Vlasov simulation of Langmuir decay instability

Takayuki Umeda[a] and Tetsuya Ito
Solar-Terrestrial Environment Laboratory, Nagoya University, Nagoya, Aichi 464-8601, Japan

(Received 12 May 2008; accepted 8 July 2008; published online 12 August 2008)

A parametric decay process of beam-generated Langmuir waves is examined by a one-dimensional Vlasov code simulation in the open system. It is confirmed that pump Langmuir waves decay into backscattered Langmuir and ion acoustic waves when the wave energy is higher than the thermal energy of background electrons. In the present Vlasov simulation, a large-amplitude pump Langmuir wave is excited by an electron beam. Then a backscattered Langmuir wave and a forward ion acoustic wave are excited by the parametric decay instability. When the wave energy of the backscattered Langmuir waves is also higher than the thermal energy of background electrons, backward ion acoustic waves are also excited. As a result, the wavenumber spectra of Langmuir and ion acoustic waves are broadened in both forward and backward directions, suggesting possible multiple parametric decay processes of Langmuir waves in a beam-plasma system. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2965494]

Langmuir waves or electron plasma waves are one of the most fundamental normal mode waves in plasmas. Langmuir waves are commonly observed in laser-produced plasma experiments in which an intense laser light wave decays into a backscattered light wave and a Langmuir wave by the stimulated Raman scattering. Langmuir waves are subject to another three-wave parametric instability called the Langmuir decay instability whereby an intense Langmuir wave decays into a backscattered Langmuir wave and an ion acoustic wave. Excitation of ion acoustic waves by the Langmuir decay instability is observed in laboratory plasma experiments with an intense laser (e.g., Ref. 1).

Spacecraft *in situ* observations have also shown that Langmuir waves are ubiquitous in space plasmas.[2,3] Langmuir waves in space plasmas are commonly excited by electron-beam-plasma interactions. Since strongly modulated waveforms of Langmuir waves were frequently observed in space plasmas, the relationship between the amplitude modulation and the Langmuir decay instability have been discussed in Ref. 4. A number of self-consistent kinetic plasma simulations of electron-beam-plasma interactions have also been performed to reproduce the amplitude modulated Langmuir waves.[5–7]

A classic picture of the Langmuir decay instability is that a large-amplitude parent Langmuir wave (L) decay into another backscattered Langmuir (L') and ion acoustic (IA') daughter waves, i.e., $L \rightarrow L' + IA'$. In this case the resonance condition of the three wave interaction is written as $\omega_L = \omega_{L'} + \omega_{IA'}$ and $\vec{k}_L = \vec{k}_{L'} + \vec{k}_{IA'}$, where ω and $\vec{k}$ are wave frequency and wavenumber vector, respectively. For the decay of Langmuir waves into ion acoustic waves, the amplitude of parent Langmuir waves should be large enough so that ion distributions can be modified. The theoretical threshold amplitude of parent Langmuir waves for the decay process was derived in Ref. 8.

It is expected from spacecraft observations that Langmuir waves may decay into ion plasma waves in the auroral ionosphere[2] in which the amplitude of Langmuir waves is very high and background electrons are strongly magnetized. In contrast, the amplitude of Langmuir waves observed in the solar wind, in the electron foreshock region, and in the magnetotail is not so high.[3] That is, the electric field energy of Langmuir waves is comparable or smaller than the thermal energy of background electrons. The background electrons are also weakly magnetized ($\omega_{ce}/\omega_{pe} < 1.0$) in these regions.

The recent kinetic simulations have confirmed that Langmuir waves are not subject to the parametric decay instability when the electric field energy is smaller than the thermal energy of background electrons.[5,6] In these simulation studies, the authors assumed a very weak electron beam with a density ratio of the beam component $R = n_b/(n_e + n_b) \ll 1.0\%$, where the subscripts b and e represent beam electrons and background electrons, respectively. Matsukiyo *et al.*[7] performed a particle-in-cell simulation with a higher density ratio $R = 1.0\%$ and reproduced an observed waveform of Langmuir waves in the auroral upward current region. However, detailed mechanisms are still unclear.

