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The strong transfer paradox in an overlapping generations framework *

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Abstract

It is shown that in an overlapping generations model, a strong transfer paradox occurs through permanent transfer in a dynamically efficient region because of international capital mobility. A graphical explanation is also provided to show how the strong paradox arises.

Key words: Transfer paradox; Overlapping generations model; Golden rule; International Capital Mobility

JEL classification: F21, F35, F43, O11, O41

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1 Introduction

This paper examines how international capital mobility brings about transfer paradoxes in Galor and Polemarchakis's (1987) overlapping generations model. The analysis is confined to the case where two countries have the same economic circumstances except time preferences, and the economy is dynamically efficient. In this instance, when a permanent transfer is carried out, a strong paradox arises such that the recipient country becomes worse off and the donor country better off. Weak paradoxes can also occur where both countries are simultaneously better or worse off.

In a static framework, Samuelson (1952, 1954) argued that a strong transfer paradox may occur in a Walrasian stable economy if the transfer can be used as a factor of production. This implies that in a dynamic framework, because the transfer affects capital accumulation (or individuals' savings), a strong paradox might occur even when the economy is dynamically efficient and Walrasian stable.

Galor and Polemarchakis (1987) first showed that the strong paradox can occur in a steady state 'away from the golden rule' through a permanent, lump-sum transfer, using a two-country overlapping generations model. Subsequently, Haaparanta (1989) proved that in a steady state, a strong paradox can occur when the 'temporary' transfer is financed by public debt in the donor country and/or is used for debt relief in the recipient country in a dynamically efficient region.

In brief, Haaparanta argued (i) "... the transfer has long-run effects only to the extent that it affects the public debt... in either of the countries." This implies that the debt-financed temporary transfer (or transfer for debt relief) has the same effect on individuals' welfare as a permanent, lump-sum transfer. Haaparanta, however, also suggested that (ii) "Galor and Polemarchakis... show that the transfer paradox... can, in the steady state, arise only if the capital stock is larger than the golden rule level." But statements (i) and (ii) are somewhat contradictory: if (i) is correct, the transfer paradox should arise through the permanent, lump-sum transfer, even if the capital stock is 'less than' the golden rule level[1].

The objective of this paper is to resolve this contradiction by showing that the strong paradox can occur in the steady state by a permanent transfer in a dynamically efficient region in a Walrasian stable economy. One source of this result can be attributed to the

[1]In addition, the definition of the capital stock is, more or less, unclear. In later sections, it will become apparent.

existence of international capital mobility, or in other words, 'foreign' debt, as indicated by Haaparanta (1989) and Buiter (1981). A simple graphical explanation is carried out to show how the strong paradox arises.

2 The source of the strong transfer paradox

The framework used in this paper is a two-country, overlapping generations model as used in Galor and Polemarchakis (1987).

Assume in each period there are two types of individuals: the young who supply their labor inelastically and earn wages, and the old who retire and consume savings accumulated in the young period. All individuals live for both periods and the population growth rate is $n > -1$. Individuals in country i (i=A, B) in period t choose consumption in their young and old periods, c^i_{1t} and c^i_{2t+1}, so as to maximize their utility, $u^i(c^i_{1t}, c^i_{2t+1})$, subject to the budget constraints in their respective young and old periods:

$$c^i_{1t} + s^i_t = w_t + t^i \quad and \quad c^i_{2t+1} = r_{t+1}s^i_t, \tag{1}$$

or the intertemporal budget constraint:

$$c^i_{1t} + \frac{1}{r_{t+1}}c^i_{2t+1} = w_t + t^i, \tag{2}$$

where r, w and s are the interest rate, wage and savings, respectively, and t^i is a permanent transfer (without loss of generality, we assume that country A is the recipient whereas country B is the donor). This maximization yields the savings function, $s^i(w_t, r_{t+1}; t^i)$, and an indirect utility function, $V^i(w_t, r_{t+1}; t^i)$. Further assuming that the marginal utility of income is one for the purpose of analytical convenience, total differentiation of this indirect utility function in the steady state gives:

$$dV^i = dw + \frac{s}{r}dr + dt^i. \tag{3}$$

The effects of the transfer on utility can be separated into two parts. The first is the direct, or transfer effect appearing in the third term on the right-hand side. The second is the indirect, or dynamic efficiency effect as shown in the first and the second terms on the right-hand side.

