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Public Capital, Economic Growth, and Welfare in an Endogenous Growth Model with the Weakest-Link Externality

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This paper develops an endogenous growth model with private and public capital accumulation under the weakest-link externality. In the model, labor productivity is subject to the weakest-link externality, composed of the Marshall-Arrow-Romer externality and public capital as pure public goods. Emphasizing discussion of the dynamic equilibrium under the Marshall-Arrow-Romer externality, this paper shows that the growth-maximizing tax rate differs from the output elasticity of public capital. Furthermore, the growth-maximizing tax rate is equivalent to the welfare-maximizing tax rate if the dynamic equilibrium is subject to the Marshall-Arrow-Romer externality.

Keywords: Public capital, Weakest-link externality, Economic growth

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Ⅰ. Introduction

Growth and welfare effects of public capital accumulation have been investigated since the pioneering theoretical study of Arrow and Kurz (1970) and the empirical study of Aschauer (1989). Many empirical studies have found that public capital is positively associated with economic growth.[1] Results of these studies imply that public capital accelerates economic growth through an increase in the marginal productivity of private capital.

On the theoretical side, Barro (1990) developed an endogenous growth model with productive public expenditure.[2] He showed that the growth maximizing tax rate on income is equal to the output elasticity of public goods (i.e., Barro tax rule), and that the growth-maximizing tax rate on income is equivalent to the welfare-maximizing tax rate on income if the output elasticity of public goods is constant.

Futagami et al. (1993) extended the Barro model by incorporating public capital instead of productive public expenditure (hereinafter, we refer to their study as FMS).[3] FMS showed that the Barro tax rule holds, but the growth-maximizing tax rate is not equivalent to the welfare-maximizing tax rate because the presence of transitional dynamics engenders the negative welfare effect of income tax through a decrease in initial consumption.

The externality of public goods is a key to maintaining sustainable growth in the models because its externality keeps the marginal productivity of private capital at a high level in response to the accumulation of private capital. Accumulation of private capital also has an externality effect, such as Marshallian externality. Marshallian externality, such as learning by doing and knowledge spillover, is also important for economic growth because it generates a situation in which the marginal productivity of private capital is not decreasing in private capital (e.g., Arrow 1962; Romer 1986).

These two externalities are both fundamentally important to accelerate economic growth. The shortage of infrastructure is the greatest bottleneck to economic development in least developed countries (LDCs). For example, the World Bank (2010) has estimated that sub-Saharan countries would need to invest US$93 billion per year in infrastructure. However, in economically developed countries, infrastructure is sufficient to maintain the industrial foundation. The main determinant of economic growth is technological progress. Knowledge spillover plays a key role in accelerating economic growth.

The weakest-link relation between knowledge spillover and public capital is reasonable to formulate realistic circumstances for economic development. This paper presents development of an endogenous growth model with private and public capital accumulation under the weakest-link externality, which comprises knowledge spillover (Marshall-Arrow-Romer externality; MAR) and public capital (FMS).

Our analysis shows that the Barro tax rule does not hold in the economy under MAR restriction, but the growth-maximizing policy is equivalent to welfare maximizing policy.[4] This result implies that the policy maker concentrates effort on a policy that cultivates economic growth because maximization of economic growth coincides with welfare maximization. This paper presents insights into public investment policy and its effects.

The remainder of this paper is organized as follows. The next section explains the basic setup of our model and provides preliminary results of dynamic properties of the model. Section 3 presents an investigation of the growth and welfare effects of public investment. Section 4 provides additional analysis, incorporating consumption tax financing. Finally, Section 5 concludes this paper.

Ⅱ. The model

We consider a closed economy with identical households and firms. The representative household lives infinitely and supplies labor inelastically. The household's budget constraint is

$$\dot{K}_t = (1-\tau)(r_t K_t + w_t) - C_t, \tag{1}$$

where K_t represents the private capital, r_t signifies the interest rate, w_t denotes the labor income, τ is the income tax rate, and C_t denotes the private consumption. The dot above letter signifies the time derivative of economic variables.

Households maximize their lifetime utility as

$$U_0 = \int_0^{\infty} \frac{C_t^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

subject to equation (1). In the utility function, θ represents the inverse of elasticity of intertemporal substitution and ρ denotes the subjective discount rate. Solving the household's optimization problem, we obtain

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\tau) r_t - \rho}{\theta}, \tag{2}$$

and the transversality condition.

