Noname manuscript No.
(will be inserted by the editor)

Shape Optimization for Suppressing Brake Squeal

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Received: date / Accepted: date

Abstract The present paper describes a solution to a non-parametric shape optimization problem of a brake model to suppress squeal noise. The brake model consists of a rotor and a pad, between which Coulomb friction occurs. The main problem is defined as a complex eigenvalue problem of the brake model obtained from the equation of motion. As an objective cost function, we use the positive real part of the complex eigenvalue generating the brake squeal. The volume of the pad is used as a constraint cost function. The Fréchet derivative of the objective cost function with respect to the domain variation, which we refer to as the shape derivative of the objective cost function, is evaluated using the solution of the main problem and the adjoint problem. A scheme by which to solve the shape optimization problem using an iterative algorithm based on the H^1 gradient method (the traction method) for reshaping is presented. Numerical results obtained using a simple rotor-pad model reveal that the real part of the target complex eigenvalue decreases monotonically, thus satisfying the volume constraint.

Keywords Shape optimization · Brake squeal · Complex eigenvalue · Self-excited vibration · Shape derivative · H1 gradient method · Traction method

This paper was presented at WCSMO10, Florida, United States.

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1 Introduction

Brake squeal is a vibration phenomenon in the frequency range of between 1 and 15 kHz caused by friction between the rotor and the pad. Brake squeal causes customer dissatisfaction, and so a method for preventing brake squeal during the design stage is strongly desired.

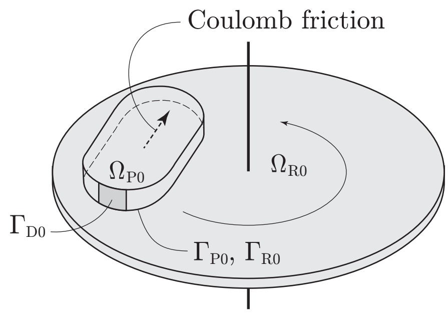
A number of studies have been conducted in order to clarify the brake squeal phenomenon. Mills (1938) explained brake squeal using the stick-slip vibration phenomenon caused by the friction force. North (1972) introduced a simple model of a rotor and a pad between which Coulomb friction occurs and considered the brake squeal to be a self-excited vibration induced by the friction force. Based on these findings, Millner (1978) revealed that the stiffness matrix becomes asymmetric in the rotor and pad model with Coulomb friction and that the natural vibrations are determined by solutions of a complex eigenvalue problem. He also reported that if the real part of a complex eigenvalue is positive, since the amplitude of the natural vaibration increases with respect to time, a dynamic instability occurs. A number of studies have analyzed dynamic instability by means of the asymmetric stiffness matrix using the finite element method (Matsushima et al. (1997); Joo et al. (2006)).

Moreover, since the 2000s, research to determine the optimum shape to minimize the positive real part of the complex eigenvalue has been started. Lee and Kikuchi (2003) and Guan et al. (2006) presented formulations of the parametric optimization problem by choosing eigenvalues of the components in the brake model as the design variables and by choosing the real part of the complex eigenvalue that generates the brake squeal as the objective cost function. Moreover, they presented numerical examples. Based on the assumption that the ideal eigenvalues reducing the positive real part of the complex eigenvalue were determined for the components in the brake model, Goto et al. (2010) presented a method by which to find the shapes of the components as a solution of the non-parametric shape optimization problem using the error of the eigenvalues from the ideal values as the objective cost function.

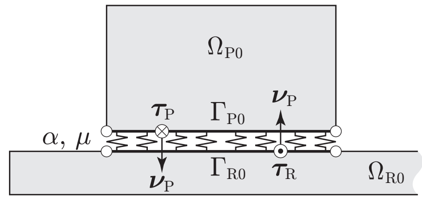
In recent years, non-parametric optimization methods have been applied to optimum design problems in the brake model to suppress the brake squeal. Nelagadde and Smith (2009) presented a method by which to obtain optimum shapes of the components of the brake model in order to increase the frequency separation between the critical modes, while constraining the frequency separation between other selected modes using commercial software. Soh and Yoo (2010) analyzed the optimum shape of the caliper housing by the topology optimization method using the real part of the complex eigenvalue as the objective function. However, an approach based on a formulation of the non-parametric shape optimization problem using the real part of the complex eigenvalue as the objective function has not yet been presented.

The objective of the present paper is to formulate a non-parametric shape optimization problem of a brake model using the real part of the complex eigenvalue as the objective function, to derive the shape derivative of the real part of the complex eigenvalue theoretically, and to show the solution of the problem by the H^1 gradient method. The brake model is assumed to consist of a rotor and a pad, between which the Coulomb friction occurs. The solution of the problem is presented by using an iterative algorithm based on the H^1 gradient method for reshaping. The H^1 gradient method was proposed by the authors as a reshaping algorithm for non-parametric shape optimization problems. This method was called the traction method in the early years (Azegami (1994); Azegami and Wu (1996); Azegami and Takeuchi (2006)). The reason why we call the method the H^1 gradient method was introduced in a previous paper (Azegami et al. (2013a)), and the primary reason for using this method and the comparison with other techniques are described in another paper (Azegami et al. (2013b)). The basic idea of the gradient method in a Hilbert space was presented by Lions (1971).

The remainder of the present paper is organized as follows. In Section 2, we define the initial domains of the brake model and choose mapping from the initial domain of the pad to the varied domain as a design variable. Using the definition of mapping and initial domain, in Section 3, we formulate the complex eigenvalue



(a) Initial domains Ω_{R0} and Ω_{P0}



(b) Friction boundaries Γ_{R0} and Γ_{P0}

Fig. 1 Brake model

problem of the natural vibrations as the main problem in the shape optimization problem. In Section 4, using the solution of the main problem, we formulate a shape optimization problem using the real part of an eigenvalue as an objective function and the volume of the pad as a constraint function. The evaluation methods for the shape derivatives of the cost functions are shown in Section 5. Using these shape derivatives of the cost functions, in Section 6, we present a method by which to obtain the domain mappings that decrease the cost functions. A scheme for solving the shape optimization problem with constraints is presented in Section 7. Finally, in Section 8, we present the numerical results for shape optimization of a simple brake model.

