ON THE STRUCTURAL SAFETY BASED ON EXTREMUM THEORY

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ABSTRACT

Errors of the measured data are discussed as the factors which influence upon the accuracy of reliability analysis of the structural member. In the structural reliability analysis, failure probability is calculated by assuming probability density functions for material strength and external force, and following two basic assumptions have been employed as a matter of course; that is, ①characteristics of the measured data are reliable and are equal to the characteristics of the population from which the measured data have been extracted by chance; ②probability density functions are estimated directly based on the frequency distributions of the measured data. The error due to the assumption ①——error of characteristics ——is deemed as an error of the most likelihood estimation, and has a strong influence on the estimation of failure probability by reason that size of the measured data is usually very small. The error due to the assumption ②——error of distributions——also has a strong influence on the estimation of failure probability by reason that theoretical endorsement of the assumed density function is not obtained.

First, the error of characteristics is mainly discussed in Chapter 2, and the influence of the error on the estimation of failure probability is evaluated for normal distribution, gamma distribution and log-normal distribution. Then the definition of the usual failure probability is modified with regard to the error. On the contrary, error of distributions is considered unavoidable, and application of the procedure described in Chapter 2 is restricted to the measured data whose frequency distribution is known in advance following to one of three distributions as a result of phenomenology and/or properties of matter.

Second, new approach to the reliability analysis without any approximation of distributions for the measured data is proposed in Chapter 3 from the standpoint that error of distributions is hardly solved by extending the usual reliability theory. The content bears no relation to the structural reliability, and describes the statistical treatments of the measured data with the sole object of introducing and demonstrating the new approach. Practical upper bound of population of the measured data is defined based on the variational method under the various restrictive conditions characterizing the

measured data.

Third, extreme procedure of Chapter 3 is extended and applied to the estimation of structural safety in Chapter 4. Since characteristics introduced in Chapter 3 are unreliable to estimate the extremum, more reliable characteristics are newly introduced. The procedure realizes the structural design where the obtained failure probability never exceed the expected one even at the worst, in other words, where a sort of guarantee for the safety can be obtained based on the variational principle. In Chapter 4, error of characteristics is estimated approximately, and graphical estimation method of extremum is developed.

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NOTATION

A = characteristic of the measured data;

A_0 = parameter corresponding to the independent variation of $\bar{R}^*$ and $\bar{S}^*$;

A_S = cross-sectional area of the chord member of truss bridge;

$\bar{A}$ = mean of characteristic A;

$\bar{A}_L$ = lower bound of $\bar{A}$;

B = characteristic of the measured data;

B_0 = parameter corresponding to the independent variation of s_R^{2*} and s_S^{2*};

$\bar{B}$ = mean of characteristic B;

$\bar{B}_L$ = lower bound of $\bar{B}$;

C = characteristic of the measured data;

C_0 = parameter corresponding to A_0+B_0 for normal distribution and gamma distribution, to $A_0 B_0$ for log-normal distribution;

$\bar{C}$ = mean of characteristic C;

$\bar{C}_L$ = lower bound of $\bar{C}$;

$\mathrm{Cov}(X,\Delta X)$ = covariance between X and ΔX;

D = dummy characteristic, which is introduced in order to modify $X(F)$ compulsorily to a monotone increasing function;

dom = parameter, an abbreviation of 'difference of means';

F, $F(X)$ = cumulative function of $f(X)$;

F_i = i-th value of F;

ΔF_i = error of F_i;

$\overline{F_i}$ = mean of F_i;

$\widehat{F_i}$ = central value of F_i;

$\widetilde{F_i}$ = mode of F_i;

$\overline{\overline{F_i}}$ = median of F_i;

$f(X)$ = probability density for X;

f_{A_0} = probability density for A_0;

f_{B_0} = probability density for B_0;

f_{C_0} = probability density for C_0;

f_{dom} = probability density for dom;

f_R = probability density for R;

f_S = probability density for S;

f_{sov} = probability density for sov;

f_t = probability density of t-distribution;

f_{χ^2} = probability density of χ^2-distribution;

f_μ = probability density for population mean;

f_{μ_R}, f_{μ_S} = probability densities for population means of R and S;

f_{σ^2} = probability density for population variance;

$f_{\sigma_R^2}$, $f_{\sigma_S^2}$ = probability densities for population variances of R and S;

$g(F_i)$ = probability density of $F_i = F(X_i)$;

g_t, $\bar{g}_s$ = equations defined by Eqs.(3.2.3) and (3.2.4);

J = functional;

L_ℓ = live load;

N = size of samples newly taken imaginatively from the population of the measured data;

n = number of the measured data, size of sample;

n_R, n_S = number of the measured data of R and S;

P = occurrence probability;

P_f = failure probability;

P_f^M = modified failure probability taking account of errors of mean and variance;

p = area of pulse part;

p_R, p_S = area of pulse part for R and S;

R = material strength, resistance;

$\bar{R}$ = sample mean of R;

R_{min} = minimum expected value for R;

S = external force, member force, load;

$\bar{S}$ = sample mean of S;

S_{max} = maximum expected value for S;

s_{F_i} = standard deviation of F_i;

s_X^2 = variance of the measured data, sample variance;

s_R^2, s_S^2 = sample variances of R and S;

$s_{X\,real}^2$ = real variance of X;

$s_{\Delta X}^2$ = variance of the error caused by the difference between X_i and $X_{i,real}$;

sov = parameter, an abbreviation of 'sum of variances';

T = structural life;

ΔT = time interval of sampling of S;

$t_1 \sim t_3$ = sizes of upper and lower blocks in case of seven restrictive conditions;

$t_{\alpha/2}$ = value of t-distribution corresponding to upper probability $\alpha/2$;

u_1 = maximum (= X_n);

u_t = mean of upper block of size t;

v_1 = minimum (absolute value) (= $-X_1$);

v_s = mean of lower block of size s;

X = normalized measured values;

$X(F)$ = probability distribution function for F, unknown function, a monotone increasing function;

X_i = order statistics of X, i.e., i-th smaller value of X;

X_i' = reversed value of X_i, i.e., $X_i' = -X_{n-i+1}$;

$X_{i,real}$ = real value of X_i;

$\overline{X_{real}}$ = real mean of X;

ΔX_i = error of X_i;

$\overline{\Delta X}$ = mean of the error caused by the difference between X_i and $X_{i,real}$;

x = measured values;

$\bar{x}$ = mean of the measured data, sample mean;

Y = estimated value of deviation from the mean;

$Y_{\max}$ = maximum of extremum $\bar{Y}_N^E$ using graphical estimation method;

$Y_{\max,U}$ = maximum of $\bar{Y}_{N,U}^E$ using graphical estimation method;

Y_R, Y_S = values of extremum $\bar{Y}_N^N$ for R and S;

$\bar{Y}_N$, $\bar{Y}_N^E$ = averaged maximum of samples of size N (averaged minimum is also expressed as $\bar{Y}_N$), and its extremum;

$\bar{Y}_{N,U}^E$ = upper bound of $\bar{Y}_N^E$ with regard to the variations of characteristic means $\bar{A}\sim\bar{C}$;

$\bar{Y}_p$, $\bar{Y}_p^E$ = averaged maximum of pulse part with area p, and its extremum;

α = upper probability of t-distribution; shape parameter;

β = upper probability of χ^2-distribution; scale parameter;

β_3 = skewness;

β_4 = kurtosis;

β_3' = characteristic corresponding to skewness approximately;

β_4' = characteristic corresponding to kurtosis approximately;

γ_0 = central safety factor;

γ_{abs} = absolute safety factor;

δ_X = coefficient of variation of the mrasured data;

δ_R, δ_S = coefficients of variation of R and S;

ε = coefficient of dangerousness;

$\lambda_1 \sim \lambda_7$ = Lagrange multipliers;

μ_X = population mean;

μ_R, μ_S = population means of R and S;

ξ = location parameter;

σ_X^2 = population variance;

σ_R^2, σ_S^2 = population variance of R and S;

Φ = upper probability of normal distribution;

$\chi^2_{\beta/2}$ = value of χ^2-distribution corresponding to upper probability $\beta/2$;

$\psi(F)$ = Dirac's δ-function defined as $\psi(F)=1/p$ for $1-p\le F\le 1$;

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Chapter 1 Introduction

Since Fredenthal proposed the structural reliability analysis in 1947 [25] by introducing probabilistic concept to the engineering decision problem, numerous applications of 'probabilistic concepts' has widely been discussed. The developments in this field up to date is roughly divided into following four subjects; that is, (1) rationalization of the design code, (2) application to the complex structure and combined load, (3) application to the prediction problem, (4) management of the uncertainty.

Subject (1) mainly aims at the rational decision of the load factor based on the reliability analysis. Some specifications have already been put to practical use such as "AISE Specification for the Design, Fabrication and Erection of Structural Steel for Buildings" (1969), "AASHTO Interim Specifications for Bridges" (1974), "CIRIA Guidance for the Drafting of Codes of Practice for Structural Safety" (1968) and "CEB-FIP International Recommendations for the Design and Construction of Concrete Structures" (1970). Subject (1) has been studied most earnestly as a direct and embodied application of the reliability theory to the structural design. Cornell, Lind et al. regarded various collapse modes and dispersions of material strength and external force as influence factors on the practical design, and evaluated them as the load factors [12,17,19,40,49,50,55,57]. Legerer et al. studied the economical design in which the optimum design is performed by minimizing the total expected cost, that is, by balancing the relationship between the initial cost for construction and the failure cost for accidental structural collapse [23,38,39,42,54]. Furthermore, Smith analyzed a complicated structure by including mean and variance of material strength [56] and Cohn et al. proposed a plastic design of frames taking account of the variations of material strength and external force [16,29].

Subject (2) aims at the extension of reliability analysis, which is initially defined for structural unit and unit load, to complex framing and combined load. The structure of real state has, in general, complex framing and combined load, and subject (2) is significant for the practical use. With respect to combined loads, Haurylkiewicz et al. proposed a set theory [15,33], where structural reliability is defined as the distance between failure region and mixed loads represented as a vector on a state space. Recently, Blockley applied a concept of fuzzy sets to the estimation of structural reliability by using a logical analysis of various factors concerning to structural failure [14,61]. Lots of applications to the repeated load problem, that is, fatigue problem, have been discussed by Ang, Moses, Shinozuka et al. [4,5,6,22,28,60].

Subject (3) aims at the time-dependent analysis of structural reliability covering a moderately varying phenomenon during a long term such as deterioration of material strength. Yang et al. managed this problem as the first passage problem where Markov process is effectively used [59,62]. Although the first passage problem has often been applied to spectrum analysis of seismic behavior, proposed application is limited to a moderately varying phenomenon.

Last, subject (4) aims at the management of uncertainties consisting of error of characteristics calculated from the measured data, error of distributions estimated from frequency distribution of the measured data, error of nominal size, of manufacture, of analysis and other errors originated by artificial factors. Cornell, Lind et al. proposed first-order probability [18,21,32], where mean and variance of the measured data is derived with regard to the first order error, and safety index is calculated by assuming the distributions for material strength and load. Rosenblueth, Turkstra et al. applied Bayesian decision theory to decide 'how the measured data should be selected and employed' for getting a desired result [20,52,58].
On the other hand, Ang et al. defined various uncertainties quantitatively by introducing some assumptions and discussed their influences on the reliability analysis [3,4,5,6]. Ang et al. also proposed the extended reliability [1,2,3] with the object of decreasing the error caused by the assumption of distributions, which was originally posed by Jorgenson [35].

In contrast to reliability analysis, extreme procedure systematized by Gumbel [31] has often been employed in the aspect of statistical management of the measured data. The procedure has been used to presume the 100 years value and the 200 years value of rainfall, discharge, wind velocity, etc.. As well as famous Gumbel's method, so-called extreme distributions such as the first, second and third asymptotes [31] have been applied directly to the measured data [26, 27,45]. Gumbel, Plackett et al. derived extremum of the averaged maximum based on the variational method [30,31,48] and Moriguti also derived it based on Schwarz's inequality [46], but these procedures have not been generally employed in the practical problems yet.

In this thesis, error of characteristics of the measured data and error of distributions are discussed. First, Freudenthal's classical reliability theory is modified by taking account of the errors of means and variances of the measured data in Chapter 2. Second, estimation procedure of extremum based on the variational method is introduced as a method without any assumption of distributions in Chapter 3. Last, the procedure of Chapter 3 is extended to the structural reliability analysis in Chapter 4.

The characteristics such as mean and variance calculated from the measured data of size $10 \sim 60$, at the most, are seldom consistent with the characteristics of the population, and error of characteristics of the measured data exerts considerable influence upon the estimation of failure probability. The error of this type was also discussed by Ang et al. [1,2,3,4,5,6], but the error was treated roughly and the error was defined based on some assumptions expediently created. On the contrary, herein, density functions of the unknown mean and variance of the population are derived analytically based on the known mean and variance of the measured data, and the modified reliability is defined by averaging all events that will happen.
Error analysis is applied to three probability distributions, that is, normal distribution, gamma distribution and log-normal distribution. When the measured data are concluded following to one of these probability distributions as a result of phenomenology and/or properties of matter, the procedure in Chapter 2 can be put in practice.

Error of distributions is more serious than error of characteristics, for the estimation of population distribution based on the measured data is difficult and unreliable in general. Failure probability calculated by using estimated distribution is quite unstable depending on the estimated distribution. The error has been deemed unavoidable for the usual reliability theory. Only one trial was proposed by Ang et al. [1,2,3] by defining the extended reliability. In the extended reliability theory, several distributions are employed to demonstrate the decrease of the error, but no guarantee can be obtained for other numerous distributions. Furthermore, if failure probability can be proposed being insensitive to the various distributions, another indefinite approximation is needed in order to introduce insensibility. By these reasons, maximization procedure of the averaged maximum proposed by Gumbel et al. [30,31,48] is adopted herein for treating the problem of error of distributions in a new aspect, and extended to the structural reliability problem. The procedure realizes the structural design where the obtained failure probability never exceed the initially expected one even at the worst, in other words, where a sort of guarantee for the safety can be obtained, based on the variational principle. The procedure also can be applied to the unilateral estimation of upper bound of load and lower bound of material strength, and especially to the estimation of the 100 years discharge, for instance, instead of the usual Gumbel's method, etc.. Besides, a graphical estimation method of extremum is developed for the practical purpose, and extremum can be estimated simply and in high accuracy by using three characteristics calculated from the measured data. Error of characteristics is also evaluated approximately, and upper bound of extremum is estimated.

Chapter 2 Modification of the Usual Reliability Analysis with regard to Error of Characteristics

In 1947 Freudenthal proposed classical reliability theory [25], where the failure probability of the structure——structural unit—— is obtained by supposing material strength——shown as R henceforth—— and external force or load——shown as S henceforth——are random variables. The distributions of R and S are approximated by using well-known distributions such as normal distribution based on the test of goodness of fit, and the failure probability is calculated by using the approximated distributions. Failure probability obtained as a result becomes unreliable on account of the use of the approximated distributions and the error of characteristics.

2.1 Definition of Errors

Following two kinds of errors are included in the usual reliability analysis.

① Characteristics, that is, mean and variance, of the measured data are so-called sample mean and sample variance, and are not equal to population mean and population variance in the most case. The error is described as 'error of characteristics' henceforth.
② Probability density functions of population are approximated by using known distributions such as normal distribution. The error is described as 'error of distributions' henceforth.

Error of characteristics is especially discussed herein from the following reasons;

a) Number of the measured data is fairly small and amounts only to $20\sim60$ at the most.
b) Mean and variance of the measured data differ from ones of the population.
c) Difference between the measured data and the population is significant in case of variance. For instance, when a set of samples of size 50 are extracted from population following to normal distribution with variance 2, the confidence interval for the population variance with confidence 0.95 is calculated as $1.40\sim2.88$ (the interval

is obtained as the range of σ_X^2 in Eq.(2.2.2) by substituting n=50, s_X^2=1, β=0.05).

d) Large difference between the measured data and the population causes large differnce on the estimation of failure probability.

e) Difference between the measured data and the population, that is, error of characteristics, can be evaluated analytically by employing t- and χ^2-distributions in the case that the population follows to normal distribution.

Errors of characteristics and distributions are treated in Chapter 2 as follows;

a) With respect to error of characteristics, the definition of failure probability is modified so as not to underestimate the failure probability, in other words, not to provide a design on dangerous side in spite of the existence of the error.

b) With respect to error of distributions, the author takes the stand that the error is unavoidable and cannot be evaluated by extending the usual reliability analysis. The error is expected to decrease in consequence of the sampling process of the measured data as described later in Section 2.5.

2.2 Error of Characteristics

In order to define the failure probability taking account of error of characteristics, probability densities of population mean and population variance must be calculated by employing mean and variance of the measured data. Although population mean and population variance are deterministic in substance and have no probability densities at all, description such as 'probability densities of population mean and population variance' is employed herein from a viewpoint that the unknown population is estimated from the known measured data.

The probability densities of population mean and population variance can be derived by using t- and χ^2-distributions, respectively, if the population follows to normal distribution. In case of log-normal distribution, probability density can be derived by performing logarithm transformation for the variables of t- and χ^2-distributions. In case of gamma distribution, probability density can be also derived approximately by performing square transformation. On the contrary, in case of other distributions such as Weibull distribution and double exponential distribution, probability densities cannot be derived because these distributions cannot be reduced to normal distribution by performing any transformation.

Supposing a set of samples of size n is obtained from a measurement, let sample mean be $\bar{x}$ and sample variance be s_x^2. And the estimated values of population mean and population variance in this case are shown as μ_x and σ_x^2, respectively.

Let probability distributions of population mean and population variance be derived in case of normal distribution to begin with. When sample mean $\bar{x}$ is obtained from the measured data of size n, the confidence interval for population mean μ_x with confidence α is expressed as;

$$\bar{x}-t_{\alpha/2}\sigma_x/(n-1)^{1/2} < \mu_x < \bar{x}+t_{\alpha/2}\sigma_x/(n-1)^{1/2} \quad \cdots\cdots (2.2.1)$$

where $t_{\alpha/2}$ is a value of t-distribution corresponding to upper probability $\alpha/2$, that is,

$$\alpha/2=\int_{t_{\alpha/2}}^{\infty} f_t(u)\,du$$

where f_t is probability density for t-distribution. On the contrary, in case of variance, the confidence interval for population variance σ_x^2 with confidence β is expressed as follows by using sample variance s_x^2;

$$(n-1)s_x^2/\chi^2_{1-\beta/2} < \sigma_x^2 < (n-1)s_x^2/\chi^2_{\beta/2} \quad \cdots\cdots (2.2.2)$$

where $\chi^2_{\beta/2}$ is a value of χ^2-distribution, f_{χ^2}, corresponding to upper probability $\beta/2$, that is,

$$\beta/2=\int_{\chi^2_{\beta/2}}^{\infty} f_{\chi^2}(u)\,du$$

The situation that population mean becomes μ_x on the occasion of sample mean being $\bar{x}$, is represented as,

$$P\{(\text{population mean})=\mu_x \mid (\text{sample mean})=\bar{x}\}=f_{\mu_x}(\mu_x)\,dx$$

where f_{μ_x} is probability density for population mean. The relationship between f_{μ_x} and f_t is expressed as,

$$\int_{\mu_x}^{\infty} f_{\mu_x}(x)\,dx = \int_{(n-1)^{1/2}(\mu_x-\bar{x})/s_x}^{\infty} f_t(u)\,du$$

f_{μ_x} is decided so as to satisfy above equation, that is;

$$f_{\mu_x}(x) = \{(n-1)^{1/2}/s_x\}\cdot f_t\{(n-1)^{1/2}(x-\bar{x})/s_x\} \quad \cdots\cdots (2.2.3)$$

With respect to variance, probability density for population variance, $f_{\sigma_x^2}$, is connected with f_{χ^2} as,

$$\int_0^{\sigma_x^2} f_{\sigma_x^2}(x)\,dx = \int_{(n-1)s_x^2/\sigma_x^2}^{\infty} f_{\chi^2}(u)\,du$$

and $f_{\sigma_x^2}$ is decided as follows;

$$f_{\sigma_x^2}(x) = \{(n-1)s_x^2/x^2\}\cdot f_{\chi^2}\{(n-1)s_x^2/x\} \quad \cdots\cdots (2.2.4)$$

Eqs.(2.2.3) and (2.2.4) are applicable only for the population following to normal distribution. In case of gamma distribution and log-normal distribution, the corresponding distributions of t- and χ^2-distributions cannot be derived analytically, but they can be estimated approximately. In case of gamma distribution, following square transformation——the derivation is expressed in Appendix 1—— is employed to convert variable u following to normal distribution into variable t following to gamma distribution, as a practical approximation;

$$u = 2(\bar{x}t)^{1/2} - (4\bar{x}^2 - s_x^2)^{1/2} + \bar{x}$$

Now, following assumption is introduced; that is, square transformation is also applicable and useful to define probability densities for population mean and population variance for gamma distribution based on Eqs.(2.2.3) and (2.2.4), respectively. Thus f_{μ_x} and $f_{\sigma_x^2}$ for gamma distribution are derived as follows by performing square transformation for Eqs.(2.2.3) and (2.2.4);

$$f_{\mu_x}(x) = \{(n-1)^{1/2}/s_x\}\cdot f_t[(n-1)^{1/2}\{2(\bar{x}x)^{1/2} - (4\bar{x}^2 - s_x^2)^{1/2}\}/s_x]\cdot(\bar{x}/x)^{1/2} \quad \cdots\cdots (2.2.5)$$

$$f_{\sigma_x^2}(x) = [(n-1)s_x^2/\{2(\bar{x}x)^{1/2} - (4\bar{x}^2 - s_x^2)^{1/2} + \bar{x}\}^2]$$
$$f_{\chi^2}[(n-1)s_x^2/\{2(\bar{x}x)^{1/2} - (4\bar{x}^2 - s_x^2)^{1/2} + \bar{x}\}]\cdot(s_x^2/x)^{1/2} \quad \cdots\cdots (2.2.6)$$

In case of log-normal distribution, following logarithm transformation

$$u = \ln t$$

is employed to convert variable u following to normal distribution into variable t following to log-normal distribution. Thus f_{μ_X} and $f_{\sigma_X^2}$ in case of log-normal distribution are derived as follows by performing logarithm transformation for Eqs.(2.2.3) and (2.2.4) and by substituting $\bar{x}$ for $ln\bar{x}$ and s_X^2 for δ_X^2;

$$f_{\mu_x}(x) = \{(n-1)^{1/2}/\delta_x\} \cdot f_t\{(n-1)^{1/2}(\ln x - \ln\bar{x})/\delta_x\} \cdot (1/x) \qquad (2.2.7)$$

$$f_{\sigma_x^2}(x) = \{(n-1)\delta_x^2/x^2\} \cdot f_{\chi^2}\{(n-1)\bar{x}^2\delta_x^2/x\} \cdot (1/\bar{x}^2) \qquad (2.2.8)$$

where δ_X is coefficient of variation of the measured data.

2.3 Classical Reliability Theory

Basic definition of failure probability in the classical reliability theory is described in this Section for the purpose of setting the proposed reliability analysis including the error of characteristics against the classical reliability theory.

Let population means of material strength, R, and load, S, be μ_R and μ_S, respectively, and population variance be σ_R^2 and σ_S^2, respectively. Let sample means of R and S be $\bar{R}$ and $\bar{S}$, respectively, sample variances be s_R^2 and s_S^2, respectively, and they are calculated from the measured data of size n_R and n_S, respectively, extracted from populations of R and S. Following suppositions underlie the classical reliability theory; that is, $\bar{R}=\mu_R$, $\bar{S}=\mu_S$, $s_R^2=\sigma_R^2$, $s_S^2=\sigma_S^2$. Variables R and S will happen to take any combination, therefore occurence probability of a certain set of R^* and S^* is expressed as,

$$P\{(R^* \leqq R \leqq R^* + dR) \cap (S^* \leqq S \leqq S^* + dS)\} = f_R(R^*) f_S(S^*)\,dS\,dR \qquad (2.3.1)$$

in which right-superscript * means a certain realized value; f_R and f_S are probability densities for R and S, respectively. Since failure probability is defined as a ratio of an event such as $R^*<S^*$ against whole event, failure probability is provided as follows by integrating Eq.(2.3.1) on the region of $R<S$ as shown in Fig.1-1;

$$P_f = \mathrm{P}(R<S) = \iint_{R<S} f_R(R) f_S(S)\,dS\,dR \quad \cdots\cdots (2.3.2)$$

Eq.(2.3.2) is expressed simply as follows if f_R and f_S are normal distributions;

$$P_f = \Phi\{(\bar{R}-\bar{S})/(s_R^2+s_S^2)^{1/2}\} \quad \cdots\cdots (2.3.3)$$

in which Φ is an upper probability of normal distribution, and is described as follows;

$$\Phi(a) = \int_a^{\infty}(1/2\pi)\exp(-x^2/2)\,dx$$

Last, central safety factor γ_0 is defined as follows purporting to an index for the practical design;

$$\gamma_0 = \bar{R}/\bar{S} \quad \cdots\cdots (2.3.4)$$

Cornell et al. connected Eq.(2.3.3) with Eq.(2.3.4) through the medium of the coefficients of variation δ_R and δ_S [2,19]. The modified reliability analysis proposed herein also aims at deriving the relationship between γ_0 and P_f finally.

2.4 Reliability Analysis with regard to Errors of Means and Variances

The modified failure probability is defined as follows so as to take account of errors of means and variances by using the probability densities for population mean and population variance defined by Eqs. (2.2.3)∿(2.2.8);

$$P_f^M = \iiiint f_{\mu_R}(\bar{R}^*) f_{\mu_S}(\bar{S}^*) f_{\sigma_R^2}(s_R^{2*}) f_{\sigma_S^2}(s_S^{2*}) \iint_{R<S} f_R(R) f_S(S)\,dS\,dR\,ds_S^2\,ds_R^2\,d\bar{S}\,d\bar{R}$$

in which right-superscript M means a failure probability taking account of error of characteristics.

Numerical integration for numerous combinations of $\bar{R}^*$, $\bar{S}^*$, s_R^{2*} and s_S^{2*} are required in order to solve the above-mentioned quadruple integral for each set of R and S. For instance, the total combination becomes 10^8 when each integral region is divided into 10^2 parts for Simpson's 1/3 rule. Now, parameter A_0, which corresponds to the independent variation of $\bar{R}^*$ and $\bar{S}^*$, is newly introduced with the sole object of decreasing the total combination of numerical integrations, and parameter B_0, which corresponds to the independent variations of s_R^{2*} and s_S^{2*}, is also newly introduced. Introducing these two parameters, total combinations decrease to 10^4.

Parameters are defined as the moved distance of the failure boundary line from the origin, when the line——originally represented as $R=S$——moves parallel along R-axis as shown in Fig.1-2. That is, parameter A_0 corresponds to the parallel movement caused by error of mean, parameter B_0 also corresponds to one caused by error of variance. The failure region is represented as follows by using parameters A_0 and B_0;

$$R < S + (A_0 + B_0) \quad \cdots\cdots (2.4.1)$$

Further, the failure region is represented more simply by introducing new parameter C_0 corresponding to $A_0 + B_0$ as,

$$R < S + C_0 \quad \cdots\cdots (2.4.2)$$

Independencies between $\bar{R}^*$ and $\bar{S}^*$, between s_R^{2*} and s_S^{2*} are not lost by introducing parameters A_0, B_0 and C_0, because the parameters are connected directly to the failure probability P_f for the fixed sets of $\bar{R}^*$ and $\bar{S}^*$, of s_R^{2*} and s_S^{2*}. The relations are expressed as follows;

(certain value of A_0)$\rightarrow$(fixed sets of $\bar{R}^*$ and $\bar{S}^*$)$\rightarrow$(certain value of P_f)

(certain value of B_0)$\rightarrow$(fixed sets of s_R^{2*} and s_S^{2*})$\rightarrow$(certain value of P_f)

(certain value of C_0)$\rightarrow$(fixed sets of A_0 and B_0)$\rightarrow$(certain value of P_f)

In the above arrow-descriptions, $\bar{R}^*$ and $\bar{S}^*$, s_R^{2*} and s_S^{2*} are employed as intermedia connecting A_0 and P_f, B_0 and P_f, respectively, and there are one-by-one correspondences between A_0 and P_f, and between B_0 and P_f.

Finally, modified failure probability taking account of error of characteristics, P_f^M, is expressed as follows by using parameter C_0;

$$P_f^{\mathrm{M}}=\int f_{\mathrm{C_0}}(C_0)\iint_{R<S+C_0} f_{\mathrm{R}}(R) f_{\mathrm{S}}(S)\,\mathrm{d}S\mathrm{d}R\mathrm{d}C_0 \qquad (2.4.3)$$

in which f_{C_0} is probability density for parameter C_0. In this case, total combinations of the numerical integrations decrease to 10^2 at the foregoing example.

