Course III: Linear Algebra
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Lesson 01
Basic Rules of Vectors

1A
- Definitions of vectors
- Basic rules
- Components of vectors
Scalars and Vectors

Scalar
• A scalar: a quantity described by a magnitude.
• Notation: normal italic type alphabet, Greek letters, etc.
  [Ex.] area $A$, temperature $t$, speed $v$, angle $\theta$

Vector
• A vector: a quantity described by magnitude and direction.
  [Ex.] force, velocity
• A vector is commonly illustrated by “an arrow”.
• Typical notation: $\mathbf{a}$, $\vec{a}$, $\overrightarrow{OP}$
• The magnitude of a vector is denoted by $|\mathbf{a}|$ or $a$. 
1. Equality \( \vec{a} = \vec{b} \)

Same magnitude and direction → they are equal.

2. Scalar Multiplication \( k\vec{a} \)

\( k \): a scalar

\( k > 0 \)

\( k < 0 \)
3. Addition

Sum = the diagonal of the parallelogram

\[
\vec{b} + \vec{a} = \vec{a} + \vec{b}
\]

Sum = the closing third side.

\[
\vec{a} + \vec{b}
\]

4. Subtraction

\[
\vec{b} + \overrightarrow{BA} = \vec{a}
\]

Therefore

\[
\overrightarrow{BA} = \vec{a} - \vec{b}
\]
Basic Laws of Vectors

1. Commutative law
   \[\vec{a} + \vec{b} = \vec{b} + \vec{a}\]

2. Associative law
   \[(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})\]

3. Distributive law
   \[k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}\]
[Examples 1-1] Let $\vec{p} = 3\vec{a} + 2\vec{b}$ and $\vec{q} = -2\vec{a} + \vec{b}$. Answer the following questions. (1) Find $\vec{x}$ which satisfies the equation. $3(\vec{x} - \vec{q}) = 2\vec{p} + \vec{x}$

(2) Find $\vec{x}$ and $\vec{y}$ which satisfy

$$
\begin{aligned}
2\vec{x} - 3\vec{y} &= \vec{p} \quad \text{(i)} \\
\vec{x} + \vec{y} &= \vec{q} \quad \text{(ii)}
\end{aligned}
$$

Ans.

(1) Substituting $\vec{p}$ and $\vec{q}$, we have

$$
3\vec{x} - 3(-2\vec{a} + \vec{b}) = 2(3\vec{a} + 2\vec{b}) + \vec{x}
\quad \therefore \quad \vec{x} = \frac{7}{2} \vec{b}
$$

(2) From (i) $-$ (ii)$\times 2$, we have

$$
-5\vec{y} = \vec{p} - 2\vec{q}
\quad \therefore \quad \vec{y} = -\frac{1}{5} \vec{p} + \frac{2}{5} \vec{q} = -\frac{1}{5} (3\vec{a} + 2\vec{b}) + \frac{2}{5} (-2\vec{a} + \vec{b}) = -\frac{7}{5} \vec{a}
$$

From (ii), we have

$$
\vec{x} = \vec{q} - \vec{y} = (-2\vec{a} + \vec{b}) - (-\frac{7}{5} \vec{a}) = -\frac{3}{5} \vec{a} + \vec{b}
$$
[Ex.1-1] Find \( \vec{x} \) and \( \vec{y} \) which satisfy the following equation

\[
\begin{align*}
3\vec{x} + 2\vec{y} &= \vec{a} \quad (i) \\
4\vec{x} - 3\vec{y} &= \vec{b} \quad (ii)
\end{align*}
\]

Ans.

Pause the video and solve the problem by yourself.
[Ex.1-1] Find $\vec{x}$ and $\vec{y}$ which satisfy the following equation

\[
\begin{aligned}
3\vec{x} + 2\vec{y} &= \vec{a} \\
4\vec{x} - 3\vec{y} &= \vec{b}
\end{aligned}
\]

Ans.

From Eq.(i) $\times 3$, we have $9\vec{x} + 6\vec{y} = 3\vec{a}$

From Eq.(ii) $\times 2$, we have $8\vec{x} - 6\vec{y} = 2\vec{b}$

Adding, we have $17\vec{x} = 3\vec{a} + 2\vec{b}$

$\therefore \vec{x} = \frac{3}{17}\vec{a} + \frac{2}{17}\vec{b}$

$\therefore \vec{y} = -\frac{3}{2}\vec{x} + \frac{1}{2}\vec{a} = -\frac{3}{2}\left(\frac{3}{17}\vec{a} + \frac{2}{17}\vec{b}\right) + \frac{1}{2}\vec{a} = \frac{4}{17}\vec{a} - \frac{3}{17}\vec{b}$
Lesson 01
Basic Rules of Vectors

1B
• Components of Vectors
Components of a Vector

Unit Vector

Vector whose length is 1

$$\vec{e} = \frac{\vec{a}}{|\vec{a}|} \quad |\vec{e}| = 1$$

Basic Unit Vector

$$\vec{i} = (1, 0) \quad \text{and} \quad \vec{j} = (0, 1)$$

Components of Vector

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} = (a_1, a_2)$$

Component
Vector Connecting Two Points

**Vector Connecting A and B**

\[ \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \]

\[ = (x_2, y_2) - (x_1, y_1) \]

\[ = (x_2 - x_1, y_2 - y_1) \]

**Length AB**

\[ |\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
Position Vector

If we select the initial point of the vector at the origin, a point is designated by a vector.

Vector Connecting A and B

$$\overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a}$$
Example

[Examples 1-2] Find the position vector of point C which divide the line connecting $A(\vec{a})$ and $B(\vec{b})$ internally in the ratio $m : n$.

Ans.

$\vec{AC}$ and $\vec{CB}$ have the same direction.

Magnitudes $|\vec{AC}| : |\vec{CB}| = m : n$

Therefore

$n(\vec{x} - \vec{a}) = m(\vec{b} - \vec{x})$

∴ $\vec{x} = \frac{n\vec{a} + m\vec{b}}{(m + n)}$
Exercise

[Ex1-2] Find the position vector $\vec{g}$ of the center of gravity of the Δ ABC. The position vectors of A, B, and C are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

[Note] The center of gravity is given by the point which divide the line AM by the ratio 2:1 where M is the center of side BC.

Ans.

Pause the video and solve the problem by yourself.
[Ex1-2] Find the position vector $\mathbf{g}$ of the center of gravity of the $\Delta ABC$. The position vectors of A, B, and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Ans.

The center of side BC is $\mathbf{m} = \frac{\mathbf{b} + \mathbf{c}}{2}$

Since the center of gravity $G$ divides the line AM internally in the ratio 2:1, we have

$$\mathbf{x} = \frac{\mathbf{a} + 2\mathbf{m}}{2 + 1} = \frac{\mathbf{a} + 2\left(\frac{\mathbf{b} + \mathbf{c}}{2}\right)}{3} = \frac{\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$$