2 Brake model

Let us define the initial domains for a brake model as depicted in Fig. 1. Let Ω_{R0} and Ω_{P0} be $d \in \{2,3\}$-dimensional bounded domains of linear elastic continua denoting a rotor and a pad, respectively. Γ_{R0} and Γ_{P0} denote contact boundaries on the boundary of rotor $\partial\Omega_{\mathrm{R0}}$ and the boundary of pad $\partial\Omega_{\mathrm{P0}}$, respectively. Let $\Gamma_{\mathrm{D0}} \subset \partial\Omega_{\mathrm{R0}} \cup \partial\Omega_{\mathrm{P0}}$ be the homogeneous Dirichlet boundary. Let $\boldsymbol{\nu}_{\mathrm{R}}$ and $\boldsymbol{\nu}_{\mathrm{P}}$ be the normals, and let $\boldsymbol{\tau}_{\mathrm{R}}$ and $\boldsymbol{\tau}_{\mathrm{P}}$ be the tangents on Γ_{R0} and Γ_{P0}, respectively. In the present paper, we assume that Ω_{P0} is variable. In order to define a shape optimization problem of Ω_{P0}, $\partial\Omega_{\mathrm{P0}}$ is required to be a Lipschitz boundary and piecewise class $C^{1,1}$.

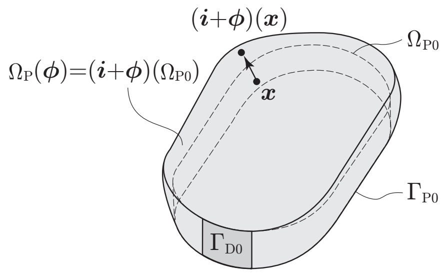


Fig. 2 Domain variation of the pad

In the present paper, we use the notation $W^{s,p}(\Omega_0;\mathbb{R}^d)$ to denote the Sobolev space for the set of functions defined in Ω_0 and having values in $\mathbb{R}^d$ that are $s \in [0,\infty]$ times differentiable and $p \in [1,\infty]$-th order Lebesgue integrable and refer to its smoothness as the $W^{s,p}$ class. The notation $H^s\left(\Omega_0;\mathbb{R}^d\right)$ and $C^{s,\alpha}$ for $\alpha \in (0,1]$ are used as $W^{s,2}\left(\Omega_0;\mathbb{R}^d\right)$ and $W^{s+\alpha,\infty}\left(\Omega_0;\mathbb{R}^d\right)$.

Moreover, we define design variable in shape optimization problem by domain variation $\boldsymbol{\phi}$, with which varied domain is created by continuous one-to-one mapping $\boldsymbol{i}+\boldsymbol{\phi} : \Omega_{\mathrm{P0}} \to \mathbb{R}^d$ by

$$\Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right) = \left\{ \left(\boldsymbol{i}+\boldsymbol{\phi}\right)\left(\boldsymbol{x}\right) \mid \boldsymbol{x} \in \Omega_{\mathrm{P0}} \right\}$$

as shown on Fig. 2. The symbol $\boldsymbol{i}$ is used as the identity mapping in the present paper. To keep the property of continuous one-to-one mapping, we define the admissible set of $\boldsymbol{\phi}$ as

$$\mathcal{D} = \left\{ \boldsymbol{\phi} \in Y \;\middle|\; \|\boldsymbol{\phi}\|_Y < \sigma,\ \boldsymbol{\phi} = \mathbf{0}_{\mathbb{R}^d} \text{ on } \Gamma_{\mathrm{P0}} \cup \Gamma_{\mathrm{D0}} \right\}, \quad (1)$$

where Y is defined by $W^{1,\infty}\left(\mathbb{R}^d;\mathbb{R}^d\right)$, and $\sigma > 0$ is chosen such that $\boldsymbol{\phi}$ becomes bijection. The domain of $\boldsymbol{\phi}$ can be extended to $\mathbb{R}^d$ by Calderón's extension theorem. In the present paper, $X = H^1\left(\mathbb{R}^d;\mathbb{R}^d\right) \supset Y$ is used as the Banach space for the perturbation $\boldsymbol{\varphi}$ of $\boldsymbol{\phi}$ in order to define the Fréchet derivatives of cost functions with respect to the domain variation as shown later.

3 Main problem

Using the domains described above for the brake model, let us define the main problem for brake squeal. First, let us consider the natural vibration of the brake model of Fig. 1.

Let $\boldsymbol{u}$ be the displacement expressing natural vibration, the admissible set of which is given for $q > d$ as

$$\mathcal{U} = \left\{ \boldsymbol{u} \in H^1\left(\mathbb{R}; W^{2,2q}\left(\mathbb{R}^d;\mathbb{R}^d\right)\right) \;\middle|\; \boldsymbol{u} = \mathbf{0}_{\mathbb{R}^d} \text{ on } \Gamma_{\mathrm{P0}} \cup \Gamma_{\mathrm{D0}} \right\}. \quad (2)$$

The condition whereby $\boldsymbol{u}$ belongs to class $W^{2,2q}$ will be used in the process of deriving the shape derivative of the objective cost function after converting $\boldsymbol{u}$ into the eigenmode $\hat{\boldsymbol{u}}_k$ for $k \in \{1,2,\cdots\}$ belonging to $\mathcal{S}$ defined in (5).

In the present paper, let $\boldsymbol{u}_{\mathrm{R}}$ and $\boldsymbol{u}_{\mathrm{P}}$ denote the displacements $\boldsymbol{u}$ in Ω_{R0} and $\Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right)$, respectively. Let

$$\boldsymbol{E}\left(\boldsymbol{u}\right) = \left(e_{ij}\left(\boldsymbol{u}\right)\right)_{ij} = \frac{1}{2}\left(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}} + \left(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\right)^{\mathrm{T}}\right),$$

$$\boldsymbol{S}\left(\boldsymbol{u}\right) = \boldsymbol{C}\boldsymbol{E}\left(\boldsymbol{u}\right) = \left(\sum_{(k,l)\in\{1,\cdots,d\}^2} c_{ijkl}e_{kl}\left(\boldsymbol{u}\right)\right)_{ij}$$

denote the strain tensor and the Cauchy stress tensor, respectively, where

$$\boldsymbol{C} = \left(c_{ijkl}\right)_{ijkl} \in W^{1,\infty}\left(\mathbb{R}^d;\mathbb{R}^{d\times d\times d\times d}\right)$$

is the stiffness of material. In the present paper, let $\left(\,\cdot\,\right)^{\mathrm{T}}$ denote the transpose. Moreover, let α denote the stiffness on the relation between the rotor and the pad, and let μ denote the coefficient of the Coulomb friction. Finally, let ρ_{R} and ρ_{P} denote the densities of the rotor and the pad, respectively. In the present paper, we assume that α, μ, ρ_{R}, and ρ_{P} are given as positive constants for simplicity.

Based on these definitions and the notation $\dot{(\,\cdot\,)}$ for the time derivative, let us define the equations of motion for the brake model.