The modified failure probability P_f^M is defined as Eq.(2.4.3) herein, but other definitions as mentioned below may be considered;

a) $P_f^{\mathrm{M}}=\iint_{R<S+C_0'} f_{\mathrm{R}}(R) f_{\mathrm{S}}(S)\,\mathrm{d}S\mathrm{d}R$, provided that parameter C_0' is decided so as to satisfy following equation; $\varepsilon=\int_{C_0'}^{\infty} f_{C_0}(C_0)\,\mathrm{d}C_0$, where ε is coefficient of dangerousness and is selected as $\varepsilon=10^{-3}$ for instance.

b) $P_f^{\mathrm{M}}=\iint_{R<S+C_0''} f_{\mathrm{R}}(R) f_{\mathrm{S}}(S)\,\mathrm{d}S\mathrm{d}R$, provided that parameter C_0'' is decided so as to maximize following integrals; $\int_{C_0''}^{\infty} f_{C_0}(C_0)\,\mathrm{d}C_0 \int_{R<S+C_0''} f_{\mathrm{R}}(R) f_{\mathrm{S}}(S)\,\mathrm{d}S\mathrm{d}R$

In the definition (a), P_f^M is considered as failure probability where upper probability of f_{C_0} is equivalent to a certain very small value ε. In other words, if the structure is designed by using P_f^M of type (a), $100(1-\varepsilon)$ percent of the error which may occur is already taken account. In actuality, ε is difficult to evaluate, and this is the weak point of the definition (a). On the contrary, in the definition (b), P_f^M is considered as failure probability where the product of P_f and upper probability of f_{C_0} is maximized. The product may be interpreted as a sort of an index corresponding to the economy of the design.

First, probability densities for parameters A_0 and B_0, f_{A_0} and f_{B_0}, are derived in the case that populations of R and S follow to normal distribution. The failure probabilities are expressed as follows for arbitrary values of parameters A_0 and B_0, respectively, based on Eq.(2.3.3);

$$\left.\begin{aligned} P_f^*&=\Phi\left\{(\bar{R}-\bar{S}-A_0^*)/(s_{\mathrm{R}}^2+s_{\mathrm{S}}^2)^{1/2}\right\} \\ P_f^*&=\Phi\left\{(\bar{R}-\bar{S}-B_0^*)/(s_{\mathrm{R}}^2+s_{\mathrm{S}}^2)^{1/2}\right\} \end{aligned}\right\} \qquad (2.4.4)$$

where sample means $\bar{R}$ and $\bar{S}$, sample variance s_R^2 and s_S^2 are deterministic values, and parameters A_0^* and B_0^* are random variables. As a preliminary arrangement of connecting failure probability P_f with parameters A_0 and B_0 through $\bar{R}^*$, $\bar{S}^*$, s_R^{2*} and s_S^{2*}, following new parameters, *dom* and *sov*, satisfying the condition P_f=const., are introduced;

$$\left.\begin{aligned} dom^* &= \bar{R}^* - \bar{S}^* \\ sov^* &= s_R^{2*} + s_S^{2*} \end{aligned}\right\} \qquad (2.4.5)$$

where *dom* is an abbreviation of 'difference of means', *sov* is an abbreviation of 'sum of variances'. The group corresponding to $\bar{R}^*-\bar{S}^*$ is extracted by using parameter dom^* among whole combinations of arbitrary $\bar{R}^*$ and $\bar{S}^*$, and the group $s_R^{2*}+s_S^{2*}$ is extracted by using parameter sov^* among whole combinations of arbitrary s_R^{2*} and s_S^{2*}. Since the group $\bar{R}^*-\bar{S}^*$ and $s_R^{2*}+s_S^{2*}$ correspond to failure probability P_f^*, parameters dom^* and sov^* can be connected with P_f^*. Relationships between parameter dom^* and $\bar{R}^*-\bar{S}^*$, and between parameter sov^* and $s_R^{2*}+s_S^{2*}$ are shown in Figs.2-1, 2-2. Let probability densities f_{μ_X} and $f_{\sigma_X^2}$ for R be f_{μ_R} and $f_{\sigma_R^2}$, respectively, and for S be f_{μ_S} and $f_{\sigma_S^2}$, respectively, based on Eqs.(2.2.3) and (2.2.4). The probability densities for parameters *dom* and *sov*, f_{dom} and f_{sov}, are expressed as follows by using f_{μ_R}, f_{μ_S}, $f_{\sigma_R^2}$ and $f_{\sigma_S^2}$;

$$\left.\begin{aligned} f_{\mathrm{dom}}(dom^*) &= \int_{dom^*} f_{\mu_R}(\bar{R}^*)\, f_{\mu_S}(\bar{R}^* - dom^*)\, \mathrm{d}\bar{R}^* \\ f_{\mathrm{sov}}(sov^*) &= \int_{sov^*} f_{\sigma_R^2}(s_R^{2*})\, f_{\sigma_S^2}(sov^* - s_R^{2*})\, \mathrm{d}s_R^{2*} \end{aligned}\right\} \qquad (2.4.6)$$

Now that failure probabilities P_f^* corresponding to parameters dom^* and sov^*, respectively, are represented as follows with reference to Eq.(2.3.3);

$$\left.\begin{aligned} P_f^* &= \Phi\{(dom^*)/(s_R^2 + s_S^2)^{1/2}\} \\ P_f^* &= \Phi\{(\bar{R} - \bar{S})/(sov^*)^{1/2}\} \end{aligned}\right\} \qquad (2.4.7)$$

Parameters dom^* and sov^* can be represented as follows by employing parameters A_0^* and B_0^* based on Eqs.(2.4.4) and (2.4.7), respectively;

$$\left.\begin{aligned} dom^* &= \bar{R} - \bar{S} - A_0^* \\ sov^* &= (s_R^2 + s_S^2)(\bar{R} - \bar{S})^2/(\bar{R} - \bar{S} - B_0^*)^2 \end{aligned}\right\} \qquad (2.4.8)$$

In consequence, imaginary parameters A_0 and B_0 having no physical meaning are connected with real parameters *dom* and *sov* representing frequencies of the errors included in $\bar{R}$, $\bar{S}$, s_R^2 and s_S^2.

Probability densities for parameters A_0 and B_0, f_{A0} and f_{B0}, are obtained by substituting Eq.(2.4.8) into Eq.(2.4.6), and probability density for parameter C_0, f_{C0}, which is required finally, is represented as follows by using f_{A0} and f_{B0};

$$f_{C_0}(C_0) = \int f_{A_0}(A_0) f_{B_0}(C_0 - A_0)\, dA_0 \qquad (2.4.9)$$

Probability densities f_{A0}, f_{B0} and f_{C0} are shown in Fig.3-1 in case of $n_R = n_S = 15$, $\delta_R = 0.1$ and $\delta_S = 0.2$.

The procedure can be extended as follows to gamma distribution and log-normal distribution. With respect to gamma distribution, expression such as Eq.(2.3.3) cannot be obtained by substituting gamma distribution directly into Eq.(2.3.2), or by substituting normal distribution which is transformed by square approximation into Eq.(2.3.2). Therefore, rough approximation such as,

(variable following to gamma distribution)
≈ (variable following to normal distribution)

is introduced and Eq.(2.3.3) is employed without any modification relative to the difference between gamma distribution and normal distribution. In that case, Eqs.(2.4.4)∿(2.4.8) also can be employed as they are. However, as for f_{μ_R}, f_{μ_S}, $f_{\sigma_R^2}$ and $f_{\sigma_S^2}$ in Eq.(2.4.6), Eqs.(2.2.5) and (2.2.6) should be employed instead of Eqs.(2.2.3) and (2.2.4), respectively. Probability densities f_{A_0}, f_{B_0} and f_{C_0} are shown in Fig.3-2 in the same case of normal distribution.

With respect to log-normal distribution, following expression is employed approximately instead of Eq.(2.3.3);

$$P_f = \Phi\{(\ln\bar{R} - \ln\bar{S}) / (\delta_R^2 + \delta_R^2)^{1/2}\} \qquad (2.4.10)$$

Parameters A_0 and B_0 are re-defined as follows; that is, parameters A_0 and B_0 are expressed as tangent of the angle between S-axis and failure boundary line $R=S$ which moves round the origin as shown in Fig.1-3. The failure region is represented as follows by using parameters A_0 and B_0;

$$R < (A_0 + B_0) \cdot S \qquad (2.4.11)$$

Since Eq.(2.4.11) is represented more simply by using parameter $C_0 = A_0 + B_0$ as,

$$R < C_0 S \qquad (2.4.12)$$

the modefied failure probability P_f^M is expressed as follows;

$$P_f^M = \int f_{C_0}(C_0) \iint_{R<C_0S} f_R(R) f_S(S)\,\mathrm{d}S\mathrm{d}R\mathrm{d}C_0 \qquad (2.4.13)$$

Failure probabilities P_f^* are expressed as follows for parameters A_0^* and B_0^* based on Eq.(2.4.10);

$$\left.\begin{aligned} P_f^* &= \Phi\{(\ln\overline{R} - \ln\overline{S} - \ln A_0^*)/(\delta_R^2 + \delta_S^2)^{1/2}\} \\ P_f^* &= \Phi\{(\ln\overline{R} - \ln\overline{S} - \ln B_0^*)/(\delta_R^2 + \delta_S^2)^{1/2}\} \end{aligned}\right\} \qquad (2.4.14)$$

Parameters *dom* and *sov* are introduced as,

$$\left.\begin{aligned} dom^* &= \ln\overline{R}^* - \ln\overline{S}^* \\ sov^* &= \delta_R^{2*} + \delta_S^{2*} \end{aligned}\right\} \qquad (2.4.15)$$

Probability densities for parameters *dom* and *sov* are expressed as follows with reference to Eq.(2.4.6);

$$\left.\begin{aligned} f_{dom}(dom^*) &= \int_{dom^*}(1/\overline{R}^*\overline{S}^*) f_{\mu_R}(\ln\overline{R}^*) f_{\mu_S}(\ln\overline{R}^* - dom^*)\,\mathrm{d}(\ln\overline{R}^*) \\ f_{sov}(sov^*) &= \int_{sov^*}(1/\overline{R}\overline{S}) f_{\sigma_R^2}(\delta_R^{2*}) f_{\sigma_S^2}(sov^* - \delta_R^{2*})\,\mathrm{d}(\delta_R^{2*}) \end{aligned}\right\} \qquad (2.4.16)$$

Since failure probability P_f^* corresponding to parameters *dom** and *sov** are represented as follows instead of Eq.(2.4.7);

$$\left.\begin{aligned} P_f^* &= \Phi\{(dom^*)/(\delta_R^2 + \delta_S^2)^{1/2}\} \\ P_f^* &= \Phi\{(\ln\overline{R} - \ln\overline{S})/(sov^*)^{1/2}\} \end{aligned}\right\} \qquad (2.4.17)$$

parameters *dom** and *sov** can be represented as follows by using parameters A_0^* and B_0^*;

$$\left.\begin{aligned} dom^* &= \ln\overline{R} - \ln\overline{S} - \ln A_0^* \\ sov^* &= (\delta_R^2 + \delta_S^2)(\ln\overline{R} - \ln\overline{S})^2/(\ln\overline{R} - \ln\overline{S} - \ln B_0^*)^2 \end{aligned}\right\} \qquad (2.4.18)$$

Probability densities for parameters A_0 and B_0 are obtained by substituting Eq.(2.4.18) into Eq.(2.4.16), and probability density for parameter C_0 is obtained based on Eq.(2.4.9). Probability densities f_{A_0}, f_{B_0} and f_{C_0} are shown in Fig.3-3 in the same cases of normal and gamma distributions.

The modified failure probability P_f^M taking account of the errors of means and variances can be calculated by substituting f_{C_0} into Eq.(2.4.3)——in case of normal and gamma distributions——and into Eq.(2.4.13)——in case of log-normal distribution. Three kinds of P_f^M are calculated for central safety factor γ_0, and shown in Fig.4 in case of normal, gamma and log-normal distributions.

2.5 Relationship among Failure Level P_f' of the Structure, Structural Life T and Sampling Process of the Measured Data for Load S

The structural life, T, is generally decided in advance at the stage of making a plan from an economical and/or political viewpoints. Failure level P_f', for which the structure is designed, acquires an engineering meaning by connecting P_f obtained from the reliability analysis with T decided in advance, because the relationship between T and P_f' is closely connected with the sampling process of the measured data (of size n_S) for load S which have been measured continuously with respect to time. Although P_f' is only connected with load S and without any relationship with R, P_f' is used as the failure level of the structural design. It is explained as follows; value of R is unknown, and therefore it must be satisfied in any case that value of S exceeds the value of R only once——not more than twice——through the whole periods numbering $T/\Delta T$, where ΔT is the time interval of sampling——that is, maximum loads are collected in each interval ΔT and used as the measured data for S. In other words, the structure which fails once during T is obtainable by using failure level P_f' corresponding to $\Delta T/T$. If the structural life T is, for instance, supposed as 100 years, following relationship between P_f' and ΔT is obtained;

P_f'	10^{-6}	10^{-5}	10^{-4}	2×10^{-4}
ΔT	1 hour	10 hours	4 days	1 week

If the time interval ΔT of the sampling process is chosen according to the above relationship, the structural life T=100 years is equally expected independently whether P_f' is selected as 10^{-6} or 10^{-4}. However in practice, the accuracy of P_f' undergoes a change according to the selection of ΔT.

Error of distribution is caused by substituting approximately f_R and f_S for the known distributions such as normal distribution, gamma distribution and log-normal distribution as described previously. Error of distributions has a tendency to decrease as P_f' increases. Adequency of the approximated distribution is substantiated generally by using test of goodness of fit, where the goodness of fit is low at the part distant from the mean, i.e., at the neighborhood of the extremum as compared with at the neighborhood of the mean. The situation is explained as follows; if P_f' is selected small, goodness of fit as a whole increases on account of the increase of sample size, and goodness of fit at the neighborhood of the extremum decreases relatively; if P_f' is selected small, the main part of the integral playing an important role in the calculation of P_f' is restricted to the neighborhood of the extremum. Going upon the comparative discussion as stated above, it is presumed to be desired that P_f' is selected in the order of $10^{-3} \sim 10^{-5}$, that is, the time interval of sampling, ΔT, is selected large. If P_f' is selected too large in the order of $10^{-2} \sim 10^{-3}$, number of samples decreases, and goodness of fit itself decreases as a whole.

On the contrary, error of characteristics has been already evaluated as shown in previous Sections. That is, the influence of the error on the estimation of failure probability is evaluated numerically, and there is no necessity for discussing the decrease of the error itself. In this case, P_f' can be chosen without restraint ——as well large in the order of $10^{-2} \sim 10^{-3}$ or small in the order of $10^{-5} \sim 10^{-8}$ ——so long as minimum number of samples for the statistical treatment is obtained.

By correlating error of characteristics and error of distributions, it is recommended that P_f' is selected in the order of $10^{-3} \sim 10^{-5}$ in so far as number of samples between 15 and 100 is obtained.

2.6 Application to the Practical Design of Truss Bridge

Means, variances and coefficients of variation for R and S are calculated by using the measured data from the tests of materials and the measurements of loads, and the modified failure probability P_f^M is calculated by using Eq.(2.4.3) or Eq.(2.4.13). By calculating P_f^M for various values of central safety factor γ_0 $(=\bar{R}/\bar{S})$, the γ_0-P_f^M curve is drawn up. By employing the curve, the structural member is designed, that is, the cross-sectional area of the member is decided. The result is compared with the usual allowable stress design based on Design Specification for Welded Steel Highway Bridge of Japan, and the absolute safety factor anticipated in the allowable stress design is obtained.

The simplest case only the tensile force acts on the structural member composed by SM41B steel is considered as a numerical example. Characteristics of SM41B steel is obtained from the measured data for yield strength tested by Society of Steel Construction of Japan in 1968 [34], and shown in Table 1.

Warren truss with parallel chords is chosen as a type of the bridge structure as shown in Fig.5-1, and the cross-sectional area is calculated for the lower chord member located at the end of the truss. The truss bridge is supposed to be a two-lane highway bridge for one-way traffic, whose cross-section as for slab is shown in Fig.5-2. Influence lines employed in the allowable stress design is also shown in Figs.5-1, 5-2 concerning to the main truss and the slab, respectively.

The measured data for S is prepared by collecting maximum values during each time interval ΔT when the traffic flow acts on the truss bridge. The traffic flow model is substituted for the random series of the axial forces generated by using computer on the basis of the traffic volume census covering over 31 weeks. These records were measured by Japan Highway Public Corporation in 1968 at Tennozan Tunnel of Meishin Highway [64]. Maximums of the tensile stress which occur in the member per week are taken as a set of the measured data for S, that is, let ΔT be one week. The characteristics for S are shown in Table 1.

Let the structural life T be 100 years. Since ΔT is chosen as one week, the failure level P_f' employed in the design criterion is indicated as,

$$P_f' = \Delta T / T = 1\ (\text{week}) / 100\ (\text{years}) \approx 2 \times 10^{-4}$$

Probability density for the population of load S is estimated on the basis of the frequency distribution of the measured data for S as shown in Fig.6. In this case, log-normal distribution is suitable for the probability density for S, f_S. Probability densities f_{A_0}, f_{B_0} and f_{C_0} are calculated as shown in Fig.7 when R and S follow to log-normal distributions with characteristics as shown in Table 1—— probability density for R is assumed being equivalent to one for S, because coefficient of variation of R is rather smaller than one of S. The modified failure probability P_f^M is calculated by using f_{C_0} based on Eq.(2.4.13) and shown in Fig.8. The cross-sectional area of the lower chord member, A_S, is determined as follows by employing the γ_0 -P_f^M curve;

a) γ_0 corresponding to P_f' is read in Fig.8 by setting $P_f^M=P_f'$. Since failure level of the member is $P_f'=2\times10^{-4}$, γ_0 is obtained as 1.77.
b) $\bar{R}$ is calculated by using γ_0. Since $\bar{S}$ is 14.44(t) in Table 1, $\bar{R}$ is obtained as 25.56(t) based on the relation $\bar{R}=\gamma_0\bar{S}$.
c) A_S is calculated as follows by using $\bar{R}$;

$$A_S = \bar{R} / (\text{yield strength of SM41B steel}) = 25.56 / 2.752 < 10\ \text{cm}^2$$

On the contrary, let the cross-sectional area of the same member be calculated based on the usual allowable stress design for the first class bridge. The concentrated load on the slab becomes $5\times4.559=22.80$(t) and the total live load, L_ℓ, acting on the lower chord member is obtained as follows with reference to Fig.5-1;

$$L_\ell = 22.80 \times 0.4375 + 1.596 \times 17.5 = 37.9\ \text{t}$$

The required cross-sectional area A_S is determined as,

$$A_S = 37.9 / 1.400 < 28\ \text{cm}^2$$

where 1.400 (t/cm^2) is the allowable stress of SM41B steel.

In consequence, the absolute safety factor, γ_{abs}, including in the allowable stress design is estimated to be nearly

$$\gamma_{abs} = 28/10 = 2.8$$

for the tension member under live load.

2.7 Conclusion

Following two kinds of errors including in the usual reliability analysis have been discussed herein;

1. Error due to means and variances——Error of characteristics
2. Error due to assumptions of distributions——Error of distributions

With respect to error of characteristics, modification of the usual reliability analysis has been proposed not to evaluate the failure probability on dangerous side even if the error will exist. With respect to error of distributions, since the error is regarded as unavoidable in the reliability analysis, larger selection of time interval ΔT has been recommended for the purpose of decreasing the influence of the error on the evaluation of failure probability.

Error analysis has been applied to normal distribution, gamma distribution and log-normal distribution. Therefore error of characteristics can be analized only when the distribution of the measured data are assumed by one of these three distributions.

Chapter 3 Estimation Procedure of Extremum

Two kinds of error being accompanied with the usual reliability analysis are discussed in Chapter 2, and the error due to 'approximation of distributions' is not evaluated analytically. It was proposed that influence of the error on the estimation of failure probability decreases by putting design criterion P_f' in the order of $10^{-3} \sim 10^{-5}$. If more strict solution is further wanted, some new approach which has no use of approximation of distributions is needed. In Chapter 3, as an initial step to the purpose, a procedure is proposed to estimate the extremum of the measured data without any approximation of distributions. The content has no immediate connection with the failure probability and the structural design yet. Only purpose is to establish a variational procedure without any approximation of distributions on the estimation of the extremum of the measured data.

3.1 Procedure without any Approximation of Distributions

Čebyšev's inequality is one of well-known procedure without any approximation of distributions. Although Čebyšev's inequality is meaningful in theory, it cannot be employed in practice on account of too much extremum——deviation——being estimated. The deviation corresponding to occurrence probability 1/1000 is calculated as 31.6 in case of normalized population with reference to following Čebyšev's inequality;

$$\mathrm{P}\{|Y-0| \geqq t\} \leqq 1/t^2 = 1/1000$$

The deviation is far too large as compared with normal distribution whose deviation corresponding to 1/1000 is nearly 3.1.

Another and widely applicable procedure is the estimation procedure of extremum based on the variational method [30,31,48], where the distribution itself is taken as an unknown function. The procedure is expressed as follows; first, characteristics such as mean and variance are calculated from the measured data of size n, and are called the restrictive conditions by which an unknown function is characterized; second, sets of samples of size N, which is $N>n$, are supposed to be taken from the same population over and over; last, an unknown function is decided so as to maximize the average value of maximums——described as the averaged maximum henceforth——of each set of sample, and so as to satisfy various characteristics based on the variational method. The derivation of extremum in this case is described in the following.

The probability density for the population, from where the measured data has been extracted, is designated by $f(X)$, and its cumulative function by $F(X)$. X is a normalized value of a measured value x, and is expressed as,

$$X = (x - \bar{x}) / s_x$$

Although $f(X)$ and $F(X)$ are taken as functions for X in ordinary cases, X is regarded as a function for F herein from a viewpoint that distribution itself is unknown. That is, expression such as $X(F)$ is employed. Using $X(F)$, the restrictive conditions that mean is 0 and variance is 1 are expressed in the forms of following integrals;

$$\int_0^1 X \, \mathrm{d}F = 0 \qquad \cdots\cdots (3.1.1)$$

$$\int_0^1 X^2 \mathrm{d}F = 1 \qquad \cdots\cdots (3.1.2)$$

The averaged maximum of samples of sizes N, $\bar{Y}_N$, is also expressed as follows;

$$\bar{Y}_N = \int_0^1 XNF^{N-1} \, \mathrm{d}F \qquad \cdots\cdots (3.1.3)$$

The unknown function $X(F)$ is obtained by solving the variational problem of maximizing $\bar{Y}_N$ of Eq.(3.1.3) under the restrictions of Eqs.(3.1.1) and (3.1.2). First, introducing Lagrange multipliers λ_1 and λ_2, functional J is defined as [31],

$$J = XNF^{N-1} - \lambda_1 X - \lambda_2 X^2 \qquad \cdots\cdots (3.1.4)$$

and function $X(F)$ is expressed as follows by taking $\partial J / \partial X = 0$;

$$X = (1/2\lambda_2)(NF^{N-1} - \lambda_1) \qquad \cdots\cdots (3.1.5)$$

Lagrange mutipliers λ_1 and λ_2 are determined by substituting Eq.(3.1.5) into Eqs.(3.1.1) and (3.1.2), and by solving the simultaneos equations for λ_1 and λ_2 such as,

$$\begin{cases} (1/2\lambda_2)(1-\lambda_1)=0 \\ (1/2\lambda_2)\{N^2/(2N-1)-2\lambda_1-\lambda_1^2\}=1 \end{cases}$$

therefore

$$\lambda_1=1, \quad 1/2\lambda_2=(A'_{NN})^{-1/2} \qquad \cdots\cdots (3.1.6)$$

where

$$A_{kl}=kl/(k+l-1), \qquad A'_{kl}=A_{kl}-1$$

Then the extremum of $\bar{Y}_N$, i.e., $\bar{Y}_N^E$, is obtained as follows by substituting Eqs.(3.1.5) and (3.1.6) into Eq.(3.1.3);

$$\bar{Y}_N^E=(1/2\lambda_2)(A'_{NN}+1-\lambda_1)=(A'_{NN})^{1/2} \qquad \cdots\cdots (3.1.7)$$

in which right-superscript E means being an extremum. Value of $\bar{Y}_N^E$ is 22.3 in case of N=1000——this is not equivalent to the occurrence probability 1/1000.

Further, another distribution $X(F)$ has been derived by Plackett [48] under the additional restriction of $f(X)$ being symmetric, and in that case extremum $\bar{Y}_N^E$ is represented approximately as,

$$\bar{Y}_N^E \fallingdotseq N/\{2(2N-1)\}^{1/2} \qquad \cdots\cdots (3.1.8)$$

that is, $\bar{Y}_N^E$ is 15.8 for N=1000. These values of $\bar{Y}_N^E$ for Eqs.(3.1.7) and (3.1.8) are fairly large, but have a physical meaning as upper bounds of the averaged maximum of the samples of size N.

3.2 Decrease of the Estimation Value of Extremum $\bar{Y}_N^E$ due to the Selection of Characteristics

The extremum of the averaged maximum, $\bar{Y}_N^E$, is still large so long as two restrictive conditions for mean and variance are considered. How can $\bar{Y}_N^E$ be estimated lower? Decrease of extremum $\bar{Y}_N^E$ is realized herein by increasing the number of restrictive conditions characterizing the measured data. Let demonstrate the results first of all.

Various values of extremum $\bar{Y}_N^E$ for N=1000, which are calculated by using samples of sizes 50 extracted from the population following to normal distribution, are shown in Table 2 with respect to various restrictive conditions described later. The values show a tendency to decrease as number of restrictive conditions increases. Although the effect has not been proved until the actual results come out, increase of the number of restrictive conditions is supposed fairly effective for decreasing the extremum $\bar{Y}_N^E$.

What kinds of characteristics are adequate as restrictive conditions in addition to mean and variance? Characteristics are necessary to satisfy the following conditions;

a) Characteristics should be expressed in the forms of definite integrals for interval between 0 and 1.
b) Integrals should be analytically possible when $X(F)$ is substituted into restrictive conditions.
c) Integrands of characteristics should be monotone increasing functions for interval between 0 and 1, because function $X(F)$——being increasing function from the character of cummulative density function ——is defined as a sum of the integrands.
d) Characteristics should represent the properties of the measured data undoubtedly.
e) Calculations of the characteristics from the measured data should be simple so that engineers may calculate without being puzzled.

Based on these properties for characteristics, following six kinds of characteristics are chosen as restrictive conditions herein; that is, ①maximum, ②minimum, ③mean of upper blocks of size t, ④mean of lower block of size s, ⑤skewness and ⑥kurtosis. Mean of upper block of size t, u_t, is expressed as $u_t=(X_{n-t+1}+\cdots+X_n)/t$ and mean of lower block of size s, $-v_s$, as $-v_s=(X_1+\cdots+X_s)/s$ based on the order statistics $X_1 \le X_2 \le \cdots \le X_n$ which are obtained as the measured data of size n. The restrictive conditions with respect to six characteristics are expressed as follows after the expressions of Eqs. (3.1.1) and (3.1.2);

$$① \quad \int_0^1 XnF^{n-1}\,\mathrm{d}F = X_n = u_1 \qquad (3.2.1)$$

$$② \quad \int_0^1 Xn(1-F)^{n-1}\,\mathrm{d}F = X_1 = -v_1 \qquad (3.2.2)$$

③ $\int_0^1 \frac{1}{t}\sum_{i=1}^{t} X \frac{n!}{(i-1)!(n-i)!} F^{n-i}(1-F)^{i-1}\,\mathrm{d}F = \int_0^1 X g_t(F)\,\mathrm{d}F = \frac{1}{t}\sum_{i=n-t+1}^{n} X_i = u_t$ .... (3.2.3)

④ $\int_0^1 \frac{1}{s}\sum_{i=1}^{s} X \frac{n!}{(i-1)!(n-i)!} F^{i-1}(1-F)^{n-i}\,\mathrm{d}F = \int_0^1 X \bar{g}_s(F)\,\mathrm{d}F = \frac{1}{s}\sum_{i=1}^{s} X_i = -v_s$ .... (3.2.4)

⑤ $\int_0^1 X^3\,\mathrm{d}F = \beta_3$ .... (3.2.5)

⑥ $\int_0^1 X^4\,\mathrm{d}F = \beta_4$ .... (3.2.6)

Since each integrand of characteristics ①~④ is simple equation for X, the equation concerning to the unknown function $X(F)$, which is obtained by taknig $\partial J/\partial X = 0$, is still a simple equation for X. Therefore $X(F)$ can be represented as easy as the derivation of Eq. (3.1.5). On the other hand, the integrands of characteristics ⑤ and ⑥ are a cubic and a quartic equations, respectively, and the unknown function $X(F)$ becomes a cubic equation when both characteristics are employed. The solution of cubic equation for X, which is solved by Ferrari's formula, becomes quite complicated and the integral becomes analytically impossible when $X(F)$ is substituted into restrictive conditions ⑤ and ⑥ in order to determine Lagrange multipliers. That is, the characteristics ⑤ and ⑥ are not adequate as the restrictive conditions——the characteristics ⑤ and ⑥ are discussed in Chapter 4 from a different standpoint. Decrease of extremum $\bar{Y}_N^{\mathrm{E}}$ is realized herein by combinating restrictive conditions ①~④.