In the present study we performed a one-dimensional electrostatic Vlasov code simulation of an electron-beam-plasma interaction to examine a basic process on the decay of Langmuir waves into ion acoustic waves. We take a one-dimensional simulation domain along an ambient magnetic field. The spatio-temporal development of position-velocity phase-space distribution function $f(x,v)$ of plasma particles is simulated by solving the Vlasov–Poisson system equations. Thus the effects of the ambient magnetic field and electromagnetic waves are completely neglected. The Vlasov simulation code uses a nonoscillatory and conservative cubic polynomial interpolation scheme[9] for stable time-integration of phase-space distribution functions.

We assume that a weak electron beam is drifting against the major background electrons and ions. A Langmuir wave is excited by an electron-beam instability due to the positive

[a]Electronic mail: umeda@stelab.nagoya-u.ac.jp.

gradient in the electron velocity distribution function. We assume that the electron beam, background electrons and ions have Maxwellian velocity distributions. The density ratio of the beam component is set as 0.5%, One may think that the density ratio $R=0.5\%$ is a small number. However, this is much higher than the density ratio used in the previous simulation studies.[5,6] We assume that the beam and background electrons have the equal thermal velocity $V_{te}=V_{tb}=1.0$. The total electron plasma frequency is assumed as $\omega_{pe}=1.0$. The beam drift velocity V_d is set as $8.0V_{te}$. Mobile ions are also evolved in order to confirm the effect of ion dynamics on the Langmuir decay instability. The ion-to-electron mass ratio is set as $m_i/m_e=1836$, and the ion-to-electron temperature ratio is set as $T_i/T_e=m_iV_{ti}^2/m_eV_{te}^2=0.1$, where the subscripts i represents background ions. The density of ions is given by $n_i=n_e+n_b$. The number of spatial grid cells is $N_x=16384$. The number of velocity grid cells is $N_{v_x}=4096$ over a velocity range from $v_{\max}=24.0V_{te}$ to $v_{\min}=-16.0V_{te}$ for electrons and from $v_{\max}=13.0V_{ti}$ to $v_{\min}=-13.0V_{ti}$ for ions. The grid spacing is equal to $\Delta x=0.2\lambda_e$ ($\lambda_e \equiv V_{te}/\omega_{pe}$), and the time step is equal to $\omega_{pe}\Delta t=0.005$. Note that $\epsilon_0=1$ is used for simplicity.

Here we use a more realistic simulation model in which an electron beam is injected into the system from an open boundary while outgoing plasmas and waves are absorbed without reflection at boundaries (e.g., Refs. 6 and 10). As the initial condition, we assume that the background plasma exists uniformly in the simulation domain without the electron beam. When a computer simulation is started, the electron beam is continuously injected from the left boundary into the background homogeneous plasma. The injected electron beam and the background electrons form an unstable velocity distribution function. In the present study, we inject the electron beam with a constant flux. The background electrons and ions are assumed to be continuous at the right boundary as if there is no boundary. The background ions are also assumed to be continuous at the left boundary, while the flux of background electrons at the left boundary is modified to keep the charge neutrality.

Figure 1 shows the energy density of electric field $|E_x|^2$ as a function of position (x) and time (t). An electron beam is continuously injected from the left boundary into the unperturbed plasma, resulting in an unstable velocity distribution function near the left boundary. Langmuir waves are excited as the electron beam propagates to the right. The Langmuir waves lead to the formation of a single wave packet in a localized region close to the left boundary, which is consistent with the previous simulation in the open system.[6] The wave packet propagates at the group velocity of the primary Langmuir mode ($v_g \sim 0.6V_{te}$), which is much slower than the beam velocity. The energy density of the electron beam is $n_bm_e(V_d^2+V_{tb}^2)=0.325n_em_eV_{te}^2$, while the wave energy density of the wave packet exceeds the thermal energy of background electrons ($>1.0n_em_eV_{te}^2$). Since the electron beam is continuously injected, the free energy source is continuously supplied to excited Langmuir waves. As a result, the amplitude of excited Langmuir waves in the open system becomes much higher than that in the uniform periodic system.[6,11]