Problem 1 (Free vibration problem) For $\boldsymbol{\phi} \in \mathcal{D}$, initial displacement $\bar{\boldsymbol{u}}_0 \in W^{2,2q}\left(\mathbb{R}^d;\mathbb{R}^d\right)$, and initial velocity $\bar{\boldsymbol{v}}_0 \in W^{2,2q}\left(\mathbb{R}^d;\mathbb{R}^d\right)$, find $\boldsymbol{u}$ such that

$$\begin{aligned}
&\rho_{\mathrm{R}}\ddot{\boldsymbol{u}}_{\mathrm{R}} - \left(\boldsymbol{\nabla}\cdot\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{R}}\right)\right)^{\mathrm{T}} = \mathbf{0}_{\mathbb{R}^d} \quad \text{in } \Omega_{\mathrm{R0}}\times\mathbb{R},\\
&\rho_{\mathrm{P}}\ddot{\boldsymbol{u}}_{\mathrm{P}} - \left(\boldsymbol{\nabla}\cdot\boldsymbol{S}(\boldsymbol{u}_{\mathrm{P}})\right)^{\mathrm{T}} = \mathbf{0}_{\mathbb{R}^d} \quad \text{in } \Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right)\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{R}}\right)\boldsymbol{\nu}_{\mathrm{R}} = \mathbf{0}_{\mathbb{R}^d} \quad \text{on } \left(\partial\Omega_{\mathrm{R0}}\setminus\bar{\Gamma}_{\mathrm{R0}}\right)\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{P}}\right)\boldsymbol{\nu}_{\mathrm{P}} = \mathbf{0}_{\mathbb{R}^d} \quad \text{on } \left(\partial\Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right)\setminus\bar{\Gamma}_{\mathrm{P0}}\right)\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{R}}\right)\boldsymbol{\nu}_{\mathrm{R}} = \alpha\left\{\left(\boldsymbol{u}_{\mathrm{R}}-\boldsymbol{u}_{\mathrm{P}}\right)\cdot\boldsymbol{\nu}_{\mathrm{R}}\right\}\boldsymbol{\nu}_{\mathrm{R}} \quad \text{on } \Gamma_{\mathrm{R0}}\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{R}}\right)\boldsymbol{\tau}_{\mathrm{R}} = \mu\alpha\left\{\left(\boldsymbol{u}_{\mathrm{R}}-\boldsymbol{u}_{\mathrm{P}}\right)\cdot\boldsymbol{\nu}_{\mathrm{R}}\right\}\boldsymbol{\tau}_{\mathrm{R}} \quad \text{on } \Gamma_{\mathrm{R0}}\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{P}}\right)\boldsymbol{\nu}_{\mathrm{P}} = \alpha\left\{\left(\boldsymbol{u}_{\mathrm{P}}-\boldsymbol{u}_{\mathrm{R}}\right)\cdot\boldsymbol{\nu}_{\mathrm{P}}\right\}\boldsymbol{\nu}_{\mathrm{P}} \quad \text{on } \Gamma_{\mathrm{P0}}\times\mathbb{R},\\
&\boldsymbol{S}\left(\boldsymbol{u}_{\mathrm{P}}\right)\boldsymbol{\tau}_{\mathrm{P}} = -\mu\alpha\left\{\left(\boldsymbol{u}_{\mathrm{P}}-\boldsymbol{u}_{\mathrm{R}}\right)\cdot\boldsymbol{\nu}_{\mathrm{P}}\right\}\boldsymbol{\tau}_{\mathrm{P}} \quad \text{on } \Gamma_{\mathrm{P0}}\times\mathbb{R}
\end{aligned} \quad (3)$$

$$\begin{aligned}
&\boldsymbol{u}_{\mathrm{R}} = \boldsymbol{u}_{\mathrm{P}} \quad \text{on } \left(\Gamma_{\mathrm{R0}}\cup\Gamma_{\mathrm{P0}}\right)\times\mathbb{R},\\
&\boldsymbol{u} = \bar{\boldsymbol{u}}_0 \quad \text{in } \Omega_{\mathrm{R0}}\cup\Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right)\times\{0\},\\
&\dot{\boldsymbol{u}} = \bar{\boldsymbol{v}}_0 \quad \text{in } \Omega_{\mathrm{R0}}\cup\Omega_{\mathrm{P}}\left(\boldsymbol{\phi}\right)\times\{0\},\\
&\boldsymbol{u} = \mathbf{0}_{\mathbb{R}^d} \text{ on } \Gamma_{\mathrm{D0}}\times\mathbb{R}.
\end{aligned}$$

Here, the negative sign on the right-hand side of (3) in front of the coefficient of the Coulomb friction makes the strain energy term in the weak form of Problem 1 asymmetric with respect to $\boldsymbol{u}$ and its adjoint function. Then, the eigenvalue problem for the natural vibrations of the brake model becomes a complex eigenvalue problem. Since this brake model is a linear system with respect to $\boldsymbol{u}$, the form of separation of variables is given for some $s \in \mathbb{C}$ as

$$\boldsymbol{u}(\boldsymbol{x}, t) = \mathrm{e}^{st} \hat{\boldsymbol{u}}(\boldsymbol{x}) + \mathrm{e}^{s^{\mathrm{c}} t} \hat{\boldsymbol{u}}^{\mathrm{c}}(\boldsymbol{x}), \tag{4}$$

where $(\,\cdot\,)^{\mathrm{c}}$ denotes complex conjugation. Based on the definition of $\mathcal{U}$ in (2), the admissible set for $\hat{\boldsymbol{u}}$ is given by

$$\mathcal{S} = \left\{ \hat{\boldsymbol{u}} \in W^{2,2q}\left(\mathbb{R}^d; \mathbb{C}^d\right) \;\middle|\; \hat{\boldsymbol{u}} = \mathbf{0}_{\mathbb{R}^d} \text{ on } \Gamma_{\mathrm{P}0} \cup \Gamma_{\mathrm{D}0} \right\}. \tag{5}$$

