section 7 deals with some simple concrete examples: (i) An economy where two goods are given stochastically, (ii) an economy where normal and handicapped are determined stochastically, and (iii) an economy where preference shocks(hungry and thirsty) are given stochastically. Here we compare the first-best, second-best and third-best solution in the idealistic, informationally centralized, informationally decentralized economies respectively.

Section 8 examines the policy implications: We should note that the informational centralized or decentralized alleviative environment is given exogenously rather than chosen. Still the resultant optimal allocation mechanism takes the form of centralized economy and decentralized one. What if mismatch arises, say we chose a centralized mechanism, while the alleviative environment is informationally decentralized? The result would be immediate collapse in theory, or a final collapse in a prolonged inefficiency in reality.

Section 9 is the concluding one and pays attention whether it is pessimistic or not to say that the decentralized economy is third-best. After all, our current economic welfare is fixed as it is now, but which is more pessimistic to say it first-best and third-best?

II. The Revelation Principle in the Standard Mechanism Design Theory

The mechanism design theory considers an institution as a set of rules of a game and seeks for the best rules of game so that the strategic equilibrium may result in the most desirable outcome. The mechanism here simply means the set of "rules", which makes a standard game when combined with types of players, then generates the equilibrium outcome, like a machine producing output (e.g. allocation) from input (e.g. player types). In choosing rules, the designer should make sure that the outcome is incentive compatible in the sense that each player should be induced to choose the equilibrium strategy out of his own interest.

For an allocation mechanism for a society with individuals I, the mechanism design proceeds in the following five steps:

(i) The planner designs a mechanism (S, g), where $S = \langle S_i \rangle$ and S_i is the strategy space of player $i \in I$ and $g: S \to Z$ is the outcome function which maps play space S into the outcome space Z.

(ii) Individual characteristics θ_i, including preferences, $\langle u_i(\cdot, \theta_i) \rangle$, are realized.

(iii) The mechanism (S, g) with $\langle u_i(\cdot, \theta_i)\rangle$ constitutes a game $(I, \langle S_i\rangle, \langle \phi_i\rangle)$ with the payoff function $\phi_i(s) = u_i[g(s), \theta_i]$.

(iv) Individuals arrive at a strategic equilibrium, $s^* = \langle s_i^*\rangle$.

(v) Outcome $z = g(s^*)$ is enforced.

We briefly review two related issues of the mechanism theory. The solution of the mechanism theory clearly depends on the equilibrium concepts of games. Three equilibrium concepts are used, each having its own strong and weak points.

2.1 Equilibrium Concepts

The dominant equilibrium is the play $s^* = \langle s_i^*\rangle$, where each player maximize his utility by choosing his equilibrium strategy regardless of others' strategies $s_{-i} = (\cdots, s_{i-1}, s_{i+1}, \cdots)$.

$$s_i^* \in \arg\max_{s_i} u_i[g(s_i, s_{-i}), \theta_i] \text{ for all } s_{-i} \text{ and } i \in I.$$

It is the most robust concept of equilibrium. The problem is that in many cases such an equilibrium does not exist. Gibbard(1973) and Satterthwaite(1975) showed that the only dominant strategy equilibrium with strategy space of three or more elements is the dictatorship.

The Nash equilibrium is the play $s^* = \langle s_i^*\rangle$, where each player maximizes his utility by choosing his equilibrium strategy, provided that other players also choose their equilibrium strategies $s_{-i}^* = (\cdots, s_{i-1}^*, s_{i+1}^*, \cdots)$.

$$s_i^* \in \arg\max_{s_i} u_i[g(s_i, s_{-i}^*), \theta_i] \text{ for all } i \in I.$$

The existence is guaranteed by the Nash theorem(1950). In fact, there are too many Nash equilibria in many cases. The main problem is that each player is assumed to know others' equilibrium strategies, which depend on their private information.

The Bayesian Nash equilibrium, proposed by Harsanyi(1967), does not require to know others' private information, but assumed to know their strategy rules ζ_i, which generates their optimal strategy if known their private information, of which one only knows their join probability distribution H. Then the Bayesian Nash equilibrium is the set of strategy rules, where each individual maximizes his expected utility for each individual shocks, provided that other players also choose their equilibrium strategy rules.

$$\zeta_i^* \in \arg\max_{\zeta_i} \int_{\theta_{-i}} u_i[g\{\zeta_i(\theta_i), \zeta_{-i}^*(\theta_{-i})\}, \theta_i]\, dH_i(\theta_{-i}) \quad \text{for all} \quad i \in I.$$

ζ_i is a strategy rule. An individual need not know others' private information, but he should know their beliefs on θ, and their beliefs on others' beliefs, *ad infinitum*, which may be more difficult. The problem is that it only pushes back problems on step into a more complex one.

In short, the robustness and applicability run in the opposite direction. It turns out, however, in our special case of the pure exchange economy with private goods, the dominant equilibrium exists and hence we need not resort to other equilibrium concepts.

2.2 Social Welfare Function

The second related issue is about the goal of the social planner. For a partial mechanism, such as employment contract, optimal taxation, or other principle-agent problem, we are only interested in the slope property of the second-best solution, and hence we can maximize the principle's utility or profit given the utility level of agents. In the general allocation problem, however, we need to specify the social welfare function $W(z)$.

But the impossibility theorem[Arrow(1951)] or the following results of Gibbard(1973) and Satterthwaite(1975), denies the existence of $W(z)$ in general. It turn out, however, that, for the constitutional problem with the veil of ignorance[Rawls(1971)], there exists $W(z)$ on which all individuals agree before any individual shocks are realized. In terms of the impossibility theorem, preference profiles are limited to be identical, or every individual becomes the dictator.

2.3 The Standard Revelation Principle

The revelation principle asserts that, for any arbitrary mechanism, there corresponds a direct mechanism such that (i) the strategy space is the set of individual characteristics and that (ii) each individual truthfully reports his characteristics at equilibrium.

The essence of the proof is as follows. (for the dominant equilibrium) For any mechanism (S, g) with an equilibrium play s^* and outcome $z^* = g(s^*)$, define a new direct mechanism $(\overline{S}, \overline{g})$ with $\overline{S} = \Theta$ and $\overline{g}(\theta) = g(\zeta^*(\theta))$, where $\zeta_i^*(\theta_i) = s_i^*$ is the strategy rule for each individual type. Then revealing the truth θ_i will become the optimal strategy for each individual i in the new direct mechanism. Suppose not. Then to pretend otherwise $\theta_i^{'}$ would be preferable, which implies that $s_i^{'} = \zeta_i^*(\theta_i^{'})$ would have been optimal, which contradicts the optimality of s^*.

Detailed proofs can be found in Dasgupta, Hammond and Maskin(1979), Laffont and Maskin(1982), Myerson(1979, 1982, 1986) and Ruppello(1990). For the allocation mechanism of our pure exchange economy, the revelation principle will be proved in Section V and VI.

III. The Economy: Physical and Implemental Properties

3.1 Physical Properties

We consider optimal mechanisms for a pure exchange economy. There are continuously many individuals $i \in I$ with measure $\nu(I)$ so that the law of large numbers is applicable [Judd(1985)]. There are $j = 1, 2, \cdots, \ell$ private goods, which are given like manna from heaven to each individual. We need not distinguish the endowment space for each individual, so let us denote the (common) set of individual endowments by $X \subset R_+^{\ell}$. We also need not distinguish the consumption space and let us denote the common set of individual consumption space by $Z \subset R_+^{\ell}$.

Individuals are *ex ante* identical before endowment and preference shocks are realized. It is not necessary but convenient to assume, without loss of generality, that the space of preference shocks, Θ, has the same dimension as X and Z so that $\Theta \subset R^{\ell}_{+}$. We assume that X and Θ are finite for the finiteness of state space *a la* Debreu(1959), while Z may be infinite, but considered to be convex and compact.

After shocks are realized, however, individuals have different preferences according to their preference shocks, $\theta \in \Theta$. Denote the meta utility function by

$$u: Z \times \Theta \rightarrow R.$$

For the utility function, we assume the following standard properties: (i) u satisfies the von Neumann-Morgenstern expected utility axioms; (ii) u is continuous on $Z \times \Theta$; and (iii) For each $\theta \in \Theta$, $u(\cdot, \theta)$ is strictly increasing, strictly concave, and continuously differentiable with respect to z.

We need to specify the endowment and preference shocks. Individual shocks (x, θ) are private information and assumed to be independent across individuals. The probability distribution functions are given as follows:

$$\Pr(x \leq x') = F(x'), \quad \Pr(\theta \leq \theta') = G(\theta').$$

This completes the specification of the physical structure of the economy. It can be summarized by $(I, X, \Theta, Z; u, F, G)$ or (u, F, G) for short.

3.2 The Resource Constraint and the Social Welfare Function

We can deduce that the social welfare function, before any individual shocks are realized, depends only on the physical properties. To be specific, like Patinkin(1965), suppose that all individuals (citizens) gathered at agora to decide the social choice rule of allocation mechanisms on "Friday" before any heterogenetic shocks are realized on next "Monday." Since they are *ex ante* homogenous, they will try to maximize the same objective, i.e. their expected utility under the proposed allocation mechanism.

$$Eu(z, \theta) = \int_{\Theta} \int_{X} u[z(x, \theta), \theta] \, dF(x) \, dG(\theta)$$

Hence they can arrive at a unanimous agreement to maximize the mean utility of *ex post* heterogeneous individuals:

$$W(z) = \int_{\Theta}\int_X u[z(x,\theta),\theta]\,dF(x)\,dG(\theta) \tag{3.1}$$

Thus, as a whole, they act as "the social planner" in choosing the social choice rule.

The resource constraint is solely dependent on physical properties. Let $z(x,\theta)$ is any allocation mechanism[2] which *eventually* allocates consumption $z \in Z$ to the individual with endowment $x \in X$ and preference $\theta \in \Theta$. Then the social planner should comply with the resource constraint,

$$\int_{\Theta}\int_X z(x,\theta)\,dF(x)\,dG(\theta) \le \bar{x} \tag{3.2}$$

where $\bar{x} = \int_X x\,dF(x) < \infty$ is the average per capita endowment. We will say an allocation $z(x,\theta)$ is *attainable*, if it satisfies the above resource constraint.

3.3 Implemental Conditions

If the planner knew individual shocks, (x,θ), his problem would be simply to maximize the social welfare function (3.1) subject to the resource constraint (3.2). However, individual shocks are private information. Hence the social planner should induce individuals to reveal their private information. In other words, the mechanism should satisfy the incentive compatibility constraint as well as the resource constraint.

To understand the importance of the incentive compatibility constraint, consider the proposal to subsidize the poor by taxing the rich (without violating the resource constraint), but there is no way for the planner to distinguish the poor from the rich. Then, since everyone pretends to be the poor, the proposal is useless, or not implementable. Being rational enough, the people at agora realize the vagueness of the proposal and seek for implementable[3] rules of the allocation game.

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2) We did not specify the strategy space yet. Still we can use the notation as a reduced form. Note also that we need not specify $z^I: (X \times \Theta)^I \to Z^I$.

3) I use the implementability here interchangeably with incentive compatibility. To be more specific, a set of rules are implementable if there exists an operational way (like step-by-step instructions like a FORTRAN program) to arrive there.

The situation becomes more complicated when we realize that the social planner literally vanishes after having designed a mechanism.[4] Hence the planner have to delegate the implementation of the chosen mechanism to social implementers. The planner should take it into account when choosing the mechanism. When the planner delegates the implementation of the designed mechanism, another incentive compatibility problem arises: the social implementors themselves may pursue their own interest out of their delegated authority. If they are loyal and coordinate with full communication, they behave like the social planner himself. However, if they are not loyal or cannot coordinate by limited communication, they will behave differently. The social planner, or the people at agora, should take this into account.

Our final concern is the alleviative environment where the social implementors are not loyal or cannot coordinate with full communication. However it is instructive to compare the result with those in the economy with honest citizen and implementors and in the economy with dishonest citizens and loyal, coordinated implementors. Hence we consider three cases of alleviative environments: (i) the honest economy with honest citizens and loyal, coordinated implementors, (ii) the informationally centralized economy with dishonest citizens and loyal, coordinated implementors, and (iii) the informationally decentralized economy with dishonest citizens and disloyal or uncoordinated implementors.

In addition, we will assume that the social planner may issue money freely, which is universally recognizable and uncounterfeitable. He may choose to use money, especially when he delegates his authority to multiple social implementors without any direct communication device.

IV. Honest Citizens and Loyal, Coordinated Implementors: The Honest, Idealistic Economy

We consider the allocation mechanism (S, δ, γ), where the outcome function $g=(\delta, \gamma)$ is decomposed into net transfers of private goods, δ, and that about the

4) Individual after shocks are realized will have different endowments and preferences and cannot agree each other.

net transfer of money, γ. We separated the two since they may behave differently. When the planner chooses not to use money, he can set $\gamma(x,\theta)\equiv 0$.

With honest citizens, the private information does not matter. The social planner or implementors can know individual types simply by asking individuals. Hence it is natural to let $S=X\times\Theta$, $\delta(s)=z(x,\theta)-x$, $\gamma(x,\theta)\equiv 0$. The implementation is straightforward. If there is one and only one central implementor, when an individual visits him:

(i) Ask individual type, (x,θ);

(ii) For any j such that $\delta_j(x,\theta)=-x_j+z_j(x,\theta)<0$ let him submit that much;

(iii) For any j such that $\delta_j(x,\theta)=-x_j+z_j(x,\theta)>0$ let him take that much.

