Physics of Plasmas

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Cite as: Phys. Plasmas **25**, 102109 (2018); https://doi.org/10.1063/1.5050542
Submitted: 31 July 2018 . Accepted: 24 September 2018 . Published Online: 12 October 2018

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PHYSICS OF PLASMAS **25**, 102109 (2018)

Electromagnetic linear dispersion relation for plasma with a drift across magnetic field revisited

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(Received 31 July 2018; accepted 24 September 2018; published online 12 October 2018)

A current across the magnetic field is formed in various situations in plasma. The relative drift between ions and electrons due to the cross-field current becomes a source of various microscopic instabilities. A fully electromagnetic and kinetic linear dispersion relation for plasma with a drift across magnetic field is derived by assuming a uniform background plasma. The dielectric permittivity tensor for shifted Maxwellian velocity distributions is also presented. Linear dispersion relations obtained by using the new dielectric permittivity tensor were confirmed by comparison with the previous studies and with particle-in-cell simulation results. *Published by AIP Publishing.*
https://doi.org/10.1063/1.5050542

I. INTRODUCTION

Plasma instabilities are driven by various sources, such as spatial inhomogeneity or velocity-space anisotropy. The linear dispersion analysis plays an essential role for studies of plasma physics. Instabilities due to spatial inhomogeneity are generally analyzed by linear electromagnetic fluid equations, while instabilities due to velocity-space anisotropy sometimes need a kinetic approach. It is known that various velocity-space instabilities are induced when a velocity distribution function $f[v_{||}, v_\perp]$ (where $v_{||}$ and $v_\perp$ are velocity components parallel and perpendicular to the ambient magnetic field, respectively) has a region of positive gradient, i.e., $\partial f/\partial v_{||} > 0$ or $\partial f/\partial v_\perp > 0$.

The positive gradient in the velocity distribution function parallel to the ambient magnetic field is formed when a beam of charged particles propagates along the ambient magnetic field or when there exists a relative velocity between ion and electron components. There are a number of textbooks on plasma physics and plasma waves that deal with the kinetic linear dispersion relation including a drift along the ambient magnetic field (hereafter, this is referred to as the "standard linear dispersion relation"). However, a few numbers of them gave the detailed derivation of the standard linear dispersion relation.[1,2]

The positive gradient in the velocity distribution function perpendicular to the ambient magnetic field is formed in various situations, such as ion reflection at shocks and magnetopauses or cross-field currents due to spatial inhomogeneity. The relative drift between ions and electrons in such situations becomes a source of various instabilities, such as the upper-hybrid drift instability (or Buneman instability[3]) the electron cyclotron drift instability (ECDI),[4] the modified two-stream instability (MTSI),[5,6] and the lower-hybrid drift instability (LHDI).[7] The former three instabilities have been identified in full kinetic numerical simulations of perpendicular collisionless shocks.[8–12] At the shock front, a part of upstream ions are reflected, which results in the deceleration of upstream electrons so that the conservation of the total current (the zero current condition in the shock normal direction) is satisfied. Consequently, there arises a relative drift velocity between the upstream electrons and the upstream/reflected ions.[13] In the linear analysis of these instabilities in the transition region of perpendicular collisionless shocks, it was assumed, by using the coordinate transformation from the shock-rest frame to the electron-rest frame, that *unmagnetized* ions drifted across the ambient magnetic field, while magnetized electrons were at rest.[13,14] Then, ion cyclotron harmonic resonance was neglected, while ion Landau damping was *enhanced*.

The diamagnetic current is a cross-field current which is formed at magnetic shear and pressure shear layers. External forces, such as gravity, across magnetic fields also result in a relative drift between ions and electrons, which forms a current. The LHDI has drawn attention by full kinetic numerical simulations of current sheets, which causes a quick triggering of magnetic reconnection and associated electron heating.[15] The LHDI is also known to play an role for the turbulent formation and the associated electron heating in thin density shear layers at the leading edge of the reconnection outflow jet.[16] In an early linear analysis of the LHDI by Davidson *et al.*,[17] the term $n = 0$ for the order of the Bessel function was retained for electrons only, while ions were assumed to be unmagnetized as Ref. 13. On the other hand, Daughton[18] numerically solved the fully electromagnetic and kinetic linear dispersion relation including spatial inhomogeneity, although his procedure was rather complex.

The purpose of the present study is to derive a fully electromagnetic and kinetic linear dispersion relation for plasma with a drift across an ambient magnetic field by modifying the standard linear dispersion relation.

II. THEORETICAL FORMULATION

Our goal is to solve the following linearized Maxwell equation for local plasma:

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$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}_1)=(\boldsymbol{k}\cdot\boldsymbol{E}_1)\boldsymbol{k}-|\boldsymbol{k}|^2\boldsymbol{E}_1=-i\tilde{\omega}\mu_0\boldsymbol{J}_1-\frac{\tilde{\omega}^2}{c^2}\boldsymbol{E}_1,$$
$$0=\left(\boldsymbol{kk}-|\boldsymbol{k}|^2\mathbf{I}\right)\frac{c^2}{\tilde{\omega}^2}\boldsymbol{E}_1+\boldsymbol{\epsilon}[\tilde{\omega},\boldsymbol{k}]\boldsymbol{E}_1, \tag{1}$$

where $\mathbf{I}$, $\boldsymbol{k}$, $\tilde{\omega}$, and c represent the unit tensor, wavenumber vector, complex frequency, and the speed of light, respectively. Note that $\boldsymbol{kk}$ denotes a dyadic tensor. The dielectric tensor $\boldsymbol{\epsilon}$ is obtained by solving

$$\boldsymbol{\epsilon}[\tilde{\omega},\boldsymbol{k}]\boldsymbol{E}_1=\boldsymbol{E}_1+i\frac{c^2}{\tilde{\omega}}\mu_0\boldsymbol{J}_1, \tag{2}$$

where $\boldsymbol{J}_1$ represents the perturbed current density given by

$$\boldsymbol{J}_1=\sum_s q_sN_{s0}\int \boldsymbol{v}f_{s1}\mathrm{d}^3\boldsymbol{v}, \tag{3}$$

with f_{s1} being the perturbed velocity distribution function for the species "s."

To evaluate the perturbed distribution function, we restart from the linearized Vlasov equation. By using the total derivative, the linearized Vlasov equation is written as

$$\frac{\partial f_{s1}}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f_{s1}}{\partial \boldsymbol{r}}+\left\{\boldsymbol{a}_{s0}+\frac{q_s}{m_s}(\boldsymbol{E}_0+\boldsymbol{v}\times\boldsymbol{B}_0)\right\}\cdot\frac{\partial f_{s1}}{\partial \boldsymbol{v}}$$
$$=\frac{\mathrm{d}f_{s1}}{\mathrm{d}t}=-\frac{q_s}{m_s}(\boldsymbol{E}_1+\boldsymbol{v}\times\boldsymbol{B}_1)\cdot\frac{\partial f_{s0}}{\partial \boldsymbol{v}}, \tag{4}$$

where $\boldsymbol{a}_{s0}$ represents an external force which includes the gravity and other (magneto-)hydro-dynamic forces due to spatial inhomogeneity that causes drift motions across the magnetic field for each particle species. The perturbed distribution function can be obtained by the method of characteristics, i.e., by the integral along a trajectory[1] as follows:

$$f_{s1}[\boldsymbol{r},\boldsymbol{v},t]=-\frac{q_s}{m_s}\int_{-\infty}^{t}\left\{\boldsymbol{E}_1[\hat{\boldsymbol{r}},\hat{t}]+\hat{\boldsymbol{v}}\times\boldsymbol{B}_1[\hat{\boldsymbol{r}},\hat{t}]\right\}$$
$$\cdot\frac{\partial f_{s0}}{\partial \hat{\boldsymbol{v}}}\exp\left[i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}-i\omega\hat{t}\right]\mathrm{d}\hat{t}, \tag{5}$$

where $(\hat{\boldsymbol{r}},\hat{\boldsymbol{v}})$ is an unperturbed trajectory of a particle which reaches the point $(\boldsymbol{r},\boldsymbol{v})$ when $\hat{t}=t$. The particle trajectory is governed by the equation of motion

$$\frac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t}=\boldsymbol{F}_{s0}+\frac{q_s}{m_s}(\hat{\boldsymbol{v}}\times\boldsymbol{B}_0), \tag{6}$$