The final good is produced using private capital and labor. The production function is

$$Y_t = \xi K_t^{1-\eta} A_t^{\eta}, \tag{3}$$

where

$$A_t = \min[\chi K_t, G_t]. \tag{4}$$

In equations (3) and (4), Y_t denotes the output of final good, A_t represents the efficient labor (labor-augmenting technological progress), G_t signifies the public capital, ξ, χ, and η are positive constants ($\xi, \chi > 0, 0 < \eta < 1$). Equation (4) presents formulation of the weakest-link relation between knowledge spillover and public capital described in the *Introduction*.[5] For each firm, A_t is as given. Profit maximization provides

$$r_t = (1-\eta)\frac{Y_t}{K_t} = (1-\eta)\xi \min[\chi^{\eta}, x_t^{\eta}], \tag{5}$$

$$w_t = \eta Y_t = \eta \xi K_t \min[\chi^{\eta}, x_t^{\eta}], \tag{6}$$

where $x_t \equiv G_t/K_t$ stands for the ratio of public capital to private capital.

The government taxes income and uses its tax revenue to invest in the public capital. Then, the government's budget constraint is

$$\dot{G}_t = \tau Y_t. \tag{7}$$

The dynamic equilibrium of this economy satisfies equations (1)-(7). Using equations (1)-(7), we obtain the following equations of

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\tau)(1-\eta)\xi \min[\chi^{\eta}, x_t^{\eta}] - \rho}{\theta}, \tag{8a}$$

$$\frac{\dot{K}_t}{K_t} = (1-\tau)\xi \min[\chi^{\eta}, x_t^{\eta}] - \frac{C_t}{K_t}, \tag{8b}$$

$$\frac{\dot{G}_t}{G_t} = \frac{\tau \xi \min[\chi^{\eta}, x_t^{\eta}]}{x_t}, \tag{8c}$$

where $z_t \equiv C_t/K_t$ denotes the ratio of private consumption to private capital. Equations (8a)-(8c) are reduced to two dynamic equations as

$$\dot{z}_t = \left[\frac{(1-\tau)(1-\eta-\theta)\xi \min\{\chi^{\eta}, x_t^{\eta}\} - \rho}{\theta} + z_t\right] z_t, \tag{9a}$$

$$\dot{x}_t = \left[\frac{\tau \xi \min\{\chi^{\eta}, x_t^{\eta}\}}{x_t} - (1-\tau)\xi \min\{\chi^{\eta}, x_t^{\eta}\} + z_t\right] x_t, \tag{9b}$$

We define a balanced growth equilibrium (BGE) as a stationary equilibrium that satisfies $\dot{z} = \dot{x} = 0$. Using the condition for balanced growth equilibrium and equations (9a) and (9b), we have

$$\dot{z} = 0 : z = \frac{\rho + (1-\tau)(\theta+\eta-1)\xi \min\{\chi^{\eta}, x^{\eta}\}}{\theta}, \tag{10a}$$

$$\dot{x} = 0 : z = \left[\frac{x - (1+x)\tau)}{x}\right] \xi \min\{\chi^{\eta}, x^{\eta}\}. \tag{10b}$$

Figure 1 and 2 portray the nullclines of (10a) and (10b). Figure 1 shows the BGE under MAR restriction. Figure 2 shows BGE under FMS restriction. In each case, there exists a unique BGE that is stable in the saddle-point sense.[6]

We specifically examine the BGE under MAR. By equalization of the growth rate of (8a) and (8c), one obtains

$$\frac{(1-\tau)(1-\eta)\xi\chi^{\eta} - \rho}{\theta} = \frac{\tau \xi \chi^{\eta}}{x}.$$

This equation engenders the stationary value of x on BGE under MAR as

$$x_{\mathrm{MAR}} = \frac{\tau \theta \xi \chi^{\eta}}{(1-\tau)(1-\eta)\xi\chi^{\eta} - \rho}. \tag{11a}$$

If and only if $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$, x_{MAR} is an equilibrium value. Inserting x_{MAR} of (11a) into (10a), one obtains

$$z_{\mathrm{MAR}} = \frac{\rho + (1-\tau)(\theta+\eta-1)\xi\chi^{\eta}}{\theta}. \tag{11b}$$

Inserting x_{MAR} of (11a) into (8a), we obtain

$$\gamma_{\mathrm{MAR}}=\frac{(1-\tau)(1-\eta)\xi\chi^{\eta}-\rho}{\theta}. \tag{11c}$$

The analysis developed above provides the following proposition:

Proposition 1. *There exists a unique BGE that is stable in the saddle-point sense. If* $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$ *holds, then the stationary values of* x *and* z *are given as* $(x, z) = (x_{\mathrm{MAR}}, z_{\mathrm{MAR}})$ *and the equilibrium growth rate is* γ_{MAR}.