The social planner will solve

$$\max_{\delta}\quad W(z)=\int_{\Theta}\int_{X}u[z(x,\theta),\theta]\,dF(x)\,dG(\theta) \qquad \text{<P4.1>}$$

$$\text{s.t.}\quad z(x,\theta)=x+\delta(x,\theta),$$

$$\int_{\Theta}\int_{X}z(x,\theta)\,dF(x)\,dG(\theta)\le \bar{x}$$

In fact, we may solve the above problem directly with respect to the optimal allocation $z^{*}(x,\theta)$, by maximizing the social welfare function subject to the resource constraint. Once z^{*} is given, the net transfer function δ^{*} can be obtained easily by $\delta^{*}(x,\theta)=-x+z^{*}(x,\theta)$.

The first-order condition will be

$$\frac{\partial u(\cdot,\theta)}{\partial z_j}=\lambda_j,\quad j=1,\cdots,\ell \qquad (4.1)$$

where λ_j is the Lagrangean multiplier for the j-th resource constraint. The condition defines the optimal allocation z^{*} uniquely. It is obvious that $z^{*}(x,\theta)$ depends only on θ(=needs), i.e. $z^{*}(x,\theta)=z^{*}(x',\theta)\ \forall x,x',\theta$. This rationalizes the socialistic idealism "from each according to his ability; to each according to his needs."

Even if there are many implementors, there is no problem as far as they coordinate their implementation with full communication. Of course, there may arise inventory problems. But it can be solved by asking (ii) to be done in the Monday morning, and (iii) in the Monday afternoon.

Can it be implemented with markets and money? By money, I mean the entity which is universally recognizable, uncounterfeitable, divisible, durable, transportable and concealable. Yes. With honest citizens, Arrow-Debreu commodities, contingent on (x, θ), can be traded on Friday. The equilibrium "price" of contingent claim $c(x, \theta)$ will be $\lambda_j \times \Pr(x, \theta)$. All trades occur on Friday, and only executing those contracts will occur on Monday. Money may be used as coupons (a record-keeping device) in order to overcome temporary inventory problems in executing contracts. In such case, $\gamma_j(x, \theta) = -\lambda_j \delta_j(x, \theta)$.

V. Dishonest Citizens and Loyal, Coordinated Implementors: Revelation Principle and Optimal Allocation Mechanism in the Informationally Centralized Economy

Now suppose that citizens are not telling the truth voluntarily unless induced otherwise. We assume, as the standard mechanism theory does, that the social planner or designer himself implements the mechanism. In fact, it does not matter the authority is delegated to a loyal, centralized implementor or to multiple loyal implementors coordinated with full communication. Consider an allocation mechanism (S, δ, γ) with any strategy or signal space S and net transfer function for goods $\delta: S \rightarrow R^n$ and for money $\gamma: S \rightarrow R$. With private goods, individuals need not care about others' strategies, hence we can use the robust notion of dominant equilibrium.

Individuals with (x, θ) will choose his strategy $s^*(x, \theta) \in S$ by solving the problem,

$$\begin{aligned} &\max_s u[z(x, \theta), \theta] \\ &\text{s.t. } z(x, \theta) = x + \delta(s) \end{aligned} \qquad \text{<P5.1>}$$

Knowing such behaviors, the social planner maximizes $W(z)$ subject to the resource constraint and the incentive compatibility constraint above to find

$z^*(s^*(x,\theta)) = x + \delta^*(s^*(x,\theta))$.

Hence the social planner's problem is to find (S, δ) which maximizes the social welfare function subject to the resource constraint and <P5.1>

$$\max_{\delta} \quad W(z) = \int_{\Theta}\int_{X} u[z(x,\theta),\theta]\, dF(x)\, dG(\theta) \qquad \text{<P5.2>}$$

$$\text{s.t.} \quad \int_{\Theta}\int_{X} z(x,\theta)\, dF(x)\, dG(\theta) \le \bar{x}$$

$$z(x,\theta) \text{ solves <P5.1>}$$

The revelation principle for this case can be proved easily. Denote by (S, δ^*) the solution which solves <P5.2>, while $s^*(x,\theta)$ solves individual problem <P5.1> given the optimal mechanism (S, δ^*). Define the direct mechanism $(\bar{S}, \bar{\delta}, \bar{\gamma})$ by making the strategy space be the space of individual types,

$$\bar{S} = X \times \Theta,$$

and by defining the new net transfer function as follows:

$$\bar{\delta}(x,\theta) = \delta^*[s^*(x,\theta)].$$

Then we assert that, in the new direct mechanism, telling the truth will be the equilibrium strategy for each individual:

$$(x,\theta) \in \arg\max_{x',\theta'} \quad u[x + \bar{\delta}(x',\theta'),\theta] \qquad \text{<P5.1'>}$$

Suppose not. Then there will be a false announcement (lie) $(\tilde{x}, \tilde{\theta})$ preferable to the true announcement such that

$$u[x + \bar{\delta}(\tilde{x},\tilde{\theta}),\theta] > u[x + \bar{\delta}(x,\theta),\theta]$$

But by construction, this implies

$$u\{x + \delta[s(\tilde{x},\tilde{\theta})],\theta\} > u\{x + \delta[s^*(x,\theta)],\theta\}$$

which contradicts the optimality of $s^*(x,\theta)$.

Hence any allocation mechanism with arbitrary strategy space can be replicated by a direct mechanism with truth-telling constraints:

$$\max_{\bar{\delta}} \int_{\Theta}\int_{X} u[z(x,\theta),\theta]\, dF(x)\, dG(\theta) \qquad \text{<P5.2'>}$$

$$\text{s.t.} \quad z(x,\theta) = x + \bar{\delta}(x,\theta)$$

$$\int_{\Theta}\int_{X} z(x,\theta)\, dF(x)\, dG(\theta) \le \bar{x}$$

$$u[x+\bar{\delta}(x,\theta),\theta] \ge u[x+\bar{\delta}(x',\theta'),\theta] \quad \forall x',\theta' \qquad (5.1)$$

The last constraint is just another way to express the truth-telling constraint, <P5.1'>. Henceforth, we will use the notation, (S,δ,γ) instead of $(\bar{S},\bar{\delta},\bar{\gamma})$, for the direct mechanism. In fact, since $S = X \times \Theta$ is obvious for the direct mechanism, we can refer any direct mechanism simply by (δ,γ).

We can detect some false announcements in the direct mechanism by asking individuals verify his announcement by submitting corresponding (negative) net transfers.

(i) Ask individual type, (x,θ);

(ii) For any j such that $\delta_j(x,\theta) = -x_j + z_j(x,\theta) < 0$ let him submit that much;

(iii) For any j such that $\delta_j(x,\theta) = -x_j + z_j(x,\theta) > 0$ let him take that much.

The second step effectively tests the nonnegativity of $x_j + \delta_j(x,\theta) \ge 0$. Hence the last constraint (5.1) in <P5.2'> can be loosened to

$$(x,\theta) \in \arg\max_{x',\theta'} u[x+\delta(x',\theta'),\theta] \ \text{whenever} \ x+\delta(x',\theta') \ge 0. \qquad (5.2)$$

We call (5.2) the truth-telling constraint. The social planner should induce individuals to tell the truth voluntarily. More specifically, once $\delta(x,\theta)$ is given, individual consumption, and hence utility depends on his announcement about his type. The planner should design delta so that each individual maximizes his utility when he truthfully reveal his type.

The social planner tries to find δ, hence z, which maximizes $W(z)$ subject to the resource constraints and the above truth-telling constraints, which we will denote z^C. Can the optimal mechanism z^C in the informationally centralized economy be implemented by money and markets? No. First of all, there will be no price. The money cannot be used even as coupons, since the optimal allocation mechanism in the informationally centralized economy is vulnerable to collusion among individuals. These points will be made clearer in those examples given in section VII.

VI. Dishonest Citizens and Disloyal, or Uncoordinated Implementors: Revelation Principle and Optimal Allocation Mechanism in the Informationally Decentralized Economy

Now we consider the economy with dishonest citizens and disloyal implementors. What can the social planner do when he cannot trust the social implementors to whom he should delegate the authority to implement the designed mechanism? At first, it may appear helpless. But we can approach the problem in two steps. Firstly, we think that all the social implementors are loyal but only have communication problem. To be specific, each social implementor will follow any instructions given by the social planner faithfully, but the only problem they cannot communicate each other to coordinate their implemental actions. Secondly, in the later part of the section, I will show how the social planner can induce disloyal implementors to behave like the loyal implementors without communication.

Before proceeding, we examine the rationale why there may be multiple implementors without direct communication. Alchian(1977) considered when the exchange with medium of exchange, consisting of two steps of selling and buying, is less costly than direct exchange of one step. According to him, it is only when there exist the need of professional skills to recognize the genuineness of each good, while money is universally recognizable. If this is the case, the social planner may need at least one implementor for each good $j=1,2,\cdots,\ell$. The need of inventory may be another reason for specialized implementors. Of course, there may be many implementors in one specialized area, especially when the economy encompasses a large geographic area. In fact, the social planner may prefer multiple implementors even in on a specialized area in a specific region, just to make them check and balance each other. Henceforth, we will assume that there are ℓ classes of implementors and that there are many implementors in each specialized class.

The reason of limited communication also needs some justification. Sometimes communication may be physically limited. Lucas(1980) considered an economy of isolated islands, with only money served as a limited communication device. Even with the technology of full communication, the social planner may prefer to induce social implementors to communicate each other only through money.

Now we are ready to consider the allocation mechanism (S, δ, γ) in the informationally decentralized economy with dishonest citizen and multiple implementors with limited communication. We first assume implementors are loyal, though they cannot communicate each other. Each class of implementors get a detailed instructions from the social planner before they are dispatched to their local offices. After arrived at their local offices they cannot communicate each other as well as with the social planner. However, they are supplied with enough inventory of currency, which should be returned to the social planner after their operation.[5)]

The detailed instruction of implementing (S, δ, γ) for j class of implementors will be as follows:

(i) Individual chooses $s \in S$;

(ii) Request to submit $\max[0, -\delta_j(s)]$ units of good j, and $\max[0, -\gamma_j(s)]$ units of currency.

(iii) If the visitor submits as requested, then make positive net transfers of $\max[0, \delta_j(s)]$ and $\max[0, \gamma_j(s)]$.

The point is to receive negative net transfers first, according to the observed strategy, and to give positive net transfers only if he received successfully the negative net transfers.

6.1 Consumer's Problem with Free Schedule of Visits to Multiple Implementors under the Mechanism (S, δ, γ)

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5) In order to express the mechanism in a proper game form, we should write the strategy space to be $\Sigma \times P$ so that it is represented by the Cartesian product of the set Σ of all possible schedules σ of visits, i.e. sequences in $J = \{1, 2, \cdots, \ell\}$, and the set P of all reporting plans ρ, i.e. sequences in the signal space S. We also need to generalize the mechanism to include individual "actions" to encompass such schedule of visits and "production" functions which translates those actions into individual holdings of goods and currency, and finally "decision function" describing how the social implementors deal with those individual holdings. But, with understanding the message game is repeated with free schedule, we still use the simple notation keeping symmetry with the idealistic and the informationally centralized economies.

Now we consider the individual consumers. With multiple implementors with the above instructions, individuals should choose both (i) visiting schedule (in what sequence he will visit each class of implementors) and (ii) reporting plan (in each visit, how to choose his signal).

For the visiting schedule, we may limit our attention on the sequence σ defined on $J=\{1,\cdots,\ell\}$, since implementors in the same class are identical. Let σ_n denote the class of enforcer at the n-th visit and let $\sigma_j^{-1}=\{n\colon \sigma_n=j\}$ denote the subset of visits to j class implementors. Finally we will denote the length of the schedule σ by $|\sigma|$, which may be arbitrarily large. For example, $\sigma=(1,2,1,2,3)$ means visiting class 1 and class 2 implementors twice in that order and then visiting class 3 implementor once. In such case, $\sigma_3=1$, $\sigma_2^{-1}=\{2,4\}$, and $|\sigma|=5$.

Along with the visiting schedule, individual should make reporting plan $\rho=\{\rho_n\}_{n=1}^{|\sigma|}$. For each visit he chooses $\rho_n(x,\theta)\in S$ according to his type. Since the consumer knows that the social implementors do not communicate each other, he need not worry about inconsistent reports. Hence he may well choose different reporting strategy for each implementor. In other words, the consumer may make inconsistent lies (pretending different types) from one implementor to other.