where $\boldsymbol{F}_{s0}\equiv\boldsymbol{a}_{s0}+q_s\boldsymbol{E}_0/m_s$. The solution to this equation considered in the present study is

$$\left.\begin{aligned}\hat{v}_x&=v'_\perp\cos\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]+u_{bs}\\ \hat{v}_y&=v'_\perp\sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]\\ \hat{v}_z&=v_{||}\end{aligned}\right\}, \tag{7}$$

where $v_{||}$ and $v'_\perp$ represent velocity components parallel and perpendicular to the ambient magnetic field, respectively, and $\Omega_{cs}\equiv q_s|\boldsymbol{B}_0|/m_s$ represents the gyro frequency. We assume that both electric field and external force are directed in the y direction, and the ambient magnetic field is directed in the z direction. Then, particles drift in the x direction across the ambient magnetic field at the drift velocity $u_{bs}=q_sF_{s0}/B_0$ as schematically illustrated in Fig. 1. Here, the velocity vector and the wavenumber vector are, respectively, defined as

$$\boldsymbol{v}=(v_x,v_y,v_z)\equiv(v'_\perp\cos\phi'_0+u_{bs},v'_\perp\sin\phi'_0,v_{||}), \tag{8}$$
$$\boldsymbol{k}=(k_x,k_y,k_z)\equiv(k_\perp\cos\theta,k_\perp\sin\theta,k_{||}). \tag{9}$$

where $v'_\perp\equiv\sqrt{(v_x-u_{bs})^2+v_y^2}$. In contrast to the standard velocity coordinate centered at $(v_{||},v_\perp)=(0,0)$, the present velocity coordinate is centered at $(v_{||},v_\perp)=(0,u_{bs})$. It should be also noted that we consider a "local" dispersion relation where the spatial scale of the perturbation is much smaller than the spatial inhomogeneity.[17] That is, it is assumed that the background field quantities such as fluid quantities and electromagnetic fields are in the equilibrium state and that the drift velocity u_{bs} is constant and independent of both position and time. With these assumptions, we can take an arbitrary drift velocity u_{bs} for each species independently without consideration of the external force $\boldsymbol{a}_{s0}$ and spatial inhomogeneity.

Integrating the velocity over the time, we find the trajectory which reaches the point $(\boldsymbol{r},\boldsymbol{v})$ when $t'=t$ as

$$\left.\begin{aligned}\hat{x}&=x-\frac{v'_\perp}{\Omega_{cs}}\left\{\sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]-\sin\phi'_0\right\}-u_{bs}(t-\hat{t})\\ \hat{y}&=y+\frac{v'_\perp}{\Omega_{cs}}\left\{\cos\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]-\cos\phi'_0\right\}\\ \hat{z}&=z-v_{||}(t-\hat{t})\end{aligned}\right\}. \tag{10}$$

Further taking the wavenumber vector $\boldsymbol{k}=(k_\perp\cos\theta,k_\perp\sin\theta,k_{||})$, we obtain the Fourier component along the unperturbed trajectory as

$$\begin{aligned}&\exp\left[i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}-i\tilde{\omega}\hat{t}\right]\\ &=\exp\left[i\boldsymbol{k}\cdot\boldsymbol{r}-i\tilde{\omega}t\right]\exp\left[i(\tilde{\omega}-v_{||}k_{||}-u_{bs}k_\perp\cos\theta)(t-\hat{t})\right]\\ &\quad\times\exp\left[-i\frac{v'_\perp k_\perp}{\Omega_{cs}}\left\{\sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0-\theta\right]-\sin\left[\phi'_0-\theta\right]\right\}\right]\\ &=\exp[i\boldsymbol{k}\cdot\boldsymbol{r}-i\tilde{\omega}t]\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}J_l\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right]J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right]\\ &\quad\times\exp\left[i(l-n)(\phi'_0-\theta)\right]\exp\Big[i(\tilde{\omega}-v_{||}k_{||}-n\Omega_{cs}\\ &\quad-u_{bs}k_\perp\cos\theta)(t-\hat{t})\Big],\end{aligned} \tag{11}$$

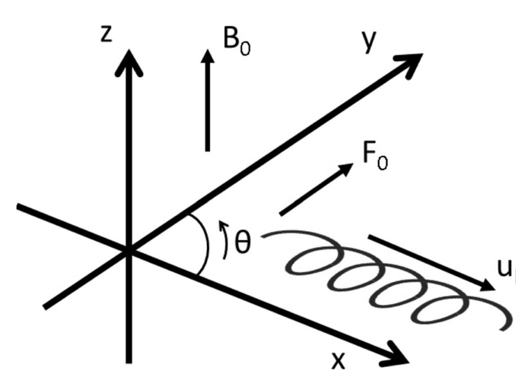

FIG. 1. Schematic illustration of the coordinate in the present study.

where θ represents the wave normal angle relative to the drift motion across the magnetic field. Here, $J_n[x]$ is the Bessel function of the first kind of order n with

$$\exp[i\lambda\sin\psi] = \sum_{n=-\infty}^{\infty} J_n[\lambda]\exp[in\psi].$$

With the Maxwell equation $\tilde{\omega}\boldsymbol{B} = \boldsymbol{k}\times\boldsymbol{E}$, we have

$$\left(\boldsymbol{E}_1 + \hat{\boldsymbol{v}}\times\frac{\boldsymbol{k}\times\boldsymbol{E}_1}{\tilde{\omega}}\right)\cdot\frac{\partial f_{s0}}{\partial\hat{\boldsymbol{v}}} = \begin{bmatrix} E_{x1}\left(1-\dfrac{\hat{v}_y k_y + \hat{v}_z k_z}{\tilde{\omega}}\right) + E_{y1}\dfrac{\hat{v}_y k_x}{\tilde{\omega}} + E_{z1}\dfrac{\hat{v}_z k_x}{\tilde{\omega}} \\ E_{x1}\dfrac{\hat{v}_x k_y}{\tilde{\omega}} + E_{y1}\left(1-\dfrac{\hat{v}_x k_x + \hat{v}_z k_z}{\tilde{\omega}}\right) + E_{z1}\dfrac{\hat{v}_z k_y}{\tilde{\omega}} \\ E_{x1}\dfrac{\hat{v}_x k_z}{\tilde{\omega}} + E_{y1}\dfrac{\hat{v}_y k_z}{\tilde{\omega}} + E_{z1}\left(1-\dfrac{\hat{v}_x k_x + \hat{v}_y k_y}{\tilde{\omega}}\right) \end{bmatrix}\cdot\begin{bmatrix} \dfrac{\hat{v}_x - u_{bs}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\hat{v}_y}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\partial f_{s0}}{\partial v_{||}} \end{bmatrix}$$

$$= \begin{bmatrix} \left(1-\dfrac{\hat{v}_y k_y + \hat{v}_z k_z}{\tilde{\omega}}\right)\dfrac{\hat{v}_x - u_{bs}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{\hat{v}_x k_y}{\tilde{\omega}}\dfrac{\hat{v}_y}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{\hat{v}_x k_z}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v_{||}} \\ \dfrac{\hat{v}_y k_x}{\tilde{\omega}}\dfrac{\hat{v}_x - u_{bs}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \left(1-\dfrac{\hat{v}_x k_x + \hat{v}_z k_z}{\tilde{\omega}}\right)\dfrac{\hat{v}_y}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{\hat{v}_y k_z}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v_{||}} \\ \dfrac{\hat{v}_z k_x}{\tilde{\omega}}\dfrac{\hat{v}_x - u_{bs}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{\hat{v}_z k_y}{\tilde{\omega}}\dfrac{\hat{v}_y}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \left(1-\dfrac{\hat{v}_x k_x + \hat{v}_y k_y}{\tilde{\omega}}\right)\dfrac{\partial f_{s0}}{\partial v_{||}} \end{bmatrix}\cdot\begin{bmatrix} E_{x1} \\ E_{y1} \\ E_{z1} \end{bmatrix} \equiv \boldsymbol{h}_s\cdot\boldsymbol{E}_1,$$

where

$$\frac{\partial f_{s0}}{\partial\hat{\boldsymbol{v}}} = \begin{bmatrix} \dfrac{\partial v'_\perp}{\partial\hat{v}_x}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\partial v'_\perp}{\partial\hat{v}_y}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\partial\hat{v}_{||}}{\partial\hat{v}_z}\dfrac{\partial f_{s0}}{\partial v_{||}} \end{bmatrix} = \begin{bmatrix} \dfrac{\hat{v}_x - u_{bs}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\hat{v}_y}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \dfrac{\partial f_{s0}}{\partial v_{||}} \end{bmatrix}.$$