By substituting (4) into Problem 1, we have a complex eigenvalue problem for natural vibrations. For compact expression of the weak form, we define the Lagrange function of the complex eigenvalue problem as

$$\mathscr{L}_{\mathrm{M}}(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}) = h(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}^{\mathrm{c}}) + h(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}) \tag{6}$$

for $(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}) \in \mathbb{C} \times \mathcal{S} \times \mathcal{S}$ for $k \in \{1, 2, \cdots\}$, where

$$\begin{aligned} h(s, \hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}^{\mathrm{c}}) &= a_{\mathrm{R}}(\hat{\boldsymbol{u}}_{\mathrm{R}}, \hat{\boldsymbol{v}}_{\mathrm{R}}^{\mathrm{c}}) + s^2 b_{\mathrm{R}}(\hat{\boldsymbol{u}}_{\mathrm{R}}, \hat{\boldsymbol{v}}_{\mathrm{R}}^{\mathrm{c}}) \\ &\quad - c_{\mathrm{R}}(\hat{\boldsymbol{u}}_{\mathrm{R}} - \hat{\boldsymbol{u}}_{\mathrm{P}}, \hat{\boldsymbol{v}}_{\mathrm{R}}^{\mathrm{c}}) - d_{\mathrm{R}}(\hat{\boldsymbol{u}}_{\mathrm{R}} - \hat{\boldsymbol{u}}_{\mathrm{P}}, \hat{\boldsymbol{v}}_{\mathrm{R}}^{\mathrm{c}}) \\ &\quad + a_{\mathrm{P}}(\hat{\boldsymbol{u}}_{\mathrm{P}}, \hat{\boldsymbol{v}}_{\mathrm{P}}^{\mathrm{c}}) + s^2 b_{\mathrm{P}}(\hat{\boldsymbol{u}}_{\mathrm{P}}, \hat{\boldsymbol{v}}_{\mathrm{P}}^{\mathrm{c}}) \\ &\quad - c_{\mathrm{P}}(\hat{\boldsymbol{u}}_{\mathrm{P}} - \hat{\boldsymbol{u}}_{\mathrm{R}}, \hat{\boldsymbol{v}}_{\mathrm{P}}^{\mathrm{c}}) + d_{\mathrm{P}}(\hat{\boldsymbol{u}}_{\mathrm{P}} - \hat{\boldsymbol{u}}_{\mathrm{R}}, \hat{\boldsymbol{v}}_{\mathrm{P}}^{\mathrm{c}}), \end{aligned} \tag{7}$$

and, for $(\,\cdot\,) \in \{\mathrm{P}, \mathrm{R}\}$,

$$\begin{aligned} a_{(\,\cdot\,)}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) &= \int_{\Omega_{(\,\cdot\,)}(\boldsymbol{\phi})} \boldsymbol{S}(\hat{\boldsymbol{u}}) \cdot \boldsymbol{E}(\hat{\boldsymbol{v}}) \, \mathrm{d}x, \\ b_{(\,\cdot\,)}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) &= \int_{\Omega_{(\,\cdot\,)}(\boldsymbol{\phi})} \rho_{(\,\cdot\,)} \hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{v}} \mathrm{d}x, \\ c_{(\,\cdot\,)}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) &= \int_{\Gamma_{(\,\cdot\,)0}} \alpha \left(\hat{\boldsymbol{u}} \cdot \boldsymbol{\nu}_{(\,\cdot\,)}\right) \left(\hat{\boldsymbol{v}} \cdot \boldsymbol{\nu}_{(\,\cdot\,)}\right) \mathrm{d}\gamma, \\ d_{(\,\cdot\,)}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) &= \int_{\Gamma_{(\,\cdot\,)0}} \mu\alpha \left(\hat{\boldsymbol{u}} \cdot \boldsymbol{\nu}_{(\,\cdot\,)}\right) \left(\hat{\boldsymbol{v}} \cdot \boldsymbol{\tau}_{(\,\cdot\,)}\right) \mathrm{d}\gamma. \end{aligned}$$

Using the above definitions, we define the weak form of the eigenvalue problem for natural vibrations of the brake model and its Lagrange formulation for convenience of later description as follows. Here, we define $U = H^1\left(\mathbb{R}^d; \mathbb{R}^d\right)$ as the Banach space for the perturbation $\boldsymbol{v}$ of $\boldsymbol{u} \in \mathcal{S}$.

Problem 2 (Natural vibration problem) For $\boldsymbol{\phi} \in \mathcal{D}$, find $(s_k, \hat{\boldsymbol{u}}_k) \in \mathbb{C} \times \mathcal{S}$ for $k \in \{1, 2, \cdots\}$ such that

$$\mathscr{L}_{\mathrm{M}}(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}) = 0$$

for all $\hat{\boldsymbol{v}} \in U$.

4 Shape optimization problem

Using the solution s_k of Problem 2, let us define a shape optimization problem for the brake model. In the present paper, referring to previous studies using the positive real part of the complex eigenvalue, we assume that the mode number k is given, and define an objective cost function as

$$f_0(\boldsymbol{\phi}, s_k) = 2\mathrm{Re}[s_k] = s_k + s_k^{\mathrm{c}}. \tag{8}$$

Moreover, we define a constraint cost function by the volume of the pad as

$$f_1(\boldsymbol{\phi}) = -\int_{\Omega_{\mathrm{P}}(\boldsymbol{\phi})} \mathrm{d}x + c_1, \tag{9}$$

where c_1 is a positive constant for which there exists $\Omega_{\mathrm{P}}(\boldsymbol{\phi})$ such that $f_1(\boldsymbol{\phi}) \leq 0$.

Using these cost functions, we define the shape optimization problem as follows.

Problem 3 (Shape optimization problem) Let $\mathcal{D}$ and $\mathcal{S}$ be defined in (1) and (5). For $\boldsymbol{\phi} \in \mathcal{D}$, let $(s_k, \hat{\boldsymbol{u}}_k) \in \mathbb{C} \times \mathcal{S}$ be the solution of Problem 2 for a given k. Let f_0 and f_1 be defined in (8) and (9), respectively. Find $\Omega_{\mathrm{P}}(\boldsymbol{\phi})$ such that

$$\begin{aligned} \min_{\boldsymbol{\phi} \in \mathcal{D}} \{ & f_0(\boldsymbol{\phi}, s_k) \mid f_1(\boldsymbol{\phi}) \leq 0, \\ & (s_k, \hat{\boldsymbol{u}}_k) \in \mathbb{C} \times \mathcal{S}, \text{ Problem } 2 \}. \end{aligned}$$

5 Shape derivative of cost functions

In order to solve Problem 3 by the gradient method, the Fréchet derivatives of the cost functions with respect to the domain variation, which we refer to as the shape derivatives, are required. Then, let us derive the shape derivatives of f_0 and f_1 here.

