

# Generating Lane-Change Trajectories of Individual Drivers

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**Abstract**—This paper describes a method to generate vehicle trajectories of lane change paths for individual drivers. Although each driver has a consistent preference in the lane change behavior, lane-changing time and vehicle trajectory are uncertain due to the presence of surrounding vehicles. To model this uncertainty, we propose a statistical driver model. We assume that a driver plans various vehicle trajectories depending on the surrounding vehicles and then selects a safe and comfortable trajectory. Lane change patterns of each driver are modeled with a hidden Markov model (HMM), which is trained using longitudinal vehicle velocity, lateral vehicle position, and their dynamic features. Vehicle trajectories are generated from the HMM in a maximum likelihood criterion at random lane-changing time and state duration. Experimental results show that vehicle trajectories generated from the HMM included a similar trajectory to that of a target driver.

## I. INTRODUCTION

The number of driver's license holders and cars is increasing each year, and cars have obviously become indispensable for our daily lives. To improve safety and road traffic efficiency, intelligent transportation system (ITS) technologies, including adaptive cruise control (ACC), lane-keeping assist systems (LKAS), and driver warning system have been developed over the last several years [1]–[5]. Many of these methods directly estimate the acceleration or throttle angle and have resulted in successful estimation accuracy. Many of conventional works on lane change behavior have focused on inferring lane change intent [6]–[8]. However, works on predicting vehicle trajectories have hardly been investigated.

In this paper, we propose a method to generate vehicle trajectories in lane-changing for individual drivers. Longitudinal vehicle velocity and lateral vehicle position patterns of a target driver are modeled using a hidden Markov model (HMM), and personalized vehicle trajectories are generated from the statistical driver model of the target driver. Given random lane-changing time and state duration, vehicle trajectories are estimated in a maximum likelihood criterion from the HMM. Contrary to conventional methods [9] that model a vehicle trajectory using several functions such as cosine and polynomial functions on the condition that the lane-changing time is fixed, our method estimates various vehicle trajectories from the HMM-based driver model on the condition that the lane-changing time is uncertain. The

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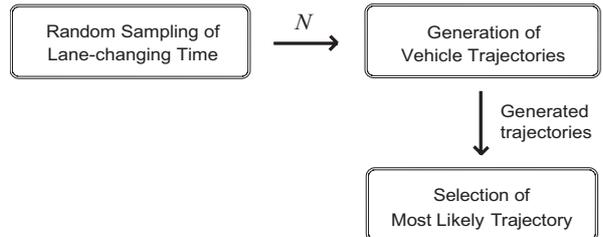


Fig. 1. Generation process of vehicle trajectory

generated vehicle trajectory is evaluated using driving signals collected in a simulator for two professional drivers who are both driving instructors.

## II. GENERATION OF VEHICLE TRAJECTORY

The process to generate vehicle trajectories is shown in Fig. 1. The process is composed of the following three parts.

- Random sampling of lane-changing time
- Generation of possible vehicle trajectories (HMM-based driver model)
- Selection of the most likely trajectory

First, lane-changing time  $N$  and state transition sequence  $\mathbf{q}$  of an HMM are determined at random. Then, various vehicle trajectories are generated from the HMM-based driver model according to  $N$  and  $\mathbf{q}$  as shown in Fig. 2. The method to generate a vehicle trajectory is the same as speech synthesis [10]. First state  $q_1$  corresponds to the period before changing lanes, second state  $q_2$  while changing lanes, and third state  $q_3$  after changing lanes. A vehicle trajectory is generated from each state  $q_1$ ,  $q_2$ , and  $q_3$  given state transition sequence  $\mathbf{q}$  determined by state duration of each state  $d_{q_1}$ ,  $d_{q_2}$ , and  $d_{q_3}$ . Next, the generated vehicle trajectories are evaluated for their relative positions. Finally, the most likely trajectory is selected based on the evaluation. This generation process corresponds to the assumption that a driver plans various vehicle trajectories and then selects a safe and comfortable trajectory.

