Quark-Lepton Correspondence and $SU(2)\times U(1)\times U(1)$ Gauge Model

(to appear in Prog. Theor. Phys. Vol.61, No.1 (Jan. 1979))

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Abstract

A gauge model based on an $SU(2)\times U(1)\times U(1)$ group is proposed to accomodate a quark-lepton correspondence in neutral current interactions as well as charged current interactions. It predicts a new type of neutral current for neutrino scattering: $I^{(3)}-2\sin^2\theta Q^{(w)}$, where $Q^{(w)}=1/2$ or $-1/2$. On the basis of a composite substructure of quarks and leptons suggested by the quark-lepton correspondence, a new charge $Q^{(w)}$ can be understood as an electric charge of a "fermion" which forms quarks and leptons together with another "boson" carrying the baryon (lepton) number.

I. Introduction

Many attempts have been made to understand observed neutral current phenomena[1] within[2] and without[3] gauge theory framework. On the basis of an $SU(2)\times U(1)$ gauge symmetry, Weinberg and Salam[4] proposed a model to unify weak and electromagnetic interactions. This model combined with the GIM construction[5] (hereafter we call it W-S model) has satisfactory accounted for the neutrino -induced neutral current data[6] although the result of an experiment on atomic parity violation[7],[8] seems to be against the model. Another point which I would like to emphasize is a symmetry which is supposed to exist between quarks and leptons. It is a quark-lepton correspondence,[9] which predicted the charm degree of freedom and which states that weak interactions are invariant under the exchanges:[10]

$$\begin{array}{llll} p\leftrightarrow\nu_e, & p'\leftrightarrow\nu_\mu, & t\leftrightarrow\nu_\tau, & \cdots, \\ n\leftrightarrow e, & \lambda\leftrightarrow\mu, & b\leftrightarrow\tau, & \cdots, \end{array} \tag{1.1}$$

neglecting the Cabibbo rotation.

The symmetry has not been considered seriously because it depends on aesthatic considerations. However, it is worthwhile to gain some understanding about the quark-lepton correspondence, which implies a subhadronic structure of quarks and leptons.[11],[12] In a composite model, two kinds of particles, "fermion" and "boson" are needed to understand the symmetry. It may suggest a clue to explaining why there are a large number of quarks and leptons.

With all this in mind, we will attempt to construct a new gauge model with the quark-lepton correspondence, which has to hold in both neutral and charged current interactions. To accomodate the correspondence, we go to a larger group[13] beyond the SU(2)×U(1). In this paper, we adopt an SU(2)×U(1)×U(1)[14] as our gauge group. In §2, we derive the Lagrangian for weak neutral current interactions and discuss the correspondence in the context of our model. For neutrino-induced reactions under the four-fermion interaction, our model provides a new type of neutral current:

$$I^{(3)}-2\sin^2\theta Q^{(w)} \tag{1.2}$$

where $Q^{(w)}=1/2$ for p, p', t, $\cdots(\nu_e,\nu_\mu,\nu_\tau,\cdots)$ and $Q^{(w)}=-1/2$ for n, λ, b, $\cdots$(e, μ, τ, $\cdots$). It should be noted that the current (1.2) does not depend on an electric charge of particle as the conventional gauge model does. In §3, we make predictions of neutral current reactions and compare them with the data. The final section is devoted to conclution. We also discuss our results on the basis of the composite model and it is shown that new quantum number $Q^{(w)}$ in Eq.(1.2) can be interpreted as an electric charge of the "fermion".

2. Quark-Lepton Correspondence

An SU(2)×U(1)×U(1) contains a triplet gauge field $W_\mu^{(i)}$ (i=1,2,3) and two singlet fields $B_\mu^{(Y)}$ and $B_\mu^{(N)}$ coupled to the weak isospin

(I), weak hypercharge (Y) and a new quantum number (N), respectively. All left handed doublets are assumed to be in the (1/2, 0, B-L) representation where the terms in the parenthesis show values: $(I^{(3)}, Y, N)$ and B(L) is the baryon (lepton) number. The other right handed fermions are assigned as follwos:

$$u_1: (0,\ 1,\ B-L)\ , \tag{2.1}$$

$$u_2: (0,\ -1,\ B-L)\ , \tag{2.2}$$

where u_1=p, p', t, ⋯(ν_e, ν_μ, ν_τ,⋯) and u_2=n, λ, b, ⋯ (e, μ, τ, ⋯). It follows that the photon field should be

$$A = \frac{g^{(Y)}g^{(N)}W^{(3)}+gg^{(N)}B^{(Y)}+gg^{(Y)}B^{(N)}}{\sqrt{(gg^{(Y)})^2+(g^{(Y)}g^{(N)})^2+(gg^{(N)})^2}}\ , \tag{2.3}$$