How will the social planner expect the consumer to choose the schedule and reporting plan? If there exist travel expenses, the consumer will try to minimize the length of schedule. But, since it is not our concern, we will ignore travel expenses. All matters is the final stock of goods left for consumption. Then we should know how the stock of goods evolves along his schedule and reporting plan. With (σ,ρ), the stock of goods after n-th visit, x^n, $n=1,2,\cdots$, evolves as follows:

$$x_j^n=\begin{cases} x_j^{n-1}+\delta_j(\rho_n(x,\theta)) & \text{if } \sigma_n=j, \\ x_j^{n-1} & \text{otherwise.} \end{cases} \tag{6.1}$$

with the initial condition, $x_j^0=x_j$. Hence the final stock of goods is given as follows:

$$z_j=x_j^{|\sigma|}=x_j+\sum_{n\in\sigma_j^{-1}}\delta_j(\rho_n(x,\theta))\ \ j=1,\cdots,\ell\,. \tag{6.2}$$

We also note that the stock of currency after n-th visit, $m^{n,}$ $n=1,2,\cdots$, evolves as follows:

$$m^n = m^{n-1} + \gamma_{\sigma_n}(\rho_n(x,\theta)) \tag{6.3}$$

with the initial condition, $m^0=0$. Though the currency holdings does not enter the utility directly, it matters since reporting ρ_n in the n-th visit is granted only if

$$x^n + \delta(\rho_n(x,\theta)) \geq 0 \tag{6.4}$$

$$m^n + \gamma(\rho_n(x,\theta)) \geq 0 \tag{6.5}$$

according to the implementation scheme (*). The final stock of currency will be

$$m^{|\sigma|} = \sum_{n=1}^{|\sigma|} \gamma_{\sigma_n}(\rho_n(x,\theta)) \tag{6.6}$$

Now we are ready to fully specify the individual problem of finding the optimal schedule and reporting plan. Given the mechanism, (S,δ,γ), each individual tries to maximize his utility by finding optimal schedule and reporting plan, while not violating the nonnegativity conditions along his schedule.

$$\max_{\sigma,\rho}\ u(z,\theta) \qquad \text{<P6.1>}$$

$$\text{s.t.}\quad z_j = x_j + \sum_{n\in\sigma_j^{-1}} \delta_j(\rho_n(x,\theta))\ \ j=1,\cdots,\ell$$

$$x_{\sigma_n}^n = x_{\sigma_n}^{n-1} + \delta_{\sigma_n}(\rho_n(x,\theta)) \geq 0\ \ n=1,\cdots,|\sigma|$$

$$m^n = m^{n-1} + \gamma_{\sigma_n}(\rho_n(x,\theta)) \geq 0\ \ n=1,\cdots,|\sigma|$$

$$x_j^0 = x_j\ \ j=1,\cdots,\ell \ \text{ and }\ m^0=0 \ \text{ given.}$$

6.2 Social Planner's Problem and the Generalized Revelation Principle

Knowing such individual behaviors, the social planner should provide detailed instructions (δ_j,γ_j) for each class of implementors so that the social welfare $W(z)$ may be maximized subject to the resource constraint and the above incentive compatibility constraint.

$$\max_{\delta,\gamma} \quad W(z) = \int_{\Theta}\int_{X} u[z(x,\theta),\theta]\, dF(x)\, dG(\theta) \qquad \text{<P6.2>}$$

$$\text{s.t.} \quad \int_{\Theta}\int_{X} z(x,\theta)\, dF(x)\, dG(\theta) \le \bar{x}$$

$$z(x,\theta) \text{ solves <P6.1> a.e.}$$

Now we can state our first theorem, which is the revelation principle generalized to our informationally decentralized economy.

Theorem 1. (Generalized Revelation Principle) For any allocation mechanism $z(x,\theta)$ implemented by (S,δ,γ) with (σ,ρ), there exists a direct mechanism $z(x,\theta)$ implemented by $(\bar{S},\bar{\delta},\bar{\gamma})$ with $(\bar{\sigma},\bar{\rho})$ such that (i) $\bar{S}=X\times\Theta$, (ii) $(x,\theta)=\bar{\rho}_n(x,\theta)$ $\forall n$ and (iii) $\bar{\sigma}(x,\theta)\in\bar{\Sigma}$, where $\bar{\Sigma}$ is a permutation of J.

Proof: Suppose (S,δ,γ) implements $z(x,\theta)$ with the equilibrium strategy rule $(\sigma(\cdot),\rho(\cdot))$. Define $(\bar{S},\bar{\delta},\bar{\gamma})$ by

$$\bar{S}=X\times\Theta$$

$$\bar{\delta}_j(x,\theta) = \sum_{n\in\sigma_j^{-1}(x,\theta)} \delta_j[\rho_n(x,\theta)]$$

$$\bar{\gamma}_j(x,\theta) = \sum_{n\in\sigma_j^{-1}(x,\theta)} \gamma_j[\rho_n(x,\theta)]$$

Notice that the latter two equations define the new net transfer functions as the sum of net transfers with optimal schedule and reporting plan under the old net transfer functions.

Define $\bar{\sigma}(x,\theta)\in\bar{\Sigma}$ by applying an arbitrary selection rule to

$$\bar{\sigma}\in\{\sigma\in\bar{\Sigma}:\ \sigma_j<\sigma_k \text{ whenever } \gamma_j(x,\theta)\ge 0 \text{ and } \gamma_k(x,\theta)<0\}.$$

In effect, the consumer visits those implementors who give net positive transfers of currency first, while visiting those implementors making negative transfers of currency later.

Also define the new reporting plan $\bar{\rho}(x,\theta)$

$$\bar{\rho}(x,\theta)=(x,\theta),\quad n=1,\cdots,\ell\,.$$

The consumer plans to tell the truth consistently to each implementors he visits.

By construction, $(\bar{\sigma}(\cdot),\bar{\rho}(\cdot))$ under $(\bar{S},\bar{\delta},\bar{\gamma})$ results the same allocation $z(x,\theta)$ as $(\sigma(\cdot),\rho(\cdot))$ under (S,δ,γ):

$$z_j(x,\theta)=x_j+\bar{\delta}_j(x,\theta)\quad j=1,\cdots,\ell$$

We must show that $(\bar{\sigma}(\cdot),\bar{\rho}(\cdot))$ is an equilibrium strategy under the new direct mechanism $(\bar{S},\bar{\delta},\bar{\gamma})$. We will first show the feasibility (i.e. satisfying the nonnegativity condition in (*) along the schedule) of $(\bar{\sigma}(\cdot),\bar{\rho}(\cdot))$ and show its optimality later. The feasibility of $(\sigma(\cdot),\rho(\cdot))$ implies $x_j+\bar{\delta}_j(x,\theta)\geq 0,\ j=1,\cdots,\ell$ and $\sum_{j=1}^{\ell}\bar{\gamma}_j(x,\theta)\geq 0$. The latter implies the nonnegativity condition of currency holds along the schedule $\bar{\sigma}$.

For the optimality of $(\bar{\sigma}(\cdot),\bar{\rho}(\cdot))$, suppose there exists $(\bar{\sigma}'(\cdot),\bar{\rho}'(\cdot))$ strictly feasible and preferable to $(\bar{\sigma}(\cdot),\bar{\rho}(\cdot))$ for some (x,θ) with a positive probability. Denote the resultant consumption by

$$z_j'(x,\theta)=x_j+\sum_{n\in\sigma_j'^{-1}}\bar{\delta}_{j'}[\,\bar{\rho}_n'(x,\theta)]\quad j=1,\cdots,\ell\,.$$

Define a new reporting plan *under* (S,δ,γ) by $\rho'(x,\theta)$ by $\rho_n'(x,\theta)=\rho_n[\,\bar{\rho}_n'(x,\theta)]$ and let $\sigma'(x,\theta)=\bar{\sigma}'(x,\theta)$. By construction,

$$z_j'(x,\theta)=x_j+\sum_{n\in\sigma_j'^{-1}}\delta_j[\rho_n'(x,\theta)]\ \ j=1,\cdots,\ell\,.$$

But we also have

$$\begin{aligned} m^{n-1}+\gamma_{\sigma_n'}[\rho_n'(x,\theta)] &= m^{n-1}+\gamma_{\sigma_n'}[\rho_n(\,\bar{\rho}_n'(x,\theta))] \\ &= m^{n-1}+\bar{\gamma}_{\sigma_n'}[\,\bar{\rho}_n'(x,\theta)] \\ &\geq 0 \quad \text{for } n=1,\cdots,|\sigma'|. \end{aligned}$$

where the last inequality follows from the strict feasibility of $(\bar{\sigma}'(\cdot),\bar{\rho}'(\cdot))$. Similarly we have

$$x^{n-1}_{\sigma_n'} + \delta_{\sigma_n'}[\rho_n'(x, \theta)] \geq 0 \quad \text{for} \quad n = 1, \cdots, |\sigma'|.$$

Hence $(\sigma'(\cdot), \rho'(\cdot))$ is feasible and strictly preferable for some individual (x, θ) with positive probability which contradicts the equilibrium property of $(\sigma(\cdot), \rho(\cdot))$.█

This is the main result of this paper. The remainder of this paper will explore the implications of this theorem. Before proceeding, we summarize the result verbally. Condition (i) simply refers the direct mechanism. Condition (ii) requires that each individual should be induced to tell the truth consistently to each implementor he visits. For future reference, we shall refer this condition as the *Multiple Truth-telling Constraint*(MTTC). Condition (iii) requires that each individual should be induced to visit each class of implementors once and only once. We shall refer this condition as the *No Repeated Visit Constraint*(NRVC).

Comparing with the standard revelation principle, the condition becomes more severe. With multiple implementors, the truth-telling constraint, MTTC, becomes more restrictive to prevent inconsistent, opportunistic lies. Also there exists additional constraint, NRVC, to prevent extra gains from repeated visits to the same class of implementors. The revelation principle in the informationally centralized economy can be considered as a special case with a single, centralized implementor. With a single implementor, it is implicitly assumed that no repeated visits are possible. Hence the necessity of NRVC is never suspected. Since there is only one implementor with single visit, the single truth-telling constraint is sufficient.

Henceforth, we will use the notation (S, δ, γ), instead of $(\overline{S}, \overline{\delta}, \overline{\gamma})$, for the direct mechanism. Similarly, we will denote the new schedule of single visits and reporting plan by (σ, ρ), instead of $(\overline{\sigma}, \overline{\rho})$, while we understand that $\sigma \in \overline{\Sigma}$ and $\rho(x, \theta) = \{(x, \theta)\}_{n=1}^{\ell}$. Under the direct mechanism, individual consumer solves the problem

$$\max_{\sigma, \rho} \ u(z, \theta) \qquad \text{<P6.1'>}$$

$$\text{s.t.}\quad z_j = x_j + \sum_{n\in\sigma_j^{-1}} \delta_j(\rho_n(x,\theta))\ \ j=1,\cdots,\ell$$

$$x_{\sigma_n}^n = x_{\sigma_n}^{n-1} + \delta_{\sigma_n}(\rho_n(x,\theta)) \geq 0\ \ n=1,\cdots,\ell$$

$$m^n = m^{n-1} + \gamma_{\sigma_n}(\rho_n(x,\theta)) \geq 0\ \ n=1,\cdots,\ell$$

$$x_j^0 = x_j\ \ j=1,\cdots,\ell \ \text{ and }\ m^0 = 0 \ \text{ given.}$$

The social planner maximizes the social welfare function subject to the resource constraint and the incentive compatibility constraints: MTTC and NRVC.

$$\max_{\delta,\gamma}\quad W(z) = \int_\Theta \int_X u[z(x,\theta),\theta]\,dF(x)\,dG(\theta) \qquad \text{<P6.2'>}$$

$$\text{s.t.}\quad \int_\Theta \int_X z(x,\theta)\,dF(x)\,dG(\theta) \leq \bar{x}$$

$$z(x,\theta)\ \text{solves <P6.2> with } \sigma\in\bar{\Sigma} \text{ and}$$
$$\rho(x,\theta) = \{(x,\theta)\}_{n=1}^{\ell}.$$

6.3 The Role of the Clower Constraints

The third constraint of the consumer's problem requires that the currency holding should be nonnegative throughout his schedule of visits. We will call it the Clower constraint, according to Clower(1967) who emphasized it as an essential characteristic of the monetary economy. Since the money does not enter into the utility function, our natural question is this: What is the role of money in the allocation mechanism?

Lemma 1. (Condition for Binding Clower Constraint) Suppose that the allocation $z(x,\theta)$ is implementable by a direct mechanism (S,δ,γ). If the Clower constraint is binding for some (x,θ) with a positive probability(p.p.), then it is binding for (x,θ) almost everywhere(a.e). Conversely, if the Clower constraint is not binding p.p., then it is not binding a.e.[6)]

The lemma asserts that it cannot be true that the Clower constraint is binding for some individuals, while it is not binding for others. The Clower constraint should be either binding for all individuals or not binding for all individuals. But when the Clower constraint should be binding?

Lemma 2. (Nonbinding Clower Constraint and the Autarky) If the Clower constraint is not binding p.p., the optimal allocation is the autarky.

The contraposition of Lemma 2 is that if the optimal allocation is not the autarky, the Clower constraint is binding p.p. But Lemma 1 implies it will be binding a.e. Hence, as corollary, we have the following:

Corollary: If the autarky is not optimal, the Clower constraint is binding a.e.

Since the optimality of the autarky is quite exceptional, we assume that the Clower constraint is always binding.[7] When the Clower constraint is binding, we conjecture that money is individually valuable in the sense that the consumer can increase his utility with extra money. The conjecture is true given the continuity of delta and gamma. An immediate consequence is the terminal condition that individual currency holdings at the end of schedule of visits should be identically zero.

6.4 The Linearity of Implicit Pricing Functions

Now we explore the relationship between δ and γ. Our natural concern is whether there exist a linear relation between δ_j and γ_j with opposite sign under the optimal mechanism, in which case we can say price, buying and selling meaningfully.