### A. Features Modeled by HMM

An HMM models feature  $\mathbf{o}_n$ , consisting of longitudinal vehicle velocity  $\dot{x}_n^{v_0}$  and lateral vehicle position  $y_n^{v_0}$  with their first and second-order dynamics  $\Delta \dot{x}_n^{v_0}$ ,  $\Delta y_n^{v_0}$  and  $\Delta^2 \dot{x}_n^{v_0}$ ,  $\Delta^2 y_n^{v_0}$ :

$$\mathbf{o}_n = (\dot{x}_n^{v_0}, y_n^{v_0}, \Delta \dot{x}_n^{v_0}, \Delta y_n^{v_0}, \Delta^2 \dot{x}_n^{v_0}, \Delta^2 y_n^{v_0})', \quad (1)$$

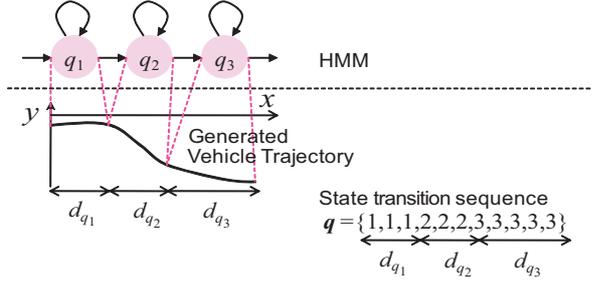


Fig. 2. Illustration of vehicle trajectory generation from HMM

where  $n$  and  $V0$  represent indices of time and a target vehicle, respectively. We define static feature  $\mathbf{c}_n$  as follows.

$$\mathbf{c}_n = (\dot{x}_n^{v0}, y_n^{v0})' \quad (2)$$

Then, the feature vector  $\mathbf{o}_n$  becomes

$$\mathbf{o}_n = (\mathbf{c}'_n, \Delta \mathbf{c}'_n, \Delta^2 \mathbf{c}'_n)', \quad (3)$$

where the dynamic features are calculated as follows.

$$\Delta \mathbf{c}_n = -0.5\mathbf{c}_{n-1} + 0.5\mathbf{c}_{n+1} \quad (4)$$

The second-order dynamic features are similarly calculated from the first-order dynamic features based on (4).

### B. Sampling Lane-changing Time and State Sequence

Since lane-changing time and state duration in an HMM are uncertain, we determine them at random.

1) *Random Lane-Changing Time*: We choose lane-changing time  $N_i$  according to a normal distribution  $\mathcal{N}(\mu, \sigma^2)$ , where  $i$  represents the index of random trials, and  $(\mu, \sigma^2)$  are the mean and variance of the lane-changing time that are calculated from training data.

2) *Random State Duration in HMM*: Furthermore, we choose the state duration at random for a fixed lane-changing time  $N_i$  according to a state duration probability distribution given as a normal distribution  $\mathcal{N}(\xi_k, \tau_k^2)$ , where  $(\xi_k, \tau_k^2)$  are the mean and variance of  $k$ -th state duration, which are estimated from a Viterbi alignment for training data.

The state duration can be estimated for a fixed lane-changing time  $N_i$  as follows:

$$d_{q_k, i} = \xi_{q_k} + \left( N_i - \sum_{k=1}^K \xi_{q_k} \right) \frac{\tau_{q_k}^2}{\sum_{k=1}^K \tau_{q_k}^2}, \quad (5)$$

where  $K$  is the number of states in an HMM. To diversify the state duration, the standard deviation  $\tau_{q_k}$  is replaced by a random number  $u_{q_k, j}$  which follows a uniform distribution  $(0, 1]$ , where  $j$  represents the index of random trials. The random state duration becomes

$$d_{q_k, i, j} = \xi_{q_k} + \left( N_i - \sum_{k=1}^K \xi_{q_k} \right) \frac{u_{q_k, j}^2}{\sum_{k=1}^K u_{q_k, j}^2}. \quad (6)$$