and the electric charge (Q^γ) is

$$Q^\gamma = I^{(3)} + \frac{Y+N}{2}\ , \tag{2.4}$$

where g, $g^{(Y)}$ and $g^{(N)}$ are the couplings associated with SU(2), U(1) and U(1), respectively. Our mixing angles are given by the following matrices:

$$\begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ -\sin\theta\cos\phi & -\cos\theta\cos\phi & \sin\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} W_\mu^{(3)} \\ B_\mu^{(Y)} \\ B_\mu^{(N)} \end{pmatrix} \tag{2.5}$$

with the definition:

$$\sin\theta = g^{(Y)}/\sqrt{g^2+(g^{(Y)})^2}\ , \tag{2.6}$$

$$\sin\phi = g^{(N)} / \sqrt{(g^{(Y)})^2\cos^2\theta + (g^{(N)})^2} \ , \tag{2.7}$$

and

$$e = g\sin\theta\sin\phi = g^{(Y)}\cos\theta\sin\phi = g^{(N)}\cos\phi \ . \tag{2.8}$$

The other parameter $\sin\alpha$ should depend on a choice of Higgs scalars. However, we do not introduce a specific Higgs scalar in our later discussion.[15)]

The Lagrangian for the basic interactions between fermions and massless gauge bosons is

$$-\mathcal{L}_{int} = gJ_\mu^{(i)}W^{\mu(i)} + g^{(Y)}J_\mu^{(Y)}B^{\mu(Y)} + g^{(N)}J_\mu^{(N)}B^{\mu(N)} \ , \tag{2.9}$$

and

$$J_\mu^{(i)} = \Sigma\, \psi_L\gamma_\mu \frac{\tau^{(i)}}{2}\psi_L \ ,$$

$$J_\mu^{(Y)} = \Sigma\, \psi_L\gamma_\mu \frac{Y}{2}\psi_L + (\ L \to R\ ) \ , \tag{2.10}$$

$$J_\mu^{(N)} = \Sigma\, \psi_L\gamma_\mu \frac{N}{2}\psi_L + (\ L \to R\ ) \ ,$$

where the summation is over the fermions according to each quantum number ($\frac{\tau^{(3)}}{2}$, Y, N) and L, R denotes left handed or right handed, respectively. The weak neutral current and electromagnetic part of the Lagrangian is expressed in a convenient form (suppressing Lorentz indices):

$$-\mathcal{L}_{int} = g\langle J|W\rangle \ , \tag{2.11}$$

where

$$|J\rangle = (J^{(3)},\ \tan\theta J^{(Y)},\ \sin\theta\tan\phi J^{(N)}), \tag{2.12}$$

$$|W\rangle = (W^{(3)},\ B^{(Y)},\ B^{(N)}). \tag{2.13}$$

After the diagonalization of the neutral fields, we obtain

$$-\mathcal{L}_{int} = g\langle J'|T^{-1}|Z\rangle + eAJ^{(em)} , \tag{2.14}$$

where

$$T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} , \tag{2.15}$$

$$|J'\rangle = (-\cos\phi\sin\theta(J^{(3)}+J^{(Y)})+\sin\theta\sin\phi\tan\phi J^{(N)} ,$$

$$\cos\theta J^{(3)}-\sin\theta\tan\theta J^{(Y)}) , \tag{2.16}$$

$$|Z\rangle = (Z_1,\ Z_2) .$$

For small-momentum transfer interactions, the effective Lagrangian can be written as:

$$-\mathcal{L}_{eff} = 4\sqrt{2}\ G_F m_W^2 \langle J'|T^{-1}MT|J'\rangle , \tag{2.18}$$

where

$$M = \begin{pmatrix} m_1^{-2} & 0 \\ 0 & m_2^{-2} \end{pmatrix} \tag{2.19}$$

The masses of Z_1 and Z_2 boson, m_1 and m_2 are given by the mixing angles. They can be determined by specifying Higgs scalars in a usual procedure and we obtain neutral-current couplings relative to charged-current ones. In our discussion, to specify Higgs scalars corresponds to imposing a condition on weak neutral currents coupled to one of leptons and quarks. It is usually applied to neutrino-induced neutral current interactions.[16)] We shall use the condition that the neutral currents coupled to neutrino have a same structure as that in the standard model as well.