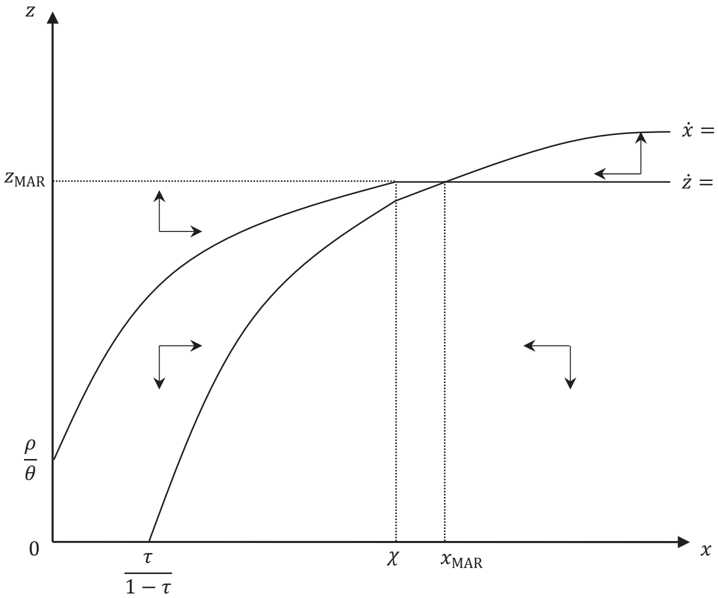

Figure 1. Phase diagram of the BGE under MAR ($\theta+\eta>1$)

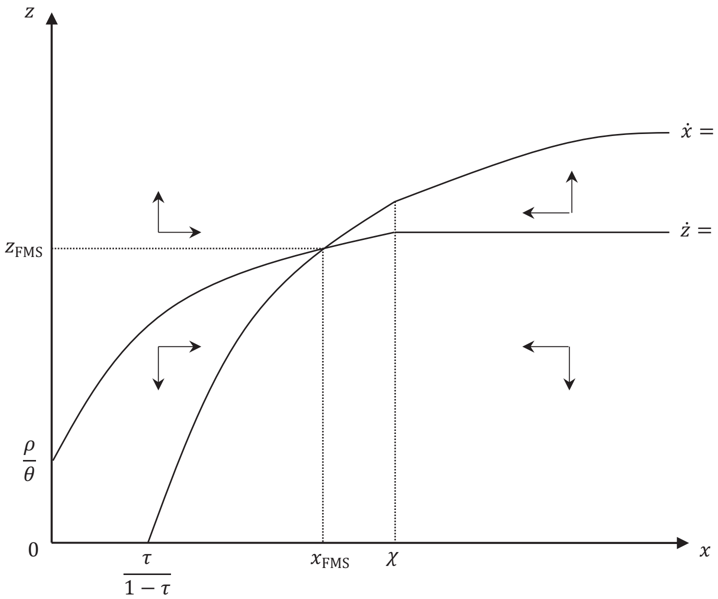

Figure 2. Phase diagram of the BGE under FMS ($\theta+\eta>1$)

Figure 1 presents the stationary equilibrium when $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$. When $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$, BGE is under the MAR restriction. Presuming that the initial state is in the neighborhood of the BGE under MAR, then the dynamics of z exhibits the initial jump onto z_{MAR} because z is controllable and set to its long-run level rationally. In contrast, the dynamics of x converges to its long-run level because x is not jumpable and should be governed by (9b) at $z = z_{\mathrm{MAR}}$. If $x_{\mathrm{MAR}} > x_{\mathrm{FMS}}$, then the dynamic properties of the stationary equilibrium are identical to those of FMS model (Figure 2). The income tax policy affects the value of x. Therefore, a large change in the tax rate presents the possibility of changing the dynamic structure of the macroeconomy. Hereinafter, we specifically examine the case where $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$, although we do not exclude the possibility of structural change.

