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# Joint Channel Decoding of Spatially and Temporally Correlated Data in Wireless Sensor Networks

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**Abstract**

In densely deployed wireless sensor networks, sensor observations are spatially correlated. Furthermore, the nature of physical phenomena constitutes a temporal correlation between transmitted observations of an individual sensor node. In this paper, we propose a joint iterative channel decoding scheme using turbo codes. The proposed decoder exploits the spatial and temporal correlations of two binary data sequences to achieve additional coding gain. Simultaneously exploiting the spatial and temporal correlation, the proposed decoder achieves large performance gain.

## 1. Introduction

In wireless sensor networks, a large number of sensor nodes are densely deployed in a sensing field of interest. The distributed sensor nodes transmit their observations over wireless noisy channels to a sink node, where appropriate processing and analyses of the collected observations are performed. Due to the high density in the sensing field, the observations are spatially correlated. Furthermore, the nature of physical phenomena constitutes a temporal correlation between the transmitted observations of the individual sensor node. Such spatial correlation and temporal correlations are significant and unique characteristics of the wireless sensor networks, which can be exploited to enhance the overall network performance [1].

One of the basic challenges in the wireless sensor networks is the reliable transmission of the correlated data available at the distributed sensor nodes. In this scenario, the spatial and temporal correlations of the transmitted data can be regarded as redundant information which can be used for source compressions and channel error correction by joint decoding [2, 3]. For the scenario of spatially correlated data, the Slepian-Wolf source coding problem [4] is actually a channel coding one and turbo codes and LDPC codes have been employed to approach the Shannon and Slepian-Wolf limit in [5] and [6]. The design of the iteratively decodable codes for the transmission of temporally correlated data has also been studied in [7], [8] and [9]. Especially in [9], the authors proposed a joint decoding scheme that allows transmit power control at the sensor nodes, which may be suitable for the sensor networks with limited resources. However, in the above mentioned schemes, the use of the spatial and temporal correlations has been discussed separately. The problem of designing good practical coding schemes to simultaneously exploit the spatial and temporal correlations of multiple sources is still open.

In this paper, we propose a joint iterative decoding scheme using turbo codes. The proposed decoder exploits the spatial and temporal correlation of two binary data sequences to achieve additional coding gain. This will be an extension of the schemes introduced in [5] and [9] (but we do not discuss the transmit power control here). In Section 2, we give a brief description of the system and the correlation structure. The algorithm of the proposed joint iterative decoder is described in Section 3. In Section 4, we present simulation results confirming the potential gain that can be obtained from our approach. Finally, Section 5 renders some conclusions.

## 2. System Description

Figure 1 depicts the outline of the distributed coding and joint decoding system for two sensor nodes and one sink node. In one time interval, two correlated data sequences will be encoded, transmitted, received and decoded. The transmitting data are spatially correlated with each other

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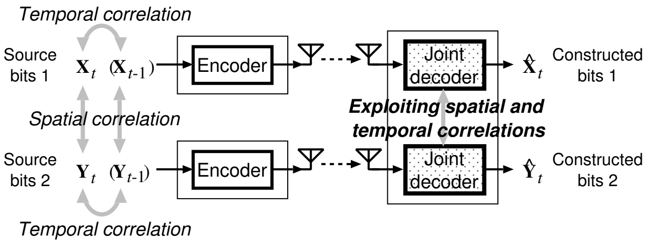


Figure 1: Outline of the distributed coding and joint decoding system.

and temporally correlated with the previously transmitted data. Both the sensor nodes encode the correlated data sequences independently, so that the encoders do not communicate with each other and have no knowledge of the correlation. The encoded data are transmitted over independent noisy channels. At the sink node, sufficient statistics for both the received sequences are processed jointly. The receiver uses an iterative soft decision decoding technique for the joint detection of each of the transmitted data. The algorithm of the proposed joint decoder is described in detail in Section 3.