The vector $\boldsymbol{h}$ is rewritten as

$$\boldsymbol{h}_s \equiv \begin{bmatrix} \cos\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]\left\{\left(1-\dfrac{v_{||}k_{||}}{\tilde{\omega}}\right)\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{v'_\perp k_{||}}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v_{||}}\right\} + \sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]\dfrac{u_{bs}k_\perp\sin\theta}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{u_{bs}k_{||}}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v_{||}} \\ \sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]\left\{\left(1-\dfrac{v_{||}k_{||}}{\tilde{\omega}}\right)\dfrac{\partial f_{s0}}{\partial v'_\perp} + \dfrac{v'_\perp k_{||}}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v_{||}}\right\} - \sin\left[\Omega_{cs}(t-\hat{t})+\phi'_0\right]\dfrac{u_{bs}k_\perp\cos\theta}{\tilde{\omega}}\dfrac{\partial f_{s0}}{\partial v'_\perp} \\ \left(1-\dfrac{u_{bs}k_\perp\cos\theta}{\tilde{\omega}}\right)\dfrac{\partial f_{s0}}{\partial v_{||}} + \cos\left[\Omega_{cs}(t-\hat{t})+\phi'_0-\theta\right]\dfrac{v'_\perp k_\perp}{\tilde{\omega}}\left(\dfrac{v_{||}}{v'_\perp}\dfrac{\partial f_{s0}}{\partial v'_\perp} - \dfrac{\partial f_{s0}}{\partial v_{||}}\right) \end{bmatrix}.$$

It follows that

$$i\frac{c^2}{\tilde{\omega}}\mu_0\boldsymbol{J}_1 = -i\sum_s\frac{\omega_{ps}^2}{\tilde{\omega}}\int\boldsymbol{v}\left\{\int_{-\infty}^{t}\boldsymbol{h}_s\cdot\boldsymbol{E}_1\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}J_l\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right]J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right]\exp\left[i(l-n)(\phi'_0-\theta)\right]\right.$$
$$\left.\times\exp\left[i(\tilde{\omega}-v_{||}k_{||}-n\Omega_{cs}-u_bk_\perp\cos\theta)(t-\hat{t})\right]\mathrm{d}\hat{t}\right\}\mathrm{d}^3\boldsymbol{v},$$

$$
\begin{aligned}
&= \sum_s \frac{\omega_{ps}^2}{\tilde{\omega}} \left\{ \int \begin{bmatrix} v'_\perp \cos\phi'_0 + u_{bs} \\ v'_\perp \sin\phi'_0 \\ v_{||} \end{bmatrix} \right.\\
&\times \sum_{n=-\infty}^{\infty} \begin{bmatrix} \left\{ J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \frac{n\Omega_{cs}}{v'_\perp k_\perp} \cos\theta - iJ'_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \sin\theta \right\} \left\{ \left(1 - \frac{v_{||}k_{||}}{\tilde{\omega}}\right) \frac{\partial f_{s0}}{\partial v'_\perp} + \frac{v'_\perp k_{||}}{\tilde{\omega}} \frac{\partial f_{s0}}{\partial v_{||}} \right\} \\
+ \left\{ J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \frac{n\Omega_{cs}}{v'_\perp k_\perp} \sin\theta + iJ'_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \cos\theta \right\} \frac{u_{bs}k_\perp \sin\theta}{\tilde{\omega}} \frac{\partial f_{s0}}{\partial v'_\perp} + J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \frac{u_{bs}k_{||}}{\tilde{\omega}} \frac{\partial f_{s0}}{\partial v_{||}} \\
\left\{ J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \frac{n\Omega_{cs}}{v'_\perp k_\perp} \sin\theta + iJ'_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \cos\theta \right\} \left\{ \left(1 - \frac{v_{||}k_{||}}{\tilde{\omega}} - \frac{u_{bs}k_\perp \cos\theta}{\tilde{\omega}}\right) \frac{\partial f_{s0}}{\partial v'_\perp} + \frac{v'_\perp k_{||}}{\tilde{\omega}} \frac{\partial f_{s0}}{\partial v_{||}} \right\} \\
J_n\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \left\{ \left(1 - \frac{n\Omega_{cs}}{\tilde{\omega}} - \frac{u_{bs}k_\perp \cos\theta}{\tilde{\omega}}\right) \frac{\partial f_{s0}}{\partial v_{||}} + \frac{n\Omega_{cs}}{\tilde{\omega}} \frac{v_{||}}{v'_\perp} \frac{\partial f_{s0}}{\partial v'_\perp} \right\} \end{bmatrix} \\
&\left. \times \sum_{l=-\infty}^{\infty} \frac{J_l\left[\frac{v'_\perp k_\perp}{\Omega_{cs}}\right] \exp\left[i(l-n)(\phi'_0 - \theta)\right]}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp \cos\theta} \mathrm{d}^3\boldsymbol{v} \right\} \cdot \boldsymbol{E}_1. \qquad (12)
\end{aligned}
$$

For the time integral in Eq. (12), we use the following formulas:

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty} J_n[\lambda] \cos\left[\Omega_{cs}(t-\hat{t}) + \phi'_0\right] \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \frac{1}{2}\cos\theta \sum_{n=-\infty}^{\infty} \left\{J_{n+1}[\lambda] + J_{n-1}[\lambda]\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&\quad - \frac{i}{2}\sin\theta \sum_{n=-\infty}^{\infty} \left\{J_{n+1}[\lambda] - J_{n-1}[\lambda]\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \sum_{n=-\infty}^{\infty} \left\{\frac{n}{\lambda} J_n[\lambda] \cos\theta - iJ'_n[\lambda] \sin\theta\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty} J_n[\lambda] \sin\left[\Omega_{cs}(t-\hat{t}) + \phi'_0\right] \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \frac{1}{2}\sin\theta \sum_{n=-\infty}^{\infty} \left\{J_{n+1}[\lambda] + J_{n-1}[\lambda]\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&\quad - \frac{1}{2i}\cos\theta \sum_{n=-\infty}^{\infty} \left\{J_{n+1}[\lambda] - J_{n-1}[\lambda]\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \sum_{n=-\infty}^{\infty} \left\{\frac{n}{\lambda} J_n[\lambda] \sin\theta + iJ'_n[\lambda] \cos\theta\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty} J_n[\lambda] \cos\left[\Omega_{cs}(t-\hat{t}) + \phi'_0 - \theta\right] \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left\{J_{n+1}[\lambda] + J_{n-1}[\lambda]\right\} \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right] \\
&= \sum_{n=-\infty}^{\infty} \frac{n}{\lambda} J_n[\lambda] \exp\left[-in\Omega_{cs}(t-\hat{t})\right] \exp\left[-in(\phi'_0 - \theta)\right],
\end{aligned}
$$

and

$$
\int_{-\infty}^{t} \exp\left[i(\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp \cos\theta)(t-\hat{t})\right] \mathrm{d}\hat{t} = \frac{i}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp \cos\theta},
$$

where $J_n'[\lambda]$ represents the differentiation of the Bessel function of the first kind with respect to argument $\lambda \equiv v'_\perp k_\perp/\Omega_{cs}$. For detailed derivation, see also Refs. 1, 2, and 19.