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6) Proofs of Lemmas are in the appendix.

7) In fact, we can make the Clower constraint binding even no trades occur when the autarky is optimal.

Lemma 3. (Parametric Relation between Goods and Currency Transfers) γ_j depends on (x, θ) only through $\delta_j(x, \theta)$ for each $j = 1, \cdots, \ell$.

Hence there should be a parametric relation between δ_j and γ_j. To examine the parametric relation, define $D_j = \{d_j \in R: d_j = \delta_j(x, \theta), x \in X, \theta \in \Theta\}$, which is simply the range of net transfers of the j-th good. Then define the pricing function $\hat{\gamma}_j: D_j \to R$ by the parametric relation between $\gamma_j(x, \theta)$ and $\delta_j(x, \theta)$ so that

$$\hat{\gamma}_j[\delta_j(x, \theta)] = \gamma(x, \theta). \tag{6.7}$$

We first confirm that the net transfers of money and good move in the opposite direction under the optimal mechanism.

Lemma 4. (The *Quid pro Quo* Condition) $\hat{\gamma}_j: D_j \to R$ is strictly decreasing with $\hat{\gamma}_j(0) = 0$, for each $j = 1, \cdots, \ell$.

Lemma 4 implies that always the net transfers of goods are always made in exchange of money. Now we can talk about "trades", or "buying" and "selling" and "prices" meaningfully, though prices may vary depending on the size of "trades". Partition D_j into $D_j^+ = D_j \cap R_{++}$, $\{0\}$ and $D_j^- = D_j \cap R_{--}$. Define $\bar{p}_j^B: D_j^+ \to R_+$ and $\bar{p}_j^S: D_j^- \to R_+$ by $\bar{p}_j^B(b) = -\hat{\gamma}_j(b)/b > 0$, and $\bar{p}_j^S(s) = \hat{\gamma}_j(-s)/s > 0$ where $b \in D_j^+$ and $-s \in D_j^-$. Though $\bar{p}_j^B$ and $\bar{p}_j^S$ denote the average price for "buying" and "selling" good j in exchange of money, from the viewpoint of individuals.

The strength of NRVC is revealed when we consider arbitrage. When there exist an opportunity of arbitrage, NRVC will be violated. We define an arbitrage route if there exists a plan (σ, ρ) such that individual currency holdings increase without decreasing any of the initial possession of commodities. To be specific, (σ, ρ) is an arbitrage route if

$$\Delta m = \sum_{n=1}^{|\sigma|} \gamma_{\sigma_n}(\tilde{x}^n, \tilde{\theta}^n) > 0, \text{ and}$$

$$\Delta x_j = \sum_{n \in \sigma_j^{-1}} \delta_{\sigma_n}(\tilde{x}^n, \tilde{\theta}^n) \geq 0 \ \forall j.$$

We will say that the mechanism (S, δ, γ) satisfy the no arbitrage condition, if there exists no arbitrage route. To prove our second main result, we need the following lemma.

Lemma 5. (No Arbitrage Condition) The no arbitrage condition holds if and only if

$$\inf \ \bar{p}_j^{\,B}(D_j^+) \geq \sup \ \bar{p}_j^{\,S}(D_j^-) \quad j = 1, \cdots, \ell. \tag{6.8}$$

Now we state the second main result about the linearity of the pricing function.

Theorem 2. (Linearity of Pricing Functions) Suppose that the initial currency holdings of individual are indentically zero. Suppose further that the no arbitrage condition holds for the economy as a whole. Then the parametric relation $\hat{\gamma}_j : D_j \rightarrow R$ should be linear with strictly negative slope for each $j = 1, \cdots, \ell$. That is, there exists a number p_j such that $\hat{\gamma}_j[\delta_j(x, \theta)] = -p_j \delta_j(x, \theta)$.

Proof: By Lemma 5, there exists $p_j > 0$ such that

$$\tilde{p}_j^{\,S}(D_j^-) \leq p_j \leq \tilde{p}_j^{\,B}(D_j^+) \tag{6.9}$$

Denote $E_+(\cdot) = E[\cdot | \delta_j(x, \theta) > 0]$ and $E_-(\cdot) = E[\cdot | \delta_j(x, \theta) < 0]$, then

$$\begin{aligned} E\gamma_j(x, \theta) &= E_-\left\{(-\delta_j)\left[\frac{\hat{\gamma}_j(\delta_j)}{-\delta_j}\right]\right\} + E_+\left\{(-\delta_j)\left[\frac{-\hat{\gamma}_j(\delta_j)}{\delta_j}\right]\right\} \\ &\leq E_-[\delta_j(x, \theta)(-p_j)] + E_+[\delta_j(x, \theta)(-p_j)] \\ &= 0, \end{aligned}$$

where the last equality follows from the resource constraint. Hence we have $E\gamma_j(x, \theta) \leq 0$, $j = 1, \cdots, \ell$.

Now suppose that the strict inequality holds for some k, $b \in D_k^+$ and $c \in D_k^-$ with a positive probability. Then similar argument yields $E\gamma_k(x, \theta) < 0$. Then together with the above inequality, it implies

$$E\sum_{j=1}^{\ell}\gamma_j(x, \theta) < 0.$$

But with no initial currency holdings, feasibility requires $\sum_{j=1}^{\ell}\gamma_j(x, \theta) \geq 0$ for almost all (x, θ), which shows the above inequality is a contradiction. Hence the weak inequality (6.9) should hold with equality a.e. for $j = i, \cdots, \ell$.∎

Since the linearity of pricing functions or the existence of prices depends on the no arbitrage condition, it is of interest how strong the no arbitrage condition is. Especially we are concerned how the no arbitrage condition is related with NRVC.

Theorem 3. (Sufficient Conditions for No Arbitrage) Suppose that the following conditions hold for each $j = 1, \cdots, \ell$:

(i) For each $b \in D_j^+$, there exists an individual who can "buy" twice as much as b; and

(ii) For each $-s \in D_j^{-1}$, there exists an individual who can "sell" twice as much as s.

Then the NRVC condition implies the no arbitrage condition.

Proof: We will show the contraposition of Lemma 5. Suppose there exist some j, $b \in D_j^+$ and $-s \in D_j^-$ such that

$$\overline{p_j^B}(b) < \overline{p_j^S}(s).$$

Then for any positive number $\varepsilon > 0$, there exists a pair (h, k) of positive integers such that $hb \geq ks$ and $|hb - ks| < \varepsilon$. Hence, for sufficiently small $\varepsilon > 0$, we have

$$\begin{aligned} \Delta m &= ks\overline{p_j^S}(s) - hb\overline{p_j^B}(b) \\ &= ks[\overline{p_j^S}(s) - \overline{p_j^B}(b)] - \varepsilon\overline{p_j^B}(b) > 0 \end{aligned}$$

$$\Delta x_j = hb - ks \geq 0.$$

Hence "buying" h times of b units and "selling" k times of s units of good j is an arbitrage route if feasible to some individual.

i) Case $b \geq s$: The individual who can "buy" b units twice, with currency not less than $2\overline{p_j^B}(b)$, reschedule his visits, according to the following rule:

If $x_j^{n-1} \geq s$ sell s units; otherwise buy b units. (*)

Repeating it enough times, i.e. buying h times and selling k times, makes the arbitrage route. By construction, the nonnegativity condition for good j is kept intact. The Clower constraint is not violated, since he needs to buy b units of good j at most, holding less than s units in good j, while the loop, once completed, can only increase his currency holdings.

2) Case $b < s$: The one who can sell s units twice, i.e. who has $2s$ units of good j, can reschedule his visits according to the above rule. The nonnegativity of good j is obvious. The maximum currency requirement is $\overline{p_j^B}(b)b$, but since $\overline{p_j^S}(s)s + \overline{p_j^B}(b)b < 2\overline{p_j^S}(s)s$, it can be met by selling the initial stock of good j.∎

The above two theorems show that NRVC under certain condition implies the linear pricing functions. It can be shown that the converse is always true: if the pricing functions are linear, the NRVC condition holds.

6.5 The Case for Disloyal Implementors

We notice that the conditions of Theorem 3 may be satisfied easily, especially when the social implementers themselves may participate to the arbitrage. Before exploring such possibility, we examine further the consequences of the linearity of pricing functions.

Lemma 6. (Sufficiency of the Nonnegativity Conditions) Suppose a reporting plan $\{\tilde{x}^j, \tilde{\theta}^j\}_{j=1}^{\ell}$ satisfies the following two nonnegativity conditions

$$x_j+\delta_j(\tilde{x}^j, \tilde{\theta}^j)\geq 0 \quad j=1,\cdots,\ell, \tag{6.10}$$

and

$$\sum_{j=1}^{\ell}\hat{\gamma}_j[\delta_j(\tilde{x}^j, \tilde{\theta}^j)]\geq 0. \tag{6.11}$$

Then there exists a schedule $\sigma\in\overline{\Sigma}$ such that $(\sigma, \{\tilde{x}^j, \tilde{\theta}^j\}j=1^{\ell})$ is feasible.

The above lemma tells that once a reporting plan is chosen to satisfy the two nonnegativity conditions, then a corresponding schedule can be found easily. Hence it may be natural to call a reporting plan feasible if it satisfies the nonnegativity conditions, without referring the schedule at all. Moreover, since the first nonnegativity condition is implied by $Z=R_+^{\ell}$ under *single visits*, the feasibility of a reporting plan can be solely represented by the nonnegativity of final currency holdings. Using the linearity result, the feasibility can be represented solely by

$$\sum_{j=1}^{\ell}p_j[\delta_j(\tilde{x}^j, \tilde{\theta}^j)]\geq 0. \tag{6.12}$$

Obviously this is the ordinary budget constraint, except that the consumer makes orders of his quantity demanded or supplied by declaring his type according to δ_j to each class of implementors. Recall that we started from arbitrary schedule and messages. Then we turn to the direct mechanism $\{\delta_j(x,\theta), \gamma_j(s,\theta)\}$ with single visits and truth-telling to each implementor. And then with the linear relation of $\gamma_j(x,\theta)=-p_j\delta_j(x,\theta)$, we arrived at the above feasibility condition for consumers. Hence, the mechanism can be summarized (S,δ,p), instead of (S,δ,γ), and the social planner need only to induce consumers to reveal consumers' excess demand truthfully at $p=\{p_j\}$.

Theorem 4. (Incentive Compatibility and Consumer Maximization) An allocation is incentive compatible with mechanism (S,δ,p) if and only if

$$z(x,\theta) \in \arg\max_z u(z,\theta) \qquad \text{<P6.1''>}$$

$$s.t. \quad \tilde{z}_j = x_j + \delta_j(\tilde{x}^j, \tilde{\theta}^j)$$

$$\sum_{j=1}^{\ell} p_j \delta_j(\tilde{x}^j, \tilde{\theta}^j) \le 0 \text{ a.e.}$$

Proof: The second condition coincides with the second nonnegativity condition in Lemma 6, while $z \in R_+^{\ell}$ satisfies the first condition. Hence they satisfy the MTTC condition. The linearity of $\hat{\gamma}$ implies the NRVC condition.

Conversely, if an attainable allocation z is incentive compatible under (S, δ, γ), gamma has the linear property and the MTTC condition implies that $z(x,\theta) = x + \delta(x,\theta)$ should be a maximizer of individual utility in the problem of Lemma 6. Hence $z(x,\theta)$ solves the above problem. ∎

The above theorem tells that the social planner should design (δ, p) such that each consumer maximize his utility by revealing his type truthfully. That is, the net transfer functions and prices should be designed so that each consumer find it optimal to report his true type consistently truthfully, $\rho = \{x^j, \theta^j\}_{j=1}^{\ell}$, among all possible reporting plan $\tilde{\rho} = \{\tilde{x}^j, \tilde{\theta}^j\}_{j=1}^{\ell}$ to solve

$$u[x + \delta(x,\theta), \theta] = \max_{\rho} u[x + \delta(\{\tilde{x}^j, \tilde{\theta}^j\}), \theta] \text{ s.t. } \sum_{j=1}^{\ell} p_j \delta_j(\tilde{x}^j, \tilde{\theta}^j) \le 0. \qquad (6.13)$$

This suggests how the social planner can make the disloyal implementors to implement voluntarily his designed mechanism. Let them pursue their own profit by allowing to set buying and selling price lower or higher than the guide prices $p^* = \{p_j^*\}_{j=1}^{\ell}$. But also let them compete each other. Consumers will also try to sell high and buy low. Then the competition will drive down their profits to zero. The check and balance will be made not only among social implementors but also by consumers.

To recapitulate, what we have proved thus far is that the optimal allocation mechanism is the allocation which allocates goods like the market economy allocates them under the perfect competition. This is the optimal mechanism that satisfy both the resource constraint and the two incentive compatibility constraints, MTTC and NRVC, under the generalized revelation principle. To derive this result, we assumed implementors without direct communication, but implements the designed mechanism faithfully according to detailed instruction given by the social planner. But the loyal, uncoordinated implementors can be replicated by disloyal implementors, when they are allowed to pursue their profits but under full competition among them and consumers' free shopping schedule. Eventually profits fall to zero and social implementors end up to serve the mechanism to operate as intended by the social planner. It is much like the cake mechanism in which the elder brother should be induced to cut the cake evenly to his own interest.

To complete this conjecture, we compare the optimal mechanism under the informationally decentralized economy with the competitive equilibrium.