Ⅲ. Macroeconomic effects of public investment

This section investigates the macroeconomic effects of public investment. We begin our analysis to examine the effects of public investment on the ratio of public capital to private capital and the ratio of private consumption to private capital. Differentiations of equations (11a) and (11b) with respect to τ lead to

$$\frac{dx_{\mathrm{MAR}}}{d\tau} = \frac{[(1-\tau)(1-\eta)\xi\chi^{\eta}-\rho]+(1-\eta)\tau\xi\chi^{\eta}}{[(1-\tau)(1-\eta)\xi\chi^{\eta}-\rho]^{2}}\theta\xi\chi^{\eta} > 0, \tag{12a}$$

$$\frac{dz_{\mathrm{MAR}}}{d\tau} = -\frac{(\theta+\eta-1)\xi\chi^{\eta}}{\theta} \lesseqgtr 0 \Leftrightarrow \theta \gtreqless 1-\eta. \tag{12b}$$

A rise in τ hastens the accumulation of public capital and inhibits the accumulation of private capital and the consumption growth rate. Therefore, a rise in τ decreases the ratio of public to private capital (12a). The effect of a rise in τ on the ratio of private consumption to private capital depends on the relative size of negative income and positive substitution effects. The relative magnitude between the former and the latter effect represents the denominator term of $(\theta+\eta-1)$ in equation (12b). Similarly, the growth effect of public investment is

$$\frac{d\gamma_{\mathrm{MAR}}}{d\tau} = -\frac{(1-\eta)\xi\chi^{\eta}}{\theta} < 0. \tag{12c}$$

Under MAR restriction, an additional increase in public capital does not affect labor productivity. An increase in public investment only increases the burden of income taxation. Then, the net return on private capital decreases by an increase in income tax rate. Therefore, public investment has a negative effect on economic growth under MAR restriction.

To maximize the equilibrium growth rate, the government sets the ratio of public investment to output onto the level that satisfies $x=\chi$. Inserting $x=\chi$ into equation (11a), we obtain

$$\chi = \frac{\tau\theta\xi\chi^{\eta}}{(1-\tau)(1-\eta)\xi\chi^{\eta}-\rho}. \tag{12c}$$

Solving this equation with respect to τ, we have

$$\tau^{*}_{\mathrm{MAR}} = \frac{(1-\eta)\xi\chi^{\eta}-\rho}{(1-\eta)\xi\chi^{\eta}+\theta\xi\chi^{\eta-1}}. \tag{13}$$

Next, we introduce the following definitions:

$$\gamma^{*}_{\mathrm{MAR}} \equiv \frac{(1-\eta)\xi\chi^{\eta}-\rho}{\theta}, \tag{14a}$$

$$r_{\mathrm{MAR}} \equiv (1-\eta)\xi\chi^{\eta}. \tag{14b}$$

Therein, $\gamma^{*}_{\mathrm{MAR}}$ denotes the maximum growth rate. Using (14a) and (14b), equation (13) is rewritten as

$$\begin{aligned}\tau^{*}_{\mathrm{MAR}} &= \frac{(1-\eta)\chi}{(1-\eta)\chi+\theta}\frac{\theta\gamma^{*}_{\mathrm{MAR}}}{\theta\gamma^{*}_{\mathrm{MAR}}+\rho} \\ &= \frac{(1-\eta)\chi}{(1-\eta)\chi+\theta}\frac{r_{\mathrm{MAR}}-\rho}{r_{\mathrm{MAR}}}.\end{aligned} \tag{15}$$

The growth-maximizing tax rate under FMS is a well-known result:[7)]

$$\tau^{*}_{\mathrm{FMS}} = \eta.$$

This tax rate is referred as the so-called Barro tax rule. Comparison of two tax rates provides

$$\begin{aligned}\tau^{*}_{\mathrm{MAR}}-\tau^{*}_{\mathrm{FMS}} &= \frac{(1-\eta)\xi\chi^{\eta}-\rho-\eta[(1-\eta)\xi\chi^{\eta}+\theta\xi\chi^{\eta-1}]}{(1-\eta)\xi\chi^{\eta}+\theta\xi\chi^{\eta-1}} \\ &= \frac{(1-\eta)^{2}\xi\chi^{\eta}-\rho-\eta\theta\xi\chi^{\eta-1}}{(1-\eta)\xi\chi^{\eta}+\theta\xi\chi^{\eta-1}} \\ &= \frac{[(1-\eta)^{2}\chi-\eta\theta]r_{\mathrm{MAR}}-(1-\eta)\chi\rho}{[(1-\eta)\chi+\theta]r_{\mathrm{MAR}}}.\end{aligned}$$