Let us consider the velocity-space integration. From the definition of velocity, we obtain the Jacobian as

$$|J(v'_\perp, \phi'_0)| = \det\begin{bmatrix} \frac{\partial v_x}{\partial v'_\perp} & \frac{\partial v_x}{\partial \phi'_0} \\ \frac{\partial v_y}{\partial v'_\perp} & \frac{\partial v_y}{\partial \phi'_0} \end{bmatrix} = \det\begin{bmatrix} \cos\phi'_0 & -v'_\perp \sin\phi'_0 \\ \sin\phi'_0 & v'_\perp\cos\phi'_0 \end{bmatrix} = v'_\perp. \tag{13}$$

Assuming that distribution functions are gyrotropic, i.e., $\frac{\partial f_{s0}}{\partial \phi'_0} = 0$ ($f_{s0}[\boldsymbol{v}] \equiv f_{s0}[v_{||}, v'_\perp]$) and that the wavenumber vector $\boldsymbol{k}$ is taken in the $x-z$ plane, i.e., $\theta = 0$ ($k_z = k_{||}$ and $k_x = k_\perp$), we have

$$\int d^3\boldsymbol{v} = \int_0^{2\pi}\int_0^\infty\int_{-\infty}^\infty v'_\perp dv_{||} dv'_\perp d\phi'_0 = \int_0^\infty\int_{-\infty}^\infty 2\pi v'_\perp dv_{||} dv'_\perp \tag{14}$$

and

$$\int_0^{2\pi} J_l[\lambda]\cos\phi'_0 \exp\left[i(l-n)\phi'_0\right]d\phi'_0 = \frac{2\pi n}{\lambda}J_n[\lambda],$$
$$\int_0^{2\pi} J_l[\lambda]\sin\phi'_0 \exp\left[i(l-n)\phi'_0\right]d\phi'_0 = -i\frac{2\pi n}{\lambda}J_n J'_n[\lambda],$$
$$\int_0^{2\pi} J_l[\lambda] \exp\left[i(l-n)\phi'_0\right]d\phi'_0 = 2\pi J_n[\lambda].$$

By using these properties, Eq. (12) is rewritten as

$$i\frac{c^2}{\tilde\omega}\mu_0 \boldsymbol{J}_1 = \sum_s \frac{\omega_{ps}^2}{\tilde\omega^2}\sum_{n=-\infty}^{\infty}\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp \mathbf{S} dv_{||}dv'_\perp}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\cdot \boldsymbol{E}_1, \tag{15}$$

where

$$\begin{aligned} S_{xx} = {} & \left(J_n^2[\lambda]\frac{n^2\Omega_{cs}^2}{v'^2_\perp k_\perp^2}\right)\left\{v'^2_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + v'_\perp(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & + \left(J_n^2[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\right)u_{bs}\left\{v'_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + (\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & + \left(J_n^2[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\right)u_{bs}\left(v'_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}\frac{\partial f_{s0}}{\partial v'_\perp}\right) \\ & + J_n^2[\lambda]u_{bs}^2\left(k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + \frac{n\Omega_{cs}}{v'_\perp}\frac{\partial f_{s0}}{\partial v'_\perp}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} S_{xy} = {} & iJ_n[\lambda]J'_n[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\left\{v'^2_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + v'_\perp(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & + iJ_n[\lambda]J'_n[\lambda]u_{bs}\left\{v'_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + (\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\}, \end{aligned} \tag{17}$$

$$\begin{aligned} S_{xz} = {} & \left(J_n^2[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\right)\left\{v'_\perp(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}v_{||}\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & + J_n^2[\lambda]u_{bs}\left\{(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}\frac{v_{||}}{v'_\perp}\frac{\partial f_{s0}}{\partial v'_\perp}\right\}, \end{aligned} \tag{18}$$

$$\begin{aligned} S_{yx} = {} & -iJ_n[\lambda]J'_n[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\left\{v'^2_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + v'_\perp(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & - iJ_n[\lambda]J'_n[\lambda]u_{bs}\left(v'_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}\frac{\partial f_{s0}}{\partial v'_\perp}\right), \end{aligned} \tag{19}$$

$$S_{yy} = J'^2_n[\lambda]\left\{v'^2_\perp k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + v'_\perp(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v'_\perp}\right\}, \tag{20}$$

$$S_{yz} = -iJ_n[\lambda]J'_n[\lambda]\left\{v'_\perp(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}v_{||}\frac{\partial f_{s0}}{\partial v'_\perp}\right\}, \tag{21}$$

$$\begin{aligned} S_{zx} = {} & J_n^2[\lambda]\frac{n\Omega_{cs}}{v'_\perp k_\perp}\left\{v'_\perp v_{||}k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + (\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)v_{||}\frac{\partial f_{s0}}{\partial v'_\perp}\right\} \\ & + J_n^2[\lambda]u_{bs}\left(v_{||}k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}\frac{v_{||}}{v'_\perp}\frac{\partial f_{s0}}{\partial v'_\perp}\right), \end{aligned} \tag{22}$$

$$S_{zy} = iJ_n[\lambda]J'_n[\lambda]\left\{v'_\perp v_{||}k_{||}\frac{\partial f_{s0}}{\partial v_{||}} + (\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)v_{||}\frac{\partial f_{s0}}{\partial v'_\perp}\right\}, \tag{23}$$

$$S_{zz} = J_n^2[\lambda]\left\{(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)v_{||}\frac{\partial f_{s0}}{\partial v_{||}} + n\Omega_{cs}\frac{v_{||}^2}{v'_\perp}\frac{\partial f_{s0}}{\partial v'_\perp}\right\}. \tag{24}$$

Inserting Eq. (15) into Eq. (2) and deleting $\boldsymbol{E}_1$, we finally obtain the dielectric permittivity tensor as

$$\boldsymbol{\epsilon}[\tilde\omega, \boldsymbol{k}] = \mathbf{I} + \sum_s \frac{\omega_{ps}^2}{\tilde\omega^2}\sum_{n=-\infty}^{\infty}\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp \mathbf{S} dv_{||}dv'_\perp}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}. \tag{25}$$

III. DIELECTRIC PERMITTIVITY TENSOR FOR SHIFTED bi-MAXWELLIAN VELOCITY DISTRIBUTION

The Maxwellian distribution is often used as a distribution of particle velocity at an equilibrium state. We use the following shifted bi-Maxwellian distribution as a velocity distribution at the present analysis:

$$f_s[v_{||}, v'_\perp] = f_{s||}[v_{||}]f_{s\perp}[v'_\perp], \tag{26}$$

$$f_{s||}[v_{||}] = \frac{1}{\sqrt{2\pi}V_{ts||}}\exp\left[-\frac{(v_{||} - u_{ds})^2}{2V_{ts||}^2}\right], \tag{27}$$

$$f_{s\perp}[v'_\perp] = \frac{1}{2\pi V_{ts\perp}^2}\exp\left[-\frac{v'^2_\perp}{2V_{ts\perp}^2}\right], \tag{28}$$

where u_{ds} is the drift velocity in the direction parallel to the ambient magnetic field and $V_{ts||} \equiv \sqrt{T_{s||}/m}$ and $V_{ts\perp} \equiv \sqrt{T_{s\perp}/m}$ in the direction parallel and perpendicular to the ambient magnetic field, respectively, with T_s being temperature of particle species. Note that such shifted bi-Maxwellian distributions are also common in laboratory plasmas.[20,21] Figure 2 shows the shifted Maxwellian velocity distribution in the $v_x - v_y$ space. The perpendicular velocity coordinates are defined as $v'_\perp \equiv \sqrt{(v_x - u_{bs})^2 + v_y^2}$. Note that the velocity component parallel to the ambient magnetic field $v_{||}$ is defined from $-\infty$ to ∞, while the perpendicular velocity $v'_\perp$ is defined from 0 to ∞. It is also noted that the perpendicular component of the velocity distribution function in the present study is not the Maxwellian "ring" velocity distribution.[22,23]

We perform the velocity space integral of **S** as

$$\mathbf{K} \equiv \int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp \mathbf{S} dv_{||} dv'_\perp}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}, \tag{29}$$

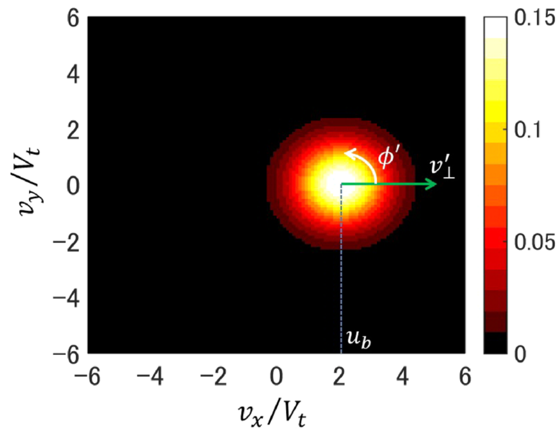

FIG. 2. The shifted Maxwellian velocity distribution. The drift velocity is set as $u_b = 2V_t$. The velocity coordinates perpendicular to the ambient magnetic field are defined as $v'_\perp \equiv \sqrt{(v_x - u_b)^2 + v_y^2}$.