We are now in position to determine neutral currents for neutrino-induced reactions. The neutrino neutral current in Eq.(2.18) is obtained by use of the current (2.16):

$$|J'\rangle = \frac{1}{4}(\bar{\nu}\gamma_\mu(1-\gamma_5)\nu)|\nu\rangle \quad , \tag{2.20}$$

$$|\nu\rangle = (-\sin\theta/\cos\phi,\ \cos\theta) \ . \tag{2.21}$$

To realize the condition, we have to set

$$m_W^2 T^{-1} M T|\nu\rangle = (0,\ \cos\theta) \ , \tag{2.22}$$

leading to

$$m_W^2 \langle\nu|T^{-1}MT|J'\rangle = \cos^2\theta J^{(3)} - \sin^2\theta J^{(Y)} \quad , \tag{2.23}$$

of which structure is identical with that in the standard SU(2)×U(1) gauge model. We then find for neutrino scattering

$$-\mathcal{L}_{eff} = \sqrt{2}G_F(\bar{\nu}\gamma_\mu(1-\gamma_5)\nu)(J_\mu^{(3)} - \sin^2\theta V_\mu^{(w)}), \tag{2.24}$$

where $$V_\mu^{(w)} = J_\mu^{(3)} + J_\mu^{(Y)} \ . \tag{2.25}$$

The vector current, $V_\mu^{(w)}$ can be expressed in terms of fermions:

$$V_\mu^{(w)} = \frac{1}{2}(\bar{u}_1\gamma_\mu u_1 - \bar{u}_2\gamma_\mu u_2) \ , \tag{2.26}$$

and it is independent of the electric charge.

The masses are now fixed through Eqs.(2.21) and (2.22) and they are

$$(m_1/m_W)^2=1 - \tan\theta/(\tan\alpha\cos\phi), \tag{2.27}$$

$$(m_2/m_W)^2=1 + \tan\theta\tan\alpha/\cos\phi , \tag{2.28}$$

where

$$m_W = (e/\sin\theta\sin\phi)/(4\sqrt{2}G_F)^{1/2} . \tag{2.29}$$

The neutral currents of Eq.(2.9) are

$$-\mathcal{L}_{int} = 2\sqrt{2}(\frac{G_F}{\sqrt{2}})^{1/2} m_W(J^{(Z1)}_{\mu} Z_1^{\mu}+J_{\mu}^{(Z2)} Z_2^{\mu}) , \tag{2.30}$$

$$J_{\mu}^{(Z1)}=J_{\mu}^{(3)}(\sin\alpha\cos\theta-\cos\alpha\cos\phi\sin\theta)-J_{\mu}^{(Y)}\sin\theta$$
$$\cdot(\cos\alpha\cos\phi+\sin\alpha\tan\theta)+J_{\mu}^{(N)}\sin\theta\cos\alpha\sin\phi\tan\phi, \tag{2.31}$$

$$J_{\mu}^{(Z2)}=J_{\mu}^{(3)}(\cos\alpha\cos\theta+\sin\alpha\cos\phi\sin\theta)+J_{\mu}^{(Y)}\sin\theta$$
$$\cdot(\sin\alpha\cos\phi-\cos\alpha\tan\theta)-J_{\mu}^{(N)}\sin\theta\sin\alpha\sin\phi\tan\phi, \tag{2.32}$$

In the above equations, the currents $J_{\mu}^{(3)}$ and $J_{\mu}^{(Y)}$ satisfy the quark-lepton correspondence on account of our quantum number assignments. On the other hand, $J_{\mu}^{(N)}$ depends on the quantum number B-L; therefore, it breaks the correspondence between quarks and leptons. Only if the interactions between Z_1 & Z_2 and $J_{\mu}^{(N)}$ are switched off, the correspondence can be realized. In other word, $\sin^2\phi= 0$ is a necessary condition to accomodate the correspondence in our model. However, in a unified theory of weak and electromagnetic interactions, there are relations between mixing angles and the electric charge. The relations are

$$\cos\phi = e/g^{(N)} \quad , \tag{2.33}$$

$$\sin\phi = e/g\sin\theta \quad . \tag{2.34}$$

in our formulation. The condition of $\sin^2\phi=0$ requires that $e=g^{(N)}$ and $g \to \infty$ while $g^{(Y)}/g$ is finite. It follows that m_W becomes infinite like:

$$\begin{aligned} m_W &= (e/\sin\theta\sin\phi)/(4\sqrt{2G_F})^{1/2} \\ &\approx (38/\sin\theta)/\sin\phi\,(\text{GeV}) \quad . \end{aligned} \tag{2.35}$$

