Axionlike Origin of the Primordial Density Perturbation

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We show that an axionlike field coupled to a new confining gauge group can generate the primordial density perturbation in the postinflation Universe. The axion decay constant and strong coupling scale are uniquely determined by observations of the density perturbation, which further suggest a temporal deconfinement of the gauge group after inflation. The resulting temperature-dependent axion potential, together with its periodic nature, gives rise to the red-tilted density perturbation, with a positive local-type non-Gaussianity of order unity.

Introduction.—The large-scale structure of our Universe was seeded by tiny density fluctuations in the early Universe. This picture of structure formation has been established by a variety of cosmological observations, which also revealed that the primordial density perturbation was adiabatic, nearly scale-invariant with a slight increase of the amplitude towards larger length scales (red tilted), and nearly Gaussian. However, the physical origin of the density perturbation still remains a mystery. One possibility is that it was produced during cosmic inflation from fluctuations of the inflaton field. Alternatively, it could have been produced after inflation by another physical degree of freedom.

In theories beyond the standard model of particle physics, the appearance of pseudo-Nambu-Goldstone bosons (PNGBs) of spontaneously broken global U(1) symmetries is ubiquitous. The most famous example is the QCD axion introduced for solving the strong *CP* problem [1]. Another well-studied example occurs in string theory, which suggests the existence of many such particles in the four-dimensional effective theory upon string compactifications [2]. Axions and axionlike particles can also play important roles in cosmology, and in particular the possibility that they constitute the dark matter of our Universe has been extensively studied.

In this Letter we argue that an axionlike field can serve as a curvaton [3] and generate the primordial density perturbation in the postinflation Universe. Considering the axion to be coupled to a new gauge force that becomes strong at low energies, we show that the observed characteristics of the density perturbation require the strong coupling scale to be higher than the de Sitter temperature of the inflationary Universe, but lower than the maximum radiation temperature during reheating, i.e., $T_{\mathrm{inf}} \ll \Lambda \ll T_{\max}$. This leads to a temporal deconfinement of the gauge group after inflation, with which the axion generates the density perturbation by virtue of its temperature-dependent and periodic potential.

Our scenario is quite distinct from previous studies on PNGB curvatons [4,5]. For a curvaton to produce the observed red spectral index $n_s - 1 \sim -10^{-2}$, one needs to require either (i) a time derivative of the Hubble rate during inflation of $-\dot{H}/H^2 \sim 10^{-2}$, and/or (ii) a tachyonic curvaton mass with amplitude $|m| \sim 10^{-1} H_{\mathrm{inf}}$ in terms of the inflationary Hubble rate. Option (i) implies that the inflaton field travels a super-Planckian distance, and thus imposes strong constraints on inflationary model building. In option (ii), the rather large tachyonic mass forces the curvaton to start oscillating soon after inflation ends, which in turn makes it challenging for the curvaton to dominate the postinflation Universe and produce the density perturbation. One way to avoid the early onset of the oscillation is to place the curvaton at a fine-tuned initial position close to a potential maximum, however this also enhances the non-Gaussianity and thus is ruled out by current observations. We will show that an axionlike field coupled to a strong sector can generate a red-tilted and nearly Gaussian perturbation via (ii), by taking advantage of a temperature-dependent potential which naturally delays the oscillation, without the need of fine-tuning the initial condition.

The axion setup.—We consider a PNGB of some global U(1) symmetry that is spontaneously broken at an energy scale f; we refer to this field as the axion and f the axion decay constant. We also assume the U(1) to be explicitly broken due to the axion being coupled to a new gauge force (not QCD) that becomes strong at low energies, yielding a periodic axion potential. Considering the periodicity to be governed by f, we write the axion Lagrangian as

$$-\frac{1}{2} f^2 \partial_\mu \theta \partial^\mu \theta - m(T)^2 f^2 (1 - \cos\theta) + \frac{\alpha}{8\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (1)$$

Here the axion is written as a dimensionless angle θ with periodicity $\theta \cong \theta + 2\pi$, and we have also added a pseudoscalar coupling to photons (or hidden photons). The axion potential arises below some strong coupling scale Λ

($< f$), and we consider the axion mass to depend on the cosmic temperature as

$$m(T) \simeq \begin{cases} m_0 \left(\dfrac{\Lambda}{T}\right)^p & \text{for } T > \Lambda, \\ m_0 & \text{for } T < \Lambda, \end{cases} \quad (2)$$

with a positive power p of order unity, although our main results are insensitive to the detailed form of the temperature dependence above Λ. The zero-temperature axion mass is given by

$$m_0 = \frac{\Lambda^2}{f}. \quad (3)$$

Without loss of generality we take the axion to lie within the range $-\pi \leq \theta \leq \pi$.