where the velocity space integral should not be performed over $v_\perp$ but should be over $v'_\perp$ because of the definition of the phase angle in the velocity space in Eq. (14). For convenience, we use the properties of the integrals of the shifted Maxwellian

$$\int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}} dv_{||} = -\frac{1}{2V_{ts||}^2}Z'_0[\zeta_n] \equiv -\boldsymbol{\mathcal{X}}_1$$

$$\int_{-\infty}^\infty \frac{n\Omega_{cs}f_{s||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} dv_{||} = -\frac{n\Omega_{cs}}{\sqrt{2}V_{ts||}k_{||}}Z_0[\zeta_n] \equiv -\boldsymbol{\mathcal{X}}_2$$

$$\int_{-\infty}^\infty \frac{(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)f_{s||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} dv_{||} = 1 - \frac{n\Omega_{cs}}{\sqrt{2}V_{ts||}k_{||}}Z_0[\zeta_n] = 1 - \boldsymbol{\mathcal{X}}_2$$

$$\int_{-\infty}^\infty \frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}} dv_{||} = -\frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{2V_{ts||}^2k_{||}}Z'_0[\zeta_n] \equiv -\boldsymbol{\mathcal{X}}_3$$

$$\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}f_{s||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} dv_{||} = -\frac{n\Omega_{cs}}{k_{||}}\left(1 + \frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{\sqrt{2}V_{ts||}k_{||}}Z_0[\zeta_n]\right) \equiv -\boldsymbol{\mathcal{X}}_4$$

$$\int_{-\infty}^\infty \frac{v_{||}k_{||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}} dv_{||} = -\frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{2V_{ts||}^2k_{||}}Z'_0[\zeta_n] = -\boldsymbol{\mathcal{X}}_3$$

$$\int_{-\infty}^\infty \frac{(\tilde\omega - v_{||}k_{||} - u_{bs}k_\perp)v_{||}f_{s||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} dv_{||} = u_{ds} - \frac{n\Omega_{cs}}{k_{||}}\left(1 + \frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{\sqrt{2}V_{ts||}k_{||}}Z_0[\zeta_n]\right) = u_{ds} - \boldsymbol{\mathcal{X}}_4$$

$$\int_{-\infty}^\infty \frac{(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)v_{||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}} dv_{||} = -\frac{(\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp)^2}{2V_{ts||}^2k_{||}^2}Z'_0[\zeta_n] \equiv -\boldsymbol{\mathcal{X}}_5$$

$$\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}^2f_{s||}}{\tilde\omega - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} dv_{||} = -\frac{n\Omega_{cs}}{k_{||}}\left\{u_{ds} + \frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{k_{||}}\left(1 + \frac{\tilde\omega - n\Omega_{cs} - u_{bs}k_\perp}{\sqrt{2}V_{ts||}k_{||}}Z_0[\zeta_n]\right)\right\} \equiv -\boldsymbol{\mathcal{X}}_6$$

and

$$\int_0^\infty 2\pi v'_\perp J_n^2[\lambda] f_\perp \mathrm{d}v'_\perp = I_n\left[\frac{a^2}{2}\right]\exp\left[-\frac{a^2}{2}\right] \equiv \mathcal{A}_n$$

$$\int_0^\infty 2\pi J_n^2[\lambda]\frac{\partial f_\perp}{\partial v'_\perp}\mathrm{d}v'_\perp = -\frac{1}{V_{t\perp}^2} I_n\left[\frac{a^2}{2}\right]\exp\left[-\frac{a^2}{2}\right] \equiv -\frac{1}{V_{t\perp}^2}\mathcal{A}_n$$

$$\int_0^\infty 2\pi v'^2_\perp J_n[\lambda] J'_n[\lambda] f_\perp \mathrm{d}v'_\perp = \frac{V_{t\perp}^2 k_\perp}{\Omega_c}\left(I'_n\left[\frac{a^2}{2}\right] - I_n\left[\frac{a^2}{2}\right]\right)\exp\left[-\frac{a^2}{2}\right] \equiv \frac{V_{t\perp}^2 k_\perp}{\Omega_c}\mathcal{B}_n$$

$$\int_0^\infty 2\pi v'_\perp J_n[\lambda] J'_n[\lambda]\frac{\partial f_\perp}{\partial v'_\perp}\mathrm{d}v'_\perp = -\frac{k_\perp}{\Omega_c}\left(I'_n\left[\frac{a^2}{2}\right] - I_n\left[\frac{a^2}{2}\right]\right)\exp\left[-\frac{a^2}{2}\right] \equiv -\frac{k_\perp}{\Omega_c}\mathcal{B}_n$$

$$\int_0^\infty 2\pi v'^3_\perp J'^2_n[\lambda] f_\perp \mathrm{d}v'_\perp = V_{t\perp}^2\left(\frac{2n^2}{a^2} I_n\left[\frac{a^2}{2}\right] - a^2 I'_n\left[\frac{a^2}{2}\right] + a^2 I_n\left[\frac{a^2}{2}\right]\right)\exp\left[-\frac{a^2}{2}\right] \equiv V_{t\perp}^2\left(\frac{2n^2}{a^2}\mathcal{A}_n - a^2\mathcal{B}_n\right)$$

$$\int_0^\infty 2\pi v'^2_\perp J'^2_n[\lambda]\frac{\partial f_\perp}{\partial v'_\perp}\mathrm{d}v'_\perp = -\left(\frac{2n^2}{a^2} I_n\left[\frac{a^2}{2}\right] - a^2 I'_n\left[\frac{a^2}{2}\right] + a^2 I_n\left[\frac{a^2}{2}\right]\right)\exp\left[-\frac{a^2}{2}\right] \equiv -\frac{2n^2}{a^2}\mathcal{A}_n + a^2\mathcal{B}_n,$$

where

$$\zeta_n = \frac{\tilde{\omega} - u_{ds}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}{\sqrt{2}k_{||}V_{ts||}}, \quad a = \frac{\sqrt{2}V_{ts\perp}k_\perp}{\Omega_{cs}},$$

where $I_n[x]$ and $Z_p[x]$ are the modified Bessel function of the first kind of order n and the plasma dispersion function,[24] respectively,

$$Z_0(x) \equiv \frac{1}{\sqrt{\pi}}\int_{-\infty}^\infty \frac{1}{v-x}\exp\left[-v^2\right]\mathrm{d}v,$$

$$Z_p(\zeta_n) \equiv -\frac{k_{||}}{\sqrt{\pi}}\int_{-\infty}^\infty \frac{v_{||}^p}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\exp\left[-\frac{(v_{||}-u_{ds})^2}{2V_{ts||}^2}\right]\mathrm{d}v_{||}.$$

Then, we obtain

$$\begin{aligned}
K_{xx} &= \int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{xx}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = \frac{n^2\Omega_{cs}^2}{k_\perp^2}\int_{-\infty}^\infty \frac{k_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n^2[\lambda] f_{s\perp}\mathrm{d}v'_\perp \\
&+ \frac{n^2\Omega_{cs}^2}{k_\perp^2}\int_{-\infty}^\infty \frac{(\tilde{\omega} - v_{||}k_{||} - u_{bs}k_\perp \cos)f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J_n^2[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp \\
&+ \frac{n\Omega_{cs}u_{bs}}{k_\perp}\int_{-\infty}^\infty \frac{k_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n^2[\lambda] f_{s\perp}\mathrm{d}v'_\perp \\
&+ \frac{n\Omega_{cs}u_{bs}}{k_\perp}\int_{-\infty}^\infty \frac{(\tilde{\omega} - v_{||}k_{||} - u_{bs}k_\perp \cos)f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J_n^2[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp \\
&+ \frac{n\Omega_{cs}u_{bs}}{k_\perp}\int_{-\infty}^\infty \frac{k_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n^2[\lambda] f_{s\perp}\mathrm{d}v'_\perp \\
&+ \frac{n\Omega_{cs}u_{bs}}{k_\perp}\int_{-\infty}^\infty \frac{(\tilde{\omega} - v_{||}k_{||} - u_{bs}k_\perp \cos)f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J_n^2[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp \\
&+ u_{bs}^2\int_{-\infty}^\infty \frac{k_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n^2[\lambda] f_{s\perp}\mathrm{d}v'_\perp \\
&+ u_{bs}^2\int_{-\infty}^\infty \frac{n\Omega_{cs}\partial f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J_n^2[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp \\
&= -\frac{n^2\Omega_{cs}^2}{k_\perp^2}\mathcal{X}_1\mathcal{A}_n - \frac{n^2\Omega_{cs}^2}{V_{ts\perp}^2 k_\perp^2}(1-\mathcal{X}_2)\mathcal{A}_n - \frac{n\Omega_{cs}u_{bs}}{k_\perp}\mathcal{X}_1\mathcal{A}_n - \frac{n\Omega_{cs}u_{bs}}{V_{ts\perp}^2 k_\perp}(1-\mathcal{X}_2)\mathcal{A}_n \\
&\quad - \frac{n\Omega_{cs}u_{bs}}{k_\perp}\mathcal{X}_1\mathcal{A}_n + \frac{n\Omega_{cs}u_{bs}}{V_{ts\perp}^2 k_\perp}\mathcal{X}_2\mathcal{A}_n - u_{bs}^2\mathcal{X}_1\mathcal{A}_n + u_{bs}^2\frac{\mathcal{X}_2}{V_{ts\perp}^2}\mathcal{A}_n \\
&= -(V_{ts\perp}^2\mathcal{X}_1 - \mathcal{X}_2)\left(n^2\frac{2}{a^2} + n\frac{4u_{bs}k_\perp}{a^2\Omega_{cs}} + \frac{u_{bs}^2}{V_{ts\perp}^2}\right)\mathcal{A}_n - n^2\frac{2}{a^2}\mathcal{A}_n + n\frac{2u_{bs}k_\perp}{a^2\Omega_{cs}}\mathcal{A}_n,
\end{aligned} \tag{30}$$