### C. Generation of Possible Vehicle Trajectories

A vehicle trajectory is estimated based on the following equation [10].

$$\hat{\mathbf{C}} = \arg \max_{\mathbf{C}} \log P(\mathbf{O} | \mathbf{q}_{i, j}, \lambda, N_i), \quad (7)$$

where  $\mathbf{q}_{i, j}$  is the state transition sequence determined by (6) for a given  $N_i$ , and  $\lambda$  represents HMM parameters. Here,  $\mathbf{O}$  and  $\mathbf{C}$  are the following vectors.

$$\mathbf{O} = (\mathbf{o}'_1, \mathbf{o}'_2, \dots, \mathbf{o}'_{N_i})' \quad (8)$$

$$\mathbf{C} = (\mathbf{c}'_1, \mathbf{c}'_2, \dots, \mathbf{c}'_{N_i})' \quad (9)$$

We also have to estimate the longitudinal position of the vehicle. The longitudinal position is calculated using the generated longitudinal vehicle velocity as follows:

$$x_{i, j}^{v0}(n+1) = x_{i, j}^{v0}(n) + \dot{x}_{i, j}^{v0}(n) \cdot T_s, \quad (10)$$

where  $T_s$  is a sampling period.

### D. Selection of The Most Likely Vehicle Trajectory

We select the most likely vehicle trajectory using relative positions to other vehicles. The relative positions are calculated using the generated trajectories as follows:

$$f_{i, j}^{v1}(n+1) = f_{i, j}^{v1}(n) + (\dot{x}^{v1}(0) - \dot{x}_{i, j}^{v0}(n)) \cdot T_s \quad (11)$$

$$f_{i, j}^{v2}(n+1) = f_{i, j}^{v2}(n) + (\dot{x}^{v2}(0) - \dot{x}_{i, j}^{v0}(n)) \cdot T_s \quad (12)$$

$$f_{i, j}^{v3}(n+1) = f_{i, j}^{v3}(n) + (\dot{x}_{i, j}^{v0}(n) - \dot{x}^{v3}(0)) \cdot T_s, \quad (13)$$

where  $f^{v1}$  represents following distance in a driving lane, and  $f^{v2}$  and  $f^{v3}$  represent distances to the leading and following vehicles in a passing lane, respectively. The values  $V1$ ,  $V2$ , and  $V3$  are indices of the surrounding vehicles, as shown in Fig. 3. We assume that longitudinal vehicle velocities of a lead vehicle in a driving lane  $\dot{x}^{v1}$ , and leading and following vehicles in a passing lane  $\dot{x}^{v2}$  and  $\dot{x}^{v3}$  are constant; i.e., longitudinal vehicle velocities are the same as those just before lane-changing  $\dot{x}^{v1}(0)$ ,  $\dot{x}^{v2}(0)$ , and  $\dot{x}^{v3}(0)$ , respectively. First, the generated trajectories for  $N_i$  and  $\mathbf{q}_{i, j}$  are penalized by the number of the points when the generated trajectories do not satisfy either one of the following three conditions,

$$\sqrt{(f_{i, j}^{v1}(n))^2 + (y_{i, j}^{v0}(n))^2} > \theta_1 \quad (14)$$

$$\sqrt{(f_{i, j}^{v2}(n))^2 + (y_{i, j}^{v0}(n) + W)^2} > \theta_2 \quad (15)$$

$$\sqrt{(f_{i, j}^{v3}(n))^2 + (y_{i, j}^{v0}(n) - W)^2} > \theta_3, \quad (16)$$

where  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are thresholds for the penalization, and lane width  $W = 3.34$  m. Then, the penalty score  $S_{i, j}$  is normalized with the lane-changing time  $N_i$  as follows.