Firms maximize their profit in aggregate terms as $F(K_t, L_t) - r_tK_t - w_tL_t$ or in per capita terms as $f(k_t) - r_tk_t - w_t$, where $f(k_t)$ is a constant-returns-to-scale, neoclassical production function (K and L represent aggregate capital and labor and k and l per capita capital and labor, respectively). As usual, the first-order conditions are:

$$f'(k_t) = r_t \quad and \quad f(k_t) - f'(k_t)k_t = w_t. \tag{4}$$

Full internatinal capital mobility is assumed: capital (or savings) flows from the lower time-preference country to the higher time-preference country, so that factor prices in the two countries are equalized. By treating international capital mobility explicitly, the world capital market equilibrium condition in the steady state can be expressed as $k^A(r) + k^B(r) = s^A(w, r; t^A) + s^B(w, r; t^B)$. Noting that the level of per capita capital in each country is the same, $k^A(r) = k^B(r) \equiv k(r)$ ($k(r)$ therefore represents per capita world capital), we obtain:

$$2(1+n)k(r) = s^A(w, r; t^A) + s^B(w, r; t^B). \tag{5}$$

Here we have to note the difference of expression for the capital market equilibrium conditions from equation (14) in Galor and Polemarchakis (1987). More particularly, with the explicit introduction of international capital mobility, the left-hand side of (5), or capital demand, becomes $2(1+n)k(r)$ instead of $(1+n)k(r)$ as in Galor and Polemarchakis (1987)[2]. Totally differentiating (5) gives:

$$\left[2(1+n) - (s_r^A + s_r^B)f_{kk} - (s_w^A + s_w^B)f_{lk}\right] dr = (s_t^A + s_t^B)f_{kk}dt, \tag{6}$$

where $s_t^A > 0$ and $s_t^B < 0$ and we use $dw = f_{lk}k_r dr$. When the economy monotonically converges to the stable steady state equilibrium, $H \equiv (s_r^A + s_r^B)f_{kk} - (s_w^A + s_w^B)f_{lk} < 2(1+n)$ will hold.

Finally, substituting (6) into (3), we obtain:

$$\frac{dV^A}{dt} = M(s_t^A + s_t^B)\left[\frac{s^A}{k} - f_k\right] + 1, \tag{7}$$

$$\frac{dV^B}{dt} = M(s_t^A + s_t^B)\left[\frac{s^B}{k} - f_k\right] - 1, \tag{8}$$

where $M \equiv \frac{1}{2(1+n)-H}\left[\frac{f_{kk}k}{f_k}\right] < 0$. It should be noted that the first terms on the right-hand side of (7) and (8) correspond to the indirect effects of the permanent transfer and the second terms (1 and -1, respectively) represent the direct effects.

[2] Buiter (1981) argued, "... it is essential to distinguish between the capital stock used in production in the home country and the value of the claims on real capital, domestic or foreign, owned by domestic residents..." The world capital market equilibrium condition, (5), is, in fact, the same as in Buiter (1981) and Haaparanta (1989).

Lemma *If the time-preference in country A is higher than (equal to, less than) country B, then* $s_t^A + s_t^B < (=, >) 0$, $\frac{s^A}{k} < (=, >) 1 + n$ *and* $\frac{s^B}{k} > (=, <) 1 + n$ *hold.*

Note *In a one-country framework where the capital market equilibrium condition,* $s/(1+n) = k$, *holds, if the economy is dynamically efficient the terms in the square bracket on the right-hand side of (7) necessarily become* $(1 + n) - f_k < 0$.