$$K_{xy}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{xy}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = i\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$+i\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{(\tilde\omega-v_{||}k_{||}-u_{bs}k_\perp)f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp + iu_{bs}\int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}$$
$$\times\int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp + iu_{bs}\int_{-\infty}^\infty \frac{(\tilde\omega-v_{||}k_{||}-u_{bs}k_\perp)f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp = -i\frac{n\Omega_{cs}}{k_\perp}\frac{V^2_{ts\perp}k_\perp}{\Omega_{cs}}\mathcal{X}_1\mathcal{B}_n$$
$$-i\frac{n\Omega_{cs}}{k_\perp}\frac{k_\perp}{\Omega_{cs}}(1-\mathcal{X}_2)\mathcal{B}_n - iu_{bs}\frac{V^2_{ts\perp}k_\perp}{\Omega_{cs}}\mathcal{X}_1\mathcal{B}_n - iu_{bs}\frac{k_\perp}{\Omega_{cs}}(1-\mathcal{X}_2)\mathcal{B}_n = -i\left(n+\frac{u_{bs}k_\perp}{\Omega_{cs}}\right)\left(V^2_{ts\perp}\mathcal{X}_1-\mathcal{X}_2\right)\mathcal{B}_n - in\mathcal{B}_n - i\frac{u_{bs}k_\perp}{\Omega_{cs}}\mathcal{B}_n, \quad (31)$$

$$K_{xz}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{xz}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = \frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{\tilde\omega-n\Omega_{cs}-u_{bs}k_\perp}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J^2_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$+\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J^2_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp + u_{bs}\int_{-\infty}^\infty \frac{\tilde\omega-n\Omega_{cs}-u_{bs}k_\perp}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}$$
$$\times\int_0^\infty 2\pi v'_\perp J^2_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp + u_{bs}\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J^2_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp$$
$$= -\frac{n\Omega_{cs}}{k_\perp}\mathcal{X}_3\mathcal{A}_n + \frac{n\Omega_{cs}}{k_\perp}\frac{\mathcal{X}_4}{V^2_{ts\perp}}\mathcal{A}_n - u_{bs}\mathcal{X}_3\mathcal{A}_n + u_{bs}\frac{\mathcal{X}_4}{V^2_{ts\perp}}\mathcal{A}_n = -\left(n\frac{2k_\perp}{a^2\Omega_{cs}}+\frac{u_{bs}}{V^2_{ts\perp}}\right)\left(V^2_{ts\perp}\mathcal{X}_3-\mathcal{X}_4\right)\mathcal{A}_n, \quad (32)$$

$$K_{yx}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{yx}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = i\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$-i\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{(\tilde\omega-v_{||}k_{||}-u_{bs}k_\perp)f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp - iu_{bs}\int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}$$
$$\times\int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp - iu_{bs}\int_{-\infty}^\infty \frac{n\Omega_{cs}f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda]\frac{\partial f_{s\perp}}{\partial v_{||}}\mathrm{d}v'_\perp$$
$$= i\frac{n\Omega_{cs}}{k_\perp}\frac{V^2_{ts\perp}k_\perp}{\Omega_{cs}}\mathcal{X}_1\mathcal{B}_n + i\frac{n\Omega_{cs}}{k_\perp}\frac{k_\perp}{\Omega_{cs}}(1-\mathcal{X}_2)\mathcal{B}_n + i\frac{u_{bs}k_\perp}{\Omega_{cs}}V^2_{ts\perp}\mathcal{X}_1\mathcal{B}_n - i\frac{u_{bs}k_\perp}{\Omega_{cs}}\mathcal{X}_2\mathcal{B}_n = -K_{xy}, \quad (33)$$

$$K_{yy}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{yy}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = \int_{-\infty}^\infty \frac{k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'^3_\perp J'^2_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$+\int_{-\infty}^\infty \frac{(\tilde\omega-v_{||}k_{||}-u_{bs}k_\perp)f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'^2_\perp J'^2_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp = -V^2_{ts\perp}\mathcal{X}_1\left(\frac{2n^2}{a^2}\mathcal{A}_n-a^2\mathcal{B}_n\right) - (1-\mathcal{X}_2)\left(\frac{2n^2}{a^2}\mathcal{A}_n-a^2\mathcal{B}_n\right)$$
$$= -\left(V^2_{ts\perp}\mathcal{X}_1-\mathcal{X}_2\right)\left(n^2\frac{2}{a^2}\mathcal{A}_n-a^2\mathcal{B}_n\right) - n^2\frac{2}{a^2}\mathcal{A}_n + a^2\mathcal{B}_n, \quad (34)$$

$$K_{yz}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{yz}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = -i\int_{-\infty}^\infty \frac{\tilde\omega-n\Omega_{cs}-u_{bs}k_\perp}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$-i\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp = i\frac{V^2_{ts\perp}k_\perp}{\Omega_{cs}}\mathcal{X}_3\mathcal{B}_n - i\frac{k_\perp}{\Omega_{cs}}\mathcal{X}_4\mathcal{B}_n = i\frac{k_\perp}{\Omega_{cs}}\left(V^2_{ts\perp}\mathcal{X}_3-\mathcal{X}_4\right)\mathcal{B}_n, \quad (35)$$

$$K_{zx}=\int_0^\infty\int_{-\infty}^\infty \frac{2\pi v'_\perp S_{zx}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\mathrm{d}v'_\perp = \frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{v_{||}k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}\int_0^\infty 2\pi v'_\perp J^2_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp$$
$$+\frac{n\Omega_{cs}}{k_\perp}\int_{-\infty}^\infty \frac{v_{||}(\tilde\omega-v_{||}k_{||}-u_{bs}k_\perp)f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J^2_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp + u_{bs}\int_{-\infty}^\infty \frac{v_{||}k_{||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\frac{\partial f_{s||}}{\partial v_{||}}\mathrm{d}v_{||}$$
$$\times\int_0^\infty 2\pi v'_\perp J^2_n[\lambda]f_{s\perp}\mathrm{d}v'_\perp + u_{bs}\int_{-\infty}^\infty \frac{n\Omega_{cs}v_{||}f_{s||}}{\tilde\omega-v_{||}k_{||}-n\Omega_{cs}-u_{bs}k_\perp}\mathrm{d}v_{||}\int_0^\infty 2\pi J^2_n[\lambda]\frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp$$
$$= -\frac{n\Omega_{cs}}{k_\perp}\mathcal{X}_3\mathcal{A}_n - \frac{n\Omega_{cs}}{V^2_{ts\perp}k_\perp}(u_{ds}-\mathcal{X}_4)\mathcal{A}_n - u_{bs}\mathcal{X}_3\mathcal{A}_n + \frac{u_{bs}}{V^2_{ts\perp}}\mathcal{X}_4\mathcal{A}_n = K_{xz} - n\frac{u_{ds}\Omega_{cs}}{V^2_{ts\perp}k_\perp}\mathcal{A}_n, \quad (36)$$