Let $\mathbf{X}_t$ and $\mathbf{Y}_t$ be transmitted data sequences of the sensor node 1 and 2 at the current time $t$. We denote the $k$-th data bit of $\mathbf{X}_t$ as $x_{t,k} \in \{+1, -1\}$ and that of $\mathbf{Y}_t$ as $y_{t,k} \in \{+1, -1\}$, $(k = 1, 2, \cdots, K)$ where $K$ is the length of the each sequence. For convenience, we map a logic-0 to the integer $-1$, logic-1 to $+1$. We assume that the each sequence is a set of equiprobable binary random variables which are independent for any value of $k$; $\mathrm{P}(x_{t,k} = \pm 1 \,|\, x_{t,l(\neq k)}) = \mathrm{P}(x_{t,k} = \pm 1) = 1/2$ and $\mathrm{P}(y_{t,k} = \pm 1 \,|\, y_{t,l(\neq k)}) = \mathrm{P}(y_{t,k} = \pm 1) = 1/2$. The same holds for the previously transmitted sequences $\mathbf{X}_{t-1}$ and $\mathbf{Y}_{t-1}$ at the previous time $t-1$.

In this paper, we consider a simple correlation structure to illustrate key concepts. The approach presented here can be extended to capture more elaborate correlation structures. We consider the specific case where the correlated data sequence are captured as following. At the current time $t$, the transmitted sequences $\mathbf{X}_t$ and $\mathbf{Y}_t$ are spatially correlated. The degree of the spatial correlation can be characterized by $\frac{1}{K}\sum_{k=1}^{K}(x_{t,k} \cdot y_{t,k})$, which takes value 1 if the identical data bits are transmitted and value 0 if half of the transmitted bits is different from each other. By invoking the ergodicity, namely that the time average for a large number $K$ can be replaced by the expectation, we define the spatial correlation coefficient as $\rho_s = \mathcal{E}\{x_{t,k} y_{t,k}\}$ for any value of $k$. Given the spatial correlation coefficient $\rho_s$, the joint probability $\mathrm{P}(x_{t,k}, y_{t,k})$ is given as $\mathrm{P}(x_{t,k} = \pm 1, y_{t,k} = \pm 1) = (1 + \rho_s)/4$ and $\mathrm{P}(x_{t,k} = \mp 1, y_{t,k} = \pm 1) = (1 - \rho_s)/4$. The same holds for the previously transmitted sequences $\mathbf{X}_{t-1}$ and $\mathbf{Y}_{t-1}$. Furthermore, the transmitted sequence $\mathbf{X}_t$ is temporally correlated with the previously transmitted sequence $\mathbf{X}_{t-1}$. We similarly define the temporal correlation coefficient as $\rho_t = \mathcal{E}\{x_{t,k} x_{t-1,k}\}$ for any value of $k$. Given the temporal correlation coefficient $\rho_t$, the joint probability $\mathrm{P}(x_{t,k}, x_{t-1,k})$ is given as $\mathrm{P}(x_{t,k} = \pm 1, x_{t-1,k} = \pm 1) = (1 + \rho_t)/4$ and $\mathrm{P}(x_{t,k} = \mp 1, x_{t-1,k} = \pm 1) = (1 - \rho_t)/4$. The same holds for the other sequences $\mathbf{Y}_t$ and $\mathbf{Y}_{t-1}$.

For brevity of notation, we will not indicate the second subscript $k$ of variables in the following (only where needed for clarification).

## 3. Joint Iterative Decoder Design

We propose a joint iterative channel decoder using turbo codes for the spatially and temporally correlated data. The structure of the proposed decoder is depicted in Figure 2 (interleavers are omitted for brevity). In the proposed decoder, two turbo decoders are jointed with each other, exchanging side information in an iterative manner. The side information is available in the form of decoder soft outputs of the other sequence and that of the previously received sequence. At every decoding iteration, the spatial and temporal correlation component is extracted from the side information and feedback to the turbo decoder as additional a priori information. At the last iteration, the side information will be stored in the buffer and utilized for decoding the following sequences. We make a minor modification to let the two turbo decoders work in parallel and let the component maximum a posteriori (MAP) decoders exploit the spatial and temporal correlation of the transmitted data. The structure of the extended MAP decoder is depicted in Figure 3.