Definition: A competitive equilibrium(CE) is a pair of a price vector $p \in R_{+}^{\ell}$ and an allocation $z: X \times \Theta \rightarrow Z$ such that

(i) Given p, $z(x, \theta)$ solves $\max_{z'} u(z', \theta)$ s.t. $pz' \leq px$; and

(ii) Markets clear: $E z_j(x, \theta) = \bar{x}_j \quad j = 1, \cdots, \ell$.

Note that CE is completely specified by the physical properties of the economy after private shocks are realized.

The following lemma guarantees the existence of the competitive equilibria.

Lemma 7. (Existence of the Competitive Equilibria) For the economy (u, F, G), the set E_p of competitive equilibrium price vectors is nonempty.

Given the result, the existence of the optimal mechanism follows easily.

Theorem 5. (Existence of the Optimal Mechanism) There exists a solution (δ^*, γ^*) for the optimal mechanism problem if and only if there exists a solution $(\hat{p}, \hat{z})$ for the corresponding CE problem. Moreover the former solution solves the latter, while the converse is not necessarily true.

Proof: (i) The only if part follows rather trivially. Define $\hat{p}_j = -\gamma_j^* / \delta_j^*$ and $\hat{z}(x, \theta) = x + \delta^*(x, \theta)$. Obviously, then, $\hat{z}(x, \theta)$ solves the consumer problem in condition (i) of CE. The resource constraint coincides exactly in both problems.

(ii) For the sufficiency, the only problem arises when CE has multiple equilibria. Define the ordinary demand function $\tilde{z}(\hat{p}, x, \theta) = \hat{z}(x, \theta)$ solves the consumer problem in condition (i) of CE. From the maximum theorem and the strict concavity of u, tilde z is well-defined, continuous and homogeneous of degree zero in prices. Hence once p^* is chosen among the set of equilibrium vectors E_p, the set of incentive compatible allocation is a singleton, and the supporting mechanism is defined uniquely by $\delta_j^*(x, \theta) = \hat{z}_j - x_j$ and $\gamma_j^*(x, \theta) = \hat{p}_j \delta_j^*(x, \theta)$, for $j = 1, \cdots, \ell$. They obviously satisfy the MTTC and NRVC conditions. It remains to show that the planner achieves the optimum in $E_p \subset \triangle_+^\ell$. Consider a sequence $\{p^n\}$ of price vectors in E_p converging to p. Then the continuity of $\tilde{z}$ implies $E\tilde{z}(p, x, \theta) = \overline{x}$ so that the limit point is also in E_p. Hence E_p is closed. Since it is also bounded, the planner achieves the optimum in E_p. ▮

The coincidence allows us to provide an algorithm if the equilibrium is unique. Suppose that the global substitutability condition holds for the economy *(u,F,G)*. This would be the case if $u(z, \theta) = \prod_{j=1}^{\ell} (z_j)^{\theta_j}$, where we restrict θ to be in the interior of $\triangle_+^\ell$. Then not only the optimal allocation coincides with the unique competitive equilibrium, but also the optimal mechanism can be found by the algorithm of the Walrasian *tâtonnement*.

VII. Examples

7.1 Example 1: Bread and Wine in the Informationally Idealistic Economy

There are two goods, bread and wine, whose endowments are given as binomial iid random variables, each with {0, 1; 1/2, 1/2}.[8] The preference shocks are degenerating. Hence, there will be only four classes of individuals: (i) those who got both bread and wine, (1, 1), (ii) those who got bread only, (1, 0); (iii) those who got wine only, (0, 1); and (iv) those who got neither, (0, 0). We may call the first class the "rich," the fourth class the "poor," and the second and third classes the "middle" classes. Each class composes the same fraction 1/4 of the population. Individual preference is given by

$$u(z, \theta) = \ln z_1 + \theta \ln z_2 \tag{7.1}$$

Since the preference shocks are degenerating, θ is a constant. We assume $\theta = 1$ for the symmetry between the two goods which simplifies the solution greatly.

(1) Idealistic Economy

We need to find $z(x_1, x_2, \theta)$ for $x_1 \in \{0,1\}$, $x_2 \in \{0,1\}$ and $\theta \in \{1\}$, or the four 2-dimensional vectors(hence, eight numbers) for $z(1,1)$, $z(1,0)$, $z(0,1)$, and $z(0,0)$.[9] The resource constraint is

$$\frac{1}{4}\sum_{a=0}^{1}\sum_{b=0}^{1} z_j(a, b) \leqq 0.5 \quad j=1,2, \tag{7.2}$$

that is, average allocation of each good among the four classes should not exceed 0.5 units. We, as the social planner, want to maximize the expected value of (7.1) subject to (7.2). The marginal condition (4.1), together with (7.2), implies that the unique solution is

$$z_j^I(x_1, x_2) = 0.5 \quad j=1,2 \tag{7.3}$$

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8) In the notation for the multinomial events, we follow the convention to enumerate possible outcomes and then corresponding probabilities in order.

9) We omit the degenerating θ for the notational brevity.

for all four classes. The welfare, or the expected utility of ex ante identical individual, thus obtained, will be $W^I \equiv W(z^I) \approx -1.39$.[10] Obviously, the optimal solution (7.3) is (ex ante) efficient and equitable.

(2) Informationally Centralized Economy

Now we consider the optimal allocation in the informationally centralized economy. The truth-telling constraints are

$$\ln z_1(a,b) + \ln z_2(a,b) \geqq \ln[z_1(c,d) - c + a] + \ln[z_2(c,d) - d + b] \tag{7.4}$$

whenever

$$z_1(c,d) - c + a \geqq 0, \; z_2(c,d) - d + b \geqq 0, \tag{7.5}$$

for each individual (a,b) and for each possible lie (c,d). Since each of (a,b,c,d) may take two values, 1 or 0, there are 16 constraints, and, among them, four hold trivially by reflexivity.

Apply the symmetry and note

$$\begin{aligned} 2\ln z_1(1,1) &\geqq 2\ln[z_1(0,0)+1] \\ 2\ln z_1(0,0) &\geqq 2\ln[z_1(1,1)-1] \end{aligned}$$

which implies

$$z_1(0,0) = z_1(1,1) - 1. \tag{7.6}$$

Also we have

$$\ln z_1(0,1) + \ln z_2(0,1) \geqq \ln z_1(0,0) + \ln[z_2(0,0)+1]. \tag{7.7}$$

It is easy to check that the above two constraints summarize the truth-telling constraints (7.4). Hence, we may maximize the welfare function subject to the resource constraints and the above two constraints. The optimal allocation mechanism for the informationally centralized economy is given by the following eight numbers:

10) The negativity of the welfare level is of no particular significance, since the welfare function is only unique up to a linear positive transformation.

$$z_1^C(0,0) = z_2^C(0,0) = 0.125,$$

$$z_1^C(0,1) = z_2^C(0,1) = 0.375,$$

$$z_1^C(1,0) = z_2^C(1,0) = 0.375,$$

$$z_1^C(1,1) = z_2^C(1,1) = 1.125.$$

With the optimal allocation mechanism, the welfare will be $W^C \equiv W(z^C) \approx -1.96$.

Several properties are noteworthy. First, with the truth-telling constraints, the idealistic allocation is not achievable, hence $W^C < W^I$. First, What if we apply z^I to the dishonest economy? Each individual in any class then would lie that they have nothing, trying to get net distribution of 0.5 of both goods, which will violate the resource constraints.[11] Second, each individual improves utility by being part of the society. There do not exist simple benevolent behaviors like the honest economy. Third, the rich get richer which seems to be pathologic. This is because the poor should be subsidized somehow, but the social implementors cannot distinguish the poor from the rich.[12] Fourth, the resources to subsidize the poor (along with the rich) come from the middle classes by "buying cheap" and "selling high." If we allow subcoalition among individuals, the middle class may trade among themselves, which will spoil our plan. The allocation is not efficient, equitable, or benevolent. Still it is the second-best, and some efforts towards equity and benevolence are made, as is seen the unilateral transfer to the poor.

(3) Informationally Decentralized Economy

We consider the incentive compatible allocation in the informationally decentralized economy with dishonest individuals and decentralized social implementors. We assume that there are two kinds of social implementors, one

11) Can't we use a group punishment(punish all for part's fault) to prevent false announcement? Even if such is legal, it is not subgame-perfect, hence does not provide a credible threats.

12) The rich always can pretend that they are the poor, while the reverse is not true. Of course, we as the social planner have taken full account of this possibility and the welfare loss from the lies of the rich. Still the benefit subsidizing the poor surpasses the cost so we decide to subsidize the poor. Note that if we can keep the rich from free-riding, we can increase the subsidy to the poor substantially.

dealing with the first good only and the other dealing with the second good only. We consider the following mechanism specified generically by $\{z_j, \gamma_j\}_{j=1}^{2}$, where $z_j : X \times \Theta \rightarrow \mathbb{R}$ and $\gamma_j : X \times \Theta \rightarrow \mathbb{R}$ are the amount of the good and money given(or taken, if negative) by the j-th kind of social implementors to each visitor. Each social implementor in kind j is instructed to "process" each visitor as follows:

(i) Ask his private shocks, to get $(\tilde{x}_1, \tilde{x}_2)$ which may or may not be the truth.

(ii) If $z_j(\tilde{x}_1, \tilde{x}_2) - \tilde{x}_j \geq 0$, request him to submit $-\gamma_j(\tilde{x}_1, \tilde{x}_2) \geq 0$. If he submits not less than requested, give him $z_j(\tilde{x}_1, \tilde{x}_2) - \tilde{x}_j \geq 0$. Otherwise, give him nothing back.

(iii) If $z_j(\tilde{x}_1, \tilde{x}_2) - \tilde{x}_j < 0$, request him to submit $\tilde{x}_j - z_j(\tilde{x}_1, \tilde{x}_2) > 0$. If he submits not less than requested, give him $\gamma_j(\tilde{x}_1, \tilde{x}_2) > 0$. Otherwise, give him nothing back.

The mechanism in this case is specified by the 8 vectors or 16 numbers,

$$z(0,0),\ z(0,1),\ z(1,0),\ z(1,1)$$
$$\gamma(0,0),\ \gamma(0,1),\ \gamma(1,0),\ \gamma(1,1)$$

First, we suppose that individuals are somehow constrained to visit each kind of social implementors once and only once. Then the truth-telling conditions are given by the generic form

$$\ln z_1(a,b) + \ln z_2(a,b) \geq \ln[a + z_1(c,d) - c] + \ln[b + z_2(e,f) - f], \tag{7.8}$$

whenever

$$a + z_1(c,d) - c \geq 0,$$

$$b + z_2(e,f) - f \geq 0, \text{ and} \tag{7.9}$$

$$\gamma_1(c,d) + \gamma_2(e,f) \geq 0,$$

i.e. an individual should not benefit by telling a lie (c,d) to the first kind of social implementors and a lie (e,f) to the second kind of social implementors whenever such lies are not detectable.[13] Since each of a to f takes two values,

13) If we allow subcoalitions among individuals, the constraints for the truth-telling constraints may not hold.

either 0 or 1, there are $2^6 = 64$ inequality constraints, among which four are reflexive. Since the truth-telling should be feasible by individuals, we also have

$$\gamma_1(a, b) + \gamma_2(a, b) \geq 0, \tag{7.10}$$

for all $a, b = 0, 1$.

Now we consider the condition for no repeated visits. Since individuals should not benefit from repeated visits or no visits,

$$\begin{aligned} z_1^D(0,0) &= z_2^D(0,0) = 0 \\ z_1^D(1,1) &= z_2^D(1,1) = 1 \end{aligned} \tag{7.11}$$

That is, there is no way to give anything to the poor. By the same reason, there is no way give something to the rich who may pretend to be the poor. Still we may have some exotic reason to give something to the rich, while not giving to the poor. But this possibility is precluded by the diminishing marginal utilities.

Then the resource constraints, together with the symmetry, implies

$$\begin{aligned} z_1^D(0,1) &= z_2^D(0,1) = 0.5 \\ z_1^D(1,0) &= z_2^D(1,0) = 0.5. \end{aligned} \tag{7.12}$$

It remains to determine γ's that satisfy (7.9) and (7.10). Define k, l, m, and n as follows:

$$k = \gamma_1(0,0) = \gamma_2(0,0),$$

$$l = \gamma_1(0,1) = \gamma_2(1,0),$$

$$m = \gamma_1(1,0) = \gamma_2(0,1),$$

$$n = \gamma_1(1,1) = \gamma_2(1,1),$$

where the latter equalities follow from symmetry.

From (7.10). we have

$$k \geq 0, \ n \geq 0 \text{ and } l + m \geq 0. \tag{7.13}$$

For (0,0) not to announce {(0,1), (0,0)} or {(0,1), (1,0)}, we should have

$$k + l < 0 \text{ and } l < 0.$$

For (0,1) not to announce {(0,1), (1,1)}, we should satisfy

$$l + n < 0.$$

Since money is individually valuable, individuals should not be able to accumulate money by repeated visits, which implies that the inequalities in (7.10) should hold strict equality. Therefore, we have

$$k = 0, \ l = -m, \ m > 0, \text{ and } n = 0, \tag{7.14}$$

where the positive number remains indeterministic. This is not surprising since we have the usual homogeneity result in absence of money illusion. If we arbitrarily fix $m = 1$, we have

$$\gamma_1^D(0,0) = \gamma_2^D(0,0) = 0,$$

$$\gamma_1^D(0,1) = \gamma_2^D(1,0) = -1,$$

$$\gamma_1^D(1,0) = \gamma_2^D(0,1) = 1,$$

$$\gamma_1^D(1,1) = \gamma_2^D(1,1) = 0.$$

It is easy to check that all other constraints in (7.9), are satisfied. Trades or cooperation occurs only between middle classes. Note the welfare thus obtained is $W^D \equiv W(z^D) = -\infty$ though it is certainly superior to the autarky. The allocation is not efficient, equitable, or benevolent. For the efficiency, it is best described as the third-best solution. No efforts towards equity or benevolence are made.