From $\omega_{pe}t \sim 1200$, there appears to be a different wave mode for $x/\lambda_e=300-500$. The wavelength of this mode is shorter than the wavelength of Langmuir waves. This wave mode propagates to the right at a very slow group velocity $V_g \sim 0.03V_{te}$ which is close to the ion acoustic speed, and this wave mode disappears around $\omega_{pe}t \sim 2000$. Note that this wave mode with the shorter wavelength was not excited in the previous simulation with a lower density ratio $R=0.1\%$,[6] in which the wave energy density of Langmuir waves was lower than the thermal energy density of background electrons ($\sim 0.4n_em_eV_{te}^2$). These results indicate the decay of Langmuir waves into ion acoustic waves. From $\omega_{pe}t \sim 2000$, another packet of this wave mode appears at $x/\lambda_e \sim 200$. The envelope of this wave packet propagates to the left. The wave energy density of the wave packet exceeds $2.0n_em_eV_{te}^2$, while the amplitude of the Langmuir waves for $x/\lambda_e>300$ becomes much smaller with the wave energy density ($\sim 10^{-4}n_em_eV_{te}^2$).

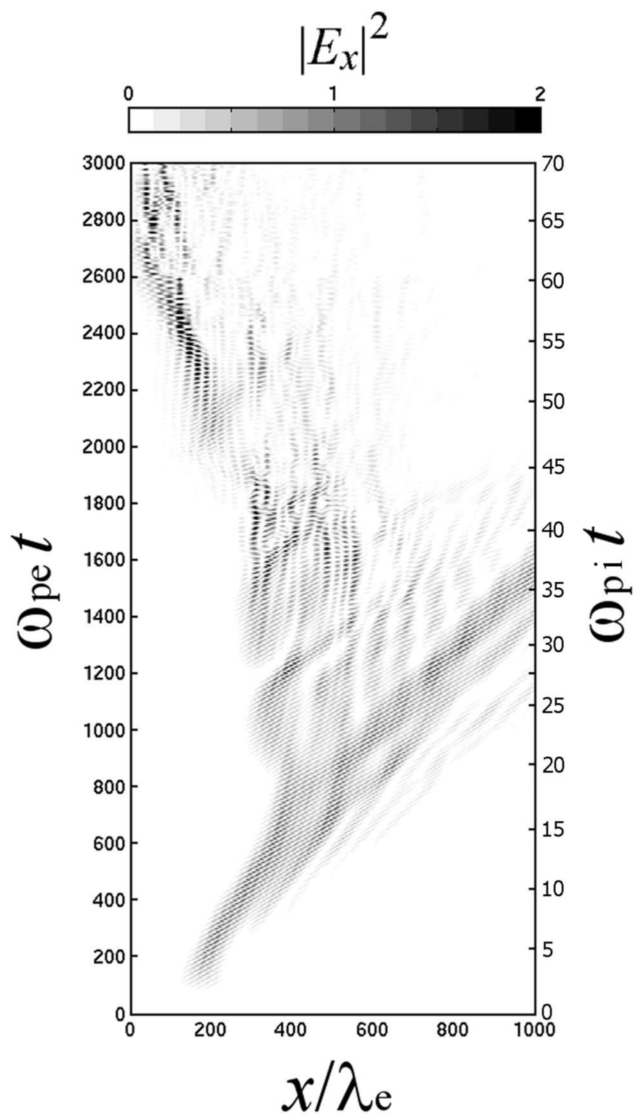

FIG. 1. Electric field energy density as a function of position and time. The electric field energy density is normalized by $n_em_eV_{te}^2$.

Phase-space distribution functions of electron and ions at different times are shown in Fig. 2. At $\omega_{pe}t=800$, we found