3.3 Extremum $\bar{Y}_N^{\mathrm{E}}$ under Three and Four Restrictive Conditions

Extremum $\bar{Y}_N^{\mathrm{E}}$ in case of three and four restrictive conditions are derived and shown below in the same manner described in Section 3.1.

① Case of three restrictive conditions for mean, variance and maximum;

$$X = (1/2\lambda_2)\{NF^{N-1} - \lambda_1 - \lambda_3 nF^{n-1}\} \quad \cdots\cdots (3.3.1)$$

$$1/2\lambda_2 = (A'_{nn} - u_1^2)^{1/2} / (A'_{NN}A'_{nn} - {A'_{Nn}}^2)^{1/2}$$

$$\lambda_3 = (A'_{Nn} - 2\lambda_2 u_1)/A'_{nn}, \quad \lambda_1 = 1 - \lambda_3$$

$$\bar{Y}_N^{\mathrm{E}} = (1/2\lambda_2)(A'_{NN} - A'_{Nn}\lambda_3) \quad \cdots\cdots (3.3.2)$$

② Case of three restrictive conditions for mean, variance and mean of upper block of size t;

$$X = (1/2\lambda_2)\{NF^{N-1} - \lambda_1 - \lambda_3 g_t(F) \quad \cdots\cdots \quad (3.3.3)$$

$$1/2\lambda_2 = (D'_{tt} - u_t^2)^{1/2} / (A'_{NN} D'_{tt} - C_t'^2)^{1/2}$$

$$\lambda_3 = (C'_t - 2\lambda_2 u_t) / D'_{tt}, \quad \lambda_1 = 1 - \lambda_3$$

$$\overline{Y}_N^{\mathrm{E}} = (1/2\lambda_2)(A'_{NN} - C'_t \lambda_3) \quad \cdots\cdots \quad (3.3.4)$$

③ Case of four restrictive conditions for mean, variance, maximum and minimum;

$$X = (1/2\lambda_2)\{NF^{N-1} - \lambda_1 - \lambda_3 nF^{n-1} - \lambda_4 n(1-F)^{n-1}\} \quad \cdots\cdots \quad (3.3.5)$$

$$1/2\lambda_2 = \{(A'^2_{nn} - B'^2_{nn}) - A'_{nn}(u_1^2 + v_1^2) - 2B'_{nn} u_1 v_1\}^{1/2} /$$

$$\{A'_{NN}(A'^2_{nn} - B'^2_{nn}) - A'_{nn}(A'^2_{Nn} + B'^2_{Nn}) + 2A'_{Nn} B'_{Nn} B'_{nn}\}^{1/2}$$

$$\lambda_3 = \{(A'_{Nn} A'_{nn} - B'_{Nn} B'_{nn}) - 2\lambda_2(A'_{nn} u_1 - B'_{nn} v_1)\} / (A'^2_{nn} - B'^2_{nn})$$

$$\lambda_4 = \{(B'_{Nn} A'_{nn} - A'_{Nn} B'_{nn}) - 2\lambda_2(A'_{nn} v_1 - B'_{nn} u_1)\} / (A'^2_{nn} - B'^2_{nn})$$

$$\lambda_1 = 1 - \lambda_3 - \lambda_4$$

$$\overline{Y}_N^{\mathrm{E}} = (1/2\lambda_2)(A'_{NN} - A'_{Nn}\lambda_3 - B'_{Nn}\lambda_4) \quad \cdots\cdots \quad (3.3.6)$$

④ Case of four restrictive conditions for mean, variance, means of upper and lower blocks of sizes t;

$$X = (1/2\lambda_2)\{NF^{N-1} - \lambda_1 - \lambda_3 g_t(F) - \lambda_4 \overline{g}_t(F)\} \quad \cdots\cdots \quad (3.3.7)$$

$$1/2\lambda_2 = \{(D'^2_{tt} - \overline{D}'^2_{tt}) - D'_{tt}(u_t^2 + v_t^2) - 2\overline{D}'_{tt} u_t v_t\}^{1/2} /$$

$$\{A'_{NN}(D'^2_{tt} - \overline{D}'^2_{tt}) - D'_{tt}(C'^2_t + \overline{C}'^2_t) + 2C'_t \overline{C}'_t \overline{D}'_{tt}\}^{1/2}$$

$$\lambda_3 = \{(C'_t D'_{tt} - \overline{C}'_t \overline{D}'_{tt}) - 2\lambda_2(D'_{tt} u_t - \overline{D}'_{tt} v_t)\} / (D'^2_{tt} - \overline{D}'^2_{tt})$$

$$\lambda_4 = \{(\overline{C}'_t D'_{tt} - C'_t \overline{D}'_{tt}) - 2\lambda_2(D'_{tt} v_t - \overline{D}'_{tt} u_t)\} / (D'^2_{tt} - \overline{D}'^2_{tt})$$

$$\lambda_1 = 1 - \lambda_3 - \lambda_4$$

$$\overline{Y}_N^{\mathrm{E}} = (1/2\lambda_2)(A'_{NN} - C'_t \lambda_3 - \overline{C}'_t \lambda_4) \quad \cdots\cdots \quad (3.3.8)$$

where

$$B_{kl} = (k!\,l!)/(k+l-1)!$$

$$C_k = \frac{N \cdot n!}{k(N+n-1)!} \sum_{i=1}^{k} \frac{(N+n-i-1)!}{(n-i)!}, \qquad \overline{C}_k = \frac{N \cdot n!}{k(N+n-1)!} \sum_{i=1}^{k} \frac{(N+i-2)!}{(i-1)!}$$

$$D_{kl}=\frac{(n!)^2}{kl(2n-1)!}\sum_{i=1}^{k}\sum_{j=1}^{l}\frac{(2n-i-j)!(i+j-2)!}{(n-i)!(n-j)!(i-1)!(j-1)!}$$

$$\overline{D}_{kl}=\frac{(n!)^2}{kl(2n-1)!}\sum_{i=1}^{k}\sum_{j=1}^{l}\frac{(n-i+j-1)!(n+i-j-1)!}{(n-i)!(n-j)!(i-1)!(j-1)!}$$

$$B'_{kl}=B_{kl}-1,\ C'_k=C_k-1,\ \overline{C}'_k=\overline{C}_k-1,\ D'_{kl}=D_{kl}-1,\ \overline{D}'_{kl}=\overline{D}_{kl}-1$$

Extremums $\bar{Y}_N^E$ are calculated for these four cases by substituting N=1000, n=50, t=5, $u_1=v_1=2.249$, $u_5=v_5=1.705$ into Eqs.(3.3.2),(3.3.4), (3.3.6) and (3.3.8) and shown in Table 2, where u_1, v_1, u_5 and v_5 are calculated based on the order statistics of normal distribution.

3.4 Extremum $\bar{Y}_N^E$ under Seven Restrictive Conditions

It is concluded from the results of Section 3.3 that extremum $\bar{Y}_N^E$ descreases as the number of restrictive conditions increases, that means of upper and lower blocks of sizes t are more effective than maximum and minimum with regard to the decrease of $\bar{Y}_N^E$. Standing on these points, standard combination of restrictive conditions is determined as seven restrictive conditions for mean, variance, means of upper blocks of sizes t_1, t_2 and t_3, means of lower blocks of sizes t_2 and t_3, where t_1, t_2 and t_3 are selected as $t_1=5$, $t_2=10$, $t_3=17$ in case of n=50, for instance.

In case of seven restrictive conditions, function $X(F)$ is expressed as follows by employing Eq.(3.2.3) for $t=t_1, t_2, t_3$ and Eq. (3.2.4) for $s=t_2, t_3$ in addition to Eqs.(3.1.1) and (3.1.2);

$$X=(1/2\lambda_2)\{NF^{N-1}-\lambda_1-\lambda_3 g_{t_1}(F)-\lambda_4 g_{t_2}(F)-\lambda_5 g_{t_3}(F)-\lambda_6\overline{g}_{t_3}(F)-\lambda_7\overline{g}_{t_2}(F)\} \quad (3.4.1)$$

Substituting Eq.(3.4.1) into Eqs.(3.1.1),(3.2.3) and (3.2.4), following simultaneous equations for λ_1, $\lambda_3\sim\lambda_7$ are set up by leaving λ_2 to be unknown;

$$[M]\{\lambda\}=\{C\}-2\lambda_2\{E\} \quad (3.4.2)$$

where

$$[\mathrm{M}] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ & D_{t_1t_1} & D_{t_1t_2} & D_{t_1t_3} & \overline{D}_{t_1t_3} & \overline{D}_{t_1t_2} \\ & & D_{t_2t_2} & D_{t_2t_3} & \overline{D}_{t_2t_3} & \overline{D}_{t_2t_2} \\ & & & D_{t_3t_3} & \overline{D}_{t_3t_3} & \overline{D}_{t_3t_2} \\ & \text{sym.} & & & D_{t_3t_3} & D_{t_3t_2} \\ & & & & & D_{t_2t_2} \end{bmatrix}$$

$$\{\lambda\}^T = \{\lambda_1 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \lambda_7\}$$
$$\{C\}^T = \{1 \quad C_{t_1} \quad C_{t_2} \quad C_{t_3} \quad \overline{C}_{t_3} \quad \overline{C}_{t_2}\}$$
$$\{E\}^T = \{0 \quad u_{t_1} \quad u_{t_2} \quad u_{t_3} \quad -v_{t_3} \quad -v_{t_2}\}$$

Solving Eq.(3.4.2), λ_1, $\lambda_3 \sim \lambda_7$ are expressed as,

$$\{\lambda\} = [\mathrm{M}]^{-1}\{C\} - 2\lambda_2[\mathrm{M}]^{-1}\{E\} \qquad (3.4.3)$$

Unknown multiplier λ_2 is derived by substituting Eq.(3.4.3) into Eq.(3.1.2) as,

$$1/2\lambda_2 = (1-\{E\}^T[\mathrm{M}]^{-1}\{E\})^{1/2}/(A_{NN}-\{C\}^T[\mathrm{M}]^{-1}\{C\})^{1/2} \qquad (3.4.4)$$

and extremum of the averaged maximum, $\bar{Y}_N^E$, is expressed as follows by substituting Eq.(3.4.4) into Eq.(3.1.3);

$$\bar{Y}_N^E = (1-\{E\}^T[\mathrm{M}]^{-1}\{E\})^{1/2}(A_{NN}-\{C\}^T[\mathrm{M}]^{-1}\{C\})^{1/2}+\{C\}^T[\mathrm{M}]^{-1}\{E\} \qquad (3.4.5)$$

Extremum $\bar{Y}_N^E$ is calculated in case of N=1000, n=50, u_5=1.705, $u_{10}=v_{10}$ = 1.372, $u_{17}=v_{17}$=1.060, and shown in Table 2, where u_5, u_{10}, v_{10}, u_{17}, v_{17} are calculated based on the order statistics of normal distribution of sizes 50.

3.5 Extremum of the Averaged Minimum

Equations concerning to the estimation of extremum $\bar{Y}_N^E$ and restrictive conditions described previously are derived in order to estimate the least upper bound of load, etc..

On the contrary, it is necessary to calculate the extremum of the averaged minimum to estimate the great lower bound of material strength, etc.. In this case, the order statistics $X'_i = -X_{n-i+1}$ $(X'_1 \leqq X'_2 \leqq \cdots \leqq X'_n)$ are newly defined by reversing the sign of X_i. Then characteristics such as u_t and v_s are calculated by using the new order statistics X'_i, and extremum of the averaged maximum, $\bar{Y}_N^{E\prime}$, is calculated by using Eq.(3.4.5). At the end, extremum of the averaged minimum, $\bar{Y}_N^E$, is obtained by taking $\bar{Y}_N^E = -\bar{Y}_N^{E\prime}$.

3.6 Applications to Various Sorts of the Measured Data

Extremums $\bar{Y}_N^E$ are calculated for various sets of the measured data. With respect to material strength, the procedure is applied for three sets of the measured data on steel tensile strength, steel buckling strength and cement compressive strength. With respect to load, the procedure is applied to a set of axial force of the lower chord member of truss bridge described in Section 2.6. Block characteriatics $u_{t_1}, u_{t_2}, u_{t_3}, v_{t_3}, v_{t_2}$ are calculated for these sets of the measured data and shown in Table 3.

The details of the measurememts are described in the following.

① Tensile strength of SM50B steel [34] : The test was carried out by Society of Steel Construction of Japan in 1968, similar to yield strength of SM41B steel discussed in Section 2.6.
② Buckling strength of SS41 steel [7] : The test was carried out by Aoki and Fukumoto at the Department of Civil Engineering of Nagoya University in 1972. The test quoted herein was performed for welded H-column with the slenderness ratio of 100.
③ Compressive strength of cement mortar [63] : The test was carried out by Cement Association of Japan in 1964. The test quoted herein was performed for 28 day's compressive strength of uniformly controlled Portland cement.
④ Axial force of the lower chord of truss bridge [64] : See Section 2.6.

Extremums $\bar{Y}_N^E$ for these sets of the measured data are shown in Table 3.

Depending upon the selection of block sizes t_1, t_2 and t_3, the part of $(1 - \{E\}^T[M]^{-1}\{E\})$ in Eq.(3.4.5) becomes negative, that is, extremum $\bar{Y}_N^E$ becomes an imaginary number. And extremum $\bar{Y}_N^E$ changes sensitively according to the selection of t_1, t_2 and t_3, even if $\bar{Y}_N^E$ is a real number. The situation is well shown in Table 3. As a result, the procedure still unsuitable for practical use. The defect is caused by the selection of the restrictive conditions, and in Chapter 4, more reliable characteristics will be introduced.

3.7 Conclusion

Maximization procedure of the averaged maximum based on the variational method has been introduced and developed in Chapter 3. The procedure has only applied to the estimations of extremums of maximum and minimum of the measured data, and has still not connected with the failure probability.

The characteristics of the procedure are as follows;

a) Restrictive conditions——characteristics——calculated from the measured data are assumed absolutely accurate based on the maximum likelihood method.
b) Extremum of the averaged maximum which can be expected by employing the measured data is obtained.
c) Distribution of $X(F)$ has a finite value at F=1——on the contrary, normal distribution becomes infinite at F=1 as often commented——and it is rational on the engineering.
d) A slight variation of the measured data has a considerable influence upon the estimation of extremum $\bar{Y}_N^E$. There is no criterion in the selection of block sizes $t_1 \sim t_3$, and the selection has a large influence upon the extremum $\bar{Y}_N^E$.
e) Although function $X(F)$ must be a monotone increasing function, this condition is often violated. And still worse, extremum $\bar{Y}_N^E$ often becomes an imaginary number.
f) Extremum $\bar{Y}_N^E$ gives sometimes an underestimated value even compared with the maximum of the measured data.

As indicated in (d)$\sim$(f), extremum $\bar{Y}_N^E$ defined herein is unreliable. These unfavorable results are caused by the unsuitable restrictive conditions. In other words, there are much negative factors in the procedure. Following improvements are required in order to correct the defects.

① Introduction of more reliable characteristics.
② Definition of the error of characteristics.

Chapter 4 Extension of the Estimation Procedure of Extremum to the Structural Safety Analysis

Applying the estimation procedure of extremum to material strength and load at the same time, the safety analysis of the structural member is proposed by connecting with maximization of the failure probability. Since maximum of the failure probability is defined—— granted that it is obtained approximately——, a sort of guarantee can be obtained as follows; that is, when a certain value A_1 is chosen as the design value, the probability that a certain value A_2 $(A_2 > A_1)$ will happen is never in excess of a certain small value ε at the worst. In other words, an arbitrary reliability level of the structural design can be guaranteed at the lowest. The concept of this design procedure is considered to be superior to the usual reliability analysis in respect that the measured data is employed more effectively and that the design is performed covering the worst state predicted from the measured data.

Three characteristics A, B and C are newly introduced as the restrictive conditions in addition to mean and variance herein. The characteristics correspond to skewness, kurtosis, etc., and can be represented as equations of low degrees for F——differing from maximum, mean of upper block, etc.. Introduction of new characteristics $A \sim C$, which is represented as equations of low degrees for F and as simple equation for X, makes the maximization of failure probability possible.

Further, following improvements are discussed herein in order to develop the estimation procedure of extremum described in Chapter 3;

a) Unknown function $X(F)$ is modified compulsorily to be a monotone increasing function. Dummy characteristic, D, is introduced for the purpose.
b) Influence of error of characteristics on the estimation of extremum $\bar{Y}_N^E$ is discussed. To be concrete, upper bound of extremum $\bar{Y}_N^E$ is presumed by defining means of characteristics $A \sim C$ and their lower bounds.
c) Graphical estimation method of extremum $\bar{Y}_N^E$ based on characteristics $A \sim C$ is developed in order to widen the utility of the procedure.

4.1 Definition of Characteristics A, B and C

The restrictive conditions that mean, variance, skewness and kurtosis are 0, 1, β_3 and β_4, respectively, are expressed as follows by employing $X(F)$ after Eqs.(3.1.1),(3.1.2),(3.2.5) and (3.2.6);

$$\int_0^1 X \, dF = 0 \qquad (4.1.1)$$

$$\int_0^1 X^2 \, dF = 1 \qquad (4.1.2)$$

$$\int_0^1 X^3 \, dF = \beta_3 \qquad (4.1.3)$$

$$\int_0^1 X^4 \, dF = \beta_4 \qquad (4.1.4)$$

The unknown function $X(F)$ is obtained by solving the variational problem of maximizing the following averaged maximum $\overline{Y}_N$;

$$\overline{Y}_N = \int_0^1 XNF^{N-1} \, dF \qquad (4.1.5)$$

However, if functional J is defined by employing Eqs.(4.1.3) and (4.1.4) as,

$$J = XNF^{N-1} - \lambda_1 X - \lambda_2 X^2 - \lambda_3 X^3 - \lambda_4 X^4$$

and taking $\partial J/\partial X = 0$, a cubic equation for X is obtained. The irrational formula, which is obtained by solving the equation for X, is analytically impossible to be integrated. Therefore, as mentioned in Section 3.2, skewness and kurtosis cannot be treated as restrictive conditions directly. Instead, the author newly introduce the following restrictive conditions, whose characteristics are similar to skewness and kurtosis, and which can be solved as a simple function for X;

$$\int_0^1 X(F-0.5)^2 \, dF = \beta_3' \qquad (4.1.6)$$

$$\int_0^1 X(F-0.5)^3 \, dF = \beta_4' \qquad (4.1.7)$$

These statistics are supposed to correspond to skewness and kurtosis, respectively. They may not be conventional statistics compared with the usual ones such as mean, variance, etc., because they are defined herein for the first time. These statistics are allowed to use as the restrictive conditiones from the following viewpoints;

a) Skewness and kurtosis are obtained by averaging the measured values X according to the weight of X^2 and X^3, respectively.
b) X can be averaged according to the weight of $(F-0.5)^2$ instead of X^2 because F is a function for X. The weight of $(F-0.5)^3$ can be substituted for X^3 in the same way.
c) $(F-0.5)^i$ and X^i $(i=1,2,\cdots)$ have the same sign in the distance from the neighborhood of the mean (or mode) except that the conditions of $(F-0.5)>0$ for $X>0$ and $(F-0.5)<0$ for $X<0$ are not always satisfied.

If these types of restrictive conditions are allowable, integral $\int_0^1 X(F-0.5)\,dF$ corresponding to variance and integral $\int_0^1 X(F-0.5)^4\,dF$ corresponding to the weighted mean which is one degree higher than kurtosis can be settled in the same way. Since the weights are expressed as a quartic equation for F, following four restrictive conditions are recommended after simplifying weighted means shown by $\int_0^1 X(F-0.5)^i\,dF$ $(i=1\sim4)$;

$$\int_0^1 XF\,dF=A \qquad (4.1.8)$$

$$\int_0^1 XF^2\,dF=B \qquad (4.1.9)$$

$$\int_0^1 XF^3\,dF=C \qquad (4.1.10)$$

$$\int_0^1 XF^4\,dF=D \qquad (4.1.11)$$

Eqs.(4.1.1),(4.1.2),(4.1.8)$\sim$(4.1.11) are employed as restrictive conditions in Chapter 4, where Eq.(4.1.11) is especially used as the dummy condition for modifying $X(F)$ to be a monotone increasing function.

4.2 Estimation of Extremum $\bar{Y}_N^E$ and the Selection of Dummy Characteristic D

The extremum of the averaged maximum, $\bar{Y}_N^E$, is derived as follows under the restrictions of Eqs.(4.1.1),(4.1.2),(4.1.8)$\sim$(4.1.11). First, functional J is defined as follows in the same way as Eq. (3.1.4);

$$J=XNF^{N-1}-\lambda_1 X-\lambda_2 X^2-\lambda_3 XF-\lambda_4 XF^2-\lambda_5 XF^3-\lambda_6 XF^4 \qquad (4.2.1)$$

and the unknown function $X(F)$ is expressed by taking $\partial J/\partial X=0$ as,

$$X=(1/2\lambda_2)(NF^{N-1}-\lambda_1-\lambda_3 F-\lambda_4 F^2-\lambda_5 F^3-\lambda_6 F^4) \qquad (4.2.2)$$

Substituting Eq.(4.2.2) into Eqs.(4.1.1),(4.1.8)∿(4.1.11), the following simultaneous equations for Lagrange multipliers λ_1, λ_3∿λ_6 are set up by leaving λ_2 to be unknown;

$$[\mathrm{M}]\{\lambda\}=\{\mathrm{C}\}-2\lambda_2\{\mathrm{E}\} \qquad (4.2.3)$$

where

$$[\mathrm{M}]=\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ & 1/3 & 1/4 & 1/5 & 1/6 \\ & & 1/5 & 1/6 & 1/7 \\ & \text{sym.} & & 1/7 & 1/8 \\ & & & & 1/9 \end{bmatrix}$$

$$\{\lambda\}^{\mathrm{T}}=\{\lambda_1 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6\}$$
$$\{\mathrm{C}\}^{\mathrm{T}}=\{1 \quad N/(N+1) \quad N/(N+2) \quad N/(N+3) \quad N/(N+4)\}$$
$$\{\mathrm{E}\}^{\mathrm{T}}=\{0 \quad A \quad B \quad C \quad D\}$$

$\{\lambda\}$ is solved as,

$$\{\lambda\}=[\mathrm{M}]^{-1}\{\mathrm{C}\}-2\lambda_2[\mathrm{M}]^{-1}\{\mathrm{E}\} \qquad (4.2.4)$$

where

$$[\mathrm{M}]^{-1}=\begin{bmatrix} 5 & -60 & 210 & -280 & 126 \\ & 960 & -3780 & 5376 & -2520 \\ & & 15876 & -23520 & 11340 \\ & \text{sym.} & & 35840 & -17640 \\ & & & & 8820 \end{bmatrix}$$

Unknown multiplier λ_2 is derived by substituting Eq.(4.2.4) into Eq. (4.1.2) as,

$$1/2\lambda_2=(1-\{\mathrm{E}\}^{\mathrm{T}}[\mathrm{M}]^{-1}\{\mathrm{E}\})^{1/2}/(A_{NN}-\{\mathrm{C}\}^{\mathrm{T}}[\mathrm{M}]^{-1}\{\mathrm{C}\})^{1/2} \qquad (4.2.5)$$

Extremum $\overline{Y}_N^{\mathrm{E}}$ is expressed as follows by substituting Eq.(4.2.5) into Eq.(4.1.5);

$$\overline{Y}_N^{\mathrm{E}}=(1-\{\mathrm{E}\}^{\mathrm{T}}[\mathrm{M}]^{-1}\{\mathrm{E}\})^{1/2}(A_{NN}-\{\mathrm{C}\}^{\mathrm{T}}[\mathrm{M}]^{-1}\{\mathrm{C}\})^{1/2}+\{\mathrm{C}\}^{\mathrm{T}}[\mathrm{M}]^{-1}\{\mathrm{E}\} \qquad (4.2.6)$$

Eqs.(4.2.3)∿(4.2.6) are similar to Eqs.(3.4.2)∿(3.4.5), and only difference is in the elements of [M], {C} and {E}.

Function $X(F)$ of Eq.(4.2.2) must be a monotone increasing function with the increase of F from 0 to 1. This demand needs to be satisfied as much as possible because the inverse function of $X(F)$ is a cumurative density under the control of Lagrange multipliers $\lambda_1 \sim \lambda_6$ in Eq.(4.2.2). Among six multipliers, λ_6 ——corresponding to dummy characteristic D——is expected to make an unique contribution to keep $X(F)$ being a monotone increasing function.

If function $X(F)$ is calculated without including Eq.(4.1.11) previously, there is a fair possibility that $X(F)$ is not a monotone increasing function. In this case, although function $X(F)$ becomes an unrealistic solution, $X(F)$ is still a strict solution by reason of being a solution of the variational problem. Extremum $\bar{Y}_N^E$ calculated by using this unrealistic solution $X(F)$ can be used as a criterion for safety, even though it is an overestimated extremum.

In order to avoid the unrealistic estimation, let $X(F)$ be derived by including Eq.(4.1.11), that is, by including dummy characteristic D. Dummy characteristic D is chosen adequately so that function $X(F)$ becomes a monotone increasing function, and so that value of $\bar{Y}_N^E$ for D becomes a maximum. That is to say, the range of D where $X(F)$ is monotonous is calculated, and after that, the largest value of $\bar{Y}_N^E$ is found in the range of D. This value is the extremum $\bar{Y}_N^E$ satisfying the condition that $X(F)$ is a monotone increasing function.

The range of dummy characteristic D can be derived as follows; Since $X'(F)$——which is obtained by differentiating Eq.(4.2.2) with respect to F——must be positive always, following inequality is obtained;

$$(1/2\lambda_2)\{N(N-1)F^{N-2}-\lambda_3-2\lambda_4 F-3\lambda_5 F^2-4\lambda_6 F^3\}>0$$

in which the coefficient of F^{N-2}, λ_2, must be positive always on account of the exponent of F, N-2, being large enough, and other coefficients including $\lambda_3 \sim \lambda_6$ must be positive as a whole. Therefore following simultaneous inequalities are obtained;

$$\left.\begin{array}{l}\lambda_2 > 0 \\ \lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2 + 4\lambda_6 F^3 < 0\end{array}\right\} \quad \cdots\cdots \quad (4.2.7)$$

The range of D is obtained by solving Eq.(4.2.7) with respect to D for $0 \le F \le 1$. First, Eq.(4.2.7) is rearranged as the simultaneous relative equations by using $\lambda_1 \sim \lambda_6$ and without using F. Since $\lambda_1 \sim \lambda_6$ are expressed by employing D through Eqs.(4.2.4),(4.2.5), Eq.(4.2.7) is expressed as the simultaneous inequalities of high degrees for D. And the range of D is obtained by solving those inequalities for D under the conditions whether the inequalities are positive or negative (See Appendix 2).

As for the calculation of extremum of the averaged minimum, the same manner is recommended as mentioned in Section 3.5.

4.3 Correlation with the Maximization of Failure Probability

A new quantity $\bar{Y}_p$ is defined below instead of $\bar{Y}_N$;

$$\bar{Y}_p = \int_0^1 X\psi(F)\,\mathrm{d}F \quad \cdots\cdots \quad (4.3.1)$$

$$(\psi(F) = 1/p \quad \text{for} \quad 1-p \leqq F \leqq 1, \quad \psi(F) = 0 \quad \text{for} \quad 0 \leqq F < 1-p)$$

New quantity $\bar{Y}_p$ implies the averaged value of the part corresponding to the upper probability p. The unknown function $X(F)$ in this case is expressed as,

$$X = (1/2\lambda_2)\{\psi(F) - \lambda_1 - \lambda_3 F - \lambda_4 F^2 - \lambda_5 F^3 - \lambda_6 F^4\} \quad \cdots\cdots \quad (4.3.2)$$

Function $X(F)$ is composed of pulse $\psi(F)$ and smooth functions of four degrees for F at the most, therefore $X(F)$ is represented as a combination of a pulse and a tail part (See Fig.9-1). The shape of a pulse part is not flat as result of the existence of $F \sim F^4$, but it is still a monotone increasing function with smooth variations. Then the probability density $f(X) = 1/(\mathrm{d}X/\mathrm{d}F)$ may probably be shown as Fig.9-2.

With respect to material strength, extremum of $\bar{Y}_{p_R}$ for $p = p_R$, $\bar{Y}^E_{p_R}$, is calculated. With respect to load, extremum of $\bar{Y}_{p_S}$ for $p = p_S$, $\bar{Y}^E_{p_S}$, is calculated. Since $\bar{Y}^E_{p_R}$ and $\bar{Y}^E_{p_S}$ are normalized deviations from the normalized mean 0, real deviations become $R_{\min}(p_R)=\bar{R}+s_R\bar{Y}^E_{p_R}$ and $S_{\max}(p_S)=\bar{S}+s_S\bar{Y}^E_{p_S}$ respectively. $\bar{Y}^E_{p_R}$ is a negative number and $\bar{Y}^E_{p_S}$ is a positive number. The state of $R_{\min}(p_R)=S_{\max}(p_S)$ is shown in Fig.9-3 imaginatively. Since failure probability is defined as,

$$P_f=\int_{R<S} f_R(R)\, f_S(S)\, dS dR$$

the failure probability corresponding to $R_{\min}(p_R)=S_{\max}(p_S)$ is shown as the shadowed portion in Fig.9-3, and evaluated approximately as $P_f = p_R\, p_S/2$.