$$K_{zy} = \int_0^\infty \int_{-\infty}^\infty \frac{2\pi v'_\perp S_{zy}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \mathrm{d}v_{||}\mathrm{d}v'_\perp = i\int_{-\infty}^\infty \frac{v_{||}k_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \frac{\partial f_{s||}}{\partial v_{||}} \mathrm{d}v_{||} \int_0^\infty 2\pi v'^2_\perp J_n[\lambda]J'_n[\lambda] f_{s\perp}\mathrm{d}v'_\perp$$
$$+i\int_{-\infty}^\infty \frac{v_{||}(\tilde{\omega} - v_{||}k_{||} - u_{bs}k_\perp) f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \mathrm{d}v_{||} \int_0^\infty 2\pi v'_\perp J_n[\lambda]J'_n[\lambda] \frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp = -i\frac{V^2_{ts\perp}k_\perp}{\Omega_{cs}}\mathcal{B}_n\mathcal{X}_3 - i\frac{k_\perp}{\Omega_{cs}}\mathcal{B}_n(u_{ds} - \mathcal{X}_4)$$
$$= -K_{yz} - i\frac{u_{ds}k_\perp}{\Omega_{cs}}\mathcal{B}_n, \tag{37}$$

$$K_{zz} = \int_0^\infty \int_{-\infty}^\infty \frac{2\pi v'_\perp S_{zz}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \mathrm{d}v_{||}\mathrm{d}v'_\perp = \int_{-\infty}^\infty \frac{(\tilde{\omega} - n\Omega_{cs} - u_{bs}k_\perp)v_{||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \frac{\partial f_{s||}}{\partial v_{||}} \mathrm{d}v_{||} \int_0^\infty 2\pi v'_\perp J^2_n[\lambda] f_{s\perp}\mathrm{d}v'_\perp$$
$$+\int_{-\infty}^\infty \frac{n\Omega_{cs}v^2_{||}f_{s||}}{\tilde{\omega} - v_{||}k_{||} - n\Omega_{cs} - u_{bs}k_\perp} \mathrm{d}v_{||} \int_0^\infty 2\pi J^2_n[\lambda] \frac{\partial f_{s\perp}}{\partial v'_\perp}\mathrm{d}v'_\perp = -\left(\mathcal{X}_5 - \frac{\mathcal{X}_6}{V^2_{ts\perp}}\right)\mathcal{A}_n. \tag{38}$$

Note that $\sum_n n\mathcal{A}_n = \sum \mathcal{B}_n = 0$ in Eqs. (30), (31), (34), (36), and (37) due to the identity of the modified Bessel function. It is also easy to extend the present dielectric permittivity tensor to Maxwellian ring velocity distributions by replacing the integrals over the velocity space perpendicular to the ambient magnetic field ($\mathcal{A}_n$ and $\mathcal{B}_n$) with the numerical ones shown in Ref. 23.

IV. NUMERICAL TESTS

A. Electron cyclotron drift and modified two-stream instabilities

As an unstable plasma, we assume that there are two ion and one electron components drifting across magnetic fields with different drift velocities. Such a situation is often observed in the transition region of perpendicular collisionless shocks where a part of upstream ions are reflected at the shock front.[13] The following physical parameters of the three plasma components for the present linear analysis were obtained in the shock foot region of a perpendicular collisionless shock in the previous particle-in-cell simulation.[14] There were incoming ions (with subscript "i"), reflected ions (with subscript "r"), and electrons (with subscript "e") in the shock foot region. In order to make a direct comparison with the previous study,[14] we normalize the velocity and angular frequency by the upstream thermal velocity and plasma frequency of electrons, V_{te1} and ω_{pe1}, respectively. The drift velocity, the thermal velocity, and the angular frequency of electrons are $u_{be}/V_{te1} = 0.32$, $V_{te}/V_{te1} = 1.75$, and $\omega_{pe}/\omega_{pe1} = 1.76$, respectively. The drift velocity, the thermal velocity, and the angular frequency of incoming ions are $u_{bi}/V_{te1} = 2.12$, $V_{ti}/V_{te1} = 0.42$, and $\omega_{pi}/\omega_{pe1} = 0.26$, respectively. The drift velocity, the thermal velocity, and the angular frequency of reflected ions are $u_{br}/V_{te1} = -1.85$, $V_{tr}/V_{te1} = 0.32$, and $\omega_{pr}/\omega_{pe1} = 0.24$, respectively. The upstream electron cyclotron frequency was $\Omega_{ce1}/\omega_{pe1} = 0.25$, but the local electron cyclotron frequency in the shock foot region was $\Omega_{ce}/\omega_{pe1} = 0.92$ by the compression of the upstream plasma. A reduced ion-to-electron mass ratio $m_i/m_e = 25$ was used in the previous study.[14] The value of the upstream thermal velocity of electrons was assumed to be $V_{te1} = 0.1c$, where c represents the speed of light. We use the secant method as a complex-root finder to solve the dispersion Eq. (25) numerically.[23]

Figure 3 shows the numerical solutions to the linear dispersion Eq. (25) for a three-component plasma in the foot region of a perpendicular collisionless shock. It is again noted that the linear dispersion relation was solved with the coordinate transformation from the shock-rest frame to the electron rest frame and that the ions were assumed to be

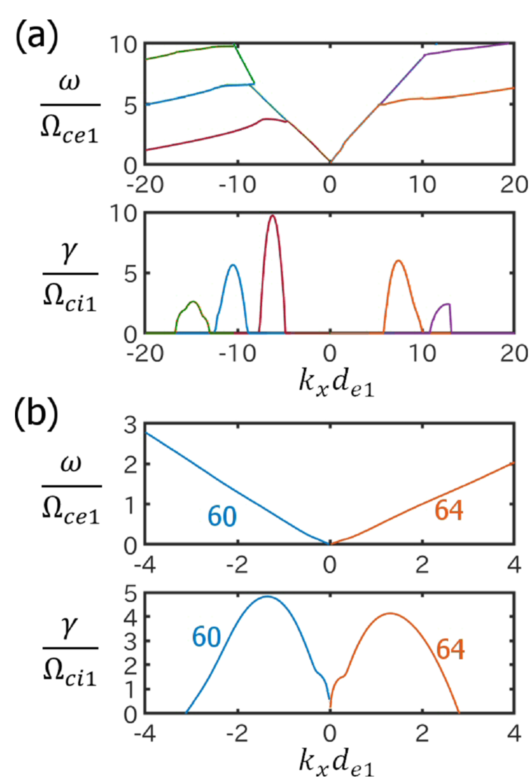

FIG. 3. Linear dispersion relations for a three-component plasma in the foot region of a perpendicular collisionless shock.[14] The x axis is parallel to the direction of the particle drifts and is perpendicular to the ambient magnetic field. (a) Higher frequency range ($\omega > \Omega_{ce}$), which corresponds to the electron cyclotron drift instability (ECDI). The angle of the wave vector relative to the ambient magnetic field is 90°. (b) Low frequency range ($\omega < \Omega_{ce}$), which corresponds to the modified two-stream instability (MTSI). The angle of the wave vector relative to the ambient magnetic field is 64° for the forward-propagating wave and 60° for the backward-propagating wave.

unmagnetized in order to apply the standard linear dispersion solvers in the previous study.[14]

Panel (a) shows the result for a higher frequency range ($\omega > \Omega_{ce}$), which corresponds to the electron cyclotron drift instability (ECDI). The angle of the wave vector relative to the ambient magnetic field is 90°. We compare panel (a) with Fig. 6(d) in Ref. 14 and find that these results are in excellent agreement.

Panel (b) in Fig. 3 shows the result for a low frequency range ($\omega < \Omega_{ce}$), which corresponds to the modified two-stream instability (MTSI). The angle of the wave vector relative to the ambient magnetic field is 64° for the forward-propagating wave and 60° for the backward-propagating wave. We also compare panel (b) with Fig. 5(d) in Ref. 14 and find that these results are in excellent agreement.