Cosmological evolution.—Upon analyzing the axion dynamics, we will make a couple of assumptions along the way. We will later show that most of the assumptions are actually required by the observed characteristics of the density perturbation.

We first assume that the de Sitter temperature during inflation, $T_{\text{inf}} = H_{\text{inf}}/2\pi$, is lower than the strong coupling scale, i.e.,

$$T_{\text{inf}} < \Lambda < f. \quad (4)$$

Hence during inflation the U(1) is already broken, and moreover the axion potential takes its zero-temperature form. Further supposing $m_0^2 \ll H_{\text{inf}}^2$, and that f is smaller than the reduced Planck scale $M_{\text{Pl}} = (8\pi G)^{-1/2}$, then $\Lambda^4 \ll M_{\text{Pl}}^2 H_{\text{inf}}^2$, i.e., the energy density of the axion is tiny compared to the total density of the Universe and thus the axion serves as a spectator. Supposing that the axion evolution is dominated by classical rolling and not quantum fluctuations, the axion slowly rolls as $3H_{\text{inf}}\dot{\theta} \simeq -m_0^2 \sin\theta$. Integrating this by ignoring any time dependence of the Hubble rate yields

$$\tan\left(\frac{\theta_{\text{end}}}{2}\right) \simeq \tan\left(\frac{\theta_1}{2}\right) \exp\left(-\frac{m_0^2}{3H_{\text{inf}}^2}\mathcal{N}_1\right). \quad (5)$$

Here θ_{end} is the field value when inflation ends, and $\mathcal{N}_1$ is the number of e folds since the time when the axion takes a value θ_1 until the end of inflation.

After inflation, the inflaton is considered to decay and trigger a radiation-dominated epoch. We assume that the maximum value of the radiation temperature lies within the range

$$\Lambda < T_{\text{max}} < f, \quad (6)$$

and thus the U(1) symmetry stays broken, however the axion potential diminishes. The energy density and temperature of radiation are related via $\rho_r = (\pi^2/30)g_* T^4$, and for simplicity we take the number of relativistic degrees of freedom as a constant. As a minimal model, we consider the inflaton to instantaneously decay at the end of inflation; then the maximum radiation temperature is

$$T_{\text{max}} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} (M_{\text{Pl}} H_{\text{inf}})^{1/2}. \quad (7)$$

The axion slow rolls until its potential diminishes, after which it freely streams for a few Hubble times and then comes to a halt due to the Hubble friction. The field excursion during the free streaming can be estimated using the slow-roll approximation just before the end of inflation as $|\Delta\theta| \sim |\dot{\theta}_{\text{end}}/H_{\text{inf}}| < (m_0^2/H_{\text{inf}}^2)|\theta_{\text{end}}| \ll |\theta_{\text{end}}|$. This shows that the axion is effectively frozen at θ_{end} while its potential is negligible.

The axion potential arises again as the Universe cools down. When the mass becomes comparable to the Hubble rate, the axion begins to oscillate about its potential minimum, and thereafter the axion number is conserved. The temperature when the oscillation begins can be computed, supposing it is above Λ and that the Universe is dominated by radiation, as

$$T_{\text{osc}} \simeq \Lambda \left(\frac{\sqrt{90}}{\pi g_*^{1/2} c} \frac{M_{\text{Pl}}}{f}\right)^{1/(p+2)}. \quad (8)$$

The subscript "osc" denotes quantities at the onset of the oscillation, and $c \equiv m(T_{\text{osc}})/H_{\text{osc}}$ is the mass-to-Hubble ratio at this time. Here, note that $m(T_{\text{osc}}) < m_0$. The number density of the axion is written as $n_\theta = \rho_\theta/m(T)$ in terms of the energy density $\rho_\theta = f^2\dot{\theta}^2/2 + m(T)^2 f^2 (1 - \cos\theta)$. Hence its value at the onset of the oscillation is

$$n_{\theta\text{osc}} \simeq m(T_{\text{osc}}) f^2 (1 - \cos\theta_{\text{end}}), \quad (9)$$

and thereafter redshifts as $n_\theta \propto a^{-3}$.