7.2 Example 2: Normal and Handicapped in the Informationally Idealistic Economy

We consider the same situation as Example I1, except that the class (0, 0) does not exist. The probability of being each of the three classes is now given by {(1, 1), (1, 0), (0, 1); 1/2, 1/4, 1/4}. We may describe those individuals with a zero endowment, either (1, 0) or (0, 1), as "handicapped," and those endowed with both as "normal."[14)]

(1) Idealistic Economy

Now the average endowment becomes 3/4 for each good. With the new resource constraints, the marginal condition (6) implies

$$z_j^I(x_1, x_2) = 0.75 \quad j=1,2 \tag{7.15}$$

and the optimal welfare level becomes $W^I \approx -0.58$. Again, the optimal solution (7.15) is *ex ante* efficient and equitable.

(2) Informationally Centralized Economy

For the informationally centralized economy, among the truth-telling constraints, only the following one is binding:

$$2 \ln z_1(1,1) \geqq \ln[z_1(0,1)+1] + \ln z_1(0,1)$$

that is, it should not be preferable for the normal to pretend to be handicapped. The optimal allocation mechanism is given by the six values:

$$z_1^C(0,1) = z_2^C(0,1) = 9/16,$$

$$z_1^C(1,0) = z_2^C(1,0) = 9/16,$$

$$z_1^C(1,1) = z_2^C(1,1) = 15/16.$$

The welfare will be $W^C \approx -0.64$. Note that the social planner exploits the nonexistence (0, 0) to narrow down the space of possible lies. Again, the allocation is not efficient, equitable, or benevolent. Still the allocation achieves the second-best efficiency, and some efforts were made towards equity and benevolence, as is seen that the normal subsidize the handicapped.

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14) If you prefer, you may use the talented for (1,1) and the normal for (1,0) or (0,1). It still seems that the descriptions of normal and handicapped may seem to be ill-used, since such characteristics is usually referring to nontransferable characteristics in θ. Still the analysis is very similar in that the handicapped will be compensated in transferable goods. That is in idealistic economy, all are insured even for the nontransferable characteristics. In this case, however, the equity property will not hold, in the usual sense referring only to the commodity bundle.

(3) Informationally Decentralized Economy

We consider now that individuals may lie and pay repeated visits now. It can be easily checked that the solution in Example 1 for the informationally decentralized economy remains valid in this case, though the distribution functions for (0,0) become redundant.[15] Trades or cooperation occur only between the "handicapped." Hence, the allocation mechanism is described by

$$z_1^D(1,1) = z_2^D(1,1) = 1,$$

$$z_1^D(0,1) = z_2^D(0,1) = 0.5,$$
$$z_1^D(1,0) = z_2^D(1,0) = 0.5.$$

and

$$\gamma_1^D(0,1) = \gamma_2^D(1,0) = -1,$$

$$\gamma_1^D(1,0) = \gamma_2^D(0,1) = 1,$$

$$\gamma_1^D(1,1) = \gamma_2^D(1,1) = 0.$$

Again the allocation is the third-best in efficiency, and no efforts towards equity are made.

7.3 Example 3: Hungry and Thirsty in the Informationally Idealistic Economy

We consider the case with a linear utility function,

$$u(z,\theta) = \sum_{j=1}^{\ell} \beta^{j-1}\theta_j z_j$$

with $\theta = (\theta_1, \cdots, \theta_\ell, \beta)$ so that θ_j represents the constant marginal utility of the j-th good.[16] Though the linear utility is only weakly concave, the solution will be unique. Besides, it gives some insight to the concept of equity, when strict concavity(or strictly diminishing marginal utility) fails. We assume that endowment

15) In this sense, the optimal scheme for the decentralized economy is more robust to the availability or transferability of goods.

16) We may imagine that the j-th good is a medicine to cure the j-th disease while θ_j is the state indicating whether the individual gets the j-th disease.

shocks are degenerating with $x_j = 1$ for $j = 1, \cdots, \ell$. The set of preference shocks is given by $\Theta = \{a, b\}$ with $b_j > a_j > 0$, $j = 1, \cdots, \ell$, where a_j means a low preference for the j-th good and b_j a high preference for the j-th good. The probability of preference shocks are given recursively by the Markov process

$$\Pr\{\theta_j \mid \theta_{j-1} = a\} = \Pr\{\theta_j \mid \theta_{j-1} = b\} = \rho \quad j = 2, \cdots, \ell,$$

where $0 < \rho < 1$ is the probability of reversal, and the initial condition

$$\theta_1 = \{a_1, b_1; 1/2, 1/2\}.$$

By construction the marginal probability satisfies

$$\Pr\{\theta_j = a_j\} = \Pr\{\theta_j = b_j\} = \frac{1}{2}, \quad j = 1, \cdots, \ell.$$

We consider the simplest case again, by restricting $\ell = 2$ and $\beta = 1$.

(1) Idealistic Economy

There are four classes of individuals again: (i) those with high preferences for both goods, (b_1, b_2), the "hungry and thirsty," (ii) those with high preference only for the first good, (b_1, a_2), the "hungry," (iii) those with high preference only for the second good, (a_1, b_2), the "thirsty," and (iv) those with no high preferences, (a_1, a_2), the "less needed". It is straightforward to see that the optimal allocation should give whatever available to those with high preference so that

$$z_1^I(1, 1; b_1, \cdot) = 2$$
$$z_2^I(1, 1; \cdot, b_2) = 2$$

while those with low preferences get nothing. The resultant welfare will be

$$W^I = b_1 + b_2.$$

(2) Informationally Centralized Economy

Note that equity in this case means that the more needed one get everything. The allocation (15) is (ex ante) efficient, but not equitable in the usual sense. This demonstrates that the concept of equity depends on the implicit assumption of diminishing marginal utility as well as interpersonal comparability.[17]

With $a_j > 0$, even those with low preference may pretend to have high preference under the honest solution. Hence, we need to give something to the low preferences as well. The solution is not unique. One optimal solution is given by the eight values:

$$z^C(a_1, a_2) = (1,1),$$

$$z^C(a_1, b_2) = (0,2),$$

$$z^C(b_1, a_2) = (2,0),$$

$$z^C(b_1, b_2) = (1,1).$$

Note that only the middle classes are improving utility. Though it is still the second-best, no efforts towards equity or benevolence are made. It shows also that the solution in the informationally centralized economy may be the same as in the informationally decentralized economy.

(3) Informationally Decentralized Economy

Now, individuals may pay multiple visits with inconsistent announcements. The symmetry implies that the "price ratio" is to be 1. Hence, the optimal solution becomes

$$z^D(a_1, a_2) = (1,1),$$

$$z^D(a_1, b_2) = (0,2),$$

$$z^D(b_1, a_2) = (2,0)$$

$$z^D(b_1, b_2) = (1,1)$$

with

$$\gamma^D(a_1, a_2) = (0,0),$$

$$\gamma^D(a_1, b_2) = (1,-1),$$

17) Though there seems to be segregation or conflict among equity and others, I think this is because of the undesirable property of the usual definition of equity.

$$\gamma^D(b_1, a_2) = (-1, 1),$$

$$\gamma^D(b_1, b_2) = (0, 0).$$

Goods flow to the most needed only on the *quid pro quo* basis.

VIII. Welfare Implications and Other Discussions

We first consider the welfare ranking of the allocation by the optimal incentive compatible mechanism (δ^*, p^*) of the informationally decentralized economy with dishonest citizen and disloyal implementors. It is obviously inferior to the optimal allocation of the idealistic economy without incentive compatibility constraint. The bottom line is the autarky, which is obviously inferior to the optimal allocation in the informationally decentralized economy. Hence we have

$$W(z^I) \geq W(z^D) \geq W(z^A) \tag{8.1}$$

where the welfare in the autarky, $W(z^A)$, is given simply by $W(z^A) = \int_\Theta \int_X x\, dF\, dG$.

8.1 Welfare Comparison Between Informationally Centralized Economy and Informationally Decentralized Economy

How does it compare with the optimal allocation z^C by the optimal incentive compatible mechanism of the informationally centralized economy? It can be guessed that z^D is inferior to the second-best solution z^C as long as the incentive compatibility constraint in the informationally decentralized economy, i.e. MTTC and NRVC, is more stringent than the single TTC in the informationally centralized economy. The following theorem confirms this conjecture.

Theorem 6. (Incentive Compatibility between Informationally Centralized vs Decentralized Economies) If there exists an incentive compatible mechanism to achieve an allocation $z(x, \theta)$ in the informationally decentralized economy, then there also exists an incentive compatible mechanism to achieve the *same* allocation in the informationally centralized economy.

Proof: Suppose $z(x,\theta)$ is implementable in the informationally decentralized economy but not in the informationally centralized economy. Then there exists $(x'\theta')$ such that consumer of type (x,θ) can increase his utility by pretending to be in the informationally centralized economy, violating the TTC condition:

$$u[x-x'+z(x',\theta'),\theta] > u[z(x,\theta),\theta] \text{ with } x+\delta(x'\theta')\geq 0.$$

But, since $\sum_{j=1}^{\ell}\gamma_j(x,\theta)\geq 0$ a.e., reporting (x',θ') consistently is feasible and strictly preferable for consumer (x,θ) in the informationally decentralized economy, which contradicts the MTTC condition.∎

Hence $W(Z^D)$ is no greater than $W(Z^C)$. With this result, we rewrite the above ranking as

$$W(z^I)\geq W(z^C)\geq W(z^D)\geq W(z^A) \tag{8.2}$$

The ranking motivates us to call z^D as the third-best solution, as z^I and z^C are frequently called the first- and the second-best solutions.

We should haste to say that it does not mean that the centralized economic system is better than the decentralized one. It only means that the social planner can design a better allocation mechanism with loyal, coordinated implementors than with disloyal, uncoordinated implementors. What happens if the social planner has mistaken disloyal, uncoordinated implementors as loyal, coordinated ones and adopted the misplaced optimal mechanism for the former? Assuming disloyal implementors loyal is perhaps as disastrous as assuming dishonest citizens honest. After all, both implementors are chosen among citizen, and it is natural they have similar (dis)honesty or (dis)loyalty. Take the example of bread and wine in 7.1 and imagine we adopted the centralized mechanism for the informationally decentralized economy. The result would be and outright bankruptcy.

Of course, in reality, the wrong placed mechanism may persist for a while, with some strengthened monitoring and auditing for private information of citizen as well as that of social implementors. But there will appear lucrative black markets where not only citizens exchange their possessions but also social implementors supply their official holdings. The latter is more destructive and feasible, since some citizen exploit the opportunity of unilateral net transfers(0.125 in the bread-wine example), which violates the resource constraint more or less gradually, and the social implementors will be given some screening authority.

Of course, there may be some measures to make implementors apparently loyal other than market competition discussed in the last section. Efficiency wage to implementors, combined with close monitoring and auditing, may be adopted. But, then, they will have incentives to maintain their position safely and possibly enlarge their authority. This will make them slow in finding and adopting technical progress or other cost-saving measures. The weakness will be more serious if monitoring is more costly like in agriculture.

On the other hand, family and other small, homogenous group, including business firms, may well do better in a centralized mechanism. It should be reminded that resource allocation even in the most advanced market economy is not solely dependent on a decentralized mechanism like markets.

8.2 Welfare Comparison Between Idealistic Economy and Informationally Decentralized Economy

Now we turn to compare allocations of the informationally decentralized economy with that of the idealistic economy. Apparently the first- and third-best solutions share certain properties: both use money and markets. In fact, though it only reveals the deficiency of the Pareto optimality, it can be shown that the allocation with the optimal mechanism in the informationally decentralized economy is also Pareto optimal as is the first-best solution.

We examine the efficiency of the optimal allocation in the informationally decentralized economy.

Definition: Consider an attainable allocation, z. Another attainable allocation z' is said to be a *weak Pareto improvement* to z, if

$$u[z'(x,\theta),\theta] \geq u[z(x,\theta),\theta]$$

a.e. It is a *strong Pareto improvement* if the inequality holds strictly for some (x,θ) p.p. An allocation z is *Pareto optimal* if there does not exist a strong Pareto improvement to it.

Though the allocation in the informationally decentralized economy is inferior to the second-best solution in the informationally centralized economy, it turns out to be Pareto optimal. We will need the following lemma. Fix an allocation z. Define

$$H_z(x,\theta) = \{\tilde{z} \in Z: u(\tilde{z},\theta) \geq u[z(x,\theta),\theta]\}$$

$H_z(x,\theta)$ is the set of consumptions which is weakly preferable to current allocation $z(x,\theta)$ for individual (x,θ). Define also

$$H_z = \{\tilde{x} \in X: \tilde{x} = Ez(x,\theta), z'(x,\theta) \in H_z(x,\theta)\}$$

H_z is the (mean) endowment which allows an allocation, which is weak Pareto improvements to z.