Lower and upper bounds, $R_{min}(p_R)$ and $S_{max}(p_S)$, are extremums in which the pulse parts (See Fig.9-2) corresponding to lower probability p_R and upper probability p_S are kept as far as possible from their means $\bar{R}$ and $\bar{S}$. The situation $P_f = p_R\, p_S/2$ corresponding to $R_{\min}(p_R)=S_{\max}(p_S)$ represents an extreme state in which the difference between $\bar{R}$ and $\bar{S}$ is taken as wider as possible. That is, if R_{min} and S_{max} are employed as the design criterion, failure probability of the design is less than $p_R\, p_S/2$ at the worst. In other words, the use of $\bar{Y}^E_{p_R}$ and $\bar{Y}^E_{p_S}$ has the same meaning of the maximization of failure probability.

Probabilities p_R and p_S are determined as follows;

① p_R is always equivalent to p_S; that is, assumed as $p_R = p_S = p$.
② p_R is not always equivalent to p_S; combination of p_R and p_S which realizes the most safe design is selected among the combinations of p_R and p_S which satisfy $P_f = p_R\, p_S/2 = \text{const.}$.

Process ① is introduced as the first approximation, though it cannot be used except in case of $p_R = p_S$. Process ② is introduced, on the contrary, as the approximation of safety-side, though it cannot be treated analytically. Following procedure is taken herein as a compromise between process ① and process ②;

③ Lower and upper bounds R_{min} and S_{max} are calculated by assuming $p_R = p_S = p$, and after that, R_{min} and S_{max} are modified by multiplying the extra coefficients according to the coefficients of variation δ_R and δ_S of the measured data, where extra coefficients are previously calculated for every combination of δ_R and δ_S by taking the maximums of ratios of the design realized by supposing $P_f = p_R p_S/2$ to the design realized by supposing $P_f = p^2/2$. Design is performed by employing the modified values for R_{min} and S_{max}. Example of extra coefficients are shown in Fig.10 in case of $p = 1/50$ ($P_f = 1/2500$).

The difference between $\bar{Y}_p^E$ and $\bar{Y}_N^E$ is discussed in the following. Substituting Eq.(4.3.2) into restrictive conditions Eqs.(4.1.1),(4.1.8) $\sim$(4.1.11), simultaneous equations for λ_1, $\lambda_3 \sim \lambda_6$ are set up as,

$$[M]\{\lambda\} = \{C'\} - 2\lambda_2\{E\} \quad \cdots\cdots \quad (4.3.3)$$

where

$$\{C'\}^T = \{1 \quad \frac{1-(1-p)^2}{2p} \quad \frac{1-(1-p)^3}{3p} \quad \frac{1-(1-p)^4}{4p} \quad \frac{1-(1-p)^5}{5p}\}$$

The only difference is in matrix {C} comparing Eq.(4.2.3) with Eq.(4.3.3). If let p be $2/N$, i.e., $p = 2/N$, the difference is described as,

$$\{C\} - \{C'\} = \{D\} = \{O(1/N^2)\}$$

where

$$\{D\}^T = \{0 \quad \frac{1}{N(N+1)} \quad \frac{8(N-1)}{3N^2(N+2)} \quad \frac{5N^2-10N+6}{N^3(N+3)} \quad \frac{8(5N^3-15N^2+18N-8)}{5N^4(N+4)}\}$$

The difference is negligible small for $N > 10$.

Further, $\bar{Y}_p^E$ is expressed as,

$$\bar{Y}_p^E = (1-\{E\}^T[M]^{-1}\{E\})^{1/2}(1/p-\{C'\}^T[M]^{-1}\{C'\})^{1/2} + \{C'\}^T[M]^{-1}\{E\} \quad \cdots\cdots \quad (4.3.4)$$

The difference between $\bar{Y}_p^E$ and $\bar{Y}_N^E$ is also represented as follows with regard to $A_{NN} = N^2/(2N-1) \simeq N/2 = 1/p$;

$$\bar{Y}_N^E - \bar{Y}_p^E = O(1/N^2)$$

As a result, extremum $\bar{Y}_p^E$ and $\bar{Y}_N^E$ can be employed in the same way for $N > 10$.

Strictly speaking, $\bar{Y}_p^E$ is employed for the estimation of failure probability and $\bar{Y}_N^E$ is employed for the estimation of extremum of the averaged maximum itself——for the estimation of maximum discharge in 100 years, for instance. The author propose to employ $\bar{Y}_N^E$ in both cases with the object of saving the trouble of keeping proper use of $\bar{Y}_N^E$ and $\bar{Y}_p^E$, because $\bar{Y}_N^E$ and $\bar{Y}_p^E$ are equivalent approximately.

4.4 Means of Characteristics A, B and C and Their Lower Bounds

Characteristcs $A \sim C$ of Eqs.(4.1.8)∿(4.1.11) are recommended to estimate in the form of $\bar{A}$, $\bar{B}$ and $\bar{C}$——means of A, B and C——based on the order statistics $X_1 \leq X_2 \leq \cdots \leq X_n$ of the measured data of size n.

Characteristic A, for instance, is expressed as,

$$A = \int_0^1 XF\mathrm{d}F$$

and can be expressed in the form of arithmetic sum as follows;

$$A = (1/n)\sum_{i=1}^{n} X_i F_i$$

Then mean of characteristic A, $\bar{A}$, is expressed as,

$$\bar{A} = \overline{(1/n)\sum X_i F_i} = (1/n)\sum \overline{X_i F_i}$$

Further characteristic mean $\bar{A}$ can be expressed more simply as follows on account of the maximum likelihood estimator of $\bar{X}_i$ being X_i ;

$$\bar{A} = (1/n)\sum X_i \bar{F}_i \qquad \text{(4.4.1)}$$

Characteristic means $\bar{B}$ and $\bar{C}$ are also expressed as follows;

$$\left.\begin{aligned} \bar{B} &= (1/n)\sum X_i \overline{F_i^2} \\ \bar{C} &= (1/n)\sum X_i \overline{F_i^3} \end{aligned}\right\} \qquad \text{(4.4.2)}$$

The probability density $g(F_i)$ for $F_i = F(X_i)$ corresponding to the i-th value of X_i is defined as [31],

$$g(F_i)=\frac{n!}{(n-i)!(i-1)!}F_i^{i-1}(1-F_i)^{n-i} \quad \cdots\cdots (4.4.3)$$

Since $\overline{F_i}$, $\overline{F_i^2}$ and $\overline{F_i^3}$ in Eqs.(4.4.1) and (4.4.2) are expressed as,

$$\left.\begin{aligned}&\overline{F_i}=\int_0^1 g(F_i)F_i\,dF_i=i/(n+1)\\&\overline{F_i^2}=i(i+1)/(n+1)(n+2),\quad \overline{F_i^3}=i(i+1)(i+2)/(n+1)(n+2)(n+3)\end{aligned}\right\}\cdots\cdots (4.4.4)$$

characteristic means $\bar{A}$, $\bar{B}$ and $\bar{C}$ are expressed as follows by substituting Eq.(4.4.4) into Eqs.(4.4.1) and (4.4.2);

$$\left.\begin{aligned}&\overline{A}=\{1/n(n+1)\}\sum iX_i\\&\overline{B}=\{1/n(n+1)(n+2)\}\sum i(i+1)X_i\\&\overline{C}=\{1/n(n+1)(n+2)(n+3)\}\sum i(i+1)(i+2)X_i\end{aligned}\right\}\cdots\cdots (4.4.5)$$

With the exception of characteristic means $\bar{A}\sim\bar{C}$, other statistics such as central values $\hat{A}\sim\hat{C}$, modes $\tilde{A}\sim\tilde{C}$, medians $\breve{A}\sim\breve{C}$ may be supposed as the estimators of characteristics $A\sim C$. Central value $\widehat{F_i}$, mode $\widetilde{F_i}$ and median $\breve{F_i}$ of F_i are expressed as [31],

$$\left.\begin{aligned}&\widehat{F_i}=(i-0.5)/n,\ \widetilde{F_i}=(i-1)/(n-1)\ (i\neq 1,n)\\&\breve{F_i}\approx(i-0.3)/(n+0.4)\end{aligned}\right\}\cdots\cdots (4.4.6)$$

where $\widehat{F_i}$ is an intuitive estimator; $\widetilde{F_i}$ is inadequate estimator because it cannot be defined at $i=1,n$. The second and third powers of F_i are represented as follows for central values and medians;

$$\left.\begin{aligned}&\widehat{F_i^2}=(\widehat{F_i})^2,\quad \widehat{F_i^3}=(\widehat{F_i})^3\\&\breve{F_i^2}=(\breve{F_i})^2,\quad \breve{F_i^3}=(\breve{F_i})^3\end{aligned}\right\}\cdots\cdots (4.4.7)$$

When characteristics $A\sim C$ are estimated by employing their means, central values and medians, the smallest estimations of $A\sim C$ are always obtained by the means $\bar{A}\sim\bar{C}$. Since $\bar{Y}_N^E$ has a tendency to increase as characteristics $A\sim C$ decrease, characteristic means $\bar{A}\sim\bar{C}$ are employed henceforth as the estimators of characteristics $A\sim C$ from a standpoint of the estimation on safety side.

The measured data is a set of samples extracted from the population, and characteristics $A\sim C$ of the measured data are not in accord with characteristics $A \sim C$ of the population——mean and variance are always 0 and 1, respectively. In the estimation procedure of extremum, $\bar{Y}_N^E$ is calculated by giving full credit to the measured data. The lower bounds of characteristics $A \sim C$ are defined in the following, and the upper bound of $\bar{Y}_N^E$ is also defined.

If there is the error of size ΔX_i in the individual value X_i of the measured data, the real i-th value becomes $X_i + \Delta X_i$. Mean and variance of real X are expressed as,

$$\left.\begin{aligned} \overline{X_{\mathrm{real}}} &= (1/n)\sum(X_i+\Delta X_i) = \overline{X} + \overline{\Delta X} \\ s^2_{X_{\mathrm{real}}} &= \{1/(n-1)\}\sum(X_i+\Delta X_i)^2 - (\overline{X_{\mathrm{real}}})^2 = s_X^2 + s_{\Delta X}{}^2 + 2\,\mathrm{Cov}(X,\ \Delta X) \end{aligned}\right\} \cdots\cdots \quad (4.4.8)$$

That is, real mean and real variance hardly become 0 and 1, respectively, and X_i must be normalized again by using $\overline{X_{real}}$ and $s^2_{X real}$. $\overline{X_{real}}$ and $s^2_{X real}$ are expressed as follows on account of $\bar{X}=0$ and $s_X^2 = 1$ in Eq.(4.4.8);

$$\overline{X_{\mathrm{real}}} = \overline{\Delta X}, \qquad s^2_{X_{\mathrm{real}}} = 1 + s_{\Delta X}{}^2 + 2\,\mathrm{Cov}(X,\ \Delta X)$$

If ΔX_i is supposed to occur evenly in positive and negative range, following approximations are obtained;

$$\Delta X = \mathrm{Cov}(X,\ \Delta X) \approx 0$$

Therefore $\overline{X_{real}}$ and $s^2_{X real}$ are expressed simply as,

$$\overline{X_{\mathrm{real}}} = 0, \qquad s^2_{X_{\mathrm{real}}} = 1 + s_{\Delta X}{}^2 \quad \cdots\cdots \quad (4.4.9)$$

Re-normalized value of X_i, $X_{i,real}$, is obtained as follows by using Eq.(4.4.9);

$$X_{i,\mathrm{real}} = (X_i - \overline{X_{\mathrm{real}}})/s_{X_{\mathrm{real}}} = X_i/(1+s_{\Delta X}{}^2)^{1/2} \quad \cdots\cdots \quad (4.4.10)$$

The lower bounds of characteristic means $\bar{A} \sim \bar{C}$, i.e., $\bar{A}_L$, $\bar{B}_L$ and $\bar{C}_L$, are calculated by substituting $X_{i,real}$ into Eq.(4.4.5) instead of X_i.

Variance $s^2_{\Delta X}$ in Eq.(4.4.10) is calculated approximately in the following. Since ΔX_i cannot be evaluated directly, ΔX_i is estimated indirectly by using ΔF_i. Standard deviation of F_i, s_{F_i}, is defined as a correspondence of the absolute value of ΔF_i, that is,

$$|\Delta F_i| = s_{F_i}$$

Standard deviation s_{F_i} is expressed as follows in the same manner for $\bar{F}_i$;

$$s_{F_i} = \{ \int_0^1 g(F_i) F_i^2 dF_i - (\bar{F}_i)^2 \}^{1/2} = \{ i(n-i+1)/(n+1)^2(n+2) \}^{1/2} \quad \cdots\cdots\cdots\cdots (4.4.11)$$

The absolute value of ΔX_i, $|\Delta X_i|$, is estimated by using s_{F_i} based on the X-F curve as shown in Fig.11. In Fig.11, the upper and lower bounds of F_i with deviation s_{F_i} are shown as a and b. Further, the upper and lower bounds for $F_1 \sim F_{i-1}$, $F_{i+1} \sim F_n$ are also plotted and interpolated, a banana-shape curve is obtained as shown in Fig.11. Deviation of X_i, $|\Delta X_i|$, which corresponds to $|\Delta F_i|$, is shown as $\overline{cd}/2$ in the figure. Variance of ΔX_i, $s_{\Delta X}^2$, is obtained by calculating the variance of $|\Delta X_i|$ as,

$$s_{\Delta X}^2 = \{ 1/(n-1) \} \sum |\Delta X_i|^2 \quad \cdots\cdots\cdots\cdots (4.4.12)$$

4.5 Application to Various Sorts of the Measured Data

Characteristic means $\bar{A}$, $\bar{B}$ and $\bar{C}$ are calculated and the extremums $\bar{Y}_N^E$ are derived for various examples of the measured data in the field of civil engineering. With respect to material strength, the measured data of tensile, buckling and compressive strengthes described in Sections 2.6 and 3.6 are discussed herein. With respect to load, the measured data of rainfalls, discharges and wind velocities are discussed herein in addition to the axial force of truss bridge described in Sections 2.6 and 3.6. Using the measured data, mean $\bar{x}$, standard deviation s_X, coefficient of variation δ_X, characteristic means $\bar{A} \sim \bar{C}$ are calculated and shown in Table 4, and also $\bar{A} \sim \bar{C}$ and their lower bounds $\bar{A}_L \sim \bar{C}_L$ are shown in Table 5.

The details of the measured data are described in the following.

① Tensile strength of steel [34] : Yield and tensile strengthes of SM41B steel, yield and tensile strengthes of SM50B steel, are discussed.

2 Buckling strength of steel [7] : See Section 3.6.

3 Comressive strength of cement mortar [63] : See Section 3.6.

4 Axial force of chord member [64] : See Sections 2.6 and 3.6. Axial force of the upper chord member is also discussed.

5 Rainfall [65] : Maximum daily rainfalls per year (unit:mm) observed over 1899-1969 at Tsukechi and over 1923-1969 at Ootaki in the water area of Kiso, are discussed.

6 Discharge [66] : Maximum daily discharges per year (unit:m^3/sec) observed over 1956-1973 at Inuyama in the water area of Kiso and at Kamo in the water area of Yodo, are discussed.

7 Wind velocity [67,68] : Maximum average wind velocities per year (unit:m/sec) observed over 1923-1974 at Nagoya, and maximum wind velocities per second and average wind velocities per year observed over 28∿33 years up to 1972 at Naha and at all-Okinawa Prefecture, are discussed.

Extremums $\bar{Y}_N^E$ are calculated for N=100 and shown in Table 4 for the above-mentioned 17 examples of seven kinds of measurements. Especially for measurements 1, 2 and 4, which are connected with the structural design in Section 4.7, the relationships between N and $\bar{Y}_N^E$ are shown in Fig.12. Further, extremum $\bar{Y}_N^E$ is calculated for N=1000 based on the order statistics of normal distribution of size 50, and shown in Table 2 by comparing it with various $\bar{Y}_N^E$ calculated in Chapter 3.

With respect to measurements 5, 6 and 7, extremum $\bar{Y}_N^E$ is also shown in Table 6 for N=100, 200. These extremums $\bar{Y}_N^E$ corresponding to so-called the 100 years values and the 200 years values being often employed in case of river engineering,etc.. The estimated values based on the usual Gumbel's method, where double exponential distribution is employed, are also shown in Table 6 in comparison with $\bar{Y}_N^E$ proposed herein. In Table 6, the 100 years value of rainfall at Tsukechi——258.4mm——is not so larger than maximum of the measured data od size 71——255.7mm——, and the 100 years value of average wind velocity at Nagoya——35.2m/sec——is evidently smaller than the maximum of the measured data of size 52——36.5m/sec. Gumbel's method has a possibility of estimating the extremum on dangerous side. On the contrary, extremum $\bar{Y}_N^E$ proposed herein never provide an estimation on dangerous side so far as Table 6 is concerned, though it still has a tendency of overestimation.

4.6 Extension to the Structural Design, and Criterion of the Structural Safety

In the preceding Sections, extremum $\bar{Y}_N^E$ (of Eq.(4.1.5)) is explained as the maximum extremum by which the failure probability can be guaranteed not to exceed $P_f = p^2/2 = 2/N^2$, when $\bar{Y}_N^E$ are calculated both for material strength and for load of the structural member. Let Y_R be $\bar{Y}_N^E$ for member strength, where lower probability is p, and Y_S be $\bar{Y}_N^E$ for member force, where upper probability is p. If the cross-sectional area, A_S, is determined by employing Y_R and Y_S so as to satisfy the following inequality,

$$A_S(\bar{R} + s_R Y_R) \geqq \bar{S} + s_S Y_S \qquad (4.6.1)$$

the probability that the structural member happens to fail is expected not to exceed

$$P_f = p^2/2 = 2/N^2 \qquad (4.6.2)$$

at the worst. This procedure is considered as a powerful standard of judgment for design on account of making efficient use of the measured data of limited supply, of including no direct supposition for the distribution of the population of the measured data, and also of creating the most dangerous situation for the structural failure based on the variational method.

Let review the meaning of N in detail. It is put in order as follows;

a) Failure probability of the design realized by $A_S(\bar{R} + s_R Y_R) \geqq \bar{S} + s_S Y_S$ is guaranteed being less than $P_f = p^2/2 = 2/N^2$ based on the variational principle.
b) If occurrence probability of the structural failure is defined as P_f', its return period T is expressed as $T = 1/P_f'$ and corresponds to the structural life. Since the measured data for S is collected in every one week, the unit of return period, ΔT, becomes one week. Then P_f' and T can be connected as $P_f' = (1\text{ week})/(T\text{ weeks}) = 1/(52\times T)$.
c) Based on (a) and (b), the structure which survives during T years

is realized by setting $P_f' = P_f = p^2/2 = 2/N^2$. Therefore N is determined as $N = (2/P_f')^{1/2}$.

d) On the contrary, N=100 directly implies 100 years, for instance, when this procedure is applied to estimate the 100 years value of rainfall, etc..

e) In conclusion, N is managed separately for the structural design and for the estimation of the 100 years value.

4.7 An Example of the Structural Design

A calculating example of the member design of truss bridge is provided by employing the test results of yield strength of SM41B steel and buckling strength of SS41 steel for the member strength, and of the simulated member force of the lower and upper chords of main truss for the member force. The member design is applied for the end lower chord (tension) and the end upper chord (compression) of the truss as shown in Fig.5-1 with thick lines. Relationship between N and $\bar{Y}_N^E$ shown in Fig.12 is employed. Y_R and Y_S are read for the value of N (=$2/p$) corresponding to the structural life of the members, P_f (=$p^2/2$). Thus the cross-sectional areas of the members, A_S, are decided so as to satisfy Eq.(4.6.1).

Let the structural life be 100 years. In this case, N is determined as nearly 100 by reason of the samples of load being consist of maximum measured values per one week, and by using Eq.(4.6.2), where

$$P_f = (\text{1 week}) / (\text{100 years}) = p^2/2 = 2/N^2 \approx 1/5200 \;, \quad N \approx 100$$

Values of Y_R and Y_S for N=100 are read on Fig.13——or obtained from Tables 4 and 5 directly——as follows;

yield strength of SM41B steel	:	$Y_R = -3.52 \;\; (-5.03)$
buckling strength of SS41 steel	:	$Y_R = -3.64 \;\; (-4.08)$
axial forces of chord members	:	$Y_S = \;\; 4.19 \;\; (\;\; 4.73)$

where numerical values in the parentheses imply the upper bounds of Y_R and Y_S (See Table 5). These values are normalized ones, and the original values are obtained with reference to Table 4 as follows;

yield strength of SM41B steel : $\overline{R} + s_R Y_R = 2.333$ (2.153) t/cm²

axial force of lower chord : $\overline{S} + s_S Y_S = 22.61$ (23.66) t

buckling strength of SS41 steel : $\overline{R} + s_R Y_R = 0.683$ (0.614) t/cm²

axial force of upper chord : $\overline{S} + s_S Y_S = 45.22$ (47.33) t

Therefore cross-sectional areas A_S of the chord members are calculated as follows;

lower chord member : $A_S = 22.61/2.333 = 10$ (11) cm²

upper chord member : $A_S = 45.22/0.683 = 67$ (78) cm²

Further, values of A_S are increased in proportion to the extra coefficients with regard to the error due to the assumption $p_R = p_S = p$. The extra coefficients for A_S are read on Fig.10 as 1.15 for lower chord and 1.6 for upper chord. Finally, cross-sectional areas A_S are decided as follows;

lower chord member : $A_S = 10 \times 1.15 = 12$ (13) cm²

upper chord member : $A_S = 67 \times 1.6 = 108$ (125) cm²

On the contrary, let the cross-sectional areas of the same members be calculated based on the allowable stress design. With respect to the lower chord member, area A_S is calculated as 28 cm² as described in Section 2.6. With respect to the upper chord member, area A_S is calculated as 109 cm² based on the allowable stress for compression, $\sigma_{ca} = 1.300 - 0.00006 \times 100^2 = 0.700$ (t/cm²) and the live load, $L_\ell = 75.80$ (t), that is;

lower chord member : $A_S = 28$ cm²

upper chord member : $A_S = 109$ cm²

In consequence, absolute safety factors, γ_{abs}, including in the allowable stress design are estimated as follows for the live load;

tension member : $\gamma_{abs} = 28/12 = 2.3$ (2.1)

compression member : $\gamma_{abs} = 109/108 = 1.0$ (0.87)

The tension member has much allowance for the structural failure as compared with the compressive member.

The member design according to the upper bound of extremum $\bar{Y}_N^E$ ——numerical values in the parenthese——creates an absolute safety factor less than 1——0.87——for the compression member. For practical use, so-called 'safe' design will be realized sufficiently by using $\bar{Y}_N^E$——without using the upper bound of $\bar{Y}_N^E$, $\bar{Y}_{N,U}^E$.

4.8 Graphical Estimation Method of Extremum $\bar{Y}_N^E$ for characteristics A, B and C, and of the Upper Bound of $\bar{Y}_N^E$

Characteristics A, B and C can be handily calculated from the measured data, but a computer is required for the calculation of extremum $\bar{Y}_N^E$. The situation is inconvenient for the practical use, and graphical estimation method of $\bar{Y}_N^E$ is developed in this Section.

First, extremum $\bar{Y}_N^E$ are calculated for the fixed combination of A and B by taking C as a variable, and let the maximum of $\bar{Y}_N^E$ be Y_{max}. Y_{max} are calculated for various combinations of A and B, and the relationships between (A,B) and Y_{max} are drawn in a graph. Next, in the same way, Y_{max} are calculated for the combinations of (B,C) and (C,A), and the relationships between (B,C) and Y_{max}, between (C,A) and Y_{max} are obtained, respectively. As a result, three graphes corresponding to $[(A,B)\text{-}Y_{max}]$, $[(B,C)\text{-}Y_{max}]$ and $[(C,A)\text{-}Y_{max}]$ are obtained.

In case of $N = 100,200,400$, i.e., $p = 1/50, 1/100, 1/200$, the relationships of $[(A,B)\text{-}Y_{max}]$, $[(B,C)\text{-}Y_{max}]$ and $[(C,A)\text{-}Y_{max}]$ are calcalated and shown in Figs.13-1∿13-3 (N=100), Figs.14-1∿14-3 (N=200), Figs.15-1∿15-3 (N=400), respectively. Value of $\bar{Y}_N^E$ is approximately defined as the minimum among three Y_{max} being obtained for (A,B), (B,C) and (C,A) from Figs.13, 14 and 15. The graphical estimation method is applied to characteristic means $\bar{A}$∿$\bar{C}$ in Tables 4 and 5, and estimated values of $\bar{Y}_N^E$ are shown in Tables 5 and 7 for N=100. The error of the method is less than 3% as shown in Table 7, and is sufficiently small for practical use.

The variation of $\bar{Y}_N^E$ caused by the error of characteristic means

$\bar{A}\sim\bar{C}$ is convenient if it is discussed on the graphes. The ranges of $\bar{A}\sim\bar{C}$ are expressed as,

$$\bar{A}_L \leqq A \leqq 2\bar{A}-\bar{A}_L \ , \quad \bar{B}_L \leqq B \leqq 2\bar{B}-\bar{B}_L \ , \quad \bar{C}_L \leqq C \leqq 2\bar{C}-\bar{C}_L$$

based on the lower bounds of $\bar{A}\sim\bar{C}$, i.e., $\bar{A}_L\sim\bar{C}_L$, defined in Section 4.4. When characteristic means $\bar{A}\sim\bar{C}$ vary between these regions, Y_{max} are represented as fan-shape regions on Figs.13∿15, respectively. For instance, the region of Y_{max} for (C,A) is shown in Fig.16 in case of buckling strength of SS41 steel. And the upper bound of Y_{max}, $Y_{max,U}$, is represented as the upper bound of the region as shown in Fig.16. In Fig.16, Y_{max} and $Y_{max,U}$ are read as 3.64 and 4.08, respectively. Finally, upper bound of extremum $\bar{Y}_N^E$, $\bar{Y}_{N,U}^E$, defined as the minimum among three $Y_{max,U}$, which are obtained for (A,B), (B,C) and (C,A). Values of $\bar{Y}_{N,U}^E$ are calculated for $\bar{A}\sim\bar{C}$ in Table 5, and shown in Table 5. The ratios of $\bar{Y}_{N,U}^E$ to $\bar{Y}_N^E$ are also shown in Table 5.

4.9 Conclusion

The extreme procedure without any approximation of distributions, which was introduced in Chapter 3, has applied to the maximization of failure probability and connected with the structural design.

The characteristics of the procedure are described in the following.

a) New characteristics A, B and C corresponding approximately to skewness, kurtosis, etc. have been defined. These are stable characteristics, that is, a slight variation of the measured data hardly exert an influence upon the estimation of extremum $\bar{Y}_N^E$.
b) Unknown function $X(F)$ has been modified compulsorily to a monotone increasing function by introducing dummy characteristic D.
c) Even if dummy characteristic D is not considered and function $X(F)$ does not become a monotone increasing function, extremum $\bar{Y}_N^E$ which is obtained mechanically remains a kind of bound value for judging the structural safety on safety side.
d) The procedure has been connected with the maximization of failure probability. And the design is performed, where the obtained failure probability of the structural member never exceed the initially expected one even at the worst.

e) The procedure has given a practical result when it was applied to the practical design as shown in Section 4.7. In Section 4.7, absolute safety factor γ_{abs} including in the allowable stress design has been also evaluated conceptionally.
f) The influence of error of characteristics $A \sim C$ on the estimation of extremum $\bar{Y}_N^E$ has been discussed. Upper bound of extremum, $\bar{Y}_{N,U}^E$, has been defined approximately. This corresponds to error of characteristics discussed in Chapter 2.
g) The graphical estimation method of extremum $\bar{Y}_N^E$ has been developed, and by using the method the extremum $\bar{Y}_N^E$ can be estimated briefly with high accuracy for the practical use.
h) In case of the estimation of the 100 years values of rainfall, discharge, etc., the procedure has been proposed to employ as a substitute for the usual Gumbel's method, etc.. And the procedure is expected not to underestimate these values.

Chapter 5 Conclusion

Two kinds of errors——error of characteristics and error of distributions——have been discussed as factors which influence upon the accuracy of the static reliability analysis. Some procedures concerning to the reliability analysis have been presented so as not to evaluate a structural reliability on dangerous side even if the errors exist.

In Chapter 2, error of characteristics has been discussed mainly. The influence of the differences between population mean and mean of the measured data and between population variance and variance of the measured data on the estimation of the failure probability, have been evaluated for normal, gamma and log-normal distributions. Then the definition of the usual failure probability has been modified with regard to the errors. With respect to error of distributions, larger selection of the time interval of the sampling for load S, ΔT, has been recommended for the purpose of decreasing the influence of the error.