In general, the growth-maximizing tax rate is not equal to the elasticity of output with respect to public capital. For large χ, we have $\tau^*_{\mathrm{MAR}} < \tau^*_{\mathrm{FMS}}$. The key relation for determining the magnitude between two tax rates is

$$r_{\mathrm{MAR}} \gtreqless \frac{(1-\eta)\chi\rho}{(1-\eta)^2\chi-\eta\theta} \Leftrightarrow \tau^*_{\mathrm{MAR}} \gtreqless \tau^*_{\mathrm{FMS}}.$$

These results are summarized as follows.

Proposition 2. *Suppose that the economy is in the stationary equilibrium such as* $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$.
(i) The growth-maximizing tax rate under MAR restriction is

$$\tau^*_{\mathrm{MAR}} = \frac{(1-\eta)\chi}{(1-\eta)\chi+\theta}\frac{\theta\gamma^*_{\mathrm{MAR}}}{\theta\gamma^*_{\mathrm{MAR}}+\rho} = \frac{(1-\eta)\chi}{(1-\eta)\chi+\theta}\frac{r_{\mathrm{MAR}}-\rho}{r_{\mathrm{MAR}}}.$$

(ii) The relation between the growth-maximizing tax rates is

$$r_{\mathrm{MAR}} \gtreqless \frac{(1-\eta)\chi\rho}{(1-\eta)^2\chi-\eta\theta} \Leftrightarrow \tau^*_{\mathrm{MAR}} \gtreqless \tau^*_{\mathrm{FMR}}.$$

Finally, we consider the welfare effect of public investment. Considering the dynamics of z, we have the indirect utility function as

$$V_{\mathrm{MAR}} = \frac{1}{1-\theta}\left[\frac{z_{\mathrm{MAR}}^{1-\theta}K_0^{1-\theta}}{(\theta-1)\gamma_{\mathrm{MAR}}+\rho} - \frac{1}{\rho}\right]. \tag{16}$$

Differentiation of equation (16) with respect to τ provides

$$\frac{dV_{\mathrm{MAR}}}{d\tau} = K_0^{1-\theta}\left[\frac{z_{\mathrm{MAR}}^{-\theta}}{(\theta-1)\gamma_{\mathrm{MAR}}+\rho}\frac{dz_{\mathrm{MAR}}}{d\tau} + \frac{z_{\mathrm{MAR}}^{1-\theta}}{\{(\theta-1)\gamma_{\mathrm{MAR}}+\rho\}^2}\frac{d\gamma_{\mathrm{MAR}}}{d\tau}\right] < 0 \text{ if } \theta \geq 1.$$

As shown in the previous section, the dynamics of z is always on the BGE under MAR. The indirect utility depends on the ratio of consumption to private capital and consumption growth rate in the BGE. From equations (12b) and (12c), a rise in τ reduces both the ratio of consumption to private capital and consumption growth rate. Therefore, a rise in τ decreases not only initial consumption level but also the long-run consumption level because the initial decrease in disposable income does not engender increases in future income.

This result establishes the following proposition:

Proposition 3. *Suppose that a change in τ is sufficiently small and the stationary equilibrium after its change is still under MAR restriction. For $\theta \geq 1$, the welfare-maximizing tax rate on income under MAR restriction is equivalent to the growth-maximizing tax rate on income under MAR restriction:*

$$\tau^*_{\mathrm{MAR}} = \tau^{**}_{\mathrm{MAR}} = \frac{(1-\eta)\xi\chi^{\eta}-\rho}{(1-\eta)\xi\chi^{\eta}+\theta\xi\chi^{\eta-1}}.$$

Ⅳ. Further analysis

In preceding sections, we assumed that the public investment is financed by the income tax. However, we have many alternatives for financing public investment. Therefore, we should examine the macroeconomic effects of public investment financed by another financial source. We consider the case of consumption tax financing. Then, some equations must be replaced by new equations. First, equation (1) is replaced by

$$\dot{K}_t = r_t K_t + w_t - (1+q)C_t, \tag{17}$$

where q is the consumption tax rate. Equation (2) is superseded by

$$\frac{\dot{C}_t}{C_t} = \frac{r_t-\rho}{\theta}. \tag{18}$$

The government's budget constraint is

$$\dot{G}_t = qC_t. \tag{19}$$

Equations (17)-(19) are rewritten as

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\eta)\xi\min[\chi^{\eta}, x_t^{\eta}]-\rho}{\theta}, \tag{20a}$$