Since the radiation density redshifts as $\rho_r \propto a^{-4}$, the axion would eventually dominate the Universe if it is sufficiently long lived. The Hubble rate when axion domination takes over, i.e., when $\rho_\theta = \rho_r$, is obtained as

$$H_{\text{dom}} \simeq \frac{60\sqrt{5}}{\pi^3 g_*^{3/2}} \frac{\Lambda^2}{M_{\text{Pl}}} \left(\frac{\Lambda}{T_{\text{osc}}}\right)^{2p+6} (1 - \cos\theta_{\text{end}})^2, \quad (10)$$

where we have considered the temperature at this time to satisfy $T_{\text{dom}} < \Lambda$ so that $\rho_\theta = m_0 n_\theta$. Whether the axion actually dominates the Universe depends on its lifetime. At temperatures below Λ, the decay width of the axion induced by the axion-photon coupling in (1) is

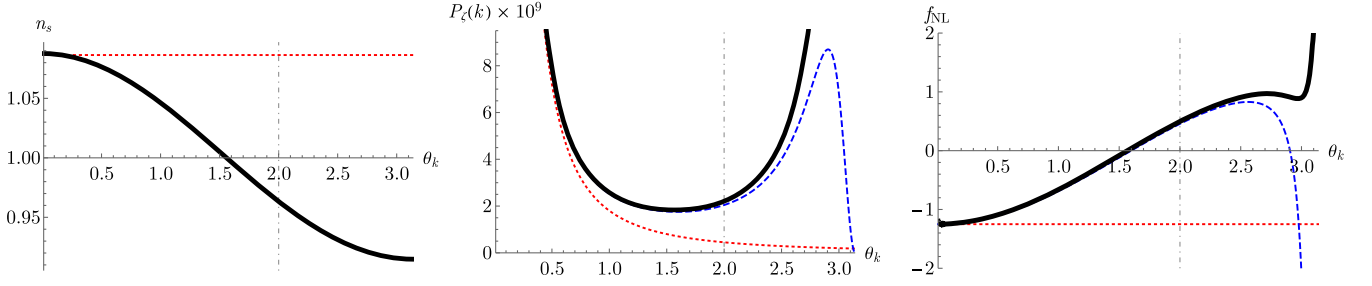$$\Gamma = \frac{\alpha^2}{256\pi^3} \frac{\Lambda^6}{f^5}. \quad (11)$$

FIG. 1. Curvature perturbation as a function of the axion angle when the pivot scale exits the horizon during inflation. The axion decay constant and strong coupling scale are taken as $f = 2500 H_{\mathrm{inf}}$ and $\Lambda = 30 H_{\mathrm{inf}}$, so that the power spectrum amplitude and the spectral index match with observations at $\theta_k \approx 2.0$. The black solid lines show the numerical results, while the blue dashed lines are analytic approximations. Results for vanilla curvatons are shown as the red dotted lines.

If $\Gamma < (>) H_{\mathrm{dom}}$, then the axion would decay after (before) dominating the Universe. Supposing that the axion decays suddenly when $H = \Gamma$ (we denote quantities at this time by the subscript "dec"), one finds a relation:

$$3 M_{\mathrm{Pl}}^2 \Gamma^2 = 3 M_{\mathrm{Pl}}^2 H_{\mathrm{osc}}^2 \left(\frac{a_{\mathrm{osc}}}{a_{\mathrm{dec}}} \right)^4 + m_0 n_{\theta\mathrm{osc}} \left(\frac{a_{\mathrm{osc}}}{a_{\mathrm{dec}}} \right)^3. \quad (12)$$

The terms in the right-hand side correspond to the radiation and axion densities right before the decay, and we write their ratio as $R \equiv (\rho_\theta/\rho_r)_{\mathrm{dec}}$ for later convenience.

Curvature perturbation.—The axion acquires super-horizon field fluctuations during inflation, which later get converted into cosmological density (or curvature) perturbations as the axion comes to dominate the Universe. The generation of the perturbation completes when the axion decays. Using the $\delta \mathcal{N}$ formalism [6], we compute the axion-induced curvature perturbation as the fluctuation in the number of e folds $\mathcal{N}$ between an initial flat slice during inflation when a comoving wave number k of interest exits the horizon ($k = aH$), and a final uniform-density slice where $H = \Gamma$. Noting that the axion field fluctuations on the initial slice is Gaussian with a power spectrum $P_{\delta\theta}(k) \simeq (H_k/2\pi f)^2$ (the subscript "k" refers to quantities when $k = aH$), the power spectrum of the curvature perturbation is written as $P_\zeta(k) \simeq (\partial \mathcal{N}/\partial \theta_k)(H_k/2\pi f)^2$. Likewise, the non-Gaussianity parameter characterizing the amplitude of the local bispectrum is $f_{\mathrm{NL}} \simeq (5/6)(\partial^2 \mathcal{N}/\partial \theta_k^2)(\partial \mathcal{N}/\partial \theta_k)^{-2}$.