$$\tilde{S}_{i, j} = \frac{S_{i, j}}{N_i} \quad (17)$$

Here, three hazardous cases can be considered as shown in Fig. 4. For these three cases, generated trajectories are penalized because the target vehicle is too close to the surrounding vehicles. Equation (14) corresponds to case 1, (15) to case 2, and (16) to case 3. Next, the most likely

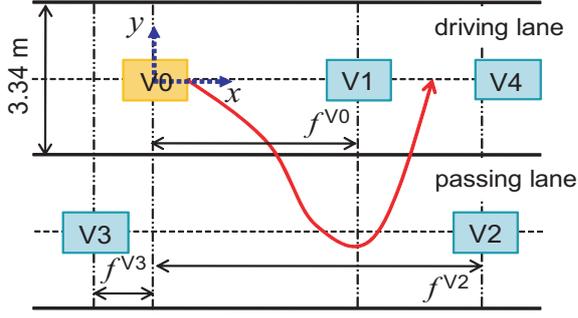


Fig. 3. Lane-change trajectory of target vehicle

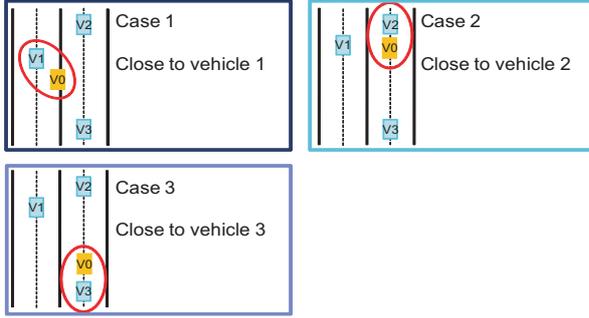


Fig. 4. Three cases of penalized trajectories

lane-changing time and the most likely state duration are estimated as follows:

$$(i^*, j^*) = \arg \min_{i,j} \tilde{S}_{i,j}, \quad (18)$$

where  $i^*$  and  $j^*$  represent the indices of the most likely lane-changing time and state duration, respectively.

### III. EXPERIMENT

#### A. Data Collection

We recorded observable driving signals using a driving simulator. The course was a straight expressway. Subjects were instructed to change lanes into the passing lane and then return to the driving lane, as depicted in Fig. 3. The subjects were allowed to change lanes whenever they wanted to. There were eighteen vehicles driving in the passing lane at different speeds. We recorded the driving signals for two different drivers thirty times.

#### B. Lane-Changing Label

We defined the endpoints of a lane-changing behavior as shown in Fig. 5. The lane-changing label starts when the relative distance  $f^{V2}$  becomes zero and ends when the lateral position  $y^{V0}$  becomes minimum. A sharp drop in the relative distance means that the target vehicle is passed by a vehicle in the passing lane.

#### C. Training of HMM

We trained an HMM for each driver in order to model characteristics of their driving behavior. The feature vector  $\mathbf{o}_n$  modeled by the HMM includes longitudinal vehicle velocity and lateral vehicle position with their first and