$$\frac{\dot{K}_t}{K_t} = \xi\min[\chi^{\eta}, x_t^{\eta}] - (1+q)\frac{C_t}{K_t}, \tag{20b}$$

$$\frac{\dot{G}_t}{G_t} = q\frac{z_t}{x_t}. \tag{20c}$$

Equations (20a)-(20c) lead to

$$\dot{z}_t = \left[\frac{(1-\eta-\theta)\xi\min\{\chi^{\eta}, x_t^{\eta}\}-\rho}{\theta} + (1+q)z_t\right]z_{t,} \tag{21a}$$

$$\dot{x} = \left[q\frac{z_t}{x_t} - \xi\min\{\chi^{\eta}, x_t^{\eta}\} + (1+q)z_t\right]z_t. \tag{21b}$$

Using the BGE condition, (21a), and (21b), we obtain

$$\dot{z}_t = 0 : z = \frac{\rho + (\theta+\eta-1)\xi \min\{\chi^\eta, x^\eta\}}{(1+q)\theta}, \tag{22a}$$

$$\dot{x} = 0 : z = \left[\frac{x}{q+(1+q)x}\right]\xi \min\{\chi^\eta, x^\eta\}. \tag{22b}$$

Regarding MAR restriction, solving equation (22a) with respect to z, we have

$$z_{\mathrm{MAR}} = \frac{\rho + (\theta+\eta-1)\xi\chi^\eta}{(1+q)\theta}. \tag{23a}$$

Inserting (23a) into (22b), we derive

$$\begin{aligned} x_{\mathrm{MAR}} &= \frac{q z_{\mathrm{MAR}}}{\xi\chi^\eta - (1+q) z_{\mathrm{MAR}}} \\ &= \frac{q}{1+q}\frac{\rho+(\theta+\eta-1)\xi\chi^\eta}{(1+\eta)\xi\chi^\eta - \rho}. \end{aligned} \tag{23b}$$

Equation (20a) leads to

$$\gamma^*_{\mathrm{MAR}} = \frac{(1-\eta)\xi\chi^\eta - \rho}{\theta}. \tag{23c}$$

The growth rate given by equation (23c) is the maximum growth rate, which is attainable, because the fixed tax rate on consumption has no distortionary effect on private capital accumulation. The consumption tax rate that attains (23c) must satisfy

$$\chi = \frac{q}{1+q}\frac{\rho + (\theta+\eta-1)\xi\chi^\eta}{(1-\eta)\xi\chi^\eta - \rho}.$$

The solution of this equation gives

$$\begin{aligned} q^*_{\mathrm{MAR}} &= \frac{\chi}{\dfrac{\rho+(\theta+\eta-1)\xi\chi^\eta}{(1-\eta)\xi\chi^\eta-\rho} - \chi} \\ &= \frac{\chi}{\dfrac{\dfrac{\gamma^*_{\mathrm{MAR}}+\rho}{1-\eta} - \gamma^*_{\mathrm{MAR}}}{\gamma^*_{\mathrm{MAR}}} - \chi} \\ &= \frac{(1-\eta)\chi\gamma^*_{\mathrm{MAR}}}{\eta\gamma^*_{\mathrm{MAR}} + \rho - (1-\eta)\chi\gamma^*_{\mathrm{MAR}}}. \end{aligned} \tag{24}$$

The standard model of infrastructure-led growth is not restricted by MAR. Therefore, an increase in public investment financed by the consumption tax monotonically increases the equilibrium growth rate because the time-invariant consumption tax does not engender distortionary effect on capital accumulation. However, in our model with weakest-link externality, it binds the labor productivity into the MAR restriction. It is vain effort that the ratio of public to private capital increases by an increase in the consumption tax rate. Therefore, the infrastructure-led growth model with weakest-link externality derives the minimum consumption tax rate to attain the maximum growth rate.