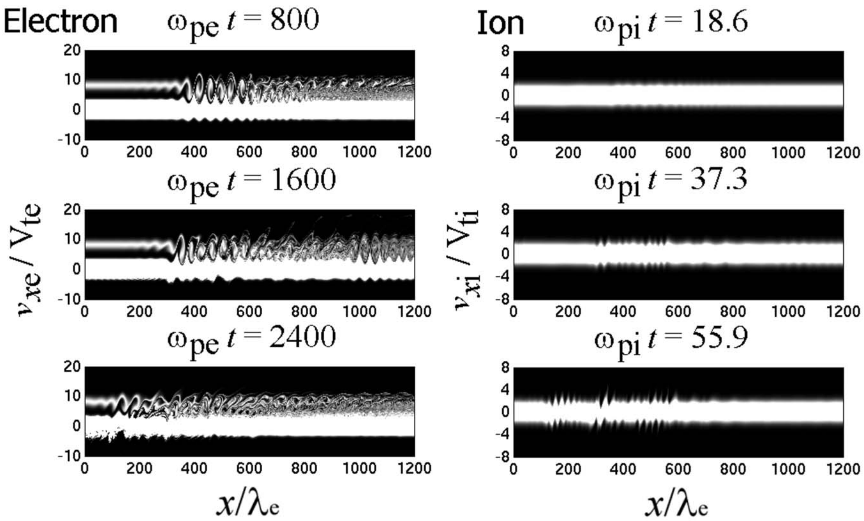

FIG. 2. The $x-v_x$ phase-space density of electrons and ions at $\omega_{pe}t$= 800, 1600, and 2400. Here $V_{ti}=0.00738V_{te}$ and $\omega_{pi}=0.0233\omega_{pe}$.

electron phase-space vortices for $x/\lambda_e=400-600$ which corresponds to the Langmuir wave packet. The ion distribution is also modulated at a spatial scale of $x/\lambda_e \sim 40$, which is equal to the wavelength of the excited Langmuir waves ($<2\pi V_d/V_{te}$). The ion distribution is easily modulated by the Langmuir wave packet because the amplitude is high enough.

At $\omega_{pe}t=1600$, there appears a strong modulation of the ion distribution at a shorter spatial scale of $x/\lambda_e \sim 20$ for $x/\lambda_e=300 \sim 600$. This structure corresponds to the wave mode with a shorter wavelength seen in Fig. 1, and is regarded as an ion acoustic wave. At the same time, we found a small modification in the negative velocity range of the electron distribution, implying that a backscattered electron plasma wave is also excited. These aspects are consistent with the classical picture of the Langmuir decay instability. At $\omega_{pe}t=2400$, the electron distribution is strongly modified in the negative velocity range. This modification is due to a packet of large-amplitude backscattered waves seen in Fig. 1. Strong modulation of the ion distribution is also broadened for $x/\lambda_e=100-600$, suggesting that a backward ion acoustic wave is excited.

Figure 3 shows the frequency-wavenumber spectra of electric field $|E(\omega,k)|$ for different time intervals, which are obtained by taking Fourier transformation of electric field data in the generation region of the short-wavelength mode where ions are strongly modulated ($x/\lambda_e=200-600$). For $\omega_{pe}t=400-1000$, we found the enhancement of Langmuir waves ($\omega/\omega_{pe} \sim 1.0$) and weak beat waves ($\omega/\omega_{pe}=0-0.1$).[7] The wavenumber range of the excited Langmuir waves is $k\lambda_e=0.12-0.25$ with a peak at $k\lambda_e=0.15$. The group velocity of the excited Langmuir waves is approximated as $v_g = \partial\omega/\partial k \sim 0.6v_{te}$, which is consistent with Fig. 1.

The $\omega-k$ spectrum for $\omega_{pe}t=1200-1800$ shows enhancement of forward low-frequency waves at $\omega/\omega_{pe} \sim 0.02$ which is close to the ion plasma frequency. The wavenumber range of the low-frequency waves is $k\lambda_e=0 \sim 0.4$. Langmuir waves are also scattered in a wide wavenumber range ($k\lambda_e=-0.2-0.1$) in Fig. 3(b). Thus the three-wave resonance condition, $\omega_L=\omega_{L'}+\omega_{IA'}$ and $k_L=k_{L'}+k_{IA'}$, can be satisfied. For $\omega_{pe}t=2400-3000$, we found strong enhancement of backscattered Langmuir waves ($\omega/\omega_{pe} \sim 1$). The spectrum of ion acoustic waves ($\omega/\omega_{pe} \sim 0.02$) is also enhanced in the negative wavenumber range. This result suggests a possible secondary decay instability of backscattered Langmuir waves, i.e., $L' \rightarrow L''+IA''$. The excitation of backward ion acoustic waves is consistent with the bottom panel of Fig. 2, in which modulation of the ion distribution is broadened to the $-x$ direction.