The derivatives of $\mathcal{N} = \ln(a_{\mathrm{dec}}/a_k)$ can be evaluated by differentiating (12) multiple times with respect to θ_k. In the vicinity of $\theta = 0$ where the axion potential is well approximated by a quadratic, the mass-to-Hubble ratio c at the onset of the oscillation is independent of the axion field value [7]; however this is not the case in the region close to the hilltop $|\theta| = \pi$ [5]. Let us for the moment suppose that θ_{end} is not too far from 0 and take $\partial c/\partial \theta_k = 0$. Then, also because the axion density is negligibly tiny compared to the total density of the Universe during $a_k \lesssim a \lesssim a_{\mathrm{osc}}$, the ratio (a_{osc}/a_k) is independent of θ_k. Derivatives of θ_{end}

can be evaluated using (5). Hence, after some manipulation we find

$$P_\zeta(k) \simeq \left(\frac{R}{3R+4} \frac{1 + \cos\theta_{\mathrm{end}}}{\sin\theta_k} \frac{H_k}{2\pi f} \right)^2,$$

$$f_{\mathrm{NL}} \simeq \frac{5}{6} \left\{ \frac{4}{R} + \frac{4}{3R+4} - \left(3 + \frac{4}{R} \right) \frac{1 - \cos\theta_{\mathrm{end}} + \cos\theta_k}{1 + \cos\theta_{\mathrm{end}}} \right\}. \quad (13)$$

In the $\theta_k \to 0$ limit, these expressions reduce to those of the vanilla curvaton with a quadratic potential. The spectral index of the power spectrum $n_s - 1 = d \ln P_\zeta/d \ln k$ is computed using $d \ln k \simeq H_k dt$ and the slow-roll approximation during inflation as

$$n_s - 1 \simeq \frac{2}{3} \frac{m_0^2}{H_k^2} \cos\theta_k + 2 \frac{\dot{H}_k}{H_k^2}. \quad (14)$$

In Fig. 1 we plot n_s, P_ζ, and f_{NL} as functions of θ_k. Here we have fixed the axion parameters as $f = 2500 H_{\mathrm{inf}}$ and $\Lambda = 30 H_{\mathrm{inf}}$, with a sufficiently small photon coupling α such that $R \gg 1$ in the entire displayed region of $10^{-2} \leq \theta_k < \pi$. The mode k exits the horizon about 50 e folds before the end of inflation, and the time variation of the inflationary Hubble rate is taken as $|\dot{H}/H^2| \ll 10^{-2}$ so that its effect on the spectral index is negligible. We note that the curvature perturbation is insensitive to the precise values of H_{inf}, α, p, and g_*, as long as the aforementioned assumptions are satisfied. The black solid lines show the exact results obtained by numerically solving the equation of motion of the axion and computing $\delta \mathcal{N}$. We have verified that the numerical results do not change if the axion and inflaton decay suddenly or smoothly, as long as $R \gg 1$ and also the inflaton decay width is not much smaller than H_{inf}. The blue dashed lines show the $R \to \infty$ limit of the analytic expressions (13), and the red dotted lines show the results for a quadratic curvaton (i.e., $\theta_k \to 0$ and $R \to \infty$). The analytic expression (14) for the spectral index fully overlaps with the numerical result, hence is not

shown in the plot. The behavior of the axion-induced perturbation reduces to that of a quadratic curvaton at $\theta_k \ll 1$. The analytic expressions in (13) are seen to match well with the numerical results up to $\theta_k \lesssim 2.5$, which corresponds to $\theta_{\rm end} \lesssim 0.5$. For larger θ_k, the inhomogeneous onset of the axion oscillation becomes relevant and thus the approximation of constant c used for deriving (13) breaks down; in particular, $f_{\rm NL}$ becomes of order 10 as θ_k approaches π [5]. The Planck best-fit values $P_\zeta(k_p) \approx 2.1 \times 10^{-9}$ and $n_s \approx 0.96$ at the pivot scale $k_p = 0.05~{\rm Mpc}^{-1}$ [8] are realized at $\theta_k \approx 2.0$. Here the local-type non-Gaussianity is also within the Planck 68% limits $f_{\rm NL} = -0.9 \pm 5.1$ [9].