We next investigate the welfare effect of the consumption tax. Differentiating (16) with respect to q, we obtain

$$\frac{dV_{\mathrm{MAR}}}{dq} = K_0^{1-\theta}\left[\frac{z_{\mathrm{MAR}}^{-\theta}}{(\theta-1)\gamma^*_{\mathrm{MAR}}+\rho}\frac{dz_{\mathrm{MAR}}}{d\tau}\right] < 0 \text{ if } \theta \geq 1.$$

Note that $dz_{\mathrm{MAR}}/dq < 0$ is derived from equation (23a). The consumption tax has no distortionary effect on private capital accumulation and no growth effect under MAR. An increase in the consumption tax rate affects indirect utility through a change in the initial consumption. A rise in the consumption tax rate increases the relative price of consumption; it therefore decreases the initial consumption. A rise in the tax rate on consumption consequently has a negative welfare effect. This result demonstrates that the minimum consumption tax rate to attain the maximum growth rate is equivalent to the welfare-maximizing consumption tax rate. Formally, we have

$$q^*_{\mathrm{MAR}} = q^{**}_{\mathrm{MAR}} = \frac{(1-\eta)\chi\gamma^*_{\mathrm{MAR}}}{\eta\gamma^*_{\mathrm{MAR}} + q - (1-\eta)\chi\gamma^*_{\mathrm{MAR}}}. \tag{25}$$

The results presented above provide the following proposition:

Proposition 4. *Suppose that the economy is in the stationary equilibrium such as* $x_{\mathrm{MAR}} < x_{\mathrm{FMS}}$.
(i) The minimum tax rate on consumption to attain the maximum growth rate under MAR is

$$q^*_{\mathrm{MAR}} = \frac{(1-\eta)\chi\gamma^*_{\mathrm{MAR}}}{\eta\gamma^*_{\mathrm{MAR}} + \rho - (1-\eta)\chi\gamma^*_{\mathrm{MAR}}}.$$

(ii) The welfare-maximizing tax rate on consumption is

$$q^*_{\mathrm{MAR}} = q^{**}_{\mathrm{MAR}} = \frac{(1-\eta)\chi\gamma^*_{\mathrm{MAR}}}{\eta\gamma^*_{\mathrm{MAR}} + \rho - (1-\eta)\chi\gamma^*_{\mathrm{MAR}}}.$$

V. Conclusion

This paper developed an endogenous growth model with weakest link externality. We clarified the necessary and sufficient condition for the stable BGE under MAR. Specifically addressing the BGE under MAR, we demonstrated that the growth-maximizing tax rate is equivalent to the welfare-maximizing tax rate. Particularly, the income tax which maximizes the equilibrium growth rate is not equal to the elasticity of public capital to output. Therefore, the Barro tax rule fails in the case presented herein. Furthermore, we analyzed the macroeconomic effects of public investment financed by consumption taxation. The analysis yielded similar results of income tax financing.

The basic model will be extendable for some applications. For example, it is interesting to investigate debt-financing and debt sustainability. Under the weakest-link externality, public investment financed by debt will provide a critical level of public debt and capital to attain the maximum growth rate (or welfare). The public debt exceeding its level will fail to ride on a desirable BGE path.[8] Furthermore, endogenous labor supply will be a natural extension of our model. In this study, the weakest-link externality was tied to labor productivity. Then, the labor supply will play a crucial role in growth rate determination. However, it ought to devote attention to treating scale effects. These topics remain as tasks for future study, for which this study provides an analytical basis.

Notes

1) See Pereira and Andraz (2013) and Bom and Ligthart (2014) for surveys of empirical studies.
2) Greiner (1998) investigated the optimal fiscal policy in the Barro model. See Irmen and Kuehnel (2009) for extended models of Barro (1990).
3) Turnovsky (1997) and Tamai (2008) investigated optimal fiscal policy in the FMS model.
4) Misch et al. (2013) investigated the relation between the growth-maximizing tax rate and the welfare-maximizing tax rate in two endogenous models of Barro and FMS.
5) These types of short-side rule are used widely in economic analysis. For example, it is important for the analysis on provision of public goods (e.g., Hirshleifer 1983, 1985). Because we examine the externality effect caused by "public goods", formulation of equation (4) is appropriate. It enables future extension of the voluntary provision of two "public goods".
6) See Futagami et al. (1993) for details of the dynamic property of BGE under FMS.
7) Barro (1990) also demonstrated that the growth-maximizing tax rate is equal to the output elasticity of public capital using an endogenous growth model with productive government expenditure.
8) With public debt, Greiner and Semmler (2000) and Ghosh and Mourmouras (2004) provide dynamic analysis similar to our study. Greiner (2007) investigates the sustainability of public debt in an endogenous growth model with public capital. This study will provide their extended models.

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