Maximization procedure of the averaged maximum based on the variational method has been proposed as a course to the design concept without any approximation of distributions, which is difficult to evaluate numerically in the usual reliability analysis. In Chapter 3, as an initial step to the purpose, estimation procedure of extremum of the averaged maximum and minimum of the measured data has been developed. However, as a result, the procedure has been indicated unsuitable for practical use on account of the unsuitable selection of the characteristics.

In Chapter 4, more reliable characteristics have been introduced, and the application to the structural design has been proposed by connecting the procedure with the maximization of the failure probability. The procedure realizes the structural design where the obtained failure probability never exceed the initially expected one even at the worst, in other words, where a kind of guarantee for the safety can be obtained. Besides, error of characteristics has been evaluated approximately, and the graphical estimation method of extremum $\bar{Y}_N^E$ has been developed for practical use.

The structural safety analysis based on the estimation method of extremum will be expected to apply to the following fields of study in the future.

1. Establishment of the safety criterion of the structure as a whole.
2. Application to the safety analysis of the structure subjected to dynamic loadings such as earthquake excitation.

The design concept based on the course proposed herein is hoped to be popularized through these extensions.

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Appendix 1 Derivation of the Square Transformation Formula

The probability density of gamma distribution, f_g, is expressed as,

$$f_g(t)\,dt = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(t-\xi)^{\alpha-1}\exp\{-(t-\xi)/\beta\}\,dt \quad (t \geqq \xi) \qquad \text{(A 1.1)}$$

where

α : shape parameter

β : scale parameter

ξ : location parameter

Mean and variance of the distribution is represented as,

mean : $\alpha\beta$, variance : $\alpha\beta^2$

Then α and β are estimated as follows by using sample mean $\bar{x}$ and sample variance s_x^2 based on the moment method [69];

$$\alpha = (\bar{x}/s_x)^2 \ , \quad \beta = s_x^2/\bar{x} \qquad \text{(A 1.2)}$$

If Eq.(A1.1) is transformed by using following parameter t',

$$t' = (t-\xi)/\beta \qquad \text{(A 1.3)}$$

standard gamma distribution is obtained as,

$$f_g(t')\,dt' = \frac{1}{\Gamma(\alpha)}\,t'^{\alpha-1}\exp(-t')\,dt' \quad (t' \geqq 0) \qquad \text{(A 1.4)}$$

Eq.(A1.4) can be transformed into χ^2-distribution with ν degrees of freedom by using the relationships as follows [69];

$$\nu = 2\alpha \ , \quad \chi^2 = 2t' \qquad \text{(A 1.5)}$$

Thus

$$f_{\chi^2}(\chi^2)\,d(\chi^2) = \frac{1}{2\Gamma(\nu/2)}\left(\frac{\chi^2}{2}\right)^{\nu/2-1} \exp\left(-\frac{\chi^2}{2}\right) d(\chi^2) \qquad \text{(A1.6)}$$

χ^2-distribution is connected with normal distribution with the aide of various approximate formulae, and Fisher's approximate formula [69] is employed herein. That is, by transforming variable χ^2 using following parameter u',

$$u' = (2\chi^2)^{1/2} - (2\nu - 1)^{1/2} \qquad \text{(A1.7)}$$

variable u' follows to (0,1) normal distribution approximately in case of $\nu > 10$.

Mean and variance of final normal distribution must be in accord with ones of original gamma distribution. Therefore, variable u' is transformed compulsorily as,

$$u = s_x u' + \bar{x} \qquad \text{(A1.8)}$$

The relationship of the transformation is shown in the same chart as,

normal distribution (u)
↓ Eq. (A1.8) .... $u = s_x u' + \bar{x}$
(0,1) normal distribution (u')
↓ Eq. (A1.7) .... $u' = (2\chi^2)^{1/2} - (2\nu - 1)^{1/2}$
χ^2-distribution (χ^2)
↓ Eq. (A1.5) .... $\chi^2 = 2t'$, $\nu = 2\alpha$
standard gamma distribution (t')
↓ Eqs. (A1.2), (A1.3) .... $t' = (t - \xi)/\beta$, $\alpha = (\bar{x}/s_x)^2$, $\beta = (s_x^2/\bar{x})$
gamma distribution (t)

In conclusion, variable t following to gamma distribution can be transformed into variable u following to normal distribution approximately based on the following formulae;

$$
\begin{aligned}
u &= s_x u' + \overline{x} \\
&= s_x \,(2 \chi^2)^{1/2} - s_x (2 \nu - 1)^{1/2} + \overline{x} \\
&= s_x \,(4\, t')^{1/2} - s_x (4 \alpha - 1)^{1/2} + \overline{x} \\
&= 2\, s_x (t / \beta)^{1/2} - s_x (4 \alpha - 1)^{1/2} + \overline{x} \\
&= 2\, s_x (t \overline{x} / s_x^2)^{1/2} - s_x (4 \overline{x}^2 / s_x^2 - 1)^{1/2} + \overline{x} \\
&= 2\, (\overline{x}\, t)^{1/2} - (4 \overline{x} - s_x^2)^{1/2} + \overline{x} \qquad \cdots\cdots \qquad (\text{A } 1.9)
\end{aligned}
$$

Appendix 2 Range of Dummy Characteristic D

Necessary condition for function $X(F)$ being a monotone increasing function is expressed as follows after Eq.(4.2.7);

$$\left.\begin{array}{l} h(F) = \lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2 + 4\lambda_6 F^3 < 0 \\ \lambda_2 > 0 \end{array}\right\} \quad \text{(A2.1)}$$

Eq.(A2.1) must be satisfied covering whole range of $0 \leq F \leq 1$.

Eq.(A2.1) can be divided into two parts such as

$\lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2$ and $4\lambda_6 F^3$

and expressed as follows separately;

$$\begin{array}{c} \lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2 < 0 \\ (\lambda_2 > 0) \end{array} - \left\{ \begin{array}{l} \lambda_6 > 0 - h(1) < 0 - \left\{ \begin{array}{l} \lambda_5 > 0 \\ \lambda_5 < 0 - \left\{ \begin{array}{l} d' > 0 - \left\{ \begin{array}{l} 0 < b < 1 - h(b) < 0 \\ 1 < b \end{array} \right. \\ d' < 0 \end{array} \right. \end{array} \right. \\ \lambda_6 < 0 \end{array} \right. \quad \text{(A2.2)}$$

In addition to Combination (A2.2), following combination also satisfies Eq.(A2.1),

$$\begin{array}{c} \lambda_6 < 0 - h(1) < 0 \\ (\lambda_3 < 0) \end{array} - \left\{ \begin{array}{l} \lambda_5 > 0 - \left\{ \begin{array}{l} d' > 0 - \left\{ \begin{array}{l} 0 < b < 1 - h(b) < 0 \\ 1 < b \end{array} \right. \\ d' < 0 \end{array} \right. \\ \lambda_5 < 0 \end{array} \right. \quad \text{(A2.3)}$$

Inequality $\lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2 < 0$ in Combination (A2.2) is expressed as,

$$\lambda_3 < 0 - \left\{ \begin{array}{l} \lambda_5 > 0 - d > 0 - 1 < a_1 \ , \ a_2 < 0 \\ \lambda_5 < 0 - \left\{ \begin{array}{l} d > 0 - \left\{ \begin{array}{l} a_1 < 0 \\ 1 < a_2 \end{array} \right. \\ d < 0 \end{array} \right. \end{array} \right. \quad \text{(A2.4)}$$

where a_1, a_2, b, d, d' are following values;

a_1, a_2 : real roots of quadratic equation $\lambda_3 + 2\lambda_4 F + 3\lambda_5 F^2 = 0$;

$$a_1 = (-\lambda_4 + d^{1/2}) / 3\lambda_5 \ , \quad a_2 = (-\lambda_4 - d^{1/2}) / 3\lambda_5$$

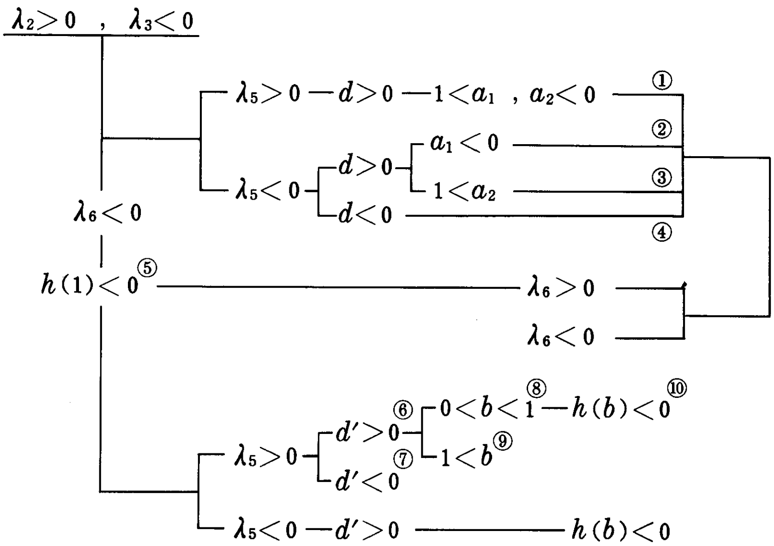
b : real root of quadratic equation $\lambda_4 + 3\lambda_5 F + 6\lambda_6 F^2 = 0$;

$$b = (-3\lambda_5 - d'^{\,1/2}) / 12\lambda_6$$

d , d' :

$$d = \lambda_4^2 - 3\lambda_3\lambda_5 \quad , \quad d' = 9\lambda_5^2 - 24\lambda_4\lambda_6$$

Composing the above-mentioned Combinations (A2.2),(A2.3) and (A2.4), following combinations are obtained;



…………………………………… (A2.5)

There are 29 combinations in Combination (A2.5), and each of them corresponds to necessary condition for $X(F)$ being a monotone increasing function.

Range of dummy characteristic D can be obtained by expressing $a_1 \sim d'$, $h(1)$, $h(b)$ in Combination (A2.5) by means of multipliers λ, and by expressing λ by means of D. Inequalities ①~⑩ in Combination (A2.5) are expressed as follows by using λ;

$$① \quad \lambda_5 > 0 \begin{cases} -3\lambda_5 < \lambda_3 \begin{cases} \lambda_4 < -(3\lambda_5 + \lambda_3)/2 \\ 0 < \lambda_4 < (3\lambda_5 + \lambda_3)/2 \end{cases} \\ \lambda_3 < -3\lambda_5 \begin{cases} \lambda_4 < (3\lambda_5 + \lambda_3)/2 \\ 0 < \lambda_4 < -(3\lambda_5 + \lambda_3)/2 \end{cases} \end{cases}$$

② $\lambda_5 < 0$ —— $\lambda_4 < -(3\lambda_3\lambda_5)^{1/2}$

③ $\lambda_5 < 0$ ┬ $0 < \lambda_4 < \{3\lambda_5(3\lambda_5+\lambda_3)\}^{1/2}$, $0 < \lambda_4 < -(3\lambda_5+\lambda_3)/2$
├ $\lambda_4 < -\{3\lambda_5(3\lambda_5+\lambda_3)\}^{1/2}$, $\lambda_4 < (3\lambda_5+\lambda_3)/2$
└ $-\{3\lambda_5(3\lambda_5+\lambda_3)\}^{1/2} < \lambda_4 < 0$

④ $\lambda_5 < 0$ —— $-(3\lambda_3\lambda_5)^{1/2} < \lambda_4 < (3\lambda_3\lambda_5)^{1/2}$

⑤ $\lambda_3 + 2\lambda_4 + 3\lambda_5 + 4\lambda_6 < 0$

⑥ $3\lambda_5^2 - 8\lambda_4\lambda_6 > 0$

⑦ $3\lambda_5^2 - 8\lambda_4\lambda_6 < 0$

⑧ ┌ $\lambda_5 + 4\lambda_6 > 0$
└ $\lambda_5 + 4\lambda_6 < 0$ —— $\lambda_4 + 3\lambda_5 + 6\lambda_6 > 0$

⑨ $\lambda_5 + 4\lambda_6 < 0$ —— $\lambda_4 + 3\lambda_5 + 6\lambda_6 < 0$

⑩ $72\lambda_3\lambda_6^2 + 9\lambda_5^3 - 36\lambda_4\lambda_5\lambda_6 + 3^{1/2}(3\lambda_5^2 - 8\lambda_4\lambda_6)^{3/2} < 0$

.... (A 2.6)

Multipliers λ are represented as follows based on Eqs.(4.2.4) and (4.2.5);

$$\{\lambda\} = [M]^{-1}\{C\} - \left(\frac{A_{NN} - \{C\}^T[M]^{-1}\{C\}}{1 - \{E\}^T[M]^{-1}\{E\}}\right)^{1/2}[M]^{-1}\{E\} \quad \cdots\cdots\cdots\cdots (A\,2.7)$$

Dummy characteristic D is included only in matrix {E}, and another matrices [M] and {C} are treated as deterministic; that is, by introducing following definitions,

$$C' = A_{NN} - \{C\}^T[M]^{-1}\{C\}$$
$$\left\{C'/(1 - \{E\}^T[M]^{-1}\{E\})\right\}^{1/2} = 1/(e_2^*D^2 + e_1^*D + e_0^*)^{1/2}$$
$$([M]^{-1}\{C\})^T = \{\, c_1^* \;\; c_3^* \;\; c_4^* \;\; c_5^* \;\; c_6^* \,\}$$
$$([M]^{-1}\{E\})^T = \{\, a_1^*D+b_1^* \;\; a_3^*D+b_3^* \;\; a_4^*D+b_4^* \;\; a_5^*D+b_5^* \;\; a_6^*D+b_6^* \,\}$$

matrix $\{\lambda_i\}^T = \{\lambda_1 \ \ \lambda_3 \ \ \lambda_4 \ \ \lambda_5 \ \ \lambda_6\}$ is expressed as a function for D only as,

$$\lambda_i = c_i^* - \frac{a_i^* D + b_i^*}{(e_2^* D^2 + e_1^* D + e_0^*)^{1/2}} \quad , \quad e_2^* D^2 + e_1^* D + e_0^* > 0 \quad \cdots\cdots \quad (A2.8)$$

Substituting Eq.(A2.8) into λ_1 , $\lambda_3 \sim \lambda_6$ in Combination (A2.5) and into ①~⑩ in Combination (A2.6), the simultaneous conditions for D are obtained.

For example, range of dummy characteristic D corresponding to $\lambda_5 > 0$ is derived in the following. $\lambda_5 > 0$ is expressed as follows by using Eq.(A2.8);

$$c_5^* - \frac{a_5^* D + b_5^*}{(e_2^* D^2 + e_1^* D + e_0^*)^{1/2}} > 0$$

This inequality is expressed as follows with regard to $a_5^* < 0$ and $c_5^* < 0$;

$$D > -\frac{b_5^*}{a_5^*} \left[\begin{array}{l} D < \dfrac{-(2a_5^* b_5^* - c_5^{*2} e_1^*) \overset{ⓐ}{-} \{(2a_5^* b_5^* - c_5^{*2} e_1^*)^2 - 4(a_5^{*2} - c_5^{*2} e_2^*)(b_5^{*2} - c_5^{*2} e_0^*)\}^{1/2}}{2(a_5^{*2} - c_5^{*2} e_2^*)} \\ D > \dfrac{-(2a_5^* b_5^* - c_5^{*2} e_1^*) \overset{ⓑ}{+} \{(2a_5^* b_5^* - c_5^{*2} e_1^*)^2 - 4(a_5^{*2} - c_5^{*2} e_2^*)(b_5^{*2} - c_5^{*2} e_0^*)\}^{1/2}}{2(a_5^{*2} - c_5^{*2} e_2^*)} \end{array} \right.$$

in which

$$(2a_5^* b_5^* - c_5^{*2} e_1^*)^2 - 4(a_5^{*2} - c_5^{*2} e_2^*)(b_5^{*2} - c_5^{*2} e_0^*) \geqq 0$$

$$a_5^{*2} - c_5^{*2} e_2^* > 0$$

and if

$$a_5^{*2} - c_5^{*2} e_2^* < 0$$

the signs of parts ⓐ and ⓑ should be reversed.

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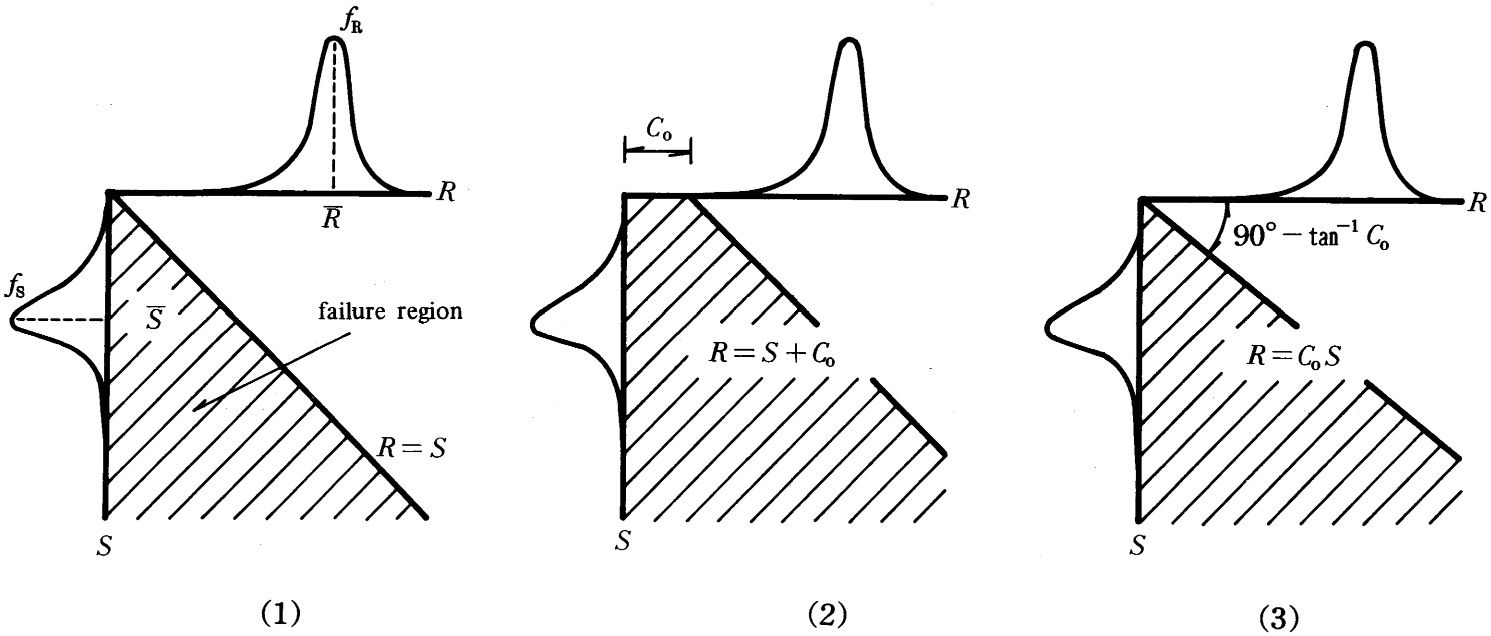


Fig. 1 Variation of Failure Region for Parameter C_o.

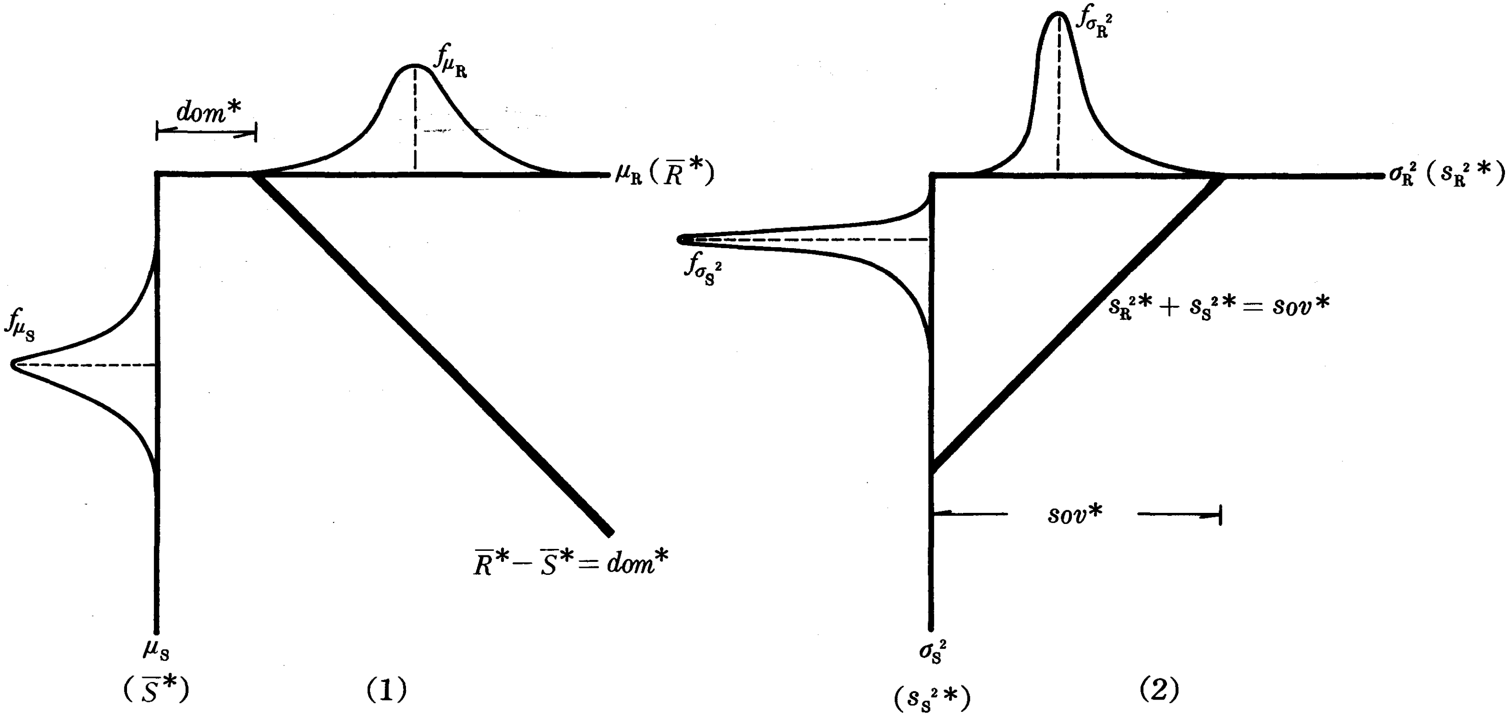


Fig. 2 Integrals along the Constant Lines for Parameters *dom** and *sov**.

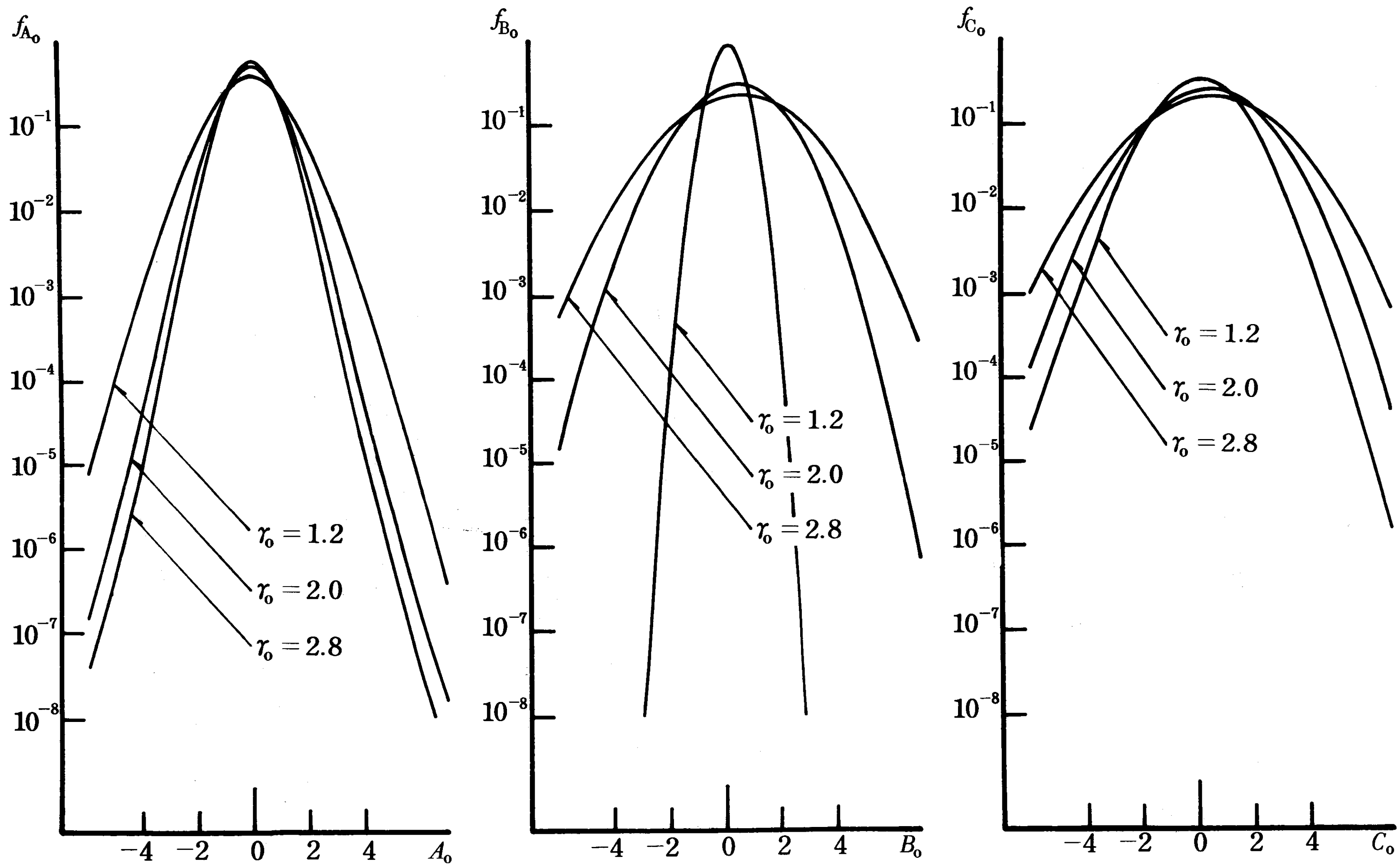


Fig. 3-1 Probability Densities f_{A_0}, f_{B_0} and f_{C_0} in case of Normal Distribution.

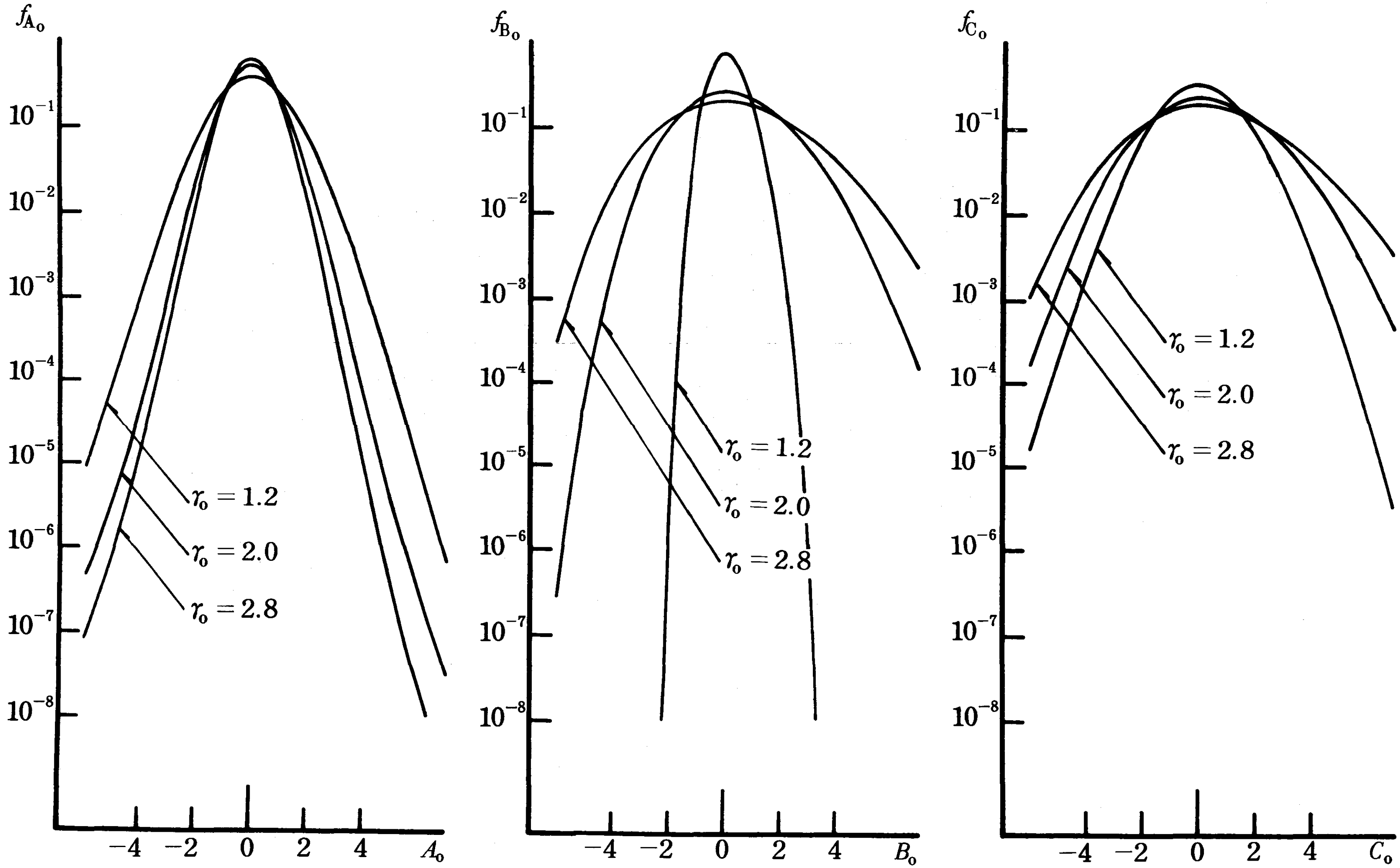


Fig. 3-2 Probability Densities f_{A_o}, f_{B_o} and f_{C_o} in case of Gamma Distribution.