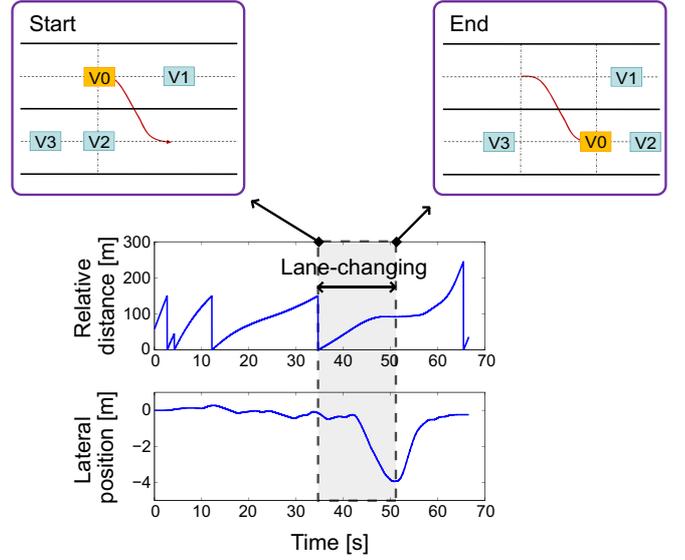


Fig. 5. Lane-changing label

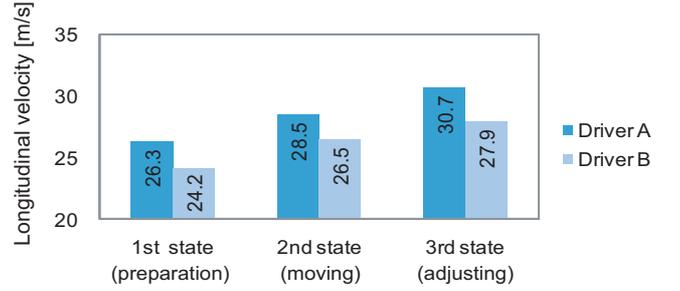


Fig. 6. Mean value of longitudinal velocity for each HMM state

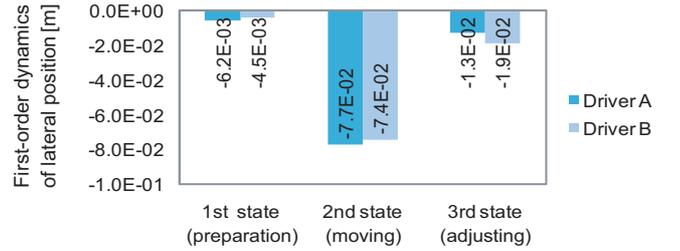


Fig. 7. Mean value of first-order dynamics of lateral position for each HMM state

second-order dynamics as shown in (1). There are three HMM states. The first state corresponds to the period before changing lanes (preparation), the second state corresponds to the period while changing lanes (moving), and the third state is the period after changing lanes (adjusting). We used full covariance Gaussians for the HMM to represent correlations among features.

The mean value of longitudinal velocity and first-order dynamics of the lateral position, i.e., lateral velocity, for each HMM state are shown in Figs. 6 and 7, respectively. Driver A tends to drive faster than driver B in the longitudinal direction. However, in the lateral direction, drivers A and B exhibit almost the same tendencies.

TABLE I  
CONDITIONS OF FIRST EXPERIMENT

# of drivers	2
Sampling period	0.16 sec.
Training data	Driving data of 20 trials
Test data	Driving data of 10 trials (3-fold cross validation)
Features	$\hat{x}_n^{v0}, y_n^{v0}, \Delta \hat{x}_n^{v0}, \Delta y_n^{v0}, \Delta^2 \hat{x}_n^{v0}, \Delta^2 y_n^{v0}$
# of HMM states	3
HMM covariance matrix	Full covariance matrix
# of trajectories	100

TABLE II  
CONDITIONS OF SECOND AND THIRD EXPERIMENTS

# of drivers	2
Sampling period	0.16 sec.
Training data	Driving data of 20 trials
Test data	Driving data of 10 trials (3-fold cross validation)
Features	$\hat{x}_n^{v0}, y_n^{v0}, \Delta \hat{x}_n^{v0}, \Delta y_n^{v0}, \Delta^2 \hat{x}_n^{v0}, \Delta^2 y_n^{v0}$
# of HMM states	3
HMM covariance matrix	Full covariance matrix
# of trajectories	20(lane-changing time) × 20(state duration)
$(\theta_1, \theta_2, \theta_3)$	(4m, 10m, 10m)

#### D. Experimental Conditions

Three experiments were conducted. The first experiment investigated whether or not a driver dependent model could represent the driver's characteristics. In this experiment, driver models for two drivers were swapped. Then, the driver dependent model and the swapped driver model were compared under the condition that a lane-changing time was the same as a reference (observed) trajectory, and the state duration was random. The second experiment investigated whether or not an appropriate trajectory was included among the generated trajectories under the condition that a lane-changing time and state duration were random. The third experiment investigated how well the selection criteria worked. The experimental conditions of the first experiment and those of the second and third experiments are respectively summarized in Tables I and II.