These considerations show that the quark-lepton correspondence holds approximately in the gauge model under the condition:

$$g,\ g^{(Y)} \gg g^{(N)} \approx e \quad , \tag{2.36}$$

or equivalently,

$$\sin^2\phi \ll 1 \quad . \tag{2.37}$$

The currents, $J_\mu^{(Z1)}$ and $J_\mu^{(Z2)}$ are expressed by

$$J_\mu^{(Z1)} = J_\mu^{(3)}\sin(\alpha-\theta) - J_\mu^{(Y)}\tan\theta\cos(\alpha-\theta) \quad , \tag{2.38}$$

$$J_\mu^{(Z2)} = J_\mu^{(3)}\cos(\alpha-\theta) + J_\mu^{(Y)}\tan\theta\sin(\alpha-\theta) \quad , \tag{2.39}$$

and the masses are

$$(m_1/m_W)^2 = \sin(\alpha-\theta)/(\cos\theta\sin\alpha) \quad , \tag{2.40}$$

$$(m_2/m_W)^2 = \cos(\alpha-\theta)/(\cos\theta\cos\alpha) \quad , \tag{2.41}$$

where $0<\theta<\alpha<\pi/2$ or $-\pi/2<\theta-\pi/2<\alpha<0$. Supposing that $\sin^2\phi=0.01$, we obtain

$$m_W \approx 380/\sin\theta \ \text{(GeV)} \ . \tag{2.42}$$

The effective Lagrangian can be written as:

$$\mathcal{L}_{eff}=8\left(\frac{G_F\cos\theta}{\sqrt{2}}\right)\left(\frac{\sin\alpha}{\sin(\alpha-\theta)}\ J_\mu^{(Z1)}J^{\mu(Z1)}+\frac{\cos\alpha}{\cos(\alpha-\theta)}\ J_\mu^{(Z2)}J^{\mu(Z2)}\right). \tag{2.43}$$

Summarizing the discussions above, we obtain the quark-lepton correspondence in the SU(2)×U(1)×U(1) gauge model with strong couplings and superheavy weak bosons.

3. Neutral Current Reactions

We predict neutral current reactions by using our Lagrangian (2.43) with the condition of $\sin^2\phi << 1$. The effective Lagrangian for neutrino- and electron-induced reactions is

$$\mathcal{L}_{eff}=\frac{G_F}{\sqrt{2}}((\bar{\nu}\gamma_\mu(1-\gamma_5)\nu)-(\bar{e}\gamma_\mu(1-\gamma_5)e)) \cdot(\mathcal{J}^{\mu(3)}-2\sin^2\theta V^{\mu(w)})$$

$$-\frac{G_F\sin^2\theta}{\sqrt{2}}((\bar{\nu}\gamma_\mu(1+\gamma_5)\nu)-(\bar{e}\gamma_\mu(1+\gamma_5)e))\cdot(\mathcal{J}^{\mu(3)}(1+\varepsilon)-2\varepsilon V^{(w)}), \tag{3.1}$$

where $\mathcal{J}_\mu^{(3)}=2J_\mu^{(3)}$ and $\varepsilon=\frac{1}{\cos\theta}\ \frac{\sin\alpha\cos^3(\alpha-\theta)+\cos\alpha\sin^3(\alpha-\theta)}{\sin(\alpha-\theta)\cos(\alpha-\theta)}$. (3.2)

Note that the ν_L- term is valid for any value of $\sin^2\phi$.

We first focus our attention on neutrino-induced reactions. In the inclusive reactions, the observed quantities, $R(\nu N)$ and $R(\bar{\nu}N)$ which are the ratios of neutral-to-charged current cross sections, can be expressed as follows:[17)]

$$R(\nu N) = \frac{1}{2} - X + \frac{X^2}{2} + \frac{X^2}{2} \cdot R_c , \tag{3.3}$$

$$R(\bar{\nu} N) = \frac{1}{2} - X + \frac{X^2}{2} + \frac{X^2}{2} \cdot \frac{1}{R_c} , \tag{3.4}$$

where $X=\sin^2\theta$ and R_c denotes the $\nu/\bar{\nu}$-charged current total cross section. The equations are valid within less than 5% for all R_c, that is all energies of incident neutrino.[18)] The predictions of the W-S model can be obtained by the replacement of $X^2/2$ with $5/9 \cdot X^2$; therefore, there are no differences in the predictions between our model and the W-S model. The result is shown in Fig.1.

Fig. 1

Expressions similar to Eqs.(3.3) & (3.4) are obtained for the elastic neutrino-electron scattering. They are

$$\sigma(\nu_\mu e) = (\frac{1}{2} - X + \frac{X^2}{2})\sigma_{V-A} + \frac{X^2}{2} \cdot \bar{\sigma}_{V-A} , \tag{3.5}$$

$$\sigma(\bar{\nu}_\mu e) = (\frac{1}{2} - X + \frac{X^2}{2})\bar{\sigma}_{V-A} + \frac{X^2}{2} \cdot \sigma_{V-A} , \tag{3.6}$$

where $\sigma_{V-A}=\bar{\sigma}_{V-A}/3 = 1.723\times10^{-41}E\ \mathrm{cm}^2/\mathrm{GeV}$. The similarity is a direct consequence of the quark-lepton correspondence. The difference between our model and the W-S model now can be stated that the former gives $\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e)<1$ for $X<1/2$ while the latter gives $\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e) \gtrsim 1$ for $X \geq 1/4$ as shown in Fig.2.