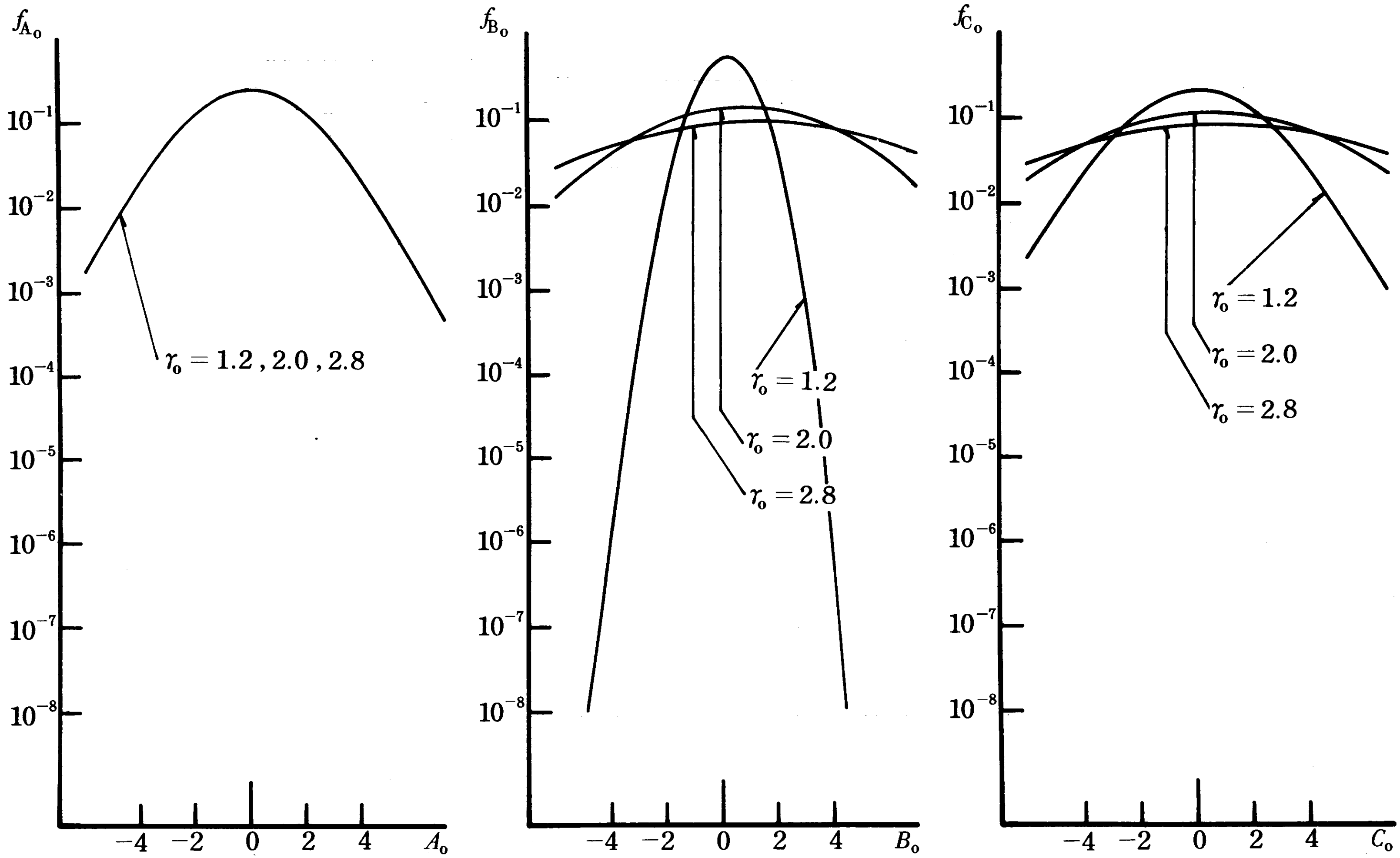


Fig. 3-3 Probability Densities f_{A_o}, f_{B_o} and f_{C_o} in case of Log-Normal Distribution.

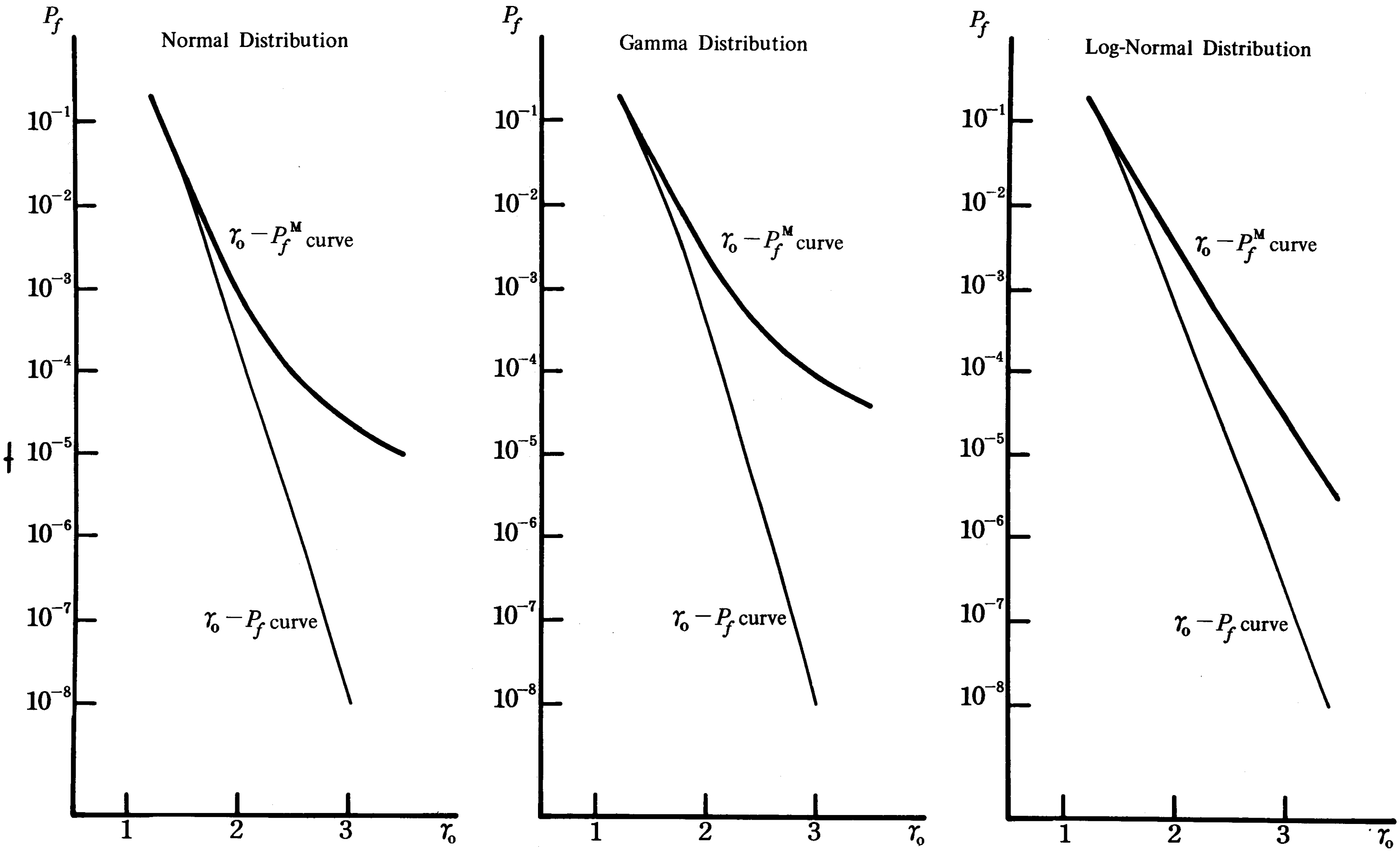
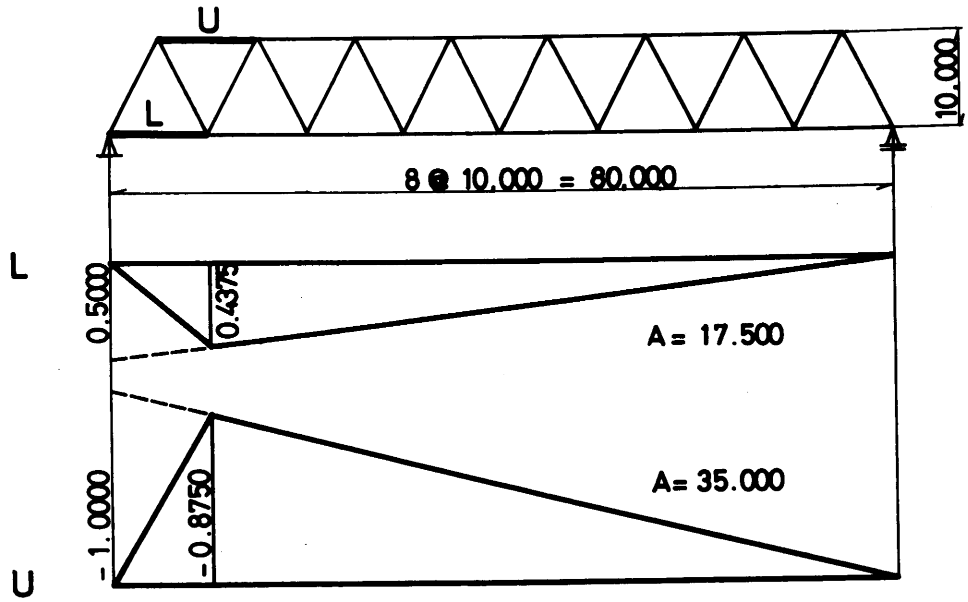
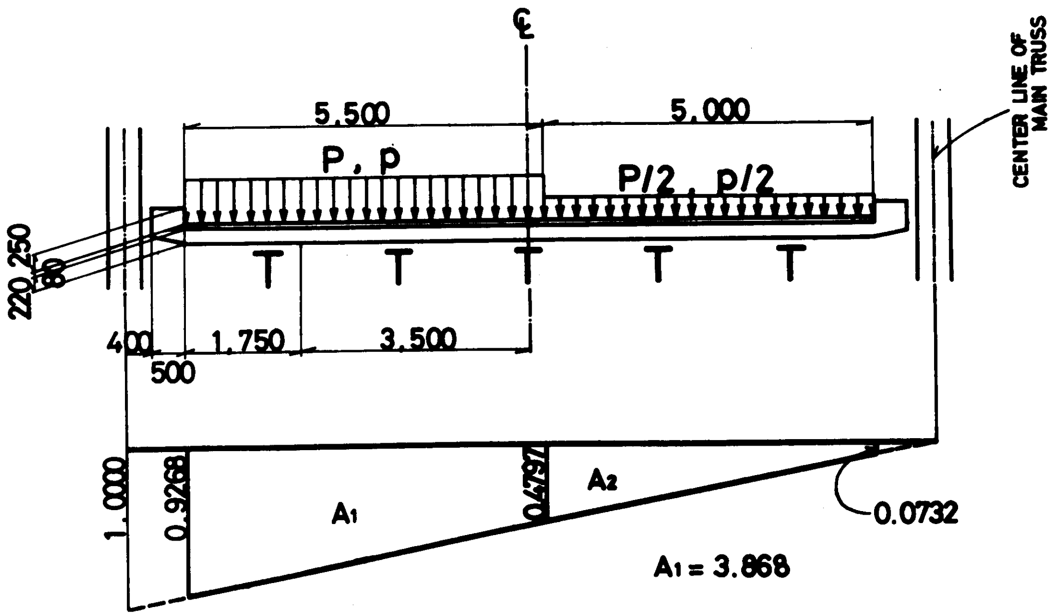


Fig. 4 The $\gamma_o - p_f^M$ Curves for Three Distributions.



(1)



(2)

Fig. 5 Truss Bridge and its Influence Lines.

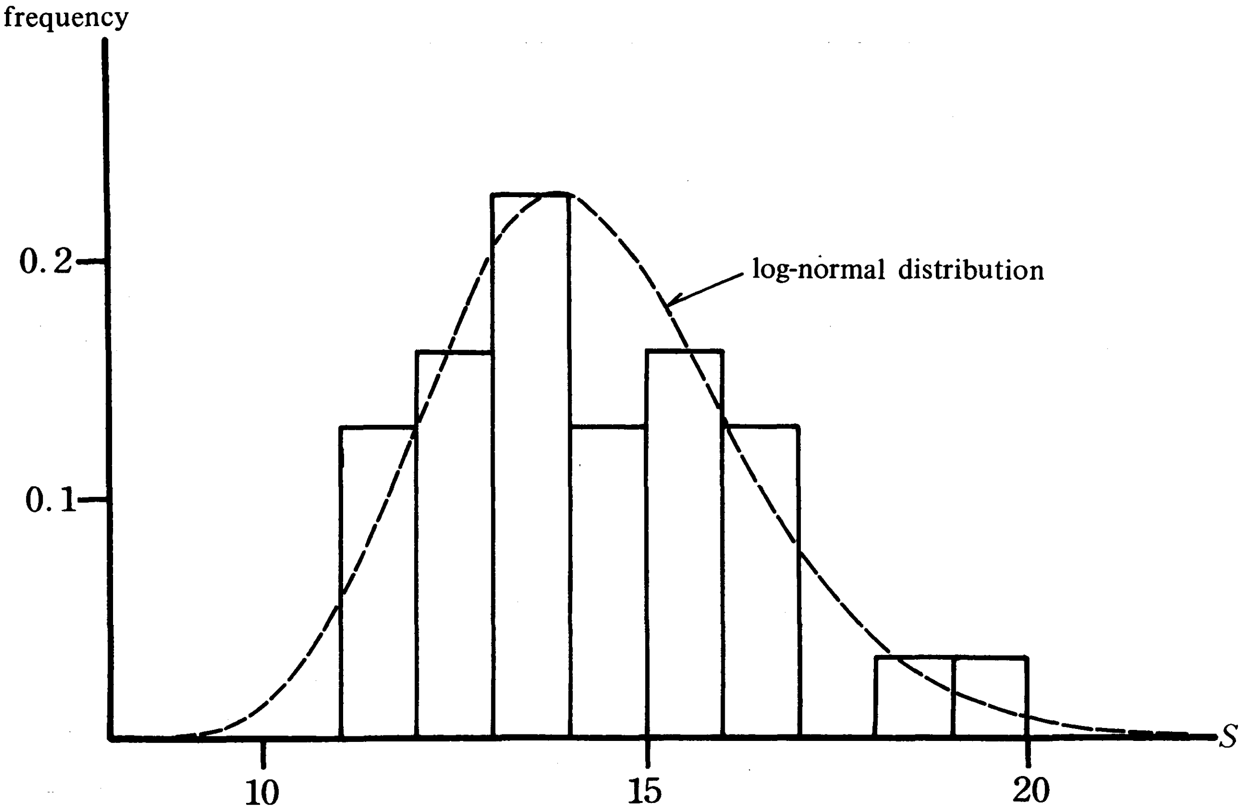


Fig. 6 Frequency Distribution of the Measured Data for Load S.

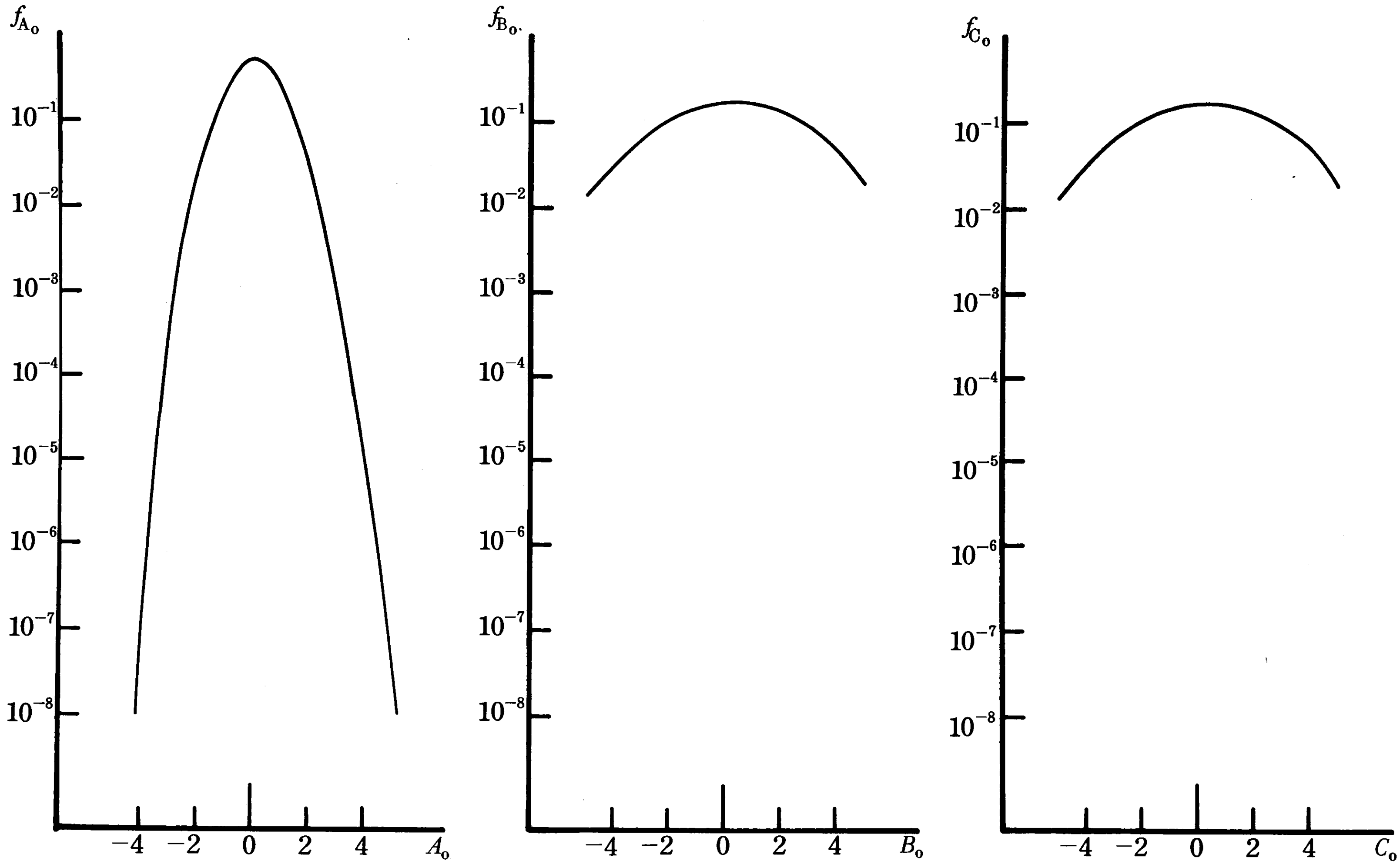


Fig. 7 Probability Dencities f_{A_o}, f_{B_o}, and f_{C_o} for the Example.

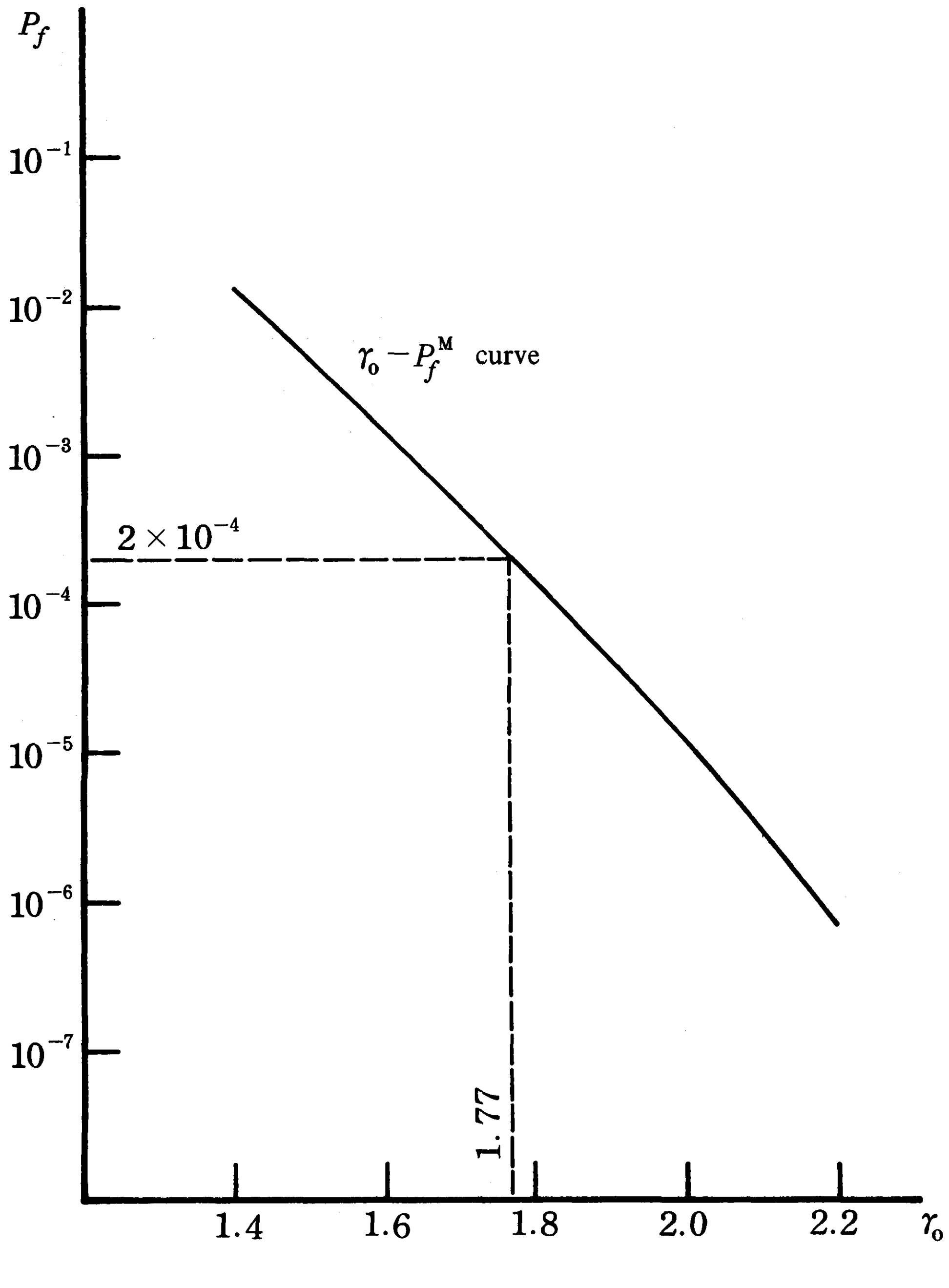


Fig. 8 The $\gamma_o - p_f^M$ Curve for the Example.

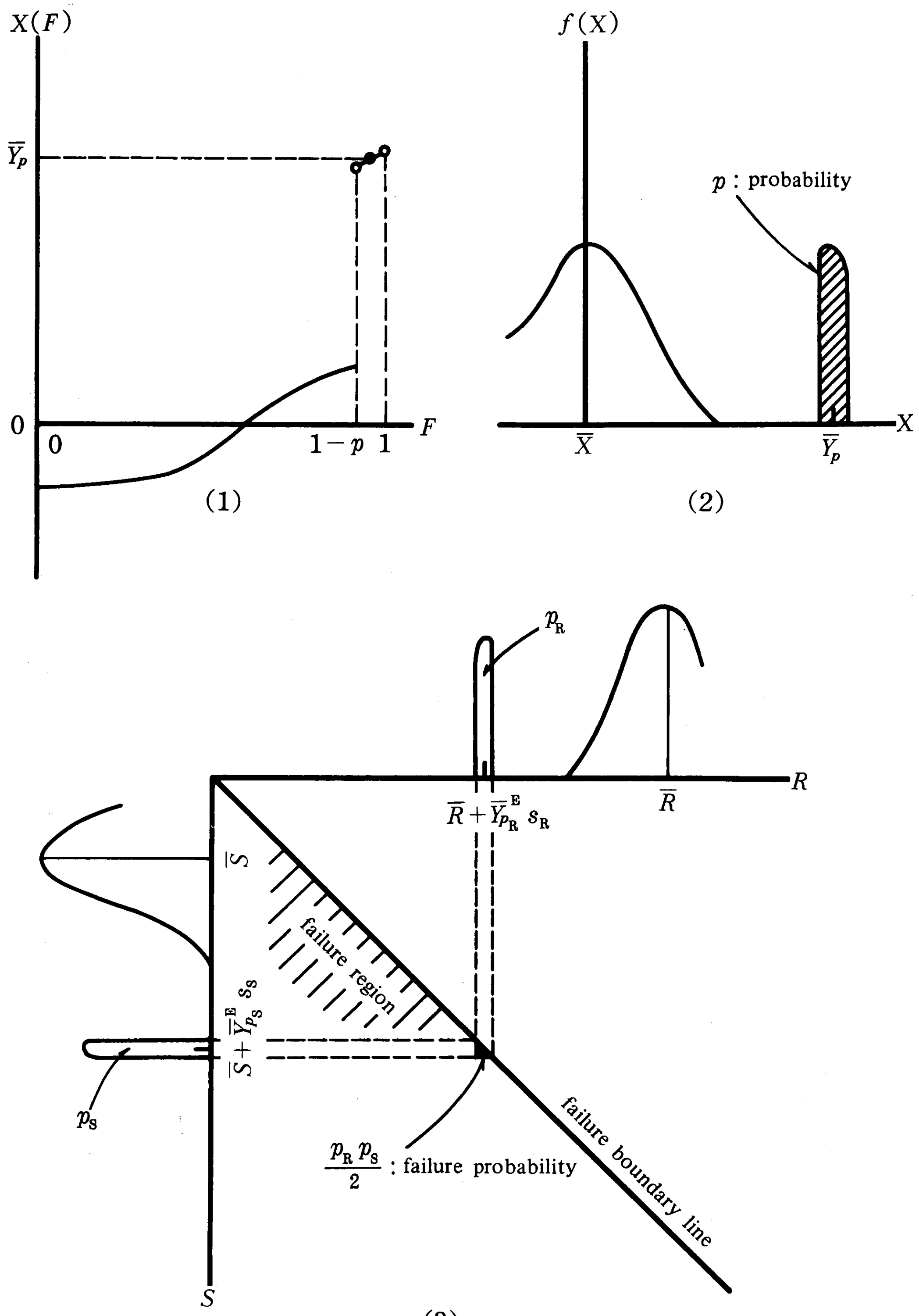


Fig. 9 Relationship between Extermum $\bar{Y}_p{}^E$ and Maximization of Failure Probability.