Lemma 8. (Minimal Properties of Cost and Endowment) For each $p \in R^{\ell}_{+}$, $z(x,\theta)$ minimizes $p\tilde{z}$ on $H_z(x,\theta)$ a.e. if and only if $Ez(x,\theta)$ minimizes px on H_z.

The following theorem asserts the Pareto optimality of the allocation of the informationally decentralized economy.

Theorem 7. (Pareto Optimality of the Third-Best Solution) The allocation $z^*(x,\theta) = \hat{z}(\hat{p},x,\theta)$ is Pareto optimal for each $\hat{p} \in E_p$.

Proof: The attainability follows from the market clearing property of $\hat{p}$. Let $z'(x,\theta)$ be and attainable allocation such that

$$u[z'(x,\theta),\theta] \geq u[z^*(x,\theta),\theta] \text{ a.e.}$$

We will show that the supposition implies

$$u[z'(x,\theta),\theta] \leq u[z^*(x,\theta),\theta] \text{ a.e.} \tag{8.3}$$

From the duality theorem, $z^*(x,\theta)$ minimizes $\hat{p}z$ on $H_{z*} = (x,\theta)$. Hence, from the above Lemma, $\bar{x} = Ez^*(x,\theta)$ minimizes $\hat{p}x$ on H_{z*}. From the attainability of z', $Ez'(x,\theta) \leq \bar{x}$. By the nonsatiation assumption, it is sufficient to show that (*) holds for z' such that $Ez'(x,\theta) = \bar{x}$.

But, then, $E(x,\theta)$ minimized $\hat{p}$ on H_z^*, which implies $z'(x,\theta)$ minimizes $\hat{p}z^*$ on $H_z^*(x,\theta)$ a.e. Thus we have $\hat{p}z'(x,\theta) \leq \hat{p}z^*(x,\theta)$ a.e. Thus we have

$$\hat{p}z'(x,\theta) \leq \hat{p}z^*(x,\theta),$$

which implies (8.3) ▮

We have shown that the Clower constraint is binding unless the autarky is optimal. The following corollary supplements the condition.

Corollary: The autarky is optimal if and only if it happens to be Pareto optimal.

Proof: The necessity follows directly from the Pareto optimality of the third-best solution. For the sufficiency, suppose that the autarky is Pareto optimal but it is not the third-best solution. Then there exists an attainable, incentive compatible allocation z such that $W(z) > W(z^A)$, which implies $u[z(x,\theta),\theta] > u(x,\theta)$ p.p. The incentive compatibility also implies that $u[z(x,\theta),\theta] > u(x,\theta)$ a.e. But these two inequalities contradicts to the Pareto optimality of the autarky, x. ▮

On the one hand, the Pareto optimality of the third-best solution is natural. Individuals engage only in Pareto-improving "trades." Otherwise, they cannot be induced to visit each class of implementors. The social planner should not pass by any pair-wise Pareto improving opportunities unexploited when each individual visit. (However, pair-wise Pareto optimality implies the overall Pareto optimality.)

On the other hand, it is problematic. The third-best solution is inferior to the second-best solution. It is well-know that the second-best solution is inferior to the first-best solution and is not Pareto optimal. How can it be true that the third-best solution is again Pareto-optimal as is the first-best solution, while the second-best solution is not Pareto optimal?

The puzzle stems from the fact that the notion of Pareto optimality depends on the commodity space and that we are dealing with different commodity spaces in the first- and third-best solutions. The Pareto optimality in the first-best solution, is relative to the contingent commodity space, while the Pareto optimality in the third-best solution is relative to the spot commodity space. In the first-best solution, trades occur before private shocks are realized and the relevant commodities are contingent commodities. In the third-best solution, "trades" occur after private shocks are realized and the relevant commodities are spot commodities. This makes a big difference. Consider there is only one good, "a loaf of bread," and two consumers, A and B, who are equally hungry. Suppose that God tosses a coin for them so that the winner gets 80% of the bread and the loser gets only 20%. If they trade before shocks are realized, they can be perfectly insured so that the Pareto optimal allocation will be each having a half loaf. If they trade after shocks are realized, the autarky of having 80% and 20% will be Pareto optimal.

Can individuals in the informationally decentralized economy achieve the Pareto optimality in the first-best sense by trading before shocks are realized so that they can be perfectly insured? No. It is not possible for the dishonest citizens due to private information, since contingent trades or insurances are only possible on public events. The informationally decentralized economy, even using the same instruments such as markets, prices and money, cannot mimic the first-best solution of the idealistic economy.

The second fundamental theorem of welfare economics asserts that any desired Pareto optimal allocation can be supported as a competitive equilibrium with suitable initial lump-sum transfers. Since the first-best solution is also Pareto optimal relative to the spot commodity space, one may suspect that the first-best solution can be imitated in the informationally decentralized economy with suitable lump-sum transfers. The answer is again negative. It is not possible for the social planner to make suitable lump-sum transfers, since the endowment itself is private informatio n.[18)]

Our final result is that the incentive compatibility constraint in the informationally decentralized economy can be represented by some weights. In other words, the optimal solution can be considered to a solution with some weighted social welfare function with the resource constraint only as in the idealistic economy.

Definition: A normalized weight is a nonnegative function $w: X\times\Theta\rightarrow R_+$ such that

$$\int w(x,\theta)\,dx\,d\theta = 1.$$

An attainable allocation $z(x,\theta)$ is said to be *idealistically efficient with respect to the weight function* $w(x,\theta)$ if it solves the problem

$$\max \int_\Theta\int_X w(x,\theta)\,u[z(x,\theta),\theta]\,dF\,dG \qquad \text{<P8.1>}$$
$$\text{s.t. } Ez(x,\theta)=\bar{x}$$

Theorem 8. (Existence of the Weight Function Representing Incentive Compatibility Constraint) Suppose that z^* is optimal with (δ^*,γ^*). Then there exists a weight w such that z^* is idealistically efficient with respect to w.

Proof: Let $H^0_{\succ z*}(x,\theta)$ be the set of consumption which is strictly preferable to $z^*(x,\theta)$ for the preference θ. Define $H^0_{\succ z*}=EH^0_{\succ z*}(x,\theta)$. It is easy to check that

18) Even if it is public, the lump-sum transfers may affect individual incentives in a sequential trades. The second welfare theorem avoids such incentive problems by assuming that all trades occur on the initial date while only implementation of contracts is occurring subsequent dates. For example, an equitable, Pareto optimal allocation can be obtained with just one lump-sum transfers in the idealistic economy, while repeated intervention is needed in the sequential economy.

$H^0_{>z*}$ is convex and that it does not contain $\overline{x}=Ez^*(x,\theta)$. Hence, by the Minkowsky theorem, there exists a separating hyperplane between $H^0_{>z*}$ and $\overline{x}$ in R^ℓ, or there exists a vector $p\in R^\ell$, $p\neq 0$, $\|p\|<\infty$ and a number $\alpha\in R$ such that $px\geq\alpha$ for all $x\in H^0_{>z*}$ and $p\overline{x}\leq\alpha$. The closure $\overline{H}_{>z*^0}$ of $H^0_{>z*}$ contains H_{z*}, hence H_{z*} lies in the closed half space *above* the hyperplane. Since $\overline{x}\in H_{z*}$, $\overline{x}$ minimizes px on H_{z*}. It follows from Lemma 8 that $z^*(x,\theta)$ minimizes $p\overline{z}$ on $H_{z*}(x,\theta)$ a.e.

Moreover we have $p_j>0$ for each $j=1,\cdots,\ell$. Suppose not, then $z_j^*(x,\theta)$ should be arbitrarily large since $u_j(\overline{z},\theta)>0$, which contradicts to the essential boundedness of x, or $\overline{x}<\infty$.

Thus the duality theorem implies that $z^*(x,\theta)$ solves

$$\max\ u(\overline{z}',\theta) \quad \text{s.t.}\ p\overline{z}'\leq pz^*$$

for each (x,θ) a.e.

Let $\mu: X\times\Theta\to R_+$ be defined by the Lagrangean multiplier $\mu(x,\theta)$ for the above maximization problem for each individual (x,θ). The non-satiation and $p>0$ implies $\mu(x,\theta)>0$ a.e. Hence we define the weight by

$$w(x,\theta)=\frac{E\mu(x,\theta)}{\mu(x,\theta)}. \tag{8.4}$$

To check the weight w in (8.4) indeed has the desired property, note that the problem <P8.1> has a unique solution for each given weight w, since the objective is strictly concave and the constraints are linear. The pointwise optimization gives

$$\frac{E\mu(x,\theta)}{\mu(x,\theta)}\frac{\partial u[z(x,\theta),\theta]}{\partial z_j}-\lambda_j=0,\ j=1,\cdots,\ell \tag{8.5}$$

a.e., where λ_j is the Lagrangean multiplier for the j-th resource constraint. Equation (8.5) and the resource constraint determine the unique solution z. From the envelope theorem, we have

$$\mu(x,\theta) = \frac{1}{p_j}\frac{\partial u[z^*(x,\theta),\theta]}{\partial z_j} \tag{8.6}$$

and

$$\lambda_j = \frac{\partial Ew(x,\theta)u[z(x,\theta),\theta]}{\partial x_j} \tag{8.7}$$

for $j = 1, \cdots, \ell$. Substituting (8.6) and (8.7), we can easily check that z^* solves (8.5) a.e., while the resource constraint is implied by the optimality of z^*. ∎

Hence the social planner in the informationally decentralized economy behaves as if he maximizes a "distorted" welfare function with the resource constraint only. The incentive compatibility constraint, MTTC and RVIC, can be represented by some distortion of weights. However, it should be noted that the weight function $w(x,\theta)$ can be found only in hindsight.

There is possible confusion between the first- and third-best solution not only by the common Pareto optimality, but also by the common use of markets, prices and money. It should be emphasized, however, that the use of money in the idealistic economy is optional, while it is essential in the informationally decentralized economy. In the idealistic economy, all trades are made in the initial date, on Friday, before private shocks are realized. This is true even in a dynamic setting. There will be no sequential trading in the idealistic economy, while it is compulsory in the informationally decentralized economy. In fact, even the markets, prices, or any institutions are not essential for the idealistic economy. Without any informational problem, without any transactional frictions, any institutions will work to produce eventually the idealistic solution.

IX. Conclusion

We analyzed the revelation principle applicable when the social planner delegates his authority to implement the allocation mechanism to informationally decentralized social implementors. Since the social implementors cannot communicate each other, individuals can choose free schedule of visits and inconsistent announcements of types to implementors they visit. Hence we expect that the incentive compatibility constraint is more severe in the informationally decentralized economy than the informationally centralized economy, where individuals are controlled to visit the central implementor once and only once. The direct mechanism in the informationally decentralized economy requires two incentive compatibility constraints (Theorem 1)--(i) each individual should be induced to tell the truth consistently to each implementor he visits (the multiple truth-telling constraint: MTTC), and (ii) each individual should be induced to visit each class of social implementors once and only once(the no repeated visit constraint: NRVC). The standard revelation principle, which consists of one truth-telling constraint, can be considered as a special case with one social implementor and individuals are controlled to visit the central implementor once and only once. This is the main result of the paper, while the remainder of the paper is devoted to examine the implications of this result.

Applied to a pure exchange economy, these two incentive compatibility conditions imply that the monetary market economy is the optimal allocation mechanism when citizen are dishonest and social implementors are informationally decentralized or disloyal. To be specific, the nonnegativity condition for goods and currency holdings during consumer schedule of visits to social implementors plays an important role to prevent a certain inconsistency of false announcements. Especially the nonnegativity condition for currency holdings, the Clower constraint, should be binding unless the autarky is optimal(Lemma 2). Moreover, the net transfer of currency should depend solely on the net transfer of good, $\gamma_j(x,\theta)=\hat{\gamma}_j[\delta_j(x,\theta)]$, in order not to violate the MTTC condition. (Lemma 3). This means that the social planner is not allowed to determine the net transfer function for good $\delta_j(x,\theta)$ and that for currency $\gamma_j(x,\theta)$ independently. Furthermore, the net transfer of the currency should be strictly decreasing with $\hat{\gamma}_j(0)=0$ (Lemma 4), which implies that net transfers between consumer and social implementers should be on the *quid pro con* basis. Since money is individually valuable in the sense that he can increase his utility with more currency holdings, the NRVC condition implies no arbitrage condition should hold under reasonable sufficiency conditions(Theorem 2). But this in turn implies that the pricing functions should be linear, $\hat{\gamma}_j[\delta_j(x, \theta)] = p_j \delta_j(x, \theta)$, or there exists a price for each good. But, then, since accommodating schedule can be found easily once the nonnegativity conditions for good and currency holdings hold (Lemma 6), the maximization problem of individuals reduced into the ordinary consumer problem with given prices(Theorem 4). Hence, there exists a solution for the optimal mechanism problem if and only if there exists a solution for the corresponding competitive equilibrium problem (Theorem 5).

Thus far, we assumed that the social implementors are loyal, though they are informationally decentralized. What the social implementors are disloyal? The above argument suggests that the social planner can make them behave as if they are loyal by allowing them to make profits but let them compete each other. Competition among them will drive their profits down to zero. With the zero-profit condition, they will behave as if they are loyal fulfilling the social planner's instructions faithfully. As Clower and Howitt(1978) conjectured, merchants are "visible fingers" of the invisible hand.