Fig. 2

A model independent analysis by Abbott and Barnett[19] finds the following values:

$$u_L = 0.33 \pm 0.07, \qquad d_L = -0.40 \pm 0.07,$$
$$u_R = -0.18 \pm 0.06, \qquad d_R = 0.0 \pm 0.11, \tag{3.7}$$

where the symbols u_L etc. stand for the couplings of relevant quarks, u=p and d=n. Comparing our values, $u_L=-d_L=(1-X)/2$ and $u_R=-d_R=-X/2$, with them, we can obtain X=0.24 which yields $u_L=-d_L=0.38$ and $u_R=-d_R=0.12$. Our model then gives a reasonable agreement with the data about neutrino-nucleon scattering experiments.

In an atomic physics experiments, a parity violation effect can be observed through an electron-quark interactions. The measurable quantity in the experiments is so-called Q_W, which expresses an effective couplings of hadronic vector current to electron axialvector current. The data on the optical rotation in the atomic bismuth are

$$R = (2.7 \pm 4.7) \times 10^{-8} \text{ in Oxford group,}^{7)}$$
$$(-0.7 \pm 3.2) \times 10^{-8} \text{ in Washington group,}^{8)} \tag{3.8}$$

where $R=2.27 \times 10^{-9} Q_W$. The Q_W-prediction of our model is

$$Q_W = -86(1 - \sin^2\theta(1+\varepsilon))/2 \,. \tag{3.9}$$

Our result is illustrated in Fig.3 as a function of $\sin^2\theta$

Fig. 3

and $\sin^2\alpha$. For $\sin^2\theta \leq 0.5$, we can set Q_W equal to zero with the appropriate value of $\sin^2\alpha$ if desired. The value, $\sin^2\theta=0.24$ suggested by the analyses on the neutrino-induced reactions provides

$$R = (-7 \sim +7) \times 10^{-8} , \tag{3.10}$$

for most of the range of $\sin^2\alpha (\geq 0.40)$, which is consistent with the experimental values quoted above.

4. Conclusion and Discussion

A possibility of a gauge model with the quark-lepton correspondence has been discussed on the basis of the $SU(2)\times U(1)\times U(1)$ gauge group where two massless gauge bosons are coupled to weak isospin and hypercharge which preserve the correspondence and one neutral boson to weak number, N which breaks it. The degree of the breaking effect is characterized by the mixing angle, $\sin^2\phi$. However, in the neutrino-induced reactions, the coupling is

$$I^{(3)} - 2\sin^2\theta Q^{(w)} , \tag{4.1}$$

without the approximation $\sin^2\phi \ll 1$. Our model predicts that $\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e) < 1$, which is subjected to some quite stringent phenomenological test. The present experimental results are in a good agreement with our model with $\sin^2\theta$ equal to 0.24.

The above feature can be interpreted from the viewpoint[20] of a composite substructure of quarks and leptons. A model[11),12] has two kinds of "subquarks": one is "fermion" carrying weak isospin and hypercharge and another ingredient is a "boson" whose quantum number is weak number (B-L). Quarks and leptons are expressed as:[12]

$$p_j = (B_j^{(1)} f_1) \ , \qquad p'_j = (B_j^{(2)} f_1), \cdots$$
$$n_j = (B_j^{(1)} f_2) \ , \qquad \lambda_j = (B_j^{(2)} f_2), \cdots \tag{4.2}$$

and

$$\nu_e = (B_0^{(1)} f_1) \ , \qquad \nu_\mu = (B_0^{(2)} f_1), \cdots$$
$$e = (B_0^{(1)} f_1), \qquad \mu = (B_0^{(2)} f_2), \cdots \tag{4.3}$$

where j(=1,2,3) and 0 denote color degree of freedom[21], or baryon and lepton number and f_1 & f_2 form a weak isospin doublet. The superscript of B shows a family like the electron family. The electric charge of quarks and leptons are found to be

$$Q^\gamma = Q^{(w)} + Q^{(b)} \ , \tag{4.4}$$

where $$Q^{(w)} = I^{(3)} + Y/2, \qquad \text{for the "fermion"}, \tag{4.5}$$

$$Q^{(b)} = (B-L)/2, \qquad \text{for the "boson"}, \tag{4.6}$$

according to the gauge-theory framework. They are presented in Table 1. It follows that the vector current, $V_\mu^{(w)}$ is an