Let us consider the problem of decoding the data sequence $\mathbf{X}_t$ in the presence of the side information present at the decoder. The following algorithm holds true for decoding $\mathbf{Y}_t$ by exchanging $\mathbf{X}$ $(x)$ and $\mathbf{Y}$ $(y)$. Since the analytical treatment of the BCJR-MAP decoder [10] is difficult, we make use of the following assumption [11]: Extrinsic outputs from the MAP decoder and a priori inputs to the MAP decoder can be modeled by an independent Gaussian random variable. This assumption suggests that for the MAP decoding of $\mathbf{X}_t$, the soft output relating to an data bit $x_t$ is given in the form of

$$S_{x_t} = \mu_{S_{x_t}} \cdot x_t + n_{S_{x_t}}\,, \quad \mu_{S_{x_t}} = \sigma^2_{S_{x_t}}/2\,, \tag{1}$$

where $n_{S_{x_t}}$ is Gaussian distributed with mean zero and variance $\sigma^2_{S_{x_t}}$. For the joint decoding of $x_t$, the side information $S_{y_t}$ contains the spatial correlation component and the side information $S_{x_{t-1}}$ contains the temporal correlation component.

In this scenario, given entire received channel outputs $\mathbf{r}_{x_t}$ (i.e., received discrete time signals for data bits and re-

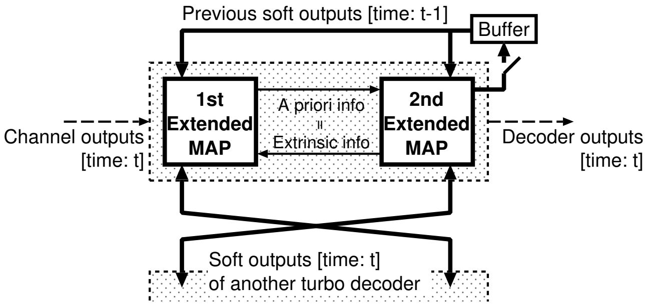


Figure 2: Structure of the proposed joint turbo decoder for the spatially and temporally correlated data.

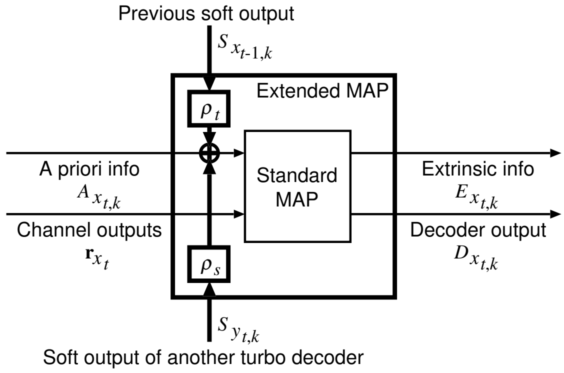


Figure 3: Structure of the extended MAP decoder exploiting the spatial and temporal correlation.

spective parity bits) and the side information $S_{y_t}$ and $S_{x_{t-1}}$, the extended MAP decoder calculates the log likelihood ratio of a posteriori probability $\mathrm{P}(x_t \,|\, \mathbf{r}_{x_t}, S_{y_t}, S_{x_{t-1}})$ for an data bit $x_t$, i.e.,

$$D_{x_t} = \log \frac{\mathrm{P}(x_t = +1 \,|\, \mathbf{r}_{x_t}, S_{y_t}, S_{x_{t-1}})}{\mathrm{P}(x_t = -1 \,|\, \mathbf{r}_{x_t}, S_{y_t}, S_{x_{t-1}})} = Z_{x_t} + A_{x_t} + A'_{x_t} + E_{x_t} \,. \tag{2}$$

The channel information $Z_{x_t}$ and the a priori information $A_{x_t}$ are defined as

$$Z_{x_t} = \log \frac{\mathrm{P}(z_{x_t} \,|\, x_t = +1)}{\mathrm{P}(z_{x_t} \,|\, x_t = -1)} \,, \quad A_{x_t} = \log \frac{\mathrm{P}(x_t = +1)}{\mathrm{P}(x_t = -1)} \,, \tag{3}$$

where $z_{x_t}$ represents the received discrete time signal of $x_t$. For the received discrete time signal from the AWGN channel, $Z_{x_t}$ is formulated similarly to Eq. (1) with variance $\sigma^2_{Z_{x_t}} = 4/\sigma^2_n$ ($\sigma^2_n$: double-sided noise power spectral density).