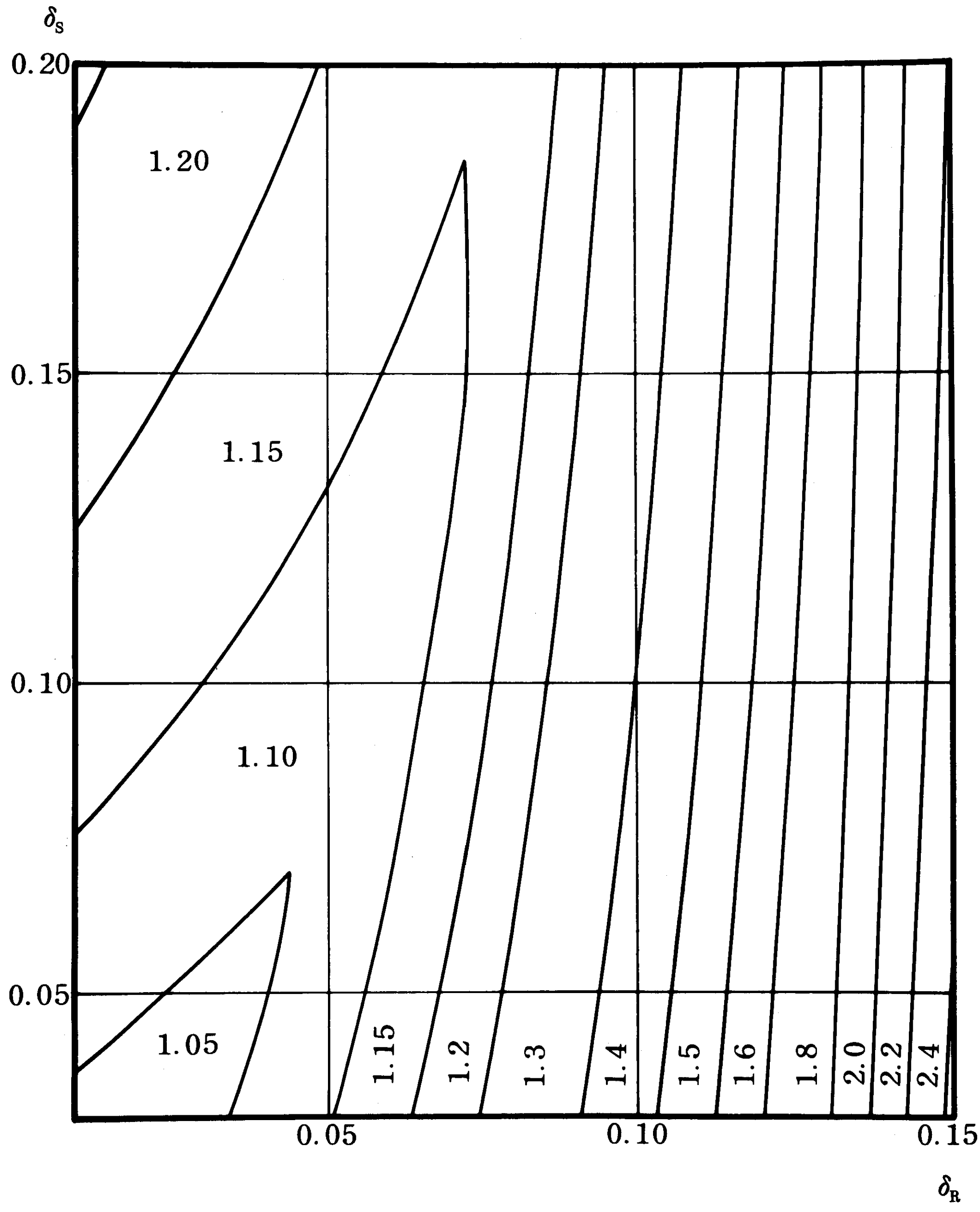


Fig. 10 Extra Coefficient of Cross-Sectional Area caused by assuming $p = p_R = p_S$ (in case of $N = 2/p = 100$).

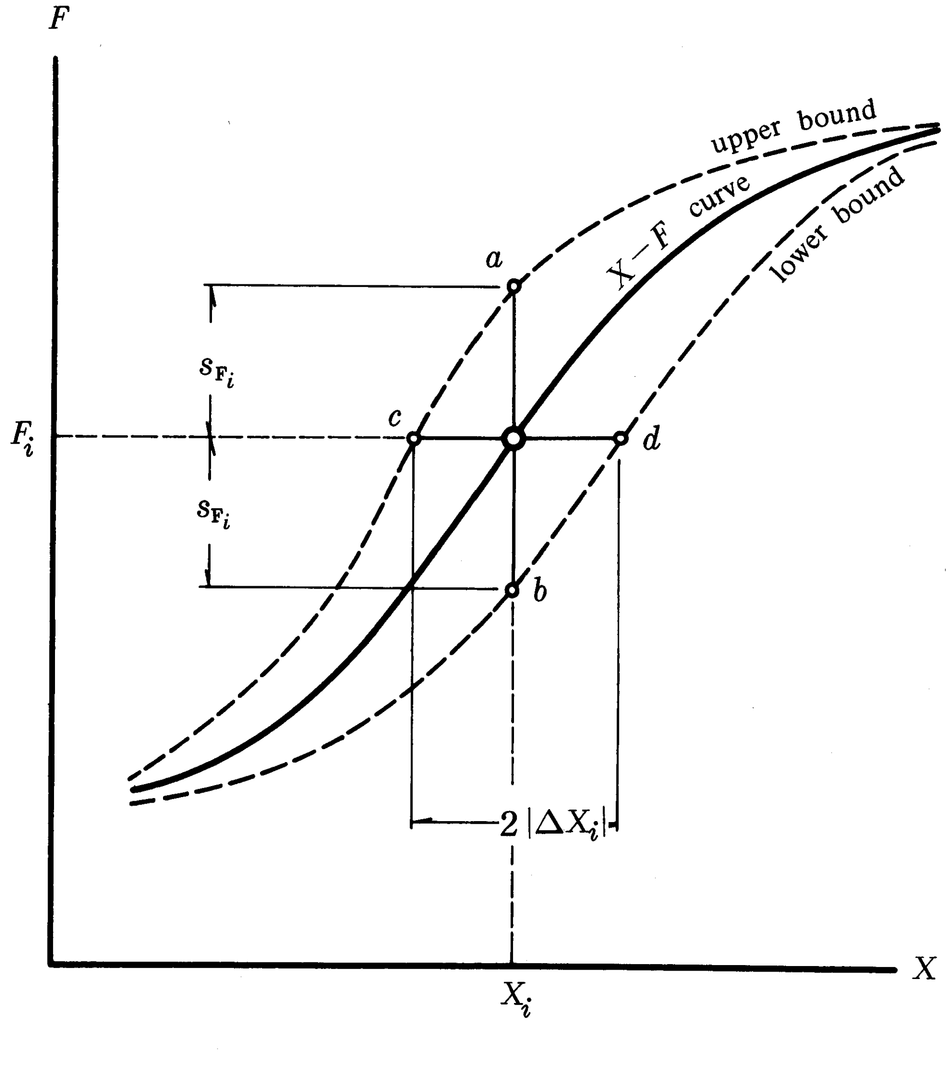


Fig. 11 Approximate Calculation of Deviation of X_i by using the $X - F$ Curve.

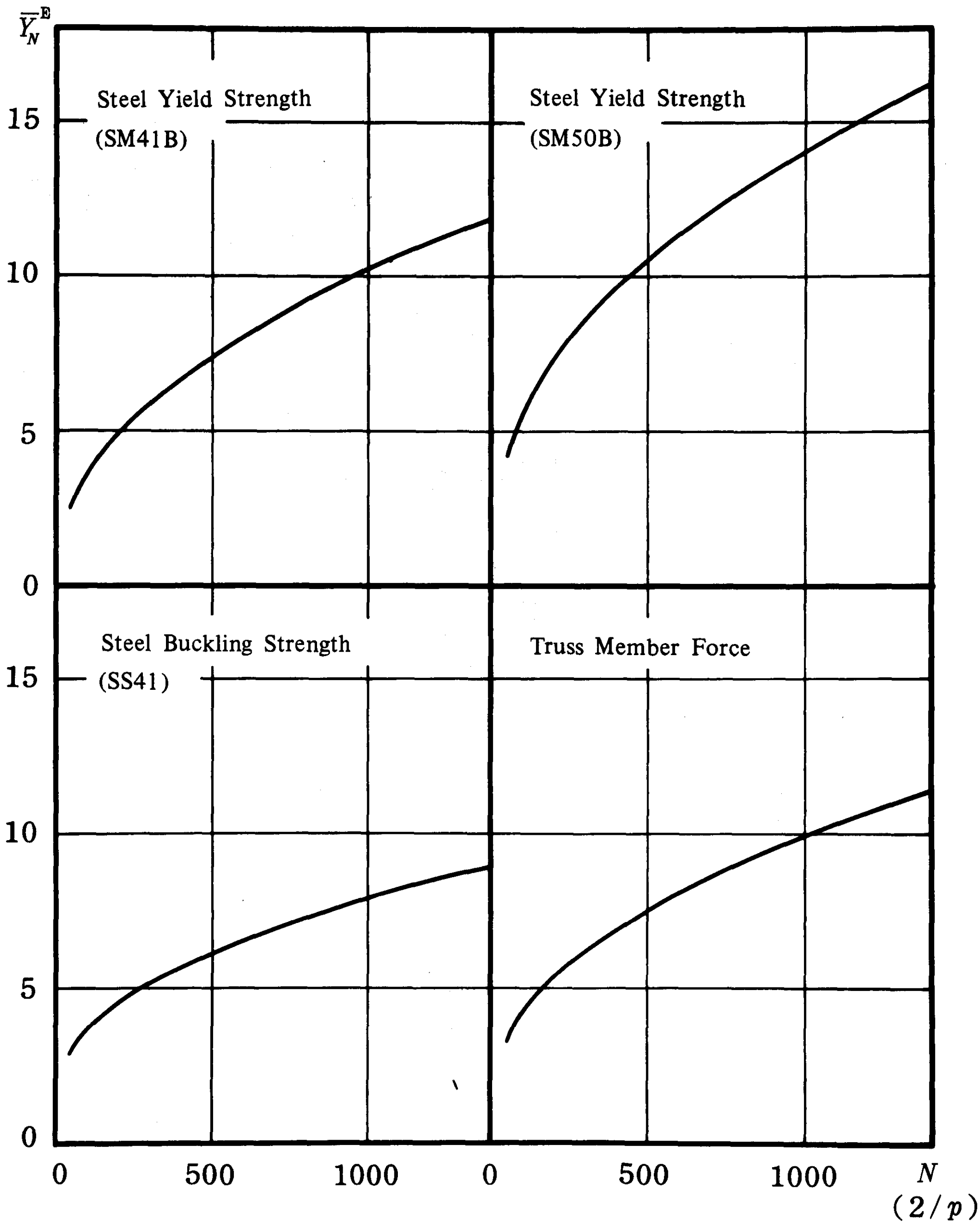


Fig. 12 The $N - \overline{Y}_N^E$ Curves for Various Sorts of the Measured Data.

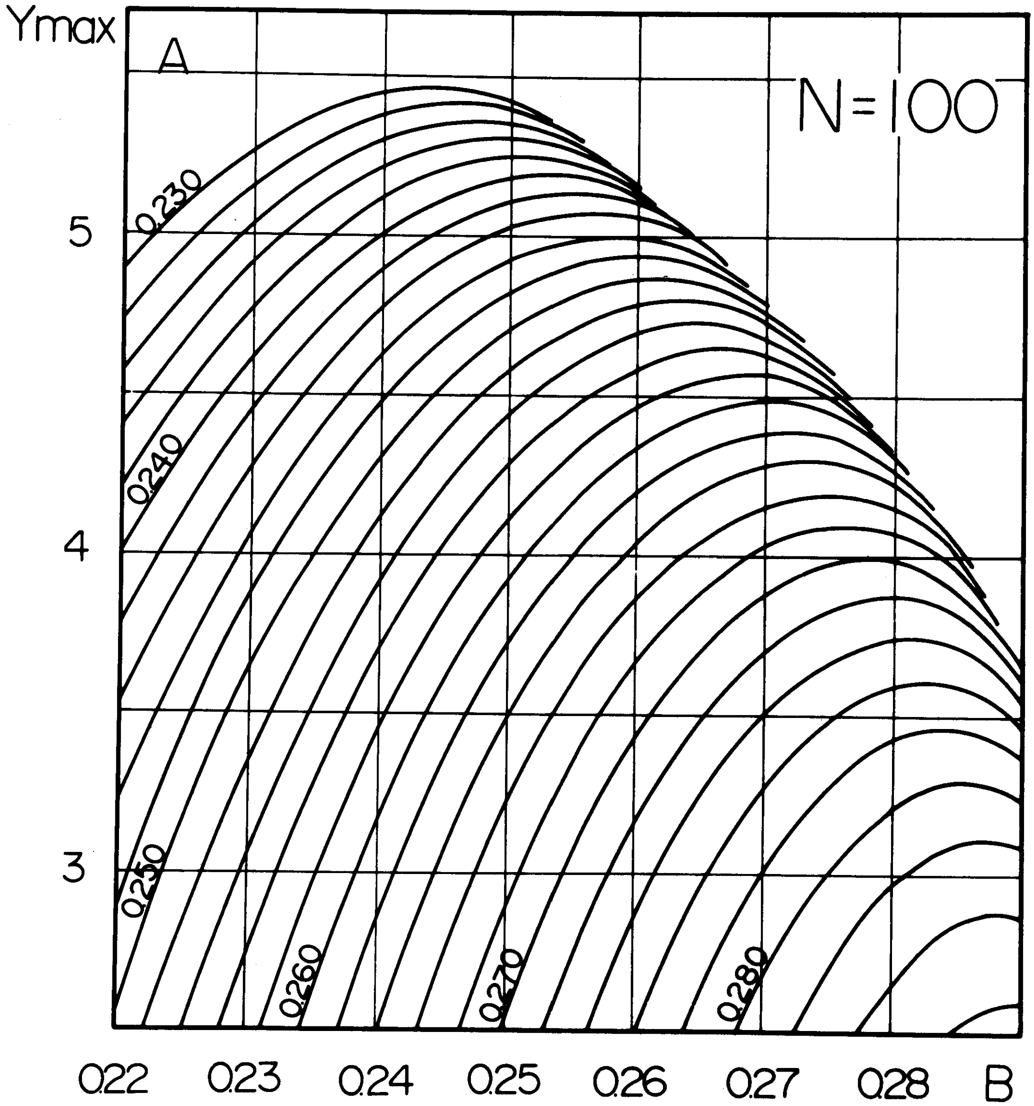


Fig. 13-1 Graphical Estimation Method of $\overline{Y}_N^E$ for $N = 100$ —— Relationship of $[(A, B) - Y_{max}]$.

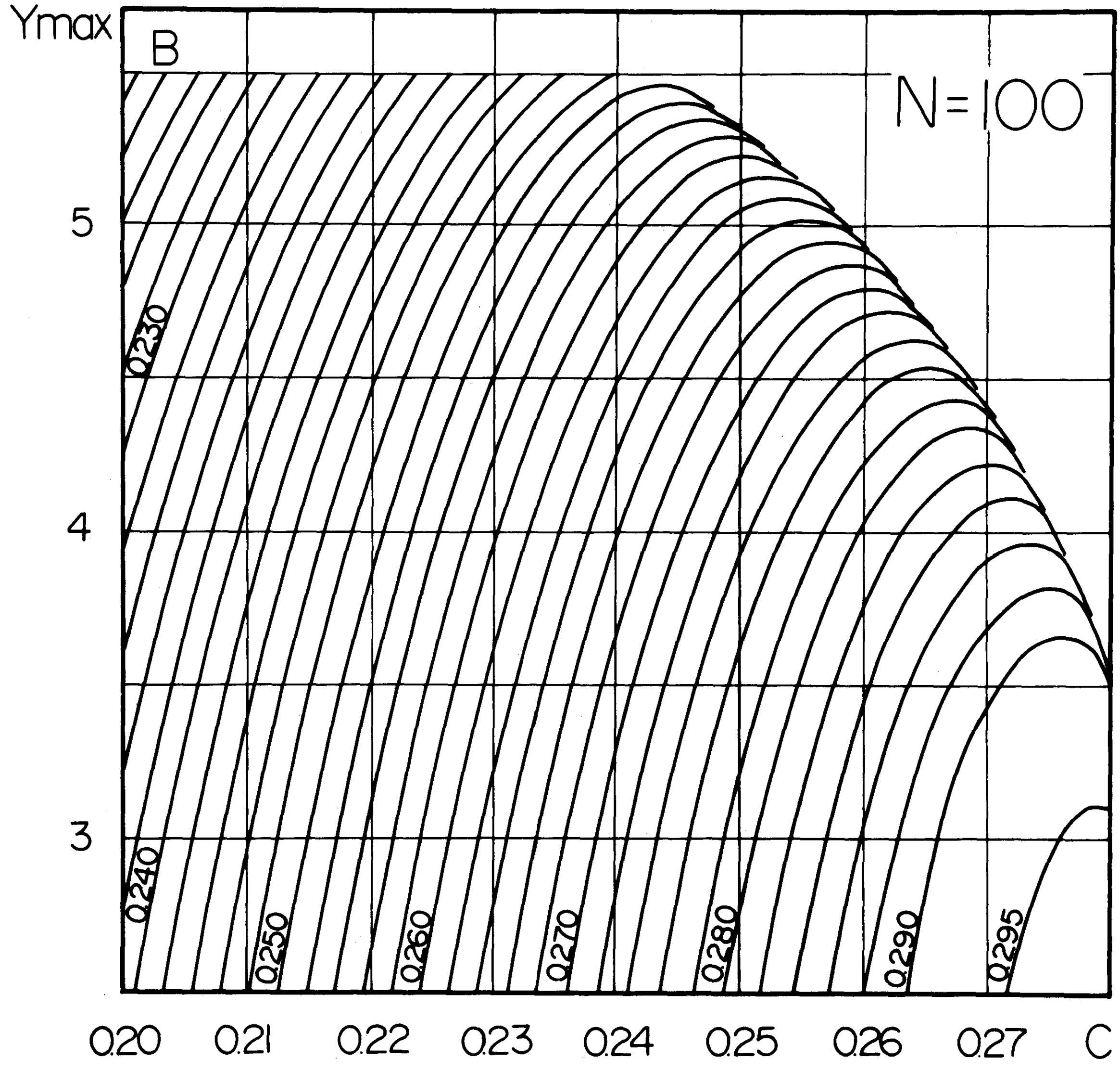


Fig. 13-2 Graphical Estimation Method of $\overline{Y}_N^{\,E}$ for $N = 100$ —— Relationship of $[(B, C) - Y_{\max}]$.

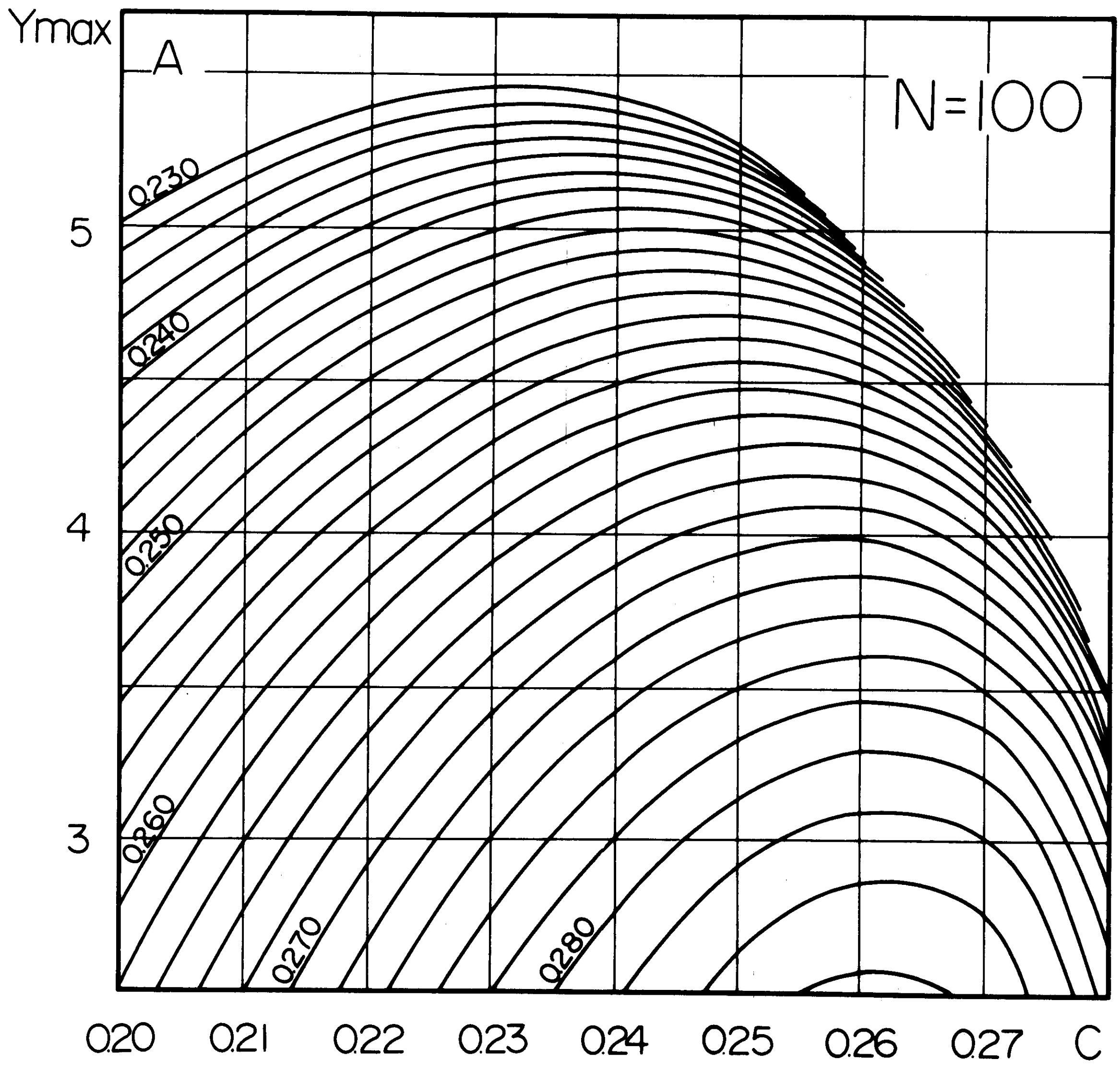


Fig. 13-3 Graphical Estimation Method of $\overline{Y}_N{}^{E}$ for $\dot{N} = 100$ —— Relationship of $[(C, A) - Y_{\max}]$.

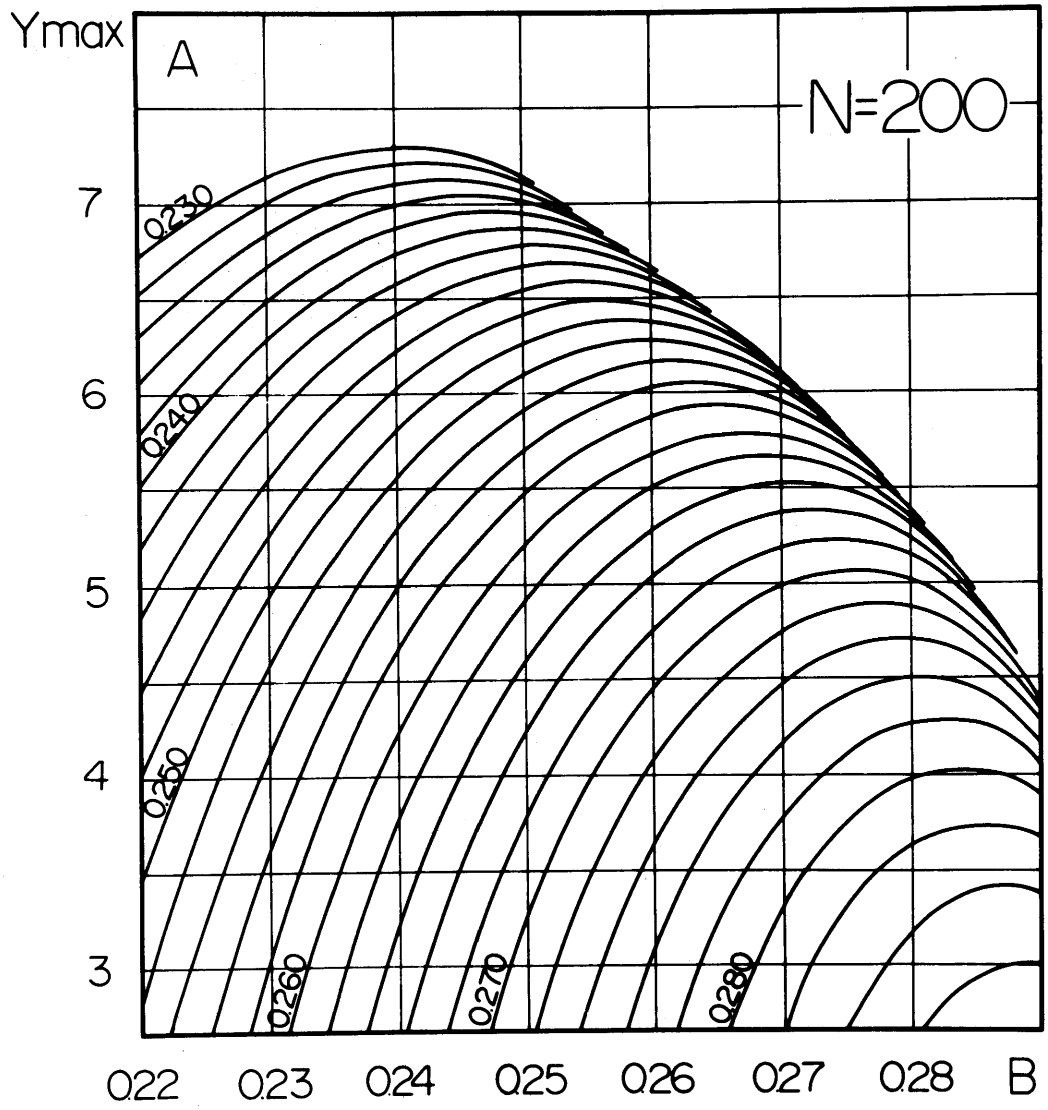


Fig. 14-1 Graphical Estimation Method of $\overline{Y}_N{}^E$ for N = 200 —— Relationship of $[(A, B) - Y_{max}]$.

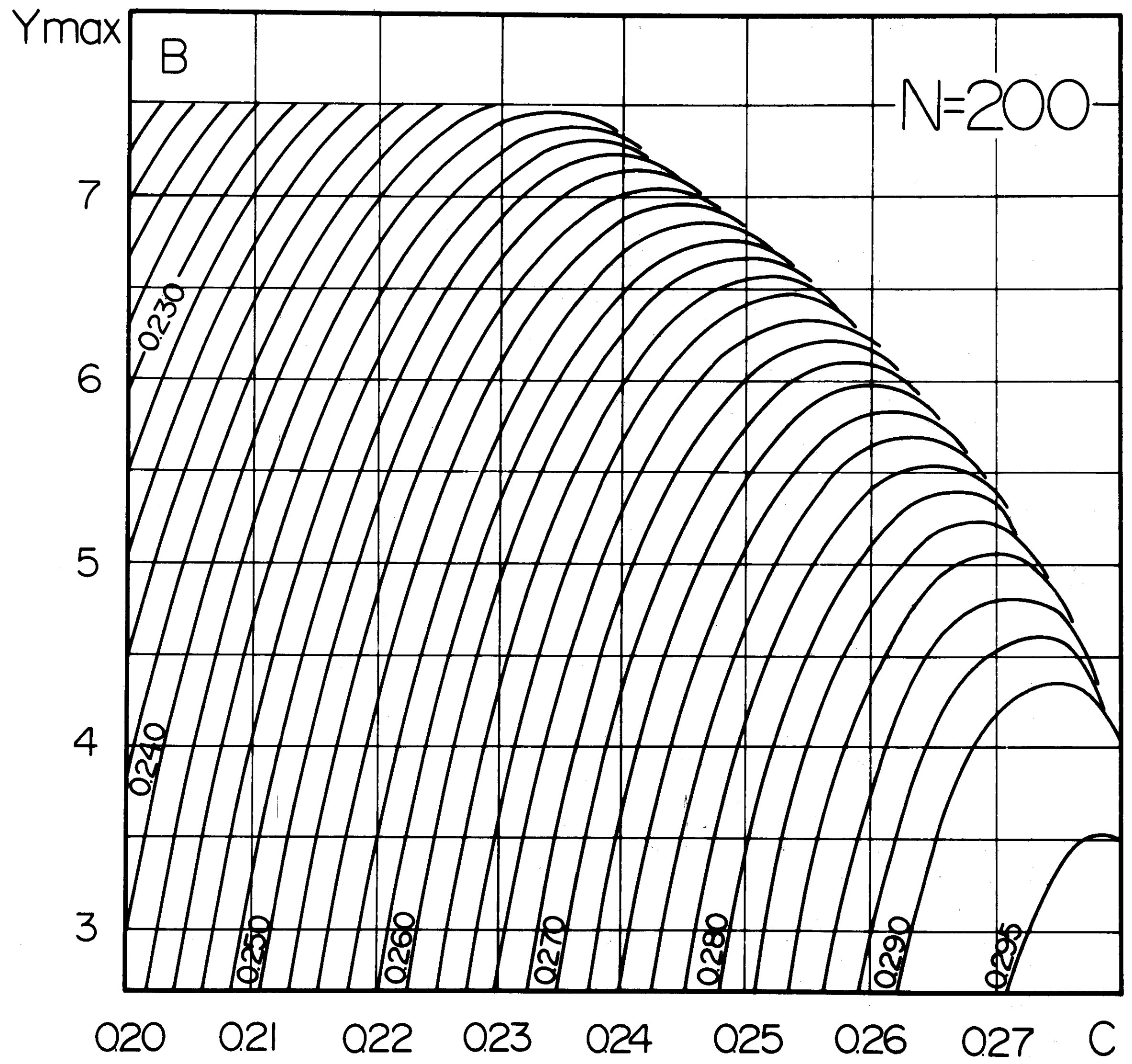


Fig. 14-2 Graphical Estimation Method of $\overline{Y}_N{}^E$ for $N = 200$ —— Relationship of $[(B, C) - Y_{max}]$.