Since the objective cost function $f_0(\boldsymbol{\phi}, s_k)$ contains s_k, we must consider the main problem to be the equality constraint. Hence, we set

$$\begin{aligned} &\mathscr{L}_0(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0) \\ &\quad = f_0(\boldsymbol{\phi}, s_k) - \mathscr{L}_{\mathrm{M}}(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0) \end{aligned} \tag{10}$$

as the Lagrange function for f_0, where $\mathscr{L}_{\mathrm{M}}$ is defined in (6), and $\hat{\boldsymbol{v}}_0$ is used as the Lagrange multiplier for f_0. The shape derivative of $\mathscr{L}_0$ with respect to arbitrary domain variation $\boldsymbol{\varphi} \in X$ can be obtained by applying the formulae of shape derivatives for domain and boundary integrals (Sokolowski and Zolésio (1992)), as follows:

$$\begin{aligned}&\dot{\mathscr{L}}_0\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)\left[\boldsymbol{\varphi}, s_k', \hat{\boldsymbol{u}}_k', \hat{\boldsymbol{v}}_0'\right]\\&\quad= \mathscr{L}_{0\boldsymbol{\phi}}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\boldsymbol{\varphi}] + \mathscr{L}_{0 s_k}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[s_k']\\&\qquad+ \mathscr{L}_{0\hat{\boldsymbol{u}}_k}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{u}}_k']\\&\qquad+ \mathscr{L}_{0\hat{\boldsymbol{v}}_0}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{v}}_0'],\end{aligned}\tag{11}$$

where $(s_k', \hat{\boldsymbol{u}}_k', \hat{\boldsymbol{v}}_0') \in \mathbb{C}\times U\times U$ denote the shape derivatives of $(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0)$ with respect to domain variation $\boldsymbol{\varphi} \in X$.

The fourth term of the right-hand side of (11), which is written as

$$\begin{aligned}&\mathscr{L}_{0\hat{\boldsymbol{v}}_0}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{v}}_0'] = -h_{\hat{\boldsymbol{v}}_0}\left(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)[\hat{\boldsymbol{v}}_0']\\&\qquad- h_{\hat{\boldsymbol{v}}_0}\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{v}}_0']\\&\quad= -h\left(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}\prime}\right) - h\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}_0'\right),\end{aligned}\tag{12}$$

becomes 0, if $(s_k, \hat{\boldsymbol{u}}_k)$ is the solution of Problem 2. On the other hand, the second term of the right-hand side of (11) is written as

$$\begin{aligned}&\mathscr{L}_{0 s_k}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[s_k'] = f_{0 s_k}\left(\boldsymbol{\phi}, s_k\right)[s_k']\\&\qquad- h_{s_k}\left(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)[s_k'] - h_{s_k^{\mathrm{c}}}\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}_0\right)[s_k^{\mathrm{c}\prime}]\\&\quad= s_k' + s_k^{\mathrm{c}\prime} - 2 s_k s_k' b_{\mathrm{R}}\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right) - 2 s_k^{\mathrm{c}} s_k^{\mathrm{c}\prime} b_{\mathrm{P}}\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)\\&\quad= s_k'\left(1 - 2 s_k b_{\mathrm{R}}\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)\right) + s_k^{\mathrm{c}\prime}\left(1 - 2 s_k^{\mathrm{c}} b_{\mathrm{P}}\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)\right).\end{aligned}\tag{13}$$

Moreover, the third term of the right-hand side of (11) is written as

$$\begin{aligned}&\mathscr{L}_{0\hat{\boldsymbol{u}}_k}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{u}}_k'] = -h_{\hat{\boldsymbol{u}}_k}\left(s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right)[\hat{\boldsymbol{u}}_k']\\&\qquad- h_{\hat{\boldsymbol{u}}_k}\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}_0\right)[\hat{\boldsymbol{u}}_k']\\&\quad= -h\left(s_k, \hat{\boldsymbol{u}}_k', \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right) - h\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}\prime}, \hat{\boldsymbol{v}}_0\right).\end{aligned}\tag{14}$$

Then, (13) and (14) become 0 if $\hat{\boldsymbol{v}}_0$ is the solution of the following weak form of the adjoint problem.

Problem 4 (Adjoint problem for f_0) For $\boldsymbol{\phi} \in \mathcal{D}$, let $(s_k, \hat{\boldsymbol{u}}_k)$ be the solution of Problem 2 for k. Find $\hat{\boldsymbol{v}}_0$ such that

$$h\left(s_k, \hat{\boldsymbol{u}}_k', \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right) + h\left(s_k^{\mathrm{c}}, \hat{\boldsymbol{u}}_k^{\mathrm{c}\prime}, \hat{\boldsymbol{v}}_0\right) = 0,\tag{15}$$

$$2 s_k b\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0^{\mathrm{c}}\right) = 2 s_k^{\mathrm{c}} b\left(\hat{\boldsymbol{u}}_k^{\mathrm{c}}, \hat{\boldsymbol{v}}_0\right) = 1\tag{16}$$

for all $\hat{\boldsymbol{u}}_k' \in U$.

For the solution $\hat{\boldsymbol{v}}_0$ of Problem 4, from (16), we have $\hat{\boldsymbol{v}}_0 = c\hat{\boldsymbol{u}}_k$ for all $c \in \mathbb{C}$. Moreover, using (15), we obtain

$$c = \frac{1}{2 s_k b\left(\hat{\boldsymbol{u}}_k, \hat{\boldsymbol{u}}_k^{\mathrm{c}}\right)}.\tag{17}$$

Then, $\hat{\boldsymbol{v}}_0$ is obtained by normalization of $\hat{\boldsymbol{u}}_k$ with c above.

Based on the results, if $\mathrm{red}(s_k, \hat{\boldsymbol{u}}_k)$ and $\hat{\boldsymbol{v}}_0$ are the solutions of Problem 2 and Problem 4, respectively, then $\dot{\mathscr{L}}_0\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)\left[\boldsymbol{\varphi}, s_k', \hat{\boldsymbol{u}}_k', \hat{\boldsymbol{v}}_0'\right]$ in (11) becomes

$$\begin{aligned}\mathscr{L}_{0\boldsymbol{\phi}}\left(\boldsymbol{\phi}, s_k, \hat{\boldsymbol{u}}_k, \hat{\boldsymbol{v}}_0\right)[\boldsymbol{\varphi}] &= \int_{\partial\Omega_{\mathrm{P}}(\boldsymbol{\phi})\setminus\left(\bar{\Gamma}_{\mathrm{P}0}\cup\bar{\Gamma}_{\mathrm{D}0}\right)} \boldsymbol{g}_{\partial\Omega 0}\cdot\boldsymbol{\varphi}\,\mathrm{d}\gamma\\&= \langle \boldsymbol{g}_0, \boldsymbol{\varphi}\rangle,\end{aligned}\tag{18}$$

where

$$\boldsymbol{g}_{\partial\Omega 0} = 2\mathrm{Re}\left[\boldsymbol{S}\left(\hat{\boldsymbol{u}}_k\right)\cdot\boldsymbol{E}\left(\hat{\boldsymbol{v}}_0^{\mathrm{c}}\right) + s_k^2 \rho_{\mathrm{P}} \hat{\boldsymbol{u}}_k\cdot\hat{\boldsymbol{v}}_0^{\mathrm{c}}\right]\boldsymbol{\nu}_{\mathrm{P}}.\tag{19}$$

Here, we used the condition $\boldsymbol{\varphi} = \mathbf{0}_{\mathbb{R}^d}$ on $\Gamma_{\mathrm{P}0}\cup\Gamma_{\mathrm{D}0}$, as given in (1).