The decoder output $D_{x_t}$ can be calculated by viewing the sum of $A_{x_t}$ and $A'_{x_t}$ as the a priori input to the standard MAP algorithm such as BCJR-MAP and Max-Log-MAP [12]. In decoding iterations, the extrinsic information $E_{x_t}(= D_{x_t} - A_{x_t} - A'_{x_t} - Z_{x_t})$ is passed through a bit interleaver to become the a priori input $A_{x_t}$ of the next MAP decoder. Since we assume the extrinsic information and the a priori information to be Gaussian distributed, the side information $S_{y_t}$ is obtained by the decoder soft output of $y_t$, i.e., $S_{y_t} = Z_{y_t} + A_{y_t} + E_{y_t}$, and updated in every iteration. Note, however, that the side information $S_{x_{t-1}}(= Z_{x_{t-1}} + A_{x_{t-1}} + E_{x_{t-1}})$ is a fixed value in all iterations because of being the previous decoder output.

The additional a priori information $A'_{x_t}$ is calculated by using the side information $S_{y_t}$ and $S_{x_{t-1}}$, as follows:

$$\begin{aligned} A'_{x_t} &= \log \frac{\mathrm{P}(S_{y_t}, S_{x_{t-1}} \,|\, x_t = +1)}{\mathrm{P}(S_{y_t}, S_{x_{t-1}} \,|\, x_t = -1)} \\ &= \log \frac{\sum_{y_t=\pm1, x_{t-1}=\pm1} e^{(y_t \cdot S_{y_t} + x_{t-1} \cdot S_{x_{t-1}})/2}\, \mathrm{P}(y_t, x_{t-1} \,|\, x_t = +1)}{\sum_{y_t=\pm1, x_{t-1}=\pm1} e^{(y_t \cdot S_{y_t} + x_{t-1} \cdot S_{x_{t-1}})/2}\, \mathrm{P}(y_t, x_{t-1} \,|\, x_t = -1)} \,, \end{aligned} \tag{4}$$

where the probability $\mathrm{P}(y_t, x_{t-1} \,|\, x_t)$ is determined by the spatio-temporal correlation of the transmitted data sequences. When the spatial and temporal correlations are characterized only by the coefficient $\rho_s$ and $\rho_t$, the additional a priori input $A'_{x_t}$ can be simplified to

$$\begin{aligned} A'_{x_t} &\approx \log \frac{e^{S_{y_t}} \cdot (1 + \rho_s) + (1 - \rho_s)}{e^{S_{y_t}} \cdot (1 - \rho_s) + (1 + \rho_s)} \\ &+ \log \frac{e^{S_{x_{t-1}}} \cdot (1 + \rho_t) + (1 - \rho_t)}{e^{S_{x_{t-1}}} \cdot (1 - \rho_t) + (1 + \rho_t)} \,. \end{aligned} \tag{5}$$

This transformation allows us to calculate the additional a priori inputs only from the correlation coefficients and the decoder soft outputs without knowing the details of the correlation structure.

## 4. Numerical Examples

We have conducted simulations of the proposed joint iterative channel decoder for spatially and temporally correlated data sequences to assess the potential gain of the approach. Simulation results are conducted for a rate-1/3 turbo code obtained from a parallel concatenation of two recursive systematic convolutional encoders with code polynomials $(G_r, G) = (025, 035)$ given in octal form, where $G_r$ stands for the feed back. The length of the transmitted data bits is fixed to $K = 1000$ and uniform random interleavers of the same length are employed for the individual turbo

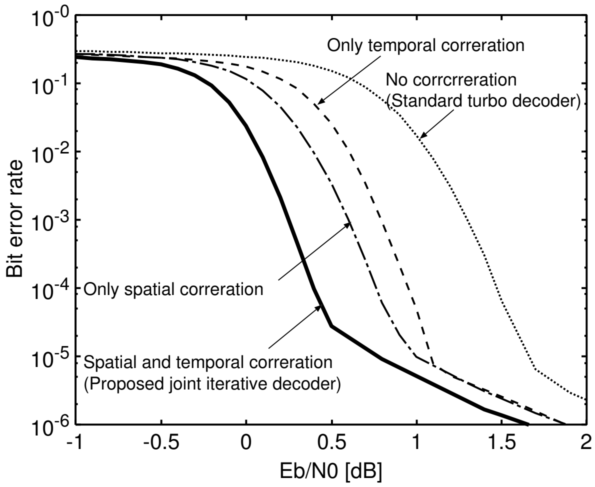


Figure 4: Bit error rate performance of the proposed decoder at time $t = 1$ in the case of the spatial correlation coefficient $\rho_s = 0.6$ and the temporal correlation coefficient $\rho_t = 0.6$.