#### E. Experimental Results

We compared the generated trajectory with the reference (observed) trajectory of the target driver using three-fold cross validation. Ten driving data out of thirty were used as reference trajectories, and the rest were used for HMM training data. To objectively evaluate how well the HMM-based driver model could generate vehicle trajectories in lane-changing, a dynamic time warping (DTW) cost  $D(L, M)$  was calculated. The DTW-cost was defined as follows:

$$D(l, m) = \min \begin{cases} D(l-1, m) \\ D(l-1, m-1) \\ D(l, m-1) \end{cases} \quad (19)$$

$$+ \frac{1}{L+M-1} \left( \frac{(x_l^{v0} - \hat{x}_m^{v0})^2}{\sum_{n=1}^L (x_n^{v0})^2} + \frac{(y_l^{v0} - \hat{y}_m^{v0})^2}{\sum_{n=1}^L (y_n^{v0})^2} \right),$$

where  $L$  and  $M$  are the lengths of the reference and generated trajectories, and  $(x_l^{v0}, y_l^{v0})$  and  $(\hat{x}_m^{v0}, \hat{y}_m^{v0})$  are the

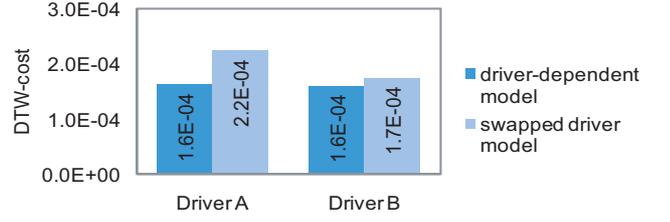


Fig. 8. DTW-cost when driver models are swapped

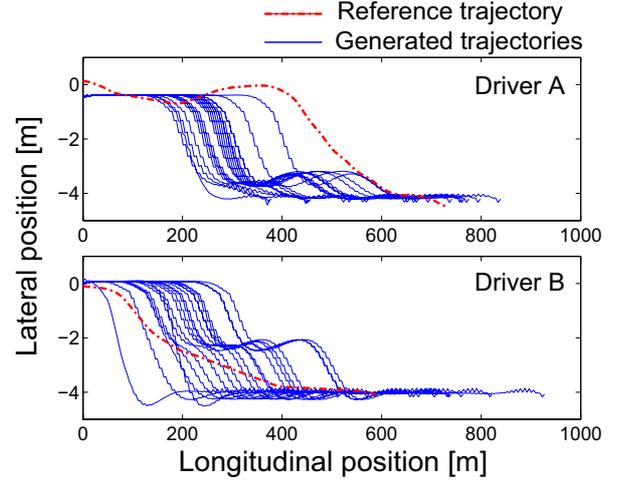


Fig. 9. Generated vehicle trajectories

vehicle positions of the reference and generated trajectories, respectively. The DTW-cost is normalized with the length of the DTW trajectory and the power of the vehicle position signal of the reference trajectory.

1) *First Experiment*: The lowest DTW-cost among all generated trajectories is shown in Fig. 8, when driver models are swapped and the lane-changing time is the same as the reference trajectory. The DTW-costs of their own driver models are lower than those of the swapped driver models. This result confirms that the HMM-based driver model for each driver can represent the driver's characteristics.