On the other hand, for $f_1(\boldsymbol{\phi})$, we have

$$\begin{aligned}f_1'\left(\boldsymbol{\phi}\right)[\boldsymbol{\varphi}] &= \int_{\partial\Omega_{\mathrm{P}}(\boldsymbol{\phi})\setminus\left(\bar{\Gamma}_{\mathrm{P}1}\cup\bar{\Gamma}_{\mathrm{D}0}\right)} \boldsymbol{g}_{\partial\Omega 0}\cdot\boldsymbol{\varphi}\,\mathrm{d}\gamma\\&= \langle \boldsymbol{g}_1, \boldsymbol{\varphi}\rangle,\end{aligned}\tag{20}$$

where

$$\boldsymbol{g}_{\partial\Omega 1} = \boldsymbol{\nu}_{\mathrm{P}}.\tag{21}$$

We refer to $\boldsymbol{g}_0$ and $\boldsymbol{g}_1$ as the shape derivatives of f_0 and f_1, respectively.

6 H^1 gradient method

The H^1 gradient method is proposed as a method for finding the variation of the design variable, such as the domain mapping or the density parameter that decreases the cost function, as a solution to a boundary value problem of an elliptic partial differential equation (Azegami and Takeuchi (2006); Azegami et al. (2011, 2013a)). For the case of shape derivative $\boldsymbol{g}_i$ of a cost function $f_i(\boldsymbol{\phi})$ for $i \in \{0, 1\}$, the H^1 gradient method can be described as follows.

Problem 5 (H^1 gradient method) Let X be a Hilbert space of $H^1\left(\mathbb{R}^d; \mathbb{R}^d\right)$, and let $a : X\times X \to \mathbb{R}$ be a coercive bilinear form on X such that there exists $\beta > 0$ that satisfies

$$a\left(\boldsymbol{w}, \boldsymbol{w}\right) \geq \beta\left\|\boldsymbol{w}\right\|_X^2$$

for all $\boldsymbol{w} \in X$. For $\boldsymbol{g}_i \in X'$ (dual space of X), which is a Fréchet derivative of cost function $f_i(\boldsymbol{\phi})$ at $\boldsymbol{\phi} \in X$, find $\boldsymbol{\varphi}_{gi} \in X$ such that

$$a\left(\boldsymbol{\varphi}_{gi}, \boldsymbol{w}\right) = -\langle \boldsymbol{g}_i, \boldsymbol{w}\rangle\tag{22}$$

for all $\boldsymbol{w} \in X$.

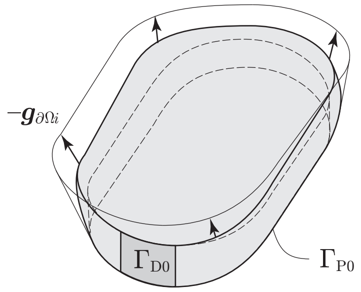


Fig. 3 Boundary conditions in Problem 6 (H^1 gradient method)

Problem 5 can be solved numerically using the standard finite element method by considering (22) to be a weak form of a boundary value problem of an elliptic partial differential equation. In the present paper, we use

$$a(\boldsymbol{\varphi}, \boldsymbol{\psi}) = c_a \int_{\Omega(\boldsymbol{\phi})} \boldsymbol{S}(\boldsymbol{\varphi}) \cdot \boldsymbol{E}(\boldsymbol{\psi})\, \mathrm{d}x \tag{23}$$

for $\boldsymbol{\varphi} \in X$ and $\boldsymbol{\psi} \in X$, where $\boldsymbol{E}(\,\cdot\,)$ and $\boldsymbol{S}(\,\cdot\,)$ are the same as in Problem 2, and c_a is a positive constant. The coerciveness is secured by the Dirichlet condition on $\Gamma_{\mathrm{P}0} \cup \Gamma_{\mathrm{D}0}$ in (5). The strong form of the H^1 gradient method using (23) is written as follows.

Problem 6 (H^1 gradient method for Problem 3) For $\boldsymbol{g}_i$, find $\boldsymbol{\varphi}_{gi}$ such that

$$\begin{aligned}
&-c_a \boldsymbol{\nabla}^{\mathrm{T}} \boldsymbol{S}(\boldsymbol{\varphi}_{gi}) = \boldsymbol{0}^{\mathrm{T}}_{\mathbb{R}^d} \quad \text{in } \Omega_{\mathrm{P}}(\boldsymbol{\phi}),\\
&c_a \boldsymbol{S}(\boldsymbol{\varphi}_{gi})\boldsymbol{\nu} = -\boldsymbol{g}_{\partial\Omega i} \quad \text{on } \partial\Omega_{\mathrm{P}}(\boldsymbol{\phi}) \setminus \left(\bar{\Gamma}_{\mathrm{P}0} \cup \bar{\Gamma}_{\mathrm{D}0}\right),\\
&\boldsymbol{\varphi}_{gi} = \boldsymbol{0}_{\mathbb{R}^d} \quad \text{on } \Gamma_{\mathrm{P}0} \cup \Gamma_{\mathrm{D}0}.
\end{aligned}$$

Figure 3 shows the boundary condition of Problem 6.

If $\hat{\boldsymbol{u}}$ satisfies the conditions in $\mathcal{S}$, we can confirm that the solution $\boldsymbol{\varphi}_{gi}$ of Problem 5 belongs to Y, which is the Banach space for the admissible set $\mathcal{D}$ defined in (1).

7 Solution to the shape optimization problem

In order to solve Problem 3, we use an iterative method based on sequential quadratic programming. The domain variation decreasing f_0 while satisfying $f_1 \le 0$ is determined with the solution of the following problem. In this section, we denote $f_0(\boldsymbol{\phi}, s_k)$ as $f_0(\boldsymbol{\phi})$ and its shape derivative as $\boldsymbol{g}_0$.