As a result, we have the following proposition:

Proposition *Assume that country A and B have the same economic circumstances except time preferences and the economy is dynamically efficient,* $1+n < f'(k)$. *If a permanent transfer occurs, both strong and weak transfer paradoxes can occur.*

Proof: Without loss of generality, we assume that the time-preference in country A is higher than that in country B. From Lemma, because $\frac{s^A}{k} < 1 + n < f_k$ holds, $\frac{s^A}{k} - f_k$ is negative. Therefore, the first term on the right-hand side of (7) as the indirect effect becomes negative. In total, the sign of (7) becomes ambiguous and depends on the configuration of the negative indirect effect and the positive direct effect. Regarding the sign of (8), because $\frac{s^B}{k}$ can be above $1 + n$, or possibly, above f_k, the first term on the right-hand side of (8) as the indirect effect can, on the contrary, be positive. In total, the sign of (8) is ambiguous.

If the negative indirect effect dominates the positive direct effect in (7), and the positive indirect effect dominates the negative direct effect in (8), a strong transfer paradox can occur. Alternatively, if the direct effect dominates the indirect effect in (7), and the positive indirect effect dominates the negative direct effect in (8), a weak (welfare-improving) transfer paradox occurs. The same procedure can be used to obtain a weak (welfare-deteriorating) transfer paradox.

As in Lemma, even if the level of capital in a lower time-preference country is less than that of the golden rule, the level of savings can exceed the capital stock associated with the golden rule. One of the main reasons why such an 'over-accumulation' of capital (or individuals' savings) is possible is the existence of international capital mobility. This corresponds to the justification given by Galor and Polemarchakis (1987), "... it is the divergence of optimal rates of interest across countries which... may allow for the transfer paradox or the phenomenon of a Pareto improving transfer."

3 A graphical explanation

Depicting the stationary consumption possibility locus in Buiter's (1981) two-country framework helps us understand how the strong paradox occurs. To draw the locus, we utilize two equations: one is the intertemporal budget constraint of individuals; the other is the world capital market equilibrium condition.

Firstly, substituting (1) and (4) into (2) and eliminating s_t^i gives the individuals' intertemporal budget constraint in both countries depending on c_{1t}^i, c_{2t+1}^i, k_t and k_{t+1}. Evaluating this intertemporal budget constraint in the steady state, we obtain:

$$2(f(k) - kf'(k)) - (c_1^A + c_1^B) = \frac{c_2^A + c_2^B}{1 + f'(k)}. \tag{9}$$

Next, substituting (1) and (4) into (5) and evaluating it in the steady state provides the world capital market equilibrium condition:

$$2(f(k) - kf'(k)) - (c_1^A + c_1^B) = 2(1 + n)k. \tag{10}$$

Finally, (9) and (10) give the stationary (average) consumption possibility locus in an open economy, OF, as shown in Figure 1[3]. Here we assume, as in Buiter (1981), that the locus is strictly concave towards the vertical axis for $c_1^i, c_2^i > 0$. As k increases, the stationary average consumption point moves from O to F and the slope of the intertemporal budget constraint through the point, $f'(k)$, becomes flatter. Point E^G represents the level of k at the steady state, where $f'(k) = 1 + n$ holds, and therefore, the slope tangent to the locus is $-(1 + n)$. The economy is dynamically efficient when the point is on the line between O and E^G, and dynamically inefficient when the point is on the line between E^G and F.

Consider the original steady state before the permanent transfer in a dynamically efficient region. If the stationary average consumption point is at E^{NW}, and the time-preference in country A is higher than in country B, then the stationary consumption points of country A and B are E^{NA} and E^{NB} respectively, on the line with slope of $-f'(k^N)$. It should be noted that the distances from E^{NA} to E^{NW} and from E^{NB} to E^{NW} are the same, because the world capital market equilibrium (or the trade balance) must hold.