B. Lower-hybrid drift instability

A linear analysis of the LHDI was conducted by Davidson *et al.*[17] In their study, ion and electron components drifting across magnetic fields with different drift velocities were assumed as an unstable plasma. In one of their numerical analysis, parameters were set as $m_i/m_e = 1836$, $T_i/T_e = 1$, $\omega_{pe}/\omega_{ce} = \sqrt{125}$, $u_{bi} = -\sqrt{2}V_{ti}$, and $u_{be} = \sqrt{2}V_{ti}$ and they found a positive growth rate at a frequency equal to several ω_{LHR}. We solved the linear dispersion Eq. (25) numerically with these parameters, but did not find any solution with a positive growth rate. We also performed a two-dimensional full particle-in-cell simulation with these parameters in a uniform and doubly periodic system and confirmed that no waves are excited. As described in Introduction, Davidson *et al.* assumed unmagnetized ions and neglected harmonic cyclotron resonance of electrons by setting $n = 0$ on the order of the Bessel function.[17] The present study suggests that the unmagnetized ions enhanced the effect of the ion Landau resonance artificially.

Next, let us consider a simple Harris current-sheet equilibrium without a background component

$$B_z[y] = B_0 \tanh\left[\frac{y}{L}\right], \tag{39}$$

$$N[y] = N_0\,\mathrm{sech}^2\left[\frac{y}{L}\right]. \tag{40}$$

The parameters are assumed to be $m_i/m_e = 512$, $T_i/T_e = 1$, $\omega_{pe0}/\Omega_{ce0} = 5$, and $r_i/L = 2$ which are identical to those of the previous study,[18] where $r_i \equiv \sqrt{2}V_{ti}/\Omega_{ci0}$ is the ion thermal gyro radius. The speed of light and the half-thickness of the current layer are obtained as $c/V_{te} = 10$ and $L/\lambda_{De} = 80$, where $\lambda_{De} \equiv V_{te}/\omega_{pe}$ is the Debye length. Daughton found a positive growth rate of the LHDI in his linear analysis at $y/L = 0.5$–2 and confirmed it by a two-dimensional particle-in-cell simulation of the current sheet.[18]

It should be noted that the present study cannot be compared with the previous study[18] directly, since the present study uses a coordinate system in the $B_0 - u_b$ plane (i.e., $z - x$ plane with $k_z = k_{||}, k_x = k_\perp, k_y = 0$ and $\theta = 0$), while the previous study[18] used a coordinate system in the $u_b - (u_b \times B_0)$ plane (i.e., $x - y$ plane with $k_z = 0$, $k_x = k_\perp \cos\theta, k_y = k_\perp \sin\theta$). In the previous study,[18] it is difficult to distinguish between fluid instabilities due to the spatial inhomogeneity and velocity-space instabilities due to $\partial f/\partial v_{||}$ or $\partial f/\partial v_\perp$, since the previous study considered the entire current sheet and both instabilities can be generated. On the other hand, the present study uses a local model where the background physical quantities are assumed to be uniform. Hence, the present study deals only with the velocity-space instabilities at a local position in the Harris sheet equilibrium.

The local drift velocity, angular plasma, and cyclotron frequencies of electrons at $y/L = 0.5$ are $u_{be}/V_{te} = 0.125$, $\omega_{pe}/\omega_{pe0} = 0.8868$, and $\Omega_{ce}/\omega_{pe0} = 0.09242$, respectively, and the local drift velocity, angular plasma, and cyclotron frequencies of ions at $y/L = 0.5$ are $u_{bi}/V_{te} = -0.125$, $\omega_{pi}/\omega_{pe0} = 0.03919$, and $\Omega_{ci}/\omega_{pe0} = 1.8051 \times 10^{-4}$, respectively. Figure 4 shows the numerical solutions to the linear dispersion Eq. (25) for a two-component plasma at the Harris current sheet. Panel (a) shows the result for wave modes with wave normal angles quasi-parallel to the ambient magnetic field. The quasi-parallel modes have the maximum growth rate at a frequency $\omega < \omega_{LHR}$ and a wavenumber $k_{||}d_i \sim 2$. The growth rate of the exactly parallel mode is maximum, but the growth rate of quasi-parallel modes is almost the same as that of the exactly parallel mode. It is suggested that these quasi-parallel modes are excited by an energy anisotropy between the parallel and perpendicular kinetic energies of particles. We do not find any solution to these modes for wave normal angles of >7° relative to the ambient magnetic field.

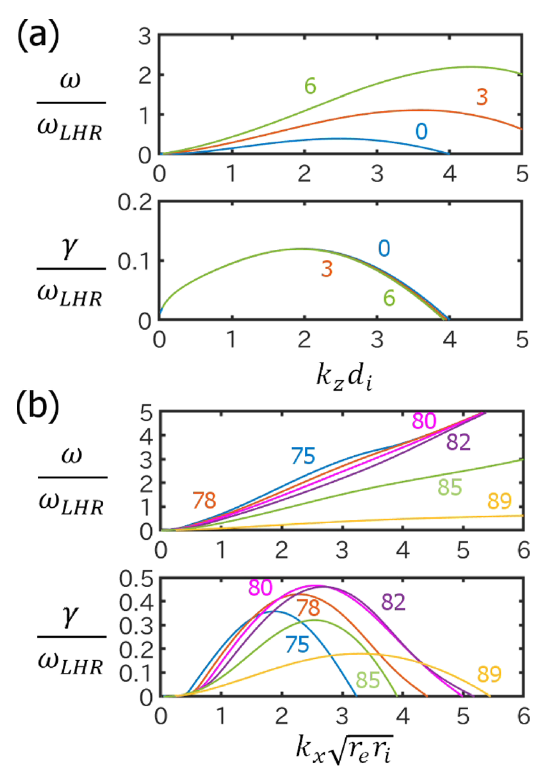

FIG. 4. Linear dispersion relations for a two-component plasma at the Harris current sheet.[18] The z axis is parallel to the ambient magnetic field. The x axis is perpendicular to the ambient magnetic field and is parallel to the direction of the particle drifts. (a) Wave modes with wave normal angles quasi-parallel to the ambient magnetic field. (b) Wave modes with wave normal angles quasi-perpendicular to the ambient magnetic field.

Panel (b) shows the result for wave modes with wave normal angles quasi-perpendicular to the ambient magnetic field. The perpendicular mode has no positive growth rate. A wave mode with a wave normal angle of $\sim 80°$ relative to the ambient magnetic field has the maximum growth rate. These modes have a large growth rate at a frequency $\omega \sim \omega_{LHR}$ and a wavenumber $k_\perp\sqrt{r_e r_i} \sim 2$, whose characteristics are similar to those of the LHDI at the intermediate scale but not the same. In the present study, unstable modes exist neither $k_\perp\sqrt{r_e r_i} \sim 1$ nor $k_\perp r_e \sim 1$. The cyclotron resonance condition of ions $\omega_{LHR} \sim n\Omega_{ci} + k_\perp u_{bi}$ is easily satisfied at $k_\perp\sqrt{r_e r_i} \sim 2$ (here, $\omega_{LHR} \sim 22.5\ \Omega_{ci}$). We found that the electron cyclotron resonance at $n = 1$ is dominant, while a wide range of the ion cyclotron resonance at $n = 1$–23 contributes to the positive growth rate of the LHDI. There is no positive growth rate when the cyclotron resonance is neglected (i.e., $n = 0$ only). It is also found that the short wavelength that mode at $k_\perp r_e \sim 1$ hardly satisfies the cyclotron resonance condition with the present parameters.

We also performed a large-scale two-dimensional full particle-in-cell simulation with these parameters in a uniform and doubly periodic system and confirmed that the unstable wave modes shown in Fig. 4 were excited.

V. CONCLUSION

A theoretical linear dispersion relation for plasma with a drift across the magnetic field for a local model was derived. Then, the dielectric permittivity tensor for shifted Maxwellian velocity distributions is obtained by using the plasma dispersion function and the modified Bessel function. The linear dispersion relations of ECDI and MTSI obtained by using the new dielectric permittivity tensor are in excellent agreement with the previous study.[14] The linear dispersion relations of the LHDI with an in-plane ambient magnetic field show the importance of the cyclotron resonance of both ions and electrons at $\omega \sim \omega_{LHR}$. The extension of the present study to $\theta \neq 0$ cases (with out-of-plane ambient magnetic field) is left as a future study.

The obtained dielectric permittivity tensor can be easily implemented to numerical linear dispersion solvers. Given the successful tests of our solvers shown in this paper, the present method would be applicable to various other targets in collisionless plasma with a drift across magnetic field.

ACKNOWLEDGMENTS

This work was supported by MEXT/JSPS under Grant-in-Aid (KAKENHI) Nos. JP26287041 and JP15K13572. Computer simulations were performed on the CIDAS supercomputer system at the Institute for Space-Earth Environmental Research in Nagoya University as a Joint Research Program.

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