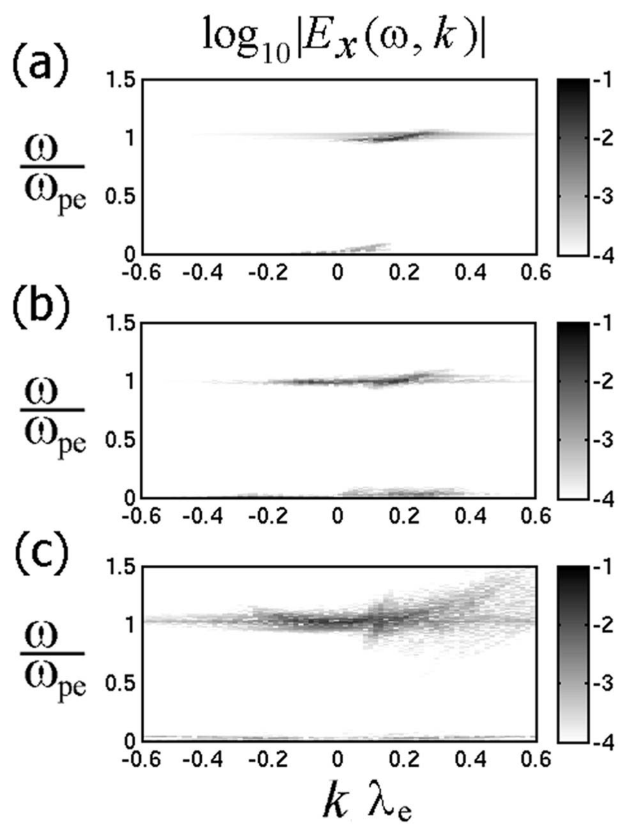

FIG. 3. Frequency-wavenumber spectra of electric field $|E_x(\omega,k)|$ for different time intervals. (a) $\omega_{pe}t=400-1000$. (b) $\omega_{pe}t=1200-1800$. (c) $\omega_{pe}t=2400-3000$. The electric field energy density is normalized by $n_e m_e V_{te}^2$.

In this letter, we have studied the decay instability of beam-driven Langmuir waves into ion acoustic waves. The simulation result is summarized as follows:

1. The decay instability of Langmuir waves into ion acoustic waves takes place when the wave energy of the Langmuir waves is higher than the thermal energy of background electrons. The decay instability evolves in the time scale of several ion plasma periods.

2. The Langmuir decay instability in a nonuniform system involves excitation of large-amplitude backscattered Langmuir waves, excitation of forward and backward ion acoustic waves, and effective damping of pump Langmuir waves. The wavenumber spectra of Langmuir and ion acoustic waves are broadened in a wide range in both forward and backward directions, suggesting a possible multiple decay instability. This might be a similar process to the broadening of wavenumber spectra observed in the laser-produced plasma experiments of stimulated Raman scattering (e.g., Ref. 1).

The excitation of large-amplitude backscattered Langmuir waves may lead to excitation of electromagnetic waves by another three-wave interaction, $L+L' \rightarrow T$.[12] The excitation of electromagnetic waves, however, is not taken into account in the present electrostatic limit, and is addressed by a future electromagnetic simulation.

This work was supported by a Grant-in-Aid for Young Scientists (Start-up) No. 19840024 from JSPS and in part by Grant-in-Aid for Creative Scientific Research No. 17GS0208 from MEXT of Japan. The computer simulation was performed on the Fujitsu PRIMEPOWER HPC2500 at ITC in Nagoya University and RISH in Kyoto University as a collaborative computational research project at STEL in Nagoya University and RISH in Kyoto University.

[1]S. Depierreux, J. Fuchs, C. Labaune, A. Michard, H. A. Baldis, D. Pesme, S. Huller, and G. Laval, Phys. Rev. Lett. **84**, 2869 (2000); S. Depierreux, C. Labaune, J. Fuchs, D. Pesme, V. T. Tikhonchuk, and H. A. Baldis, *ibid.* **89**, 045001 (2002).