When a permanent transfer is carried out from country B to country A, the stationary average consumption point moves towards the southwest, E^{TW}, and the slope through

[3]Strictly speaking, the locus OF represents the stationary consumption locus for the hypothetical average individual in the economy with a lifetime consumption pattern $((c_1^A + c_1^B)/2,\ (c_2^A + c_2^B)/2)$.

the point becomes steeper, $-f'(k^T)$. As country A receives the transfer permanently, the intertemporal budget constraint of that country locates to the right of the stationary average consumption locus with the distance of t^A. Therefore, the stationary consumption point of country A becomes E^{TA} and, in the same way, that of country B becomes E^{TB}[4].

As shown in Figure 1, when E^{TA} is southwest of E^{NA} and E^{TB} is northeast of E^{NB}, the strong paradox occurs. Such a situation possibly occurs when the distance between E^{NW} and E^{NB} (or equivalently, between E^{NW} and E^{NA}) is sufficiently long. This implies a large volume of international capital movement.

4 Conclusion

The main source of transfer paradoxes, both strong and weak, is the existence of international capital mobility. International lending will permit the 'over-accumulation' of individuals' savings compared to the golden rule level in a lower time-preference country, and as a result, will bring about paradoxical effects.

[4]The distances from E^{TA} to E^{TW} and from E^{TB} to E^{TW} should be the same because world capital market equilibrium must again hold following the permanent transfer.

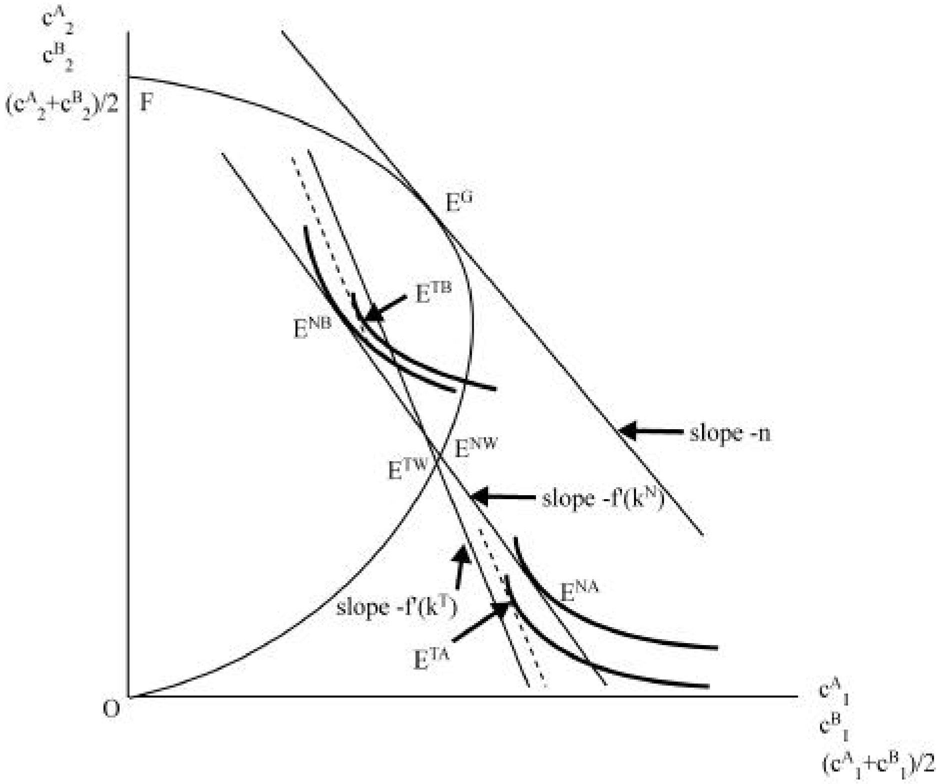

Figure 1: The stationary consumption possibility locus in a two-country framework

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