Table 1

electromagnetic current whose coupling is just equal to $Q^{(w)}$, the charge of the "fermion". Furthermore, our choice,[22] Eq.(4.6) yields a "universality" in the neutral current interactions:[11]

$$G^2_{p\to p} = G^2_{n\to n} = G^2_{\nu_e\to\nu_e} = G^2_{e\to e} = \cdots , \qquad (4.7)$$

where G's are appropriate products of G_F and $\sin^2\theta$ & $\sin^2\alpha$, in the same sense as in the charged current interactions.

The gauge bosons, $W^{(i)}$ and $B^{(Y)}$ whose interactions conserve the quark-lepton correspondence only couple to the "fermion" while another "boson" interacts with $B^{(N)}$-boson and yields the breakdown of the correspondence. The assumption that we have made in order to obtain the correspondence can be stated as follows: the coupling constant of the "fermion" to the gauge bosons $W^{(i)}$ and $B^{(Y)}$ is much larger than that of the "boson" to $B^{(N)}$. Therefore, the only "fermion" is responsible for the weak interactions in a gauge theory with the quark-lepton correspondence. In the composite model, according to the discussion of Ref.20) the effective couplings of quarks and leptons to gauge bosons take the same forms as those in the SU(2)×U(1)×U(1) model discussed in §2. We should expect that new consequence specific to composite substructure of quarks and leptons appears in weak interactions which go beyond the level of low energy phenomenological four-fermion interaction.[12]

Acknowledgements

I would like to thank Professor S. Ogawa for his valuable suggestion. I am also grateful to Professor M. Nakagawa for communicating his idea to me. Finally, I would like to thank Professor M.K. Sundaresan for the hospitality of the department, where part of this work was done.

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Table 1: Quantum numbers for the "fermion" and "boson". The electric charge is defined as $I^{(3)}+(Y+N)/2$.

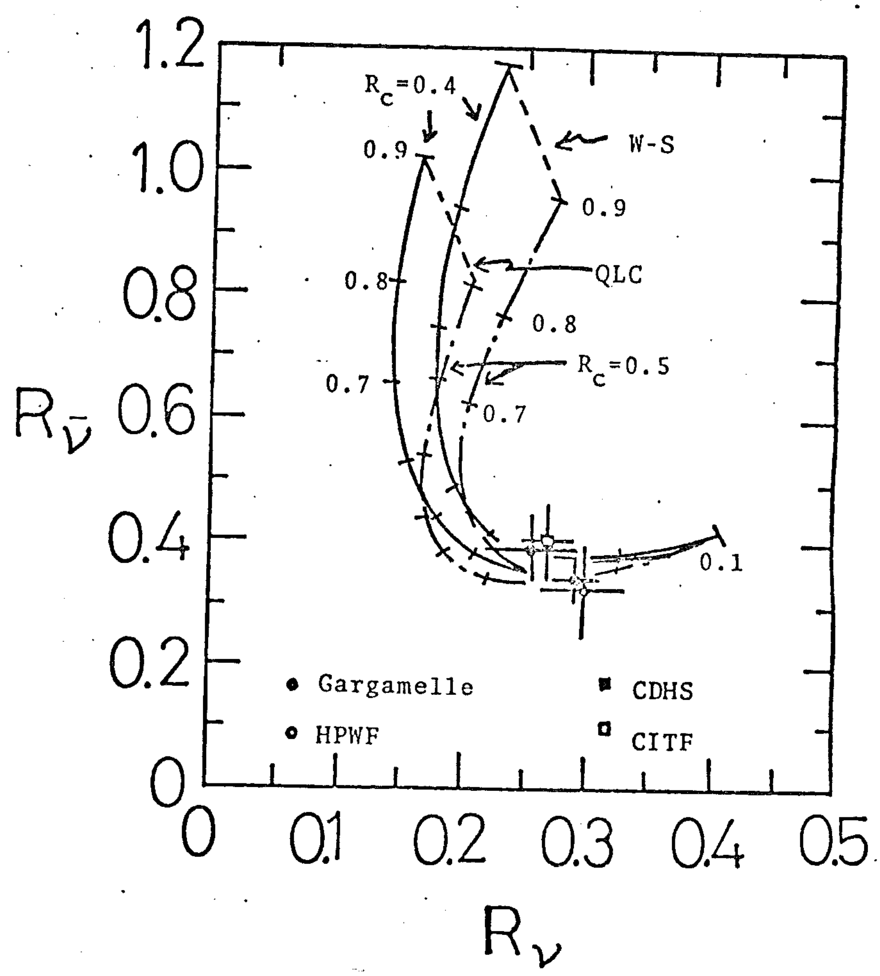
	$I^{(3)}$	Y	N	Q^{γ}
$(f_1)_L$	1/2	0	0	1/2
$(f_2)_L$	-1/2	0	0	-1/2
$(f_1)_R$	0	1	0	1/2
$(f_2)_R$	0	-1	0	-1/2
$B_j^{(a)}$	0	0	1/3	1/6
$B_0^{(a)}$	0	0	-1	-1/2