Problem 7 (SQ approximation) For $\boldsymbol{\phi} \in \mathcal{D}$, let $\boldsymbol{g}_i$ be the shape derivatives of $f_i(\boldsymbol{\phi})$ for $i \in \{0, 1\}$, and let $f_1(\boldsymbol{\phi}) \le 0$. Let $a(\,\cdot\,,\,\cdot\,)$ be given as in (22). Find $\boldsymbol{\varphi}$ such that

$$\min_{\boldsymbol{\varphi}\in X} \left\{ q(\boldsymbol{\varphi}) = \frac{1}{2} a(\boldsymbol{\varphi}, \boldsymbol{\varphi}) + \langle \boldsymbol{g}_0, \boldsymbol{\varphi} \rangle \;\middle|\; f_1(\boldsymbol{\phi}) + \langle \boldsymbol{g}_1, \boldsymbol{\varphi} \rangle \le 0 \right\}.$$

The Lagrange function of Problem 7 is defined as

$$\mathscr{L}_{\mathrm{SQ}}(\boldsymbol{\varphi}, \lambda_1) = q(\boldsymbol{\varphi}) + \lambda_1 (f_1(\boldsymbol{\phi}) + \langle \boldsymbol{g}_1, \boldsymbol{\varphi} \rangle)$$

where $\lambda_1 \in \mathbb{R}$ is the Lagrange multiplier for the constraint $f_1(\boldsymbol{\varphi}) \le 0$. The Karush–Kuhn–Tucker conditions for Problem 7 are given as

$$a(\boldsymbol{\varphi}, \boldsymbol{\varphi}) + \langle \boldsymbol{g}_0 + \lambda_1 \boldsymbol{g}_1, \boldsymbol{\varphi} \rangle = 0, \tag{24}$$
$$f_1(\boldsymbol{\phi}) + \langle \boldsymbol{g}_1, \boldsymbol{\varphi} \rangle \le 0, \tag{25}$$
$$\lambda_1 (f_1(\boldsymbol{\phi}) + \langle \boldsymbol{g}_1, \boldsymbol{\varphi} \rangle) = 0, \tag{26}$$
$$\lambda_1 \ge 0 \tag{27}$$

for all $\boldsymbol{\varphi} \in X$. Here, let $\boldsymbol{\varphi}_{gi}$ for $i \in \{0, 1\}$ be the solutions to Problem 5, and set

$$\boldsymbol{\varphi}_g = \boldsymbol{\varphi}_{g0} + \lambda_1 \boldsymbol{\varphi}_{g1}. \tag{28}$$

Then, by substituting $\boldsymbol{\varphi}_g$ of (28) for $\boldsymbol{\varphi}$ in (24), (24) holds. If the constraint in (25) is active, i.e., (25) holds with the equality, we have

$$\langle \boldsymbol{g}_1, \boldsymbol{\varphi}_{g1} \rangle \lambda_1 = -f_1(\boldsymbol{\phi}) + \langle \boldsymbol{g}_1, \boldsymbol{\varphi}_{g0} \rangle. \tag{29}$$

Equation (29) has a unique solution of λ_1. Moreover, if $f_1(\boldsymbol{\phi}) = 0$, we have

$$\langle \boldsymbol{g}_1, \boldsymbol{\varphi}_{g1} \rangle \lambda_1 = -\langle \boldsymbol{g}_1, \boldsymbol{\varphi}_{g0} \rangle. \tag{30}$$

Since (30) is independent of the magnitude of $\boldsymbol{\varphi}_{g0}$ and $\boldsymbol{\varphi}_{g1}$ to determine λ_1, (30) is used in the numerical scheme for the initial domain Ω_0 in which we assume that $f_1(\boldsymbol{\phi}) = 0$ is satisfied. If $\lambda_1 < 0$ in the solution λ_1 to (29) or (30), by setting $\lambda_1 = 0$, we have λ_1 satisfying (24) to (27).

The magnitude of $\boldsymbol{\varphi}_g$ in (28), which means the step size for domain variation, is adjusted by selection of c_a in (23) using criteria such as the Armijo and Wolfe's criteria to ensure the global convergence in Problem 7. An outline of the numerical scheme is shown in Fig. 4. The details of the numerical scheme are shown in the previous paper (Azegami et al. (2013a))

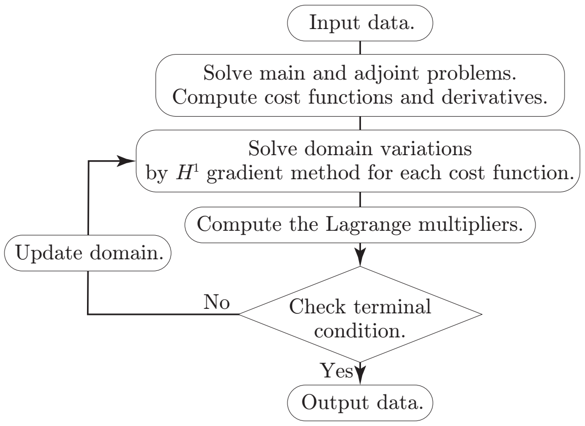


Fig. 4 Flowchart of program to solve shape optimization problem

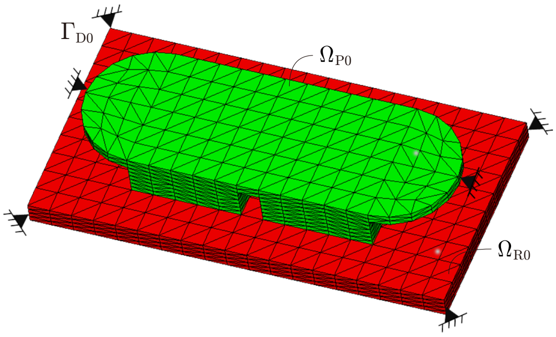
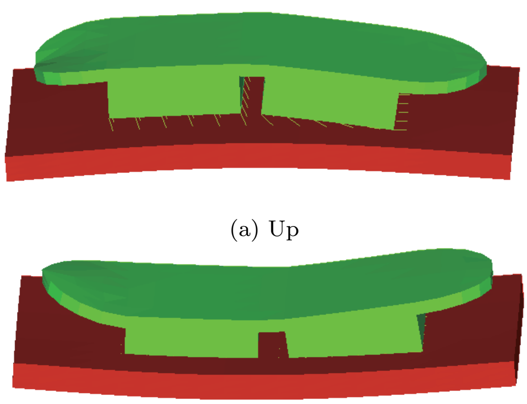


Fig. 5 Finite element model



(a) Up

(b) Down

Fig. 6 3rd natural vibration mode of initial shape

8 Numerical example

We developed a computer program for solving Problem 3 based on the numerical scheme described above. A multidisciplinary finite element solver RADIOSS developed by Altair Engineering, Inc. was used to solve Problem 2 and Problem 4.