Observational constraints.—We now investigate the parameter space. The periodic axion potential comes equipped with negatively curved regions where the red-tilted perturbation can be sourced without relying on large-field inflation. The observed value of n_s indicates that the axion angle lies within $\pi/2 < |\theta_k| < \pi$ when the cosmological scales exit the horizon, and also that the zero-temperature axion mass is of order $10^{-1}H_{\rm inf}$ [10]. Current limits on non-Gaussianity favor $R \gg 1$ (i.e., axion domination before decay), and also $|\theta_k|$ not too close to π where the inhomogeneous onset of the oscillation enhances $f_{\rm NL}$. Hence, supposing the axion to roll down to the region $|\theta_{\rm end}| < \pi/2$ by the end of inflation, we take $-\cos \theta_k \sim |\sin \theta_k| \sim \cos \theta_{\rm end} \sim 1$ to make a rough estimate of the axion parameters. Then (13) and (14) give $P_\zeta \sim (H_{\rm inf}/3\pi f)^2$, $n_s - 1 \sim -2m_0^2/3H_{\rm inf}^2$, and $f_{\rm NL} \sim 5/4$. Thus we find that the observed values of P_ζ and n_s fix the decay constant and strong coupling scale in terms of the inflation scale as

$$f \sim 2000 H_{\rm inf}, \qquad \Lambda \sim 20 H_{\rm inf}. \qquad (15)$$

One can then check, taking $c \sim 1$, that most of the assumptions in the previous sections are automatically satisfied as long as the inflation scale does not exceed the observational upper bound $H_{\rm inf} \lesssim 10^{14}$ GeV [8]. On the other hand, the condition of $T_{\rm max} < f$ [cf. (6)], without which the axion would temporarily vanish after inflation and thus would not be able to source the perturbation, imposes a lower bound on the inflation scale as

$$H_{\rm inf} \gtrsim 10^{11}~{\rm GeV}. \qquad (16)$$

Hence our scenario is compatible not only with high scale inflation producing observably large primordial gravitational waves, but also with inflation happening at lower scales. We remark that the lower bound (16) is further relaxed if the inflaton does not decay instantaneously and yields $T_{\rm max}$ lower than (7), or if the symmetry breaking scale is larger than the axion periodicity (possibly with a large domain wall number).

The decay width of the axion is constrained by the assumption of $R \gg 1$, and also from the requirement that the cosmic temperature after the decay be higher than ~ 4 MeV so as not to spoil big bang nucleosynthesis [11]. The former condition restricts the axion-photon coupling α from above, and the latter from below. These bounds become more stringent for lower inflation scales; for $H_{\rm inf} = 10^{11}(10^{14})$ GeV the coupling is bounded as

$$10^{-11}(10^{-13}) \lesssim \alpha \lesssim 10^{-2}(10^3). \qquad (17)$$

We also note that if α greatly exceeds unity then the effective theory would break down at $T = T_{\rm max}$.

Discussion.—We showed that an axionlike particle coupled to a new confining sector can generate the primordial density perturbation of our Universe. The axion decay constant and strong coupling scale are uniquely determined by observations in terms of the inflation scale as (15), which, in the minimal model, entail a temporal deconfinement of the gauge group after inflation.

Let us comment on the observational consequences. The axionlike scenario predicts a *positive* local-type non-Gaussianity of order unity, $f_{\rm NL} \sim 1$, on large scales. This is in contrast to single-field inflation which yields a much smaller local $f_{\rm NL}$, and also to a vanilla curvaton which produces $f_{\rm NL} = -5/4$ in the dominating limit ($R \to \infty$). These values of non-Gaussianity are within reach of upcoming large-scale structure surveys [12]. We also remark that the axion lies within $\pi/2 < |\theta| < \pi$ and sources the red spectral tilt when the cosmological scales exit the horizon, and then rolls down typically to the region $|\theta| < \pi/2$ by the end of inflation. This implies that the density perturbation spectrum becomes blue tilted at small scales and thus is enhanced. Likewise, $f_{\rm NL}$ runs towards negative values at small scales. These features could be tested observationally using, for instance, ultracompact minihalos [13].

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