Figure Captions

Fig.1 The neutral-to-charged current cross section ratio for antineutrinos versus that ratio for neutrinos. The tenth values of $\sin^2\theta$ are shown with tick marks on the theoretical curves. Our model is referred to "QLC". The data is from Ref.1).

Fig.2 The total cross section for the elastic neutrino electron scattering as function of $\sin^2\theta$.

Fig.3 The Q_W prediction of our model versus $\text{sun}^2\alpha$. The data is from Refs. 7) and 8). The values on the curves show the values of $\sin^2\theta$.



1.2
1.0
0.8
0.6
0.4
0.2
0
$R_{\bar{\nu}}$
0
0.1
0.2
0.3
0.4
0.5
R_{ν}
R_c=0.4
0.9
W-S
0.9
0.8
QLC
0.8
R_c=0.5
0.7
0.7
0.1
Gargamelle
HPWF
CDHS
CITF

Fig.1

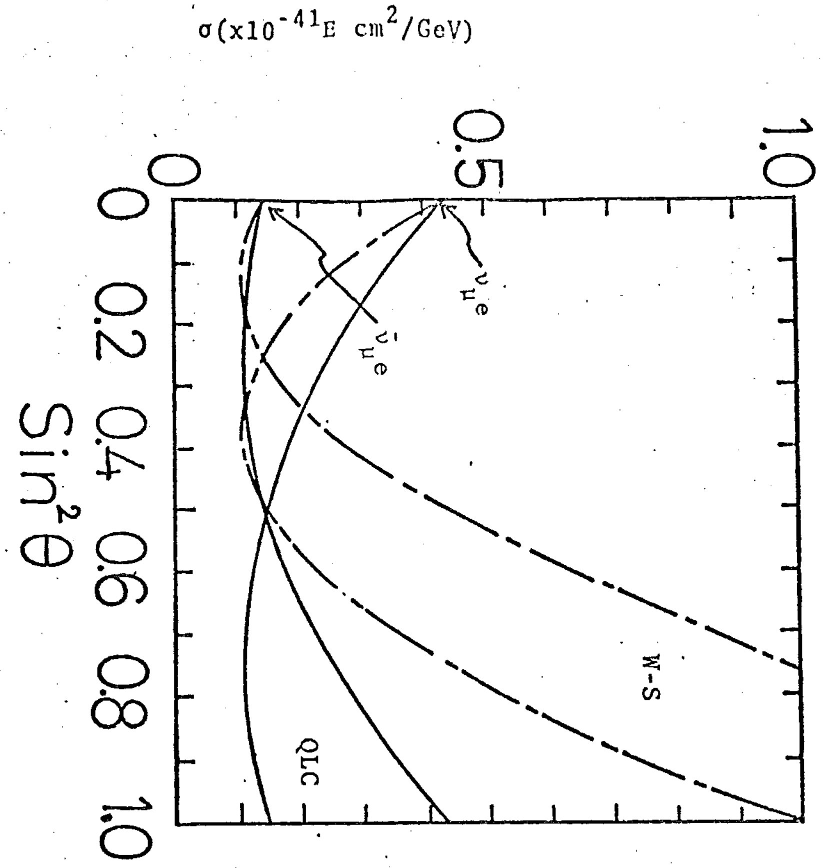

σ(x10^-41 E cm^2/GeV)
0
0.5
1.0
0 0.2 0.4 0.6 0.8 1.0
Sin^2 θ
ν̄μe
νμe
W-S
QLC

Fig 2

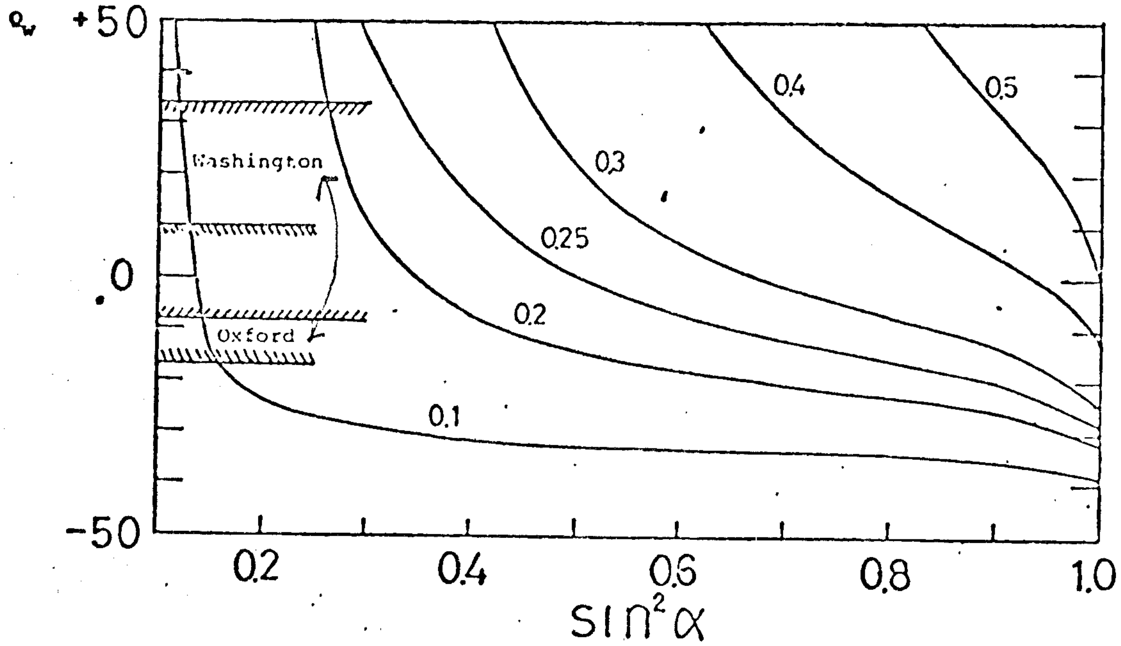

Qw
+50
0
-50
Washington
Oxford
0.1
0.2
0.25
0.3
0.4
0.5
0.2 0.4 0.6 0.8 1.0
sin^2 α

Fig.3

DEPARTMENT OF PHYSICS
NAGOYA UNIVERSITY

DATE

NO.

Lesson from the standard SU(2)×U(1) model

ゲージ不変な coupling (minimal coupling)

・SU(2) $g\left(W^{(1)}I^{(1)}+W^{(2)}I^{(2)}+W^{(3)}I^{(3)}\right)$