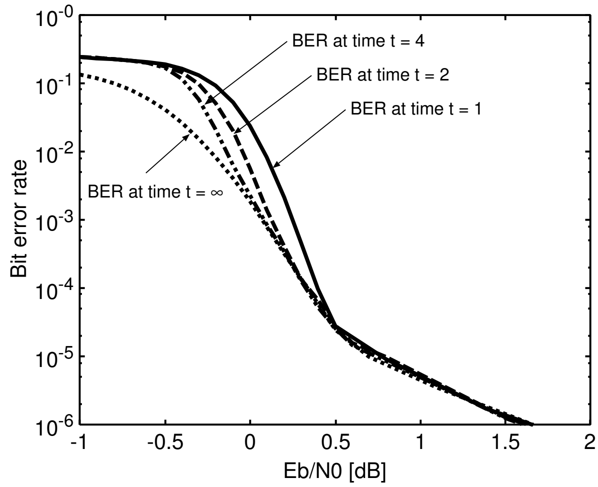


Figure 5: Bit error rate performance of the proposed decoder for various time $t = 1, 2, 4, \infty$ in the case of the spatial correlation coefficient $\rho_s = 0.6$ and the temporal correlation coefficient $\rho_t = 0.6$.

codes. For iterative decoding, five iterations of the Max-Log-MAP algorithm are applied. To assess the average performance, energy per transmitted data bit in the individual AWGN channels is set to the same so that the channel capacity will be the same. The spatial correlation coefficient $\rho_s$ and the temporal correlation coefficient $\rho_t$ are assumed to be known at the decoder.

We present simulation results of the bit error rate in the case of the spatial correlation coefficient $\rho_s = 0.6$ and the temporal correlation coefficient $\rho_t = 0.6$. Figure 4 shows the bit error rate performance of the proposed decoder at time $t = 1$. The performance of the joint decoder exploiting only spatial correlation and that of the joint decoder exploiting only temporal correlation are depicted in the same figure. At time $t = 0$ (the first data transmission), there are no previously received sequences, so that the proposed decoder exploits only the spatial correlation. At time $t = 1$ (the second data transmission), the received sequences are temporally correlated with the received sequences at time $t = 0$. In this case, the proposed decoder exploits both the spatial and temporal correlation and achieves larger performance gain than that achieved at time $t = 0$. As shown in Figure 4, simultaneously exploiting the spatial and temporal correlation, the proposed decoder achieves larger performance gain than that achieved by exploiting only either the spatial or temporal correlation.

To have an idea of the maximum achievable performance of the proposed decoder, we show the bit error rate performance for various time $t = 1, 2, 4, \infty$ in Fig. 5. At time $t = \infty$, the previously received data are perfectly decoded and the most reliable side information about the temporal correlation are exploited for the joint decoding. In this case, the second term of Eq. (5) is modified as

$$\mathrm{sign}(S_{x_{t-1}}) \cdot \log \frac{1 + \rho_t}{1 - \rho_t} \,. \tag{6}$$

As shown in Fig. 5, near maximum performance can be achieved in the region of BER $< 10^{-3}$ at time $t = 2$ (the third data transmission).

## 5. Conclusions

We have presented a joint iterative channel decoding scheme for decoding turbo encoded correlated data. The proposed decoder exploits the spatial and temporal correlation of the data sequences to achieve additional coding gain. Simulation results suggest that simultaneously exploiting the spatial and temporal correlation, the proposed decoder achieves large performance gain. Furthermore, simulations suggest that near maximum performance can be achieved at a relatively small number of data transmissions. We have considered a simple correlation structure to illustrate key concepts. The approach presented here can be extended to capture more elaborate correlation structures.


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