[2]R. E. Ergun, C. W. Carlson, J. P. McFadden, J. H. Clemmons, and M. H. Boehm, Geophys. Res. Lett. **18**, 1177, DOI:10.1029/91GL01563 (1991); K. Stasiewicz, B. Holback, V. Krasnoselskikh, M. Boehm, and P. Kinter, J. Geophys. Res. **101**, 21515, DOI:10.1029/96JA01747 (1996); J. Bonnell, P. Kintner, J.-E. Wahlund, and J. A. Holtet, *ibid.* **102**, 17233, DOI:10.1029/97JA01499 (1997); G. V. Lizunov, Y. Khotyaintsev, and K. Stasiewicz, *ibid.* **106**, 24755, DOI:10.1029/2001JA900074 (2001); R. Pottelette, R. E. Ergun, R. A. Treumann, M. Berthomier, C. W. Carlson, J. P. McFadden, and I. Roth, Geophys. Res. Lett. **26**, 2629, DOI:10.1029/1999GL900462 (1999).

[3]D. A. Gurnett, G. B. Hospodarsky, W. S. Kurth, D. J. Williams, and S. J. Bolton, J. Geophys. Res. **98**, 5631, 10.1029/92JA02838 (1993); P. J. Kellogg, S. J. Monson, K. Goetz, R. L. Howard, J.-L. Bougeret, and M. L. Kaiser, Geophys. Res. Lett. **23**, 1243, DOI:10.1029/96GL01067 (1996); P. J. Kellogg, K. Goetz, S. J. Monson, and S. D. Bale, J. Geophys. Res. **104**, 17069, DOI:10.1029/1999JA900163 (1999); H. Kojima, H. Furuya, H. Usui, and H. Matsumoto, Geophys. Res. Lett. **24**, 3049, DOI:10.1029/97GL03043 (1997); J. Soucek, V. Krasnoselskikh, T. Dudok de Wit, J. Pickett, and C. Kletzing, J. Geophys. Res. **110**, A08102, DOI:10.1029/2004JA010977 (2005).

[4]I. H. Cairns and P. A. Robinson, Geophys. Res. Lett. **19**, 2187, DOI:10.1029/92GL02632 (1992); I. H. Cairns, P. A. Robinson, and N. I. Smith, J. Geophys. Res. **103**, 287, DOI:10.1029/97JA02871 (1998); B. Li, A. J. Willes, P. A. Robinson, and I. H. Cairns, Phys. Plasmas **10**, 2748 (2003).

[5]L. Muschietti, I. Roth, and R. E. Ergun, J. Geophys. Res. **100**, 17481, 10.1029/95JA00595 (1995); **101**, 15605, DOI:10.1029/96JA00926 (1996); K. Akimoto, Y. Omura, and H. Matsumoto, Phys. Plasmas **3**, 2559 (1996); H. Usui, H. Furuya, H. Kojima, H. Matsumoto, and Y. Omura, J. Geophys. Res. **110**, A06203, 10.1029/2004JA010703 (2005); T. Umeda, Phys. Plasmas **13**, 092304 (2006); I. Silin, R. Sydora, and K. Sauer, *ibid.* **14**, 012106 (2007).

[6]T. Umeda, Nonlinear Processes Geophys. **14**, 671 (2007).

[7]S. Matsukiyo, R. A. Treumann, and M. Scholer, J. Geophys. Res. **109**, A06212, DOI:10.1029/2004JA010477 (2004).

[8]P. A. Robinson, A. J. Willes, and I. H. Cairns, Astrophys. J. **5**, 720 (1993); P. A. Robinson and I. H. Cairns, Geophys. Res. Lett. **22**, 2657, DOI:10.1029/95GL01779 (1995).

[9]T. Umeda, Earth, Planets Space **60**, 773 (2008); T. Umeda, M. Ahsour-Abdalla, and D. Schriver, J. Plasma Phys. **72**, 1057 (2006).

[10]T. Umeda, Y. Omura, H. Matsumoto, and H. Uusi, J. Geophys. Res. **107**, 1449, DOI:10.1029/2001JA000286 (2002).

[11]Y. S. Sigov and V. D. Levchenko, Plasma Phys. Controlled Fusion **38**, A49 (1996).

[12]B. Li, A. J. Willes, P. A. Robinson, and I. H. Cairns, Phys. Plasmas **12**, 012103 (2005); B. Li, P. A. Robinson, and I. H. Cairns, Phys. Rev. Lett. **96**, 145005 (2006).