2) *Second Experiment*: Various trajectories were generated for different lane-changing times, as shown in Fig. 9. The best trajectory, which has the lowest DTW-cost among the generated trajectories, is plotted in Fig. 10, and the lowest DTW-cost among the generated trajectories is shown in Fig. 11. The generated trajectories include a similar trajectory to that of the target driver, and the DTW-cost is also low.

3) *Third Experiment*: The selected trajectory based on our selection criterion is shown in Fig. 12, and the DTW-cost of the selected trajectory is shown in Fig. 13. Although the DTW-cost of the selected trajectory is higher than that of the best of the generated trajectories, the selected vehicle trajectory follows the reference trajectory of the target driver.

## IV. CONCLUSION

We generated vehicle trajectories in lane-changing for individual drivers. The experimental results confirmed that

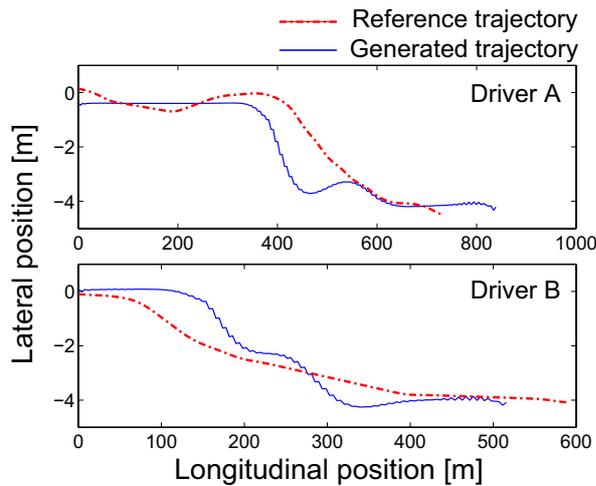


Fig. 10. Best of the generated vehicle trajectories



Fig. 11. DTW-cost of the best vehicle trajectories

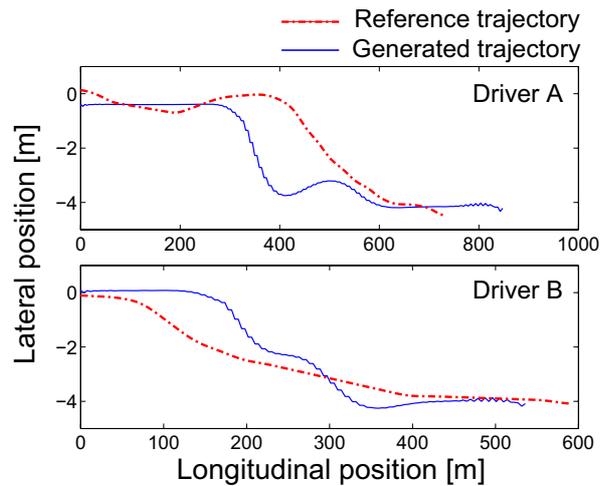


Fig. 12. Selected vehicle trajectories

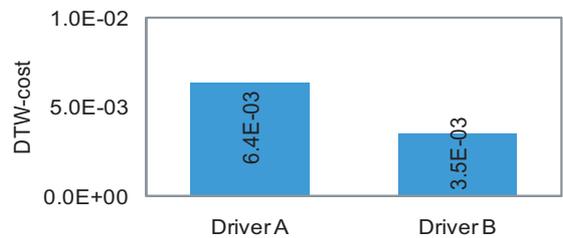


Fig. 13. DTW-cost of the selected vehicle trajectories

the HMM-based driver model was able to represent a driver's characteristics and generate lane-changing trajectories similar to those of the target drivers. In this paper, we estimated driver-specific parameters in the HMM-based driver models and confirmed their effectiveness. However, ad-hoc parameters were used for the selection of a vehicle trajectory. The selection of an appropriate trajectory is still brings difficulties. We plan to utilize driver-dependent penalties that are appropriate for individual drivers to help select vehicle trajectories and evaluate their effectiveness.

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