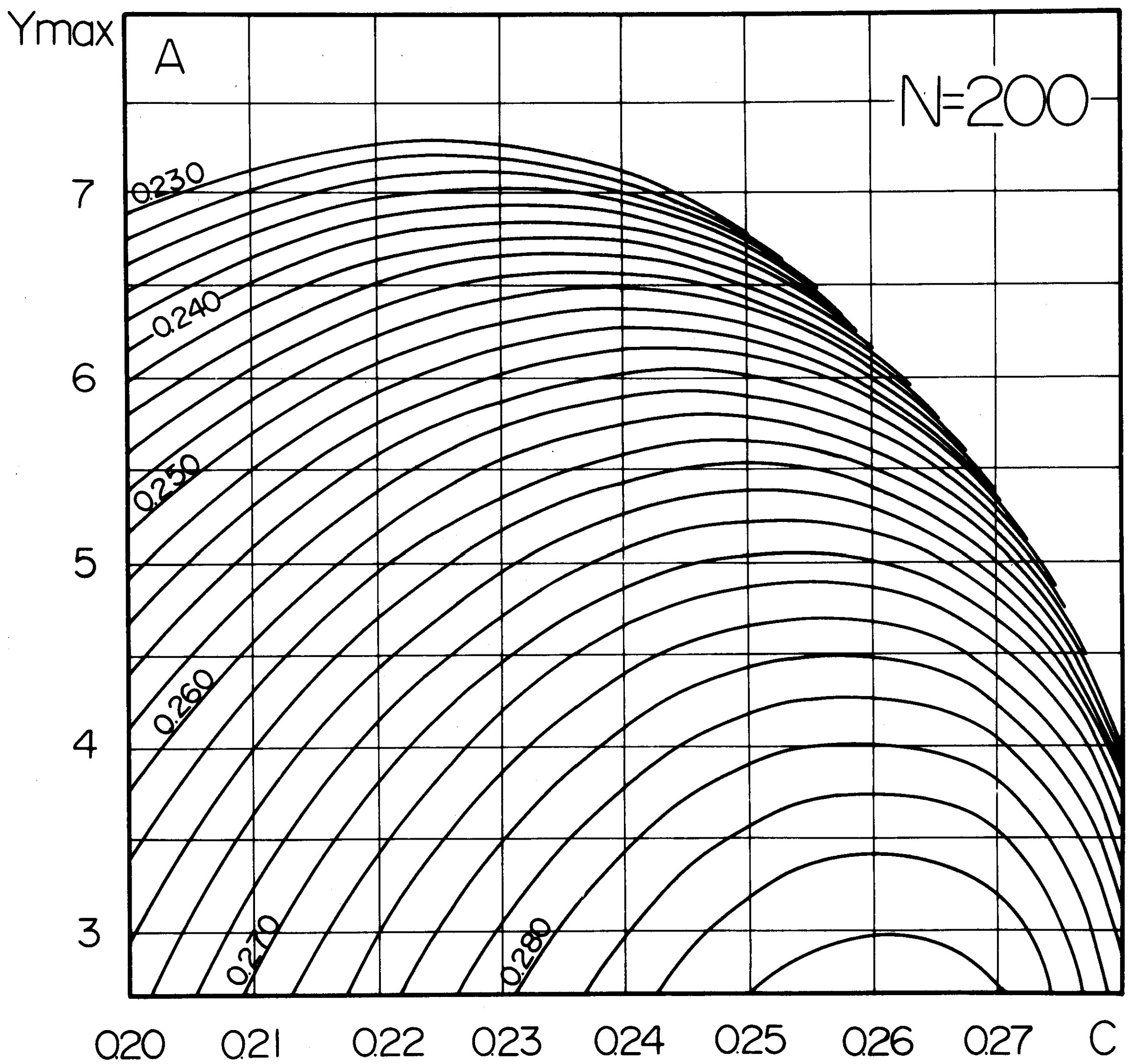


Fig. 14-3 Graphical Estimation Method of $\overline{Y}_N^{\,E}$ for $N = 200$ —— Relationship of $[(C, A) - Y_{max}]$.

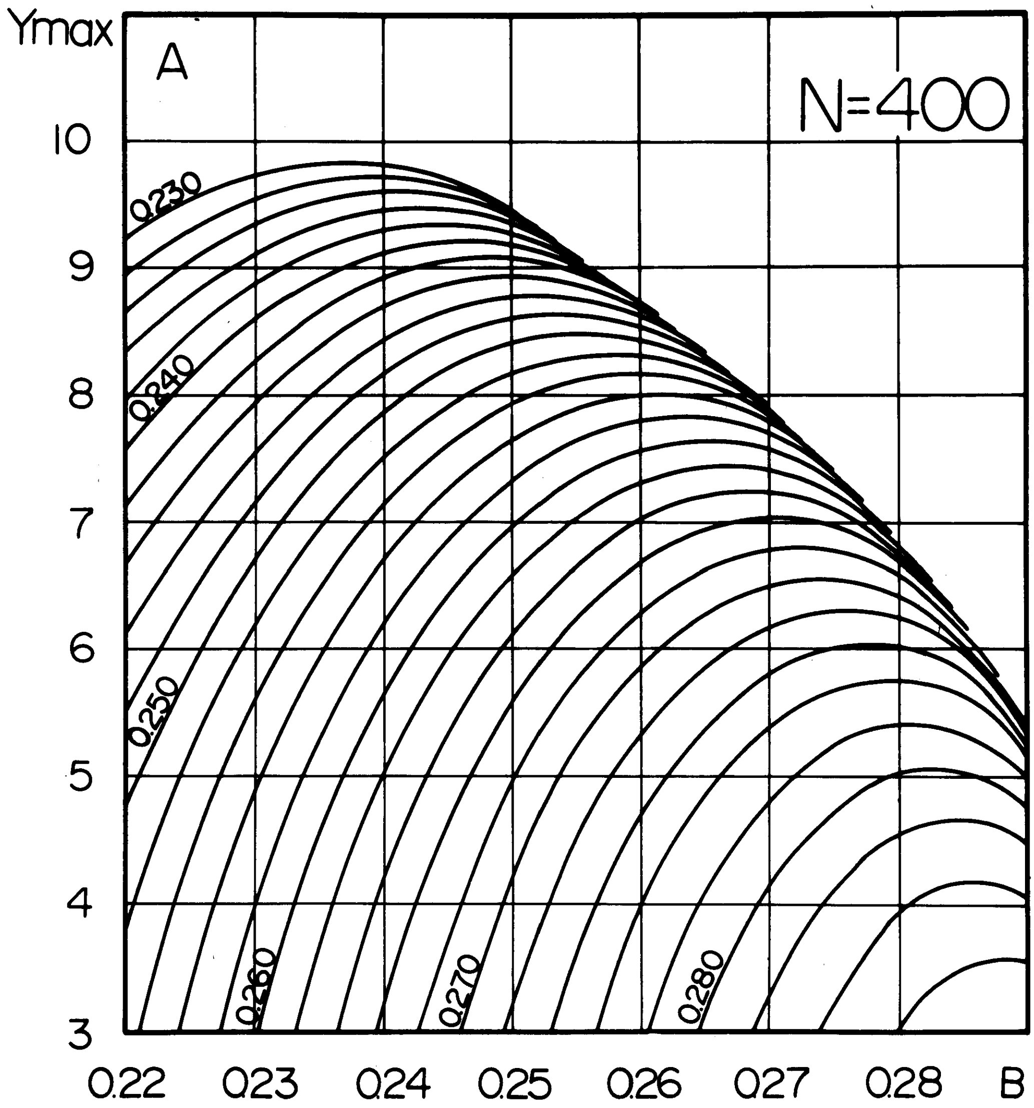


Fig. 15-1 Graphical Estimation Method of $\overline{Y}_N{}^E$ for $N = 400$ —— Relationship of $[(A, B) - Y_{max}]$.

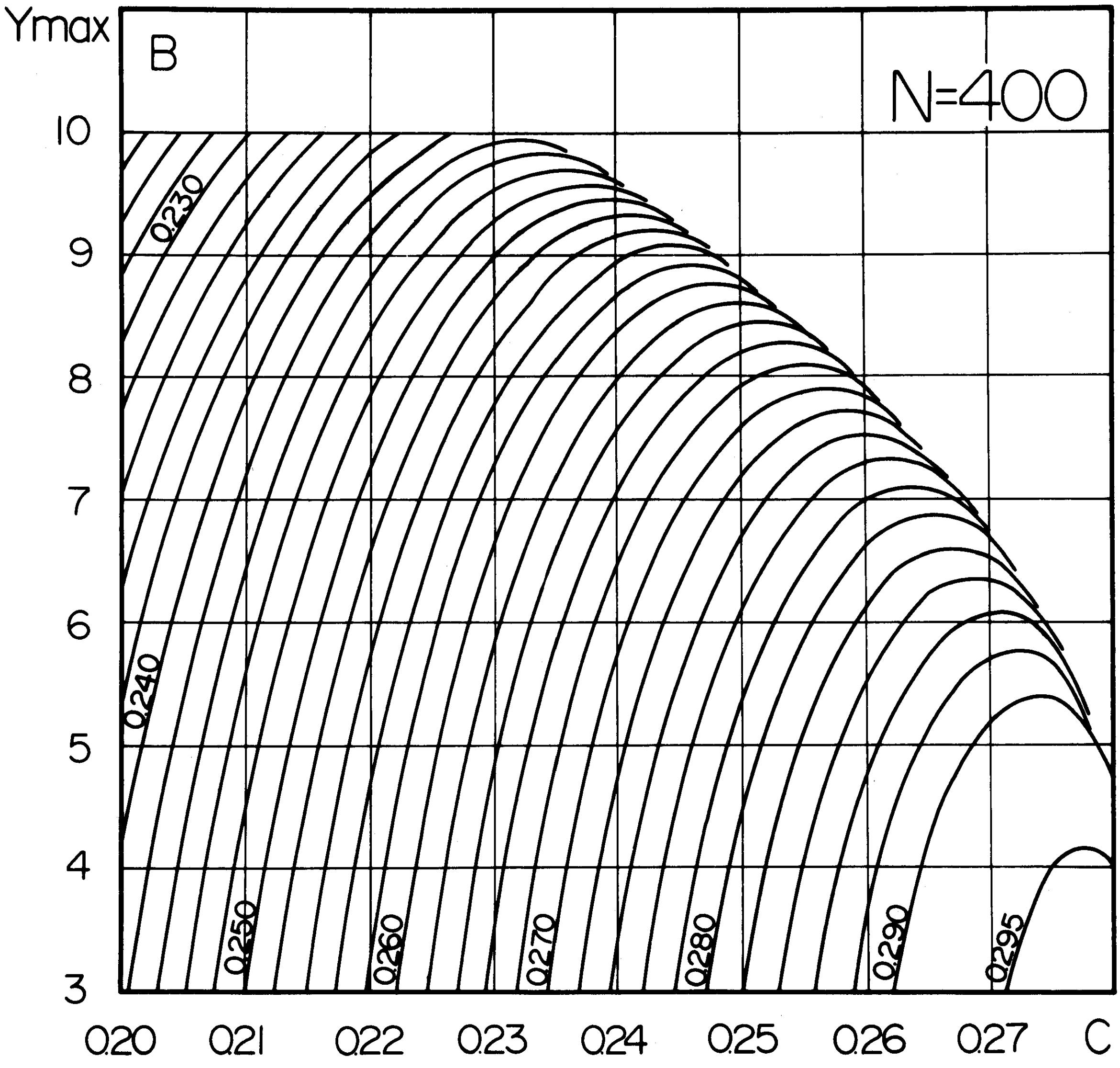


Fig. 15-2 Graphical Estimation Method of $\overline{Y}_N{}^E$ for $N = 400$ —— Relationship of $[(B, C) - Y_{max}]$.

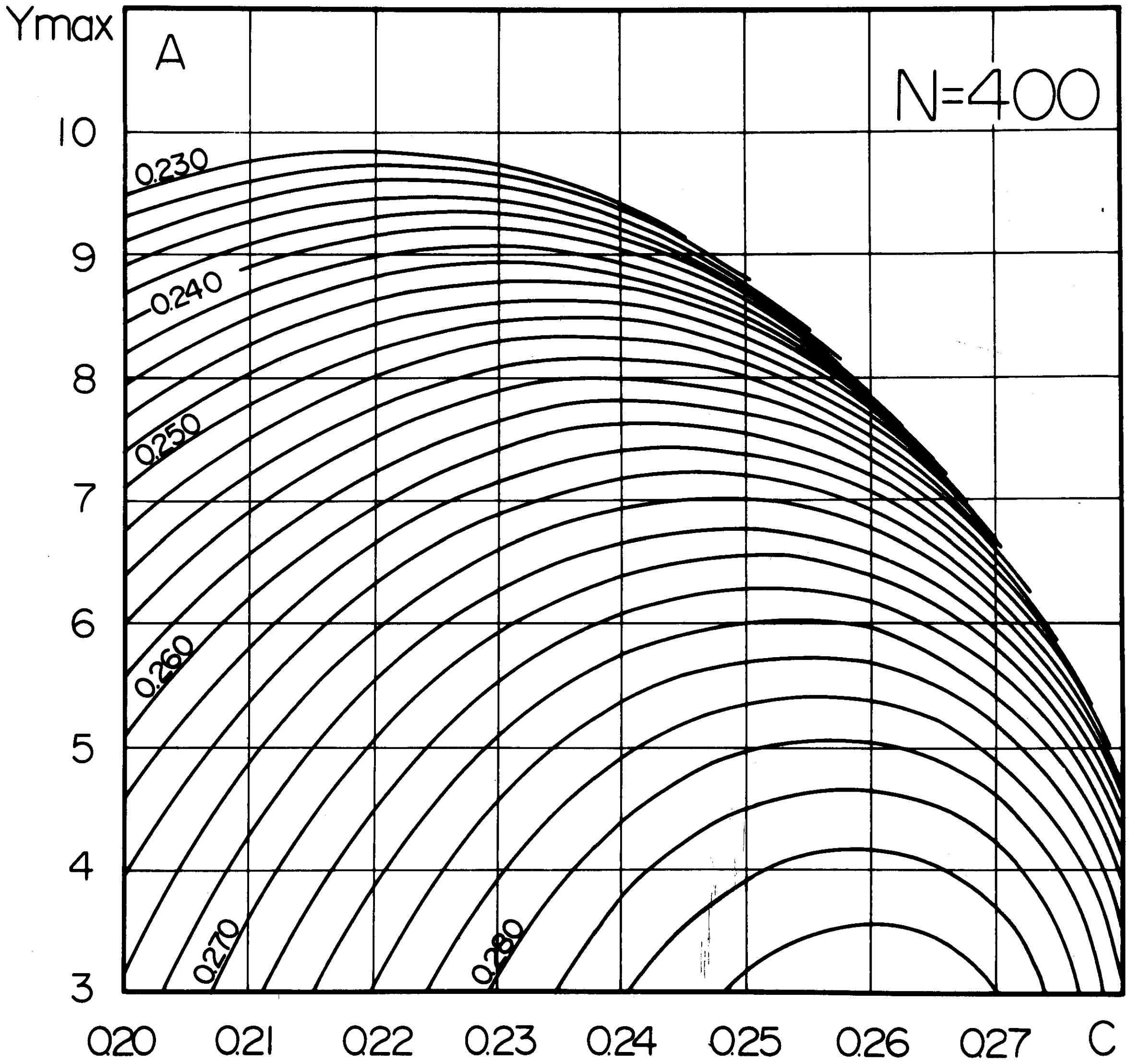


Fig. 15-3 Graphical Estimation Method of $\overline{Y}_N{}^E$ for $N = 400$ —— Relationship of $[(C, A) - Y_{\max}]$.

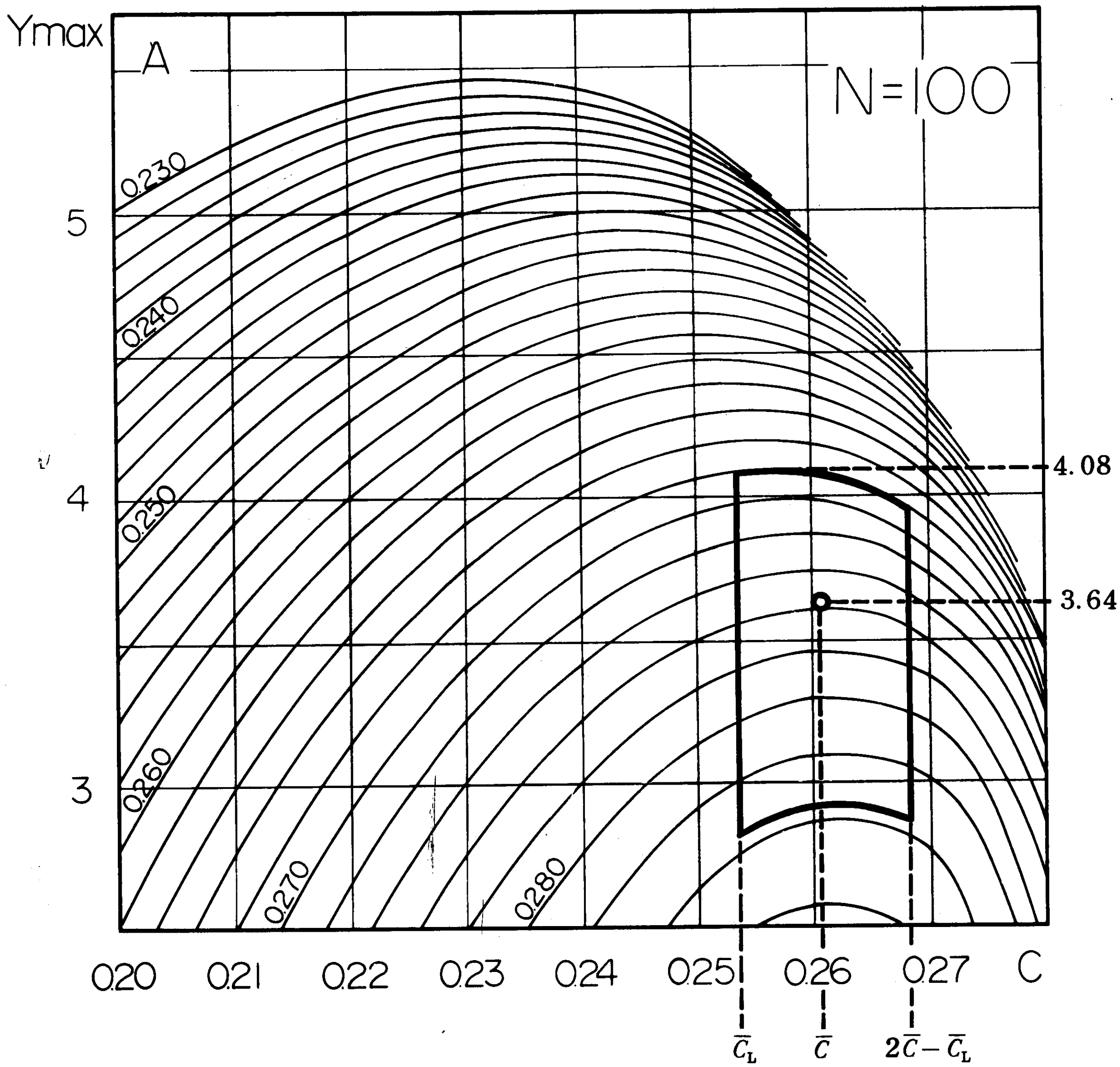


Fig. 16 Fan Shape Region of Y_{max} —— Estimation of Upper Bound of $\bar{Y}_N{}^E$ in case of Buckling Strength of SS41 Steel.

Measured Data	n	$\overline{x}$	s_x^2	s_x	δ_x
Steel Yield Strength (SM 41B) (t/cm²)	21	2.752	0.01414	0.1189	0.0432
Truss Member Force (t)	31	14.44	3.818	1.954	0.1353

Table 1 Characteristics of the Measured Data.

Restrictive Conditions	$\overline{Y}_N^{E}$
$\overline{x}$, s_X^2	22. 3
$\overline{x}$, s_X^2 , symmetry	15. 8
$\overline{x}$, s_X^2 , u_1	22. 3
$\overline{x}$, s_X^2 , u_5	18. 9
$\overline{x}$, s_X^2 , u_1 , v_1	20. 1
$\overline{x}$, s_X^2 , u_5 , v_5	13. 3
$\overline{x}$, s_X^2 , u_5 , u_{10} , u_{17} , v_{17} , v_{10}	6. 5
$\overline{x}$, s_X^2 , A , B , C , (D)	7. 8

Table 2 Comparison of Various $\overline{Y}_N^{E}$ with respect to the Measured Data of Size 50 following to Normal Distribution.

Measured Data	n	t			means of upper blocks			means of lower blocks		$\overline{Y}_N{}^E$
		t_1	t_2	t_3	u_{t_1}	u_{t_2}	u_{t_3}	v_{t_3}	v_{t_2}	
Steel Yield Strength (SM41B)	21	2	4	7	1.223	1.094	0.977	−1.238	−1.469	imaginary number
Steel Tensile Strength (SM50B)	48	4	9	16	1.860	1.349	1.012	−0.997	−1.330	9.35
* * * *	48	4	11	18	1.841	1.210	0.933	−0.913	−1.216	9.93
Steel Buckling Strength (SS41)	48	4	9	16	1.911	1.572	1.163	−1.061	−1.325	imaginary number
* * * *	47	3	11	18	1.891	1.383	1.049	−0.995	−1.230	2.36
Cement Compressive Strength	56	5	10	17	2.030	1.584	1.195	−1.117	−1.334	imaginary number
* * * *	50	5	10	17	1.683	1.312	1.006	−1.141	−1.428	4.07
Truss Member Force	31	3	6	10	1.998	1.480	1.124	−1.088	−1.288	imaginary number

Table 3 Extremums $\overline{Y}_N{}^E$ for Various Sorts of the Measured Data by using Means of Upper and Lower Blocks.

Measured Data	unit	n	$\overline{x}$	s_X	δ_X	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{Y_N}^{E}$ $(N=100)$
Steel Yield Strength (SM 41B)	t/cm²	21	2.752	0.119	0.043	0.2577	0.2467	0.2142	−3.52
* * (SM 50B)	*	48	3.637	0.207	0.057	0.2238	0.2426	0.2343	−5.59
Steel Tensile Strength (SM 41B)	*	21	4.321	0.143	0.033	0.2573	0.2704	0.2502	−4.25
* * (SM 50B)	*	48	5.377	0.057	0.011	0.2565	0.2571	0.2361	−4.47
Steel Buckling Strength (SS 41)	*	48	1.251	0.156	0.124	0.2758	0.2829	0.2608	−3.64
Cement Compressive Strength	*	56	0.3905	0.0109	0.028	0.2746	0.2823	0.2611	−3.71
Truss Member Force (Lowor Chord)	t	31	14.44	1.95	0.135	0.2663	0.2736	0.2528	4.19
* * (Upper Chord)	*	31	28.88	3.91	0.135	*	*	*	*
Rainfall (Tsukechi)	mm	71	121.1	40.3	0.333	0.2615	0.2815	0.2688	4.16
* (Ootaki)	*	47	117.6	33.3	0.284	0.2702	0.2797	0.2603	3.97
Discharge (Inuyama)	m³/sec	18	4,386	1,516	0.346	0.2599	0.2643	0.2424	4.42
* (Kamo)	*	18	1,027	711	0.692	0.2463	0.2640	0.2497	4.89
Wind Velocity per 10 min. (Nagoya)	m/sec	52	18.6	4.8	0.257	0.2378	0.2664	0.2612	4.86
* * (Okinawa)	*	33	40.4	8.2	0.202	0.2678	0.2702	0.2473	4.05
* * (Naha)	*	33	32.9	8.5	0.257	0.2735	0.2759	0.2506	3.68
Wind Velocity per second (Okinawa)	*	30	57.0	11.8	0.207	0.2656	0.2706	0.2497	4.19
* * (Naha)	*	28	46.8	12.7	0.272	0.2698	0.2715	0.2465	3.89

Table 4 Extremums $\overline{Y_N}^{E}$ for Various Sorts of the Measured Data by using Three Characteristics A, B and C.

Measured Data	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{Y_N}^{E}$	lower bound of A	lower bound of B	lower bound of B	upper bound of $\overline{Y_N}^{E}$
Steel Yield Strength (SM 41 B)	0.2577	0.2467	0.2142	3.60	0.2423	0.2320	0.2014	5.03
* * (SM 50 B)	0.2238	0.2426	0.2343	5.60	0.1810	0.1962	0.2343	—
Steel Tensile Strength (SM 41 B)	0.2573	0.2704	0.2502	4.37	0.2378	0.2499	0.2312	5.25
* * (SM 50 B)	0.2565	0.2571	0.2361	4.47	0.2294	0.2300	0.2537	5.48
Steel Buckling Strength (SS 41)	0.2758	0.2829	0.2608	3.64	0.2682	0.2752	0.2537	4.08
Cement Compressive Strength	0.2746	0.2823	0.2611	3.71	0.2681	0.2756	0.2549	4.09
Truss Member Force	0.2663	0.2736	0.2528	4.19	0.2536	0.2605	0.2408	4.73
Rainfall (Tsukechi)	0.2615	0.2815	0.2688	4.19	0.2533	0.2726	0.2604	4.63
* (Ootaki)	0.2702	0.2797	0.2603	3.97	0.2599	0.2690	0.2503	4.48
Discharge (Inuyama)	0.2599	0.2643	0.2424	4.43	0.2434	0.2475	0.2269	5.09
* (Kamo)	0.2463	0.2640	0.2497	4.90	0.2177	0.2334	0.2208	—
Wind Velocity per 10 min. (Nagoya)	0.2378	0.2664	0.2612	4.86	0.2264	0.2536	0.2486	5.34
* * (Okinawa)	0.2678	0.2702	0.2473	4.05	0.2537	0.2560	0.2343	4.73
* * (Naha)	0.2735	0.2759	0.2506	3.70	0.2637	0.2659	0.2415	4.31
Wind Velocity per second (Okinawa)	0.2656	0.2706	0.2497	4.19	0.2510	0.2557	0.2360	4.84
* * (Naha)	0.2698	0.2715	0.2465	3.90	0.2579	0.2595	0.2356	4.57

Table 5 Graphical Estimation Method of Extremum $\overline{Y_N}^{E}$.

Measured Data	Data		Proposed Procedure		Gumbel's Method	
	n	maximum	100 years	200 years	100 years	200 years
Rainfall (Tsukechi)	71	255. 7	288. 6	326. 1	258. 4	282. 0
* (Ootaki)	47	213. 4	250. 0	282. 3	234. 5	254. 6
Discharge (Inuyama)	18	7, 064	11, 087	13, 179	10, 281	11, 286
* (Kamo)	18	2, 926	4, 504	5, 577	3, 792	4, 263
Wind Velocity per 10 min. (Nagoya)	52	36. 5	41. 8	47. 5	35. 2	38. 0
* * (Okinawa)	33	60. 8	73. 4	82. 7	69. 9	75. 0
* * (Naha)	33	49. 5	64. 0	72. 8	63. 5	68. 7
Wind Velocity per second (Okinawa)	30	85. 3	106. 5	120. 2	100. 2	107. 6
* * (Naha)	28	73. 6	96. 2	110. 7	93. 6	101. 6

Table 6 Comparison between Proposed Procedure and Gumbel's Method.

Measured Data	$\lvert\overline{Y}_N^{\mathrm{E}}\rvert$	graphical estimation of $\lvert\overline{Y}_N^{\mathrm{E}}\rvert$	estimation error (%)	n	upper bound of $\lvert\overline{Y}_N^{\mathrm{E}}\rvert$	ratio of $\lvert\overline{Y}_N^{\mathrm{E}}\rvert$ and its upper bound
Steel Yield Strength (SM41B)	3.52	3.60	2.3	21	5.03	1.40
* * (SM50B)	5.59	5.60	0.2	48	—	—
Steel Tensile Strength (SM41B)	4.25	4.37	2.8	21	5.25	1.20
* * (SM50B)	4.47	4.47	0	48	5.48	1.25
Steel Buckling Strength (SS41)	3.64	3.64	0	48	4.08	1.12
Cement Compressive Strength	3.71	3.71	0	56	4.09	1.10
Truss Member Force	4.19	4.19	0	31	4.73	1.13
Rainfall (Tsukechi)	4.16	4.19	0.7	71	4.63	1.11
* (Ootaki)	3.97	3.97	0	47	4.48	1.13
Dischange (Inuyama)	4.42	4.43	0.2	18	5.09	1.15
* (Kamo)	4.89	4.90	0.2	18	—	—
Wind Velocity per 10 min. (Nagoya)	4.86	4.86	0	52	5.34	1.10
* * (Okinawa)	4.05	4.05	0	33	4.73	1.17
* * (Naha)	3.68	3.70	0.5	33	4.31	1.16
Wind Velocity per second (Okinawa)	4.19	4.19	0	30	4.84	1.16
* * (Naha)	3.89	3.90	0.3	28	4.57	1.17

Table 7 Estimation Errors of Graphical Estimation Method.