After analyzing three examples of allocation problems, each of which being solved for the idealistic, informationally centralized and decentralized economies, we considered welfare implications of the optimal allocation of the informationally decentralized economy. Obviously, it is inferior to the first-best solution of the idealistic economy without the incentive compatibility constraint. As much as the incentive compatibility constraint of the informationally decentralized economy is more severe than that of the informationally centralized economy, it is also inferior to the second-best solution (Theorem 6). I hasted to add that it does not mean that the centralized economic system is better than the decentralized one. It only means that the social planner can design a better allocation mechanism with loyal, coordinated implementors than with disloyal, uncoordinated implementors. If the planner misconceived the implemental conditions and misplaced a wrong mechanism, the consequences may be devastating.

We also compared the third-best solution with the first-best solution of the idealistic economy. While the allocation is ranked third, it resembles the first-best solution in that both use prices, markets and money. We noted that the third-best solution is also Pareto optimal(Theorem 7), while the second-best solution is not. The puzzling result stems from the fact that the notion of Pareto optimality depends on the commodity space. In the idealistic economy (and in the informationally centralized economy), it was the space of contingent commodities, while it is the space of spot commodities in the informationally decentralized economy. The social cannot achieve the first-best solution, either by insurance or by lump-sum transfers as the second fundamental theorem of welfare economics may suggest. However, it is true that the incentive compatibility constraint in the informationally decentralized economy can be represented by some weight function, so that the social planner can be modelled as if he maximizes the weighted social welfare subject to the resource constraint only(Theorem 8).

It may be very challenging and controversial question to ask whether the actual monetary market economy is the first- or the third-best solution. For such a direct comparison, we should admit that our strict assumption of private information should be loosened flexibly. The endowments of some goods may be public, and those of some other goods may be monitored with some varying costs, while those of some cannot be known at all. The third-best solution, then, includes some contingent commodities for the first and some of the second class of goods provided the monitoring cost is less than gains in the social welfare. While we do not pursue further on this line, it suggests that the existence of some contingency commodities is not sufficient to prove the actual market economy is the first-best solution.

One obvious test for the third-best solution is to check whether there exists a superior second-best solution, i.e. whether it is possible to design a better mechanism provided that there are loyal, coordinated implementors. If the actual economy is the third-best solution, we can devise one; if it is the first-best solution, we cannot. I think there are plenty of evidences that we can. Especially, the allocation in a family or in a group of good friends seems to be obviously superior to the real market solution. In fact, the limited availability of the contingency commodities or Arrow's contingent securities will be another evidence.

One may criticize it to be pessimistic. But which is pessimistic? The current status remains the same, regardless whether the current market economy belongs to the first-best or the third-best solution. Then the only difference is whether we have room to enhance welfare or not.

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Appendix: Proofs of Lemmas

Proof of Lemma 1. (Condition for Binding Clower Constraint)

Suppose that the Clower constraint is binding for some (x', θ') p.p. This can be true only if $\delta_j(x'\theta')>0$ for some j. If the Clower constraint were not binding for some (x', θ'), everyone could increase z_j indefinitely by announcing (x', θ') with repeated visits, violating MTTC or NRVC. This proves the first part of the lemma. The converse is just the contraposition. ∎

*Proof of Lemma 2. (N*onbinding Clower Constraint and the Autarky)

Suppose that the Clower constraint is not binding p.p., but the optimal allocation is not the autarky. The former implies that the Clower constraint is binding a.e. by Lemma 1. The latter implies $z_j(x', \theta') \neq x_j$ for some (x', θ') p.p., which in turn implies, together with the resource constraint, $\delta_j(x'', \theta'') > 0$ for some (x'', θ'') p.p. But then repeated visits announcing (x'', θ'') would be strictly feasible and preferable a.e., which contradicts MTTC or NRVC.∎

*Proof of Lemma 3. (P*arametric Relation between Goods and Currency Transfers)

Suppose not. Then there exist some j, (x, θ), (x', θ') such that $\delta_j(x, \theta) = \delta_j(x', \theta')$ but $\gamma_j(x, \theta) \neq \gamma_j(x', \theta')$. Without loss of generality, we may assume $\gamma_j(x', \theta') > \gamma_j(x, \theta)$. Then announcing (x', θ') is strictly feasible and preferable for individual (x, θ), violating MTTC condition.∎

*Proof of Lemma 4. (T*he Quid *pro Quo Con*dition)

Suppose $\hat{\gamma}_j$ is not strictly decreasing. Then there exist some (x, θ) and (x', θ') such that

$$\delta_j(x', \theta') > \delta_j(x, \theta) \text{ and } \hat{\gamma}_j[\delta_j(x', \theta')] \geq \hat{\gamma}_j[\delta_j(x, \theta)].$$

But then reporting (x', θ') at the j-th class of implementor, while telling the truth elsewhere, is feasible and strictly preferable for (x, θ), which violates MTTC. The latter assertion $\hat{\gamma}_j(0) = 0$ follows from NRVC.∎

*Proof of Lemma 5. (N*o Arbitrage Condition)

1) Sufficiency: Suppose that $\overline{p}_j^B(b) < \overline{p}_j^S(s)$ holds for some j, $b \in D_j^+$ and $-s \in D_j^-$. Then there exist two positive integers h and k such that $h/k \geq s/b$ and arbitrarily close to s/b. Then "buying" h times of good j in b units and "selling" k times in s units result in

$$\Delta m = k\,\overline{p}_j^S(s)s - h\,\overline{p}_j^B(b)b > 0.$$

Since $\Delta x_j = hb - ks \geq 0$ also, it is an arbitrage route.

2) Necessity: It is clear that (6.8) shows the no arbitrage condition for good j. We must show that if no arbitrage condition holds for each good, then the same holds for the economy as a whole. We will show the contraposition. Suppose that there exists an arbitrage route (sigma, rho) for the economy. Then we have

$$\Delta m=\sum_{n=1}^{|\sigma|}\gamma_{\sigma_n}(\tilde{x}^n,\ \tilde{\theta}^n)=\sum_{j=1}^{\ell}\sum_{n\in\sigma_j^{-1}}\gamma_j(\tilde{x}^n,\ \tilde{\theta}^n)>0$$

which in turn implies $\sum_{n\in\sigma_j^{-1}}\gamma_j(\tilde{x}^n,\ \tilde{\theta}^n)>0$ for some j.

We also have $\Delta x_j=\sum_{n\in\sigma_j^{-1}}\delta_j(\tilde{x}^n,\ \tilde{\theta}^n)>=0$ for any j. Extract sigma' in sigma whenever $\sigma_n=j$, and extract ρ' conformingly, it is obvious that (σ',ρ') is feasible, which proves (σ',ρ') is an arbitrage route in good j. ∎

Proof of Lemma 6. (Sufficiency of the Nonnegativity Conditions)

We only need to check the Clower constraint. Partition J into J_B and J_S according to $\delta_j(\tilde{x}^j,\ \tilde{\theta}^j)\geq 0$ or not. Define the subset $\overline{\Sigma}^*$ of bar $\overline{\Sigma}$ so that $\sigma_j>\sigma_k$ whenever $j\in J_B$ and $k\in J_S$. Obviously $\overline{\Sigma}^*$ is not empty, hence we can pick $\sigma^*\in\overline{\Sigma^*}$. For any σ^*, the Clower constraint is satisfied along the schedule.∎

Proof of Lemma 7. (Existence of the Competitive Equilibria)

Let $\overline{x}$ be the least essential upper bound of X and $\triangle_+^{\ell}$ be the simplex in R_+^{ℓ}. Define the pseudo-demand function $\overline{z}^n:\triangle_+^{\ell}\times X\times\Theta\to Z$ by the unique solution of the problem

$$\begin{aligned}\max_{\overline{z}}\ & u(\overline{z},\theta)\\ s.t.\ & p\overline{z}\leq px\\ & z\leq 2^{n-1}\overline{x}.\end{aligned}$$

$\overline{z}^n$ is well-defined and continuous. Define the aggregate demand function bar $\overline{z_n}:\triangle_+^{\ell}\to Z$ by

$$\overline{z_n}(p)=E\,\overline{z}^n(p,x,\theta).$$

We apply the Brouwer's fixed-point theorem following the standard process. Define $g^n: \triangle^\ell_+ \to R^\ell_+$ by

$$g^n_j(p) = \max[p_j + \bar{z}_{j^n}(p) - \bar{x}_j, 0.5p_j]$$

for $j = 1, \cdots, \ell$ and $h_n: \triangle^\ell_+ \to R_+$ by

$$h_n(p) = \sum_{j=1}^{\ell} g^n_j(p).$$

Finally, define $f^n: \triangle^\ell_+ \to \triangle^\ell_+$ by

$$f^n_j = \frac{g^n_j(p)}{h_n(p)}$$

for $j = 1, \cdots, \ell$. The non-satiation implies $g^n_j(p) > 0$, for $j = 1, \cdots, \ell$. hence $h_n(p) > 0$, so that f^n is well-defined. It is easy to check that f^n maps $\triangle^\ell_+$ continuously into itself.

Hence the Brouwer's fixed-point theorem asserts the existence of a vector $\hat{p}_j^{\,n} \in \triangle^\ell_+$ such that

$$\hat{p}_j^{\,n} = \frac{g^n_j(\hat{p}_j^{\,n})}{h_n(\hat{p}_j^{\,n})}, \quad j = 1, \cdots, \ell.$$

It is immediate that $g^n_j(p) > 0$ for all p implies $\hat{p}_j^{\,n} > 0$ for $j = 1, \cdots, \ell$. First suppose that $g^n_j(\hat{p}_j^{\,n}) = 0.5\ \hat{p}_j^{\,n}$ for all $j = 1, \cdots, \ell$, so that $h_n(\hat{p}_j^{\,n}) = 0.5$. Then $\sum_{j=1}^{\ell} \hat{p}_j^{\,n}[\bar{x}_j^{\,n}(h\ \hat{p}_j^{\,n}) - \bar{x}_j] \le -0.5\sum_{j=1}^{\ell}(h\ \hat{p}_j^{\,n})^2 < 0$, which contradicts the Walras' law or the nonsatiation assumption. Hence we have $h_n(\hat{p}_j^{\,n}) > 0.5$. Now suppose $g^n_j(\hat{p}_j^{\,n} = 0.5\ \hat{p}_j^{\,n}$ for some j. Then $\hat{p}_j^{\,n} = .5\ \hat{p}_j^{\,n}/h_n(\hat{p}_j^{\,n}) < \hat{p}_j^{\,n}$, which is again a contradiction.

Thus we can rewrite (A.1) as

$$\hat{p}_j^{\,n} = \frac{\hat{p}_j^{\,n} + \bar{z}_j^{\,n}(\hat{p}_j^{\,n}) - \bar{x}_j}{h_n(\hat{p}_j^{\,n})}$$

for $j=1,\cdots,\ell$. Multiply by $\bar{z}_j^{\,n}(\hat{p}_j^{\,n})-\bar{x}_j$, sum over j, then apply the Walras' law again, and we have

$$\sum_{j=1}^{\ell}[\bar{z}_j^{\,n}(\hat{p}_j^{\,n})-\bar{x}]^2=0,$$

since $h_n(\hat{p}_j^{\,n})>0$. This shows that $\hat{p}^n$ is indeed an equilibrium price vector with the pseudo-demand function for each n.

Finally, we claim that there exists an integer $k<\infty$ such that the pseudo-demand function coincides with the ordinary demand function at equilibrium, i.e.

$$\bar{z}_j(\hat{p}^k,x,\theta)=\hat{z}_j(\hat{p}^k,x,\theta),\ j=1,\cdots,\ell.$$

Suppose not, then the constraint $\bar{z}_j\le 2^{n-1}\bar{x}_j$ is binding for some j and (x,θ), however large n may be. But this is impossible, since $\bar{z}_j\le\frac{px}{\hat{p}_j^{\,n}}$ with $\hat{p}_j^{\,n}>0$ and $p\in\triangle_+^{\ell}$ from the budget constraint and since x is essentially bounded. Hence $\hat{p}=\hat{p}^k$ is an equilibrium price that has been sought for.▮

Proof of Lemma 8. (Minimal Properties of Cost and Endowment)

(i) Necessity: Suppose not. Then there exist an allocation z' such that $Ez'(x,\theta)\in H_z$ and $pEz'(x,\theta)<pEz(x,\theta)$. But this implies $pz'(x,\theta)<pz(x,\theta)$ p.p., which is a contradiction.

(ii) Sufficiency: Suppose not. Then there exists and allocation z' and a subset of $\Lambda\subset X\times\Theta$ with $\nu(\Lambda)>0$ such that $z'(x,\theta)\in H_z(x,\theta)$ and $pz'(x,\theta)<pz(x,\theta)$ for all $(x,\theta)\in\Lambda$. Define z'' by

$$z''(x,\theta)=\begin{cases} z'(x,\theta) & \text{if } (x,\theta)\in\Lambda, \\ z(x,\theta) & \text{otherwise.}\end{cases}$$

Clearly $Ez''(x,\theta)$ in H_z. But we also have $pEz''(x,\theta)<pEz(x,\theta)$, which contradicts the minimalist of $pEz(x,\theta)$. ▮