Table 1 Complex eigenvalues for the initial shape

k	Re	Im
1	-1.692E+01	8.022947E+03
2	-1.444E+01	9.438261E+03
3	8.613E+00	1.249724E+04
4	-2.944E+01	1.437360E+04
5	-5.783E+01	1.629984E+04
6	-5.356E+01	2.168113E+04
7	-5.195E+01	2.394771E+04
8	-6.593E+01	2.573753E+04
9	-6.325E+01	2.711726E+04
10	-6.896E+01	2.893466E+04

In order to demonstrate the effectiveness of the present method, we solved a shape optimization problem involving a simple brake model. Figure 5 shows the finite element model of the brake model. In this figure, nodal points with fixed signs are assumed to be fixed in Problem 2 and Problem 4, i.e., the Dirichlet condition is assigned. The length of the largest edge in Ω_{R0} is 0.15 [m]. The Young's modulus, Poisson's ratio, and the density were 210 [GPa], 0.3, and 7.8×10^3 [kg/m^3], respectively, for the rotor and 16 [GPa], 0.3, and 2.1×10^3 [kg/m^3], respectively, for the pad. Moreover, the contact stiffness α, the friction coefficient μ and structural damping ratio were 5.0×10^6 [N/m], 0.1 and 0.005, respectively.

The numerical results for the complex eigenvalues for the initial shape, i.e., the numerical solution of Problem 2, are shown in Table 1. Among these results, the 3rd eigenvalue has a positive real part. As such, we set $k = 3$ in Problem 3. Figure 6 shows the 3rd natural vibration mode of the initial shape. We can observed that primary bending mode is occurred.

The iteration histories of cost functions f_0 and f_1 with respect to the number of reshaping iterations are shown in Fig. 7. In this figure, $f_{0\,\mathrm{init}}$ and c_1 denote the value of f_0 and the volume of Ω_{P0}, respectively, for the initial shape. Note that f_0 decreases monotonically while satisfying the domain measure constraint of f_1. Table 2 shows the numerical results for the complex eigenvalues after 60 reshaping iterations. The 3rd eigenvalue has a negative value in the real part. Figure 8 compares the initial and optimized shapes. Figure 9 shows the 3rd natural vibration mode of the optimized shape. From those results, a remarkable change in the shape is observed at the center and both ends. Those parts correspond to the antinode of the natural vibration mode and the fixed part, respectively, and were reinforced by shape optimization. This change means that the stiffness with respect to the deformation of the 3rd natural vibration mode is improved. By this change, we can consider that the cause of the brake squeal is suppressed.

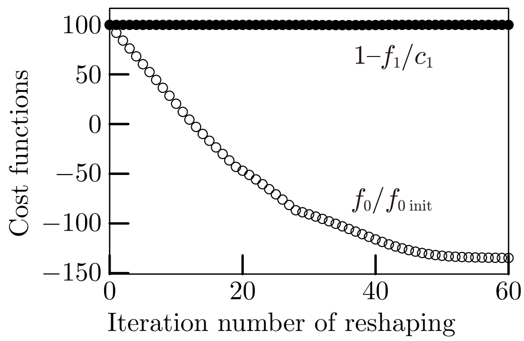
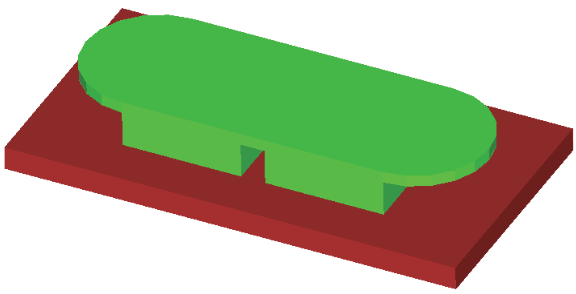
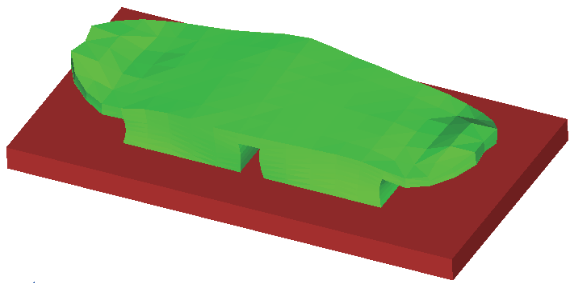


Fig. 7 Iteration histories of cost functions with respect to reshaping



(a) Initial shape

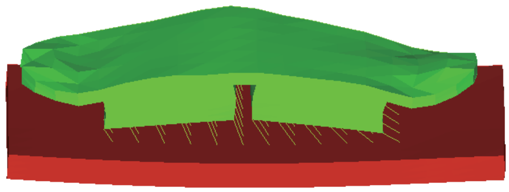


(b) Optimized shape

Fig. 8 Comparison of shapes

9 Conclusions

In the present paper, we have introduced a numerical solution to shape optimization problems of a brake model consisting of a rotor and a pad, between which Coulomb friction occurs. The main problem was constructed as a complex eigenvalue problem of the brake model obtained from the equation of motion. The shape optimization problem was formulated using the positive real part of the complex eigenvalue assigned as the cause of brake squeal as an objective cost function and the volume of the pad as a constraint cost function. The method of evaluating the shape derivative of the objective cost function was derived using the stationary conditions of the Lagrange function. A standard scheme by which to solve the shape optimization problem using the H^1 gradient method for reshaping was used to construct the numerical scheme. Finally, numerical results obtained using a simple rotor-pad model were presented, in which the real part of the target complex eigenvalue decreases monotonically, satisfying the constraint for the volume of the pad. Based on these results, in the present paper, a methodology for finding the optimum shape that minimizes the real part of the complex eigenvalue representing the cause of brake squeal was presented.



(a) Up

(b) Down

Fig. 9 3rd natural vibration mode of optimized shape

Table 2 Complex eigenvalues for the optimized shape

k	Re	Im
1	-1.647E+01	7.745197E+03
2	-1.765E+01	1.027973E+04
3	-1.163E+01	1.110440E+04
4	-3.048E+01	1.503565E+04
5	-4.185E+01	2.092213E+04
6	-5.070E+01	2.186379E+04
7	-6.588E+01	2.671747E+04
8	-7.522E+01	2.756015E+04
9	-7.540E+01	3.137934E+04
10	-7.658E+01	3.320161E+04

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