・U(1) $g'B\frac{Y}{2}$

電荷 $Q^{\gamma}=I^{(3)}+\frac{Y}{2}$

中性ボソン, $W^{(3)}$ と B の間に mixing がおこり photon A ともう一つのボソン Z になる。A は Q^{γ} と couple するように g と g' が決定される。

$$W^{(3)}=\cos\theta Z+\sin\theta A$$
$$B=-\sin\theta Z+\cos\theta A$$

から

$$gW^{(3)}I^{(3)}+g'B\frac{Y}{2}=\left(g\sin\theta I^{(3)}+g'\cos\theta\frac{Y}{2}\right)A+\left(g\cos\theta I^{(3)}-g'\sin\theta\frac{Y}{2}\right)Z$$

で $eQ^{\gamma}=g\sin\theta I^{(3)}+g'\cos\theta\frac{Y}{2}$ より

$$\boxed{e=g\sin\theta=g'\cos\theta}$$

Z-field はその結果

$$g\cos\theta I^{(3)}-g'\sin\theta\frac{Y}{2}=\sqrt{g^2+g'^2}\left(\cos^2\theta I^{(3)}-\sin^2\theta\frac{Y}{2}\right)$$

となり. 対応するカレントで表わせば

$$\boxed{\cos^2\theta J_{\mu}^{(3)}-\sin^2\theta J_{\mu}^{(Y)}}$$

となる。右巻き，左巻きと考慮してやれば Q^{γ} に対比すべき表式は

$$\boxed{I^{(3)}-2\sin^2\theta Q^{\gamma}}$$

この結果は簡単に一般化することができる
(1) photon field A の定義

$$A = \cos\theta\, B + \sin\theta\, W^{(3)}$$

$$= \left(\frac{e}{g}\right) W^{(3)} + \left(\frac{e}{g'}\right) B$$

$$\Longrightarrow \sum_i \left(\frac{e}{g^{(i)}}\right) B^{(i)}$$ i は異なる中性ボソンを示す

SU(2)×U(1)×U(1) においては、それゆえ

$$A = \left(\frac{e}{g}\right) W^{(3)} + \left(\frac{e}{g^{(Y)}}\right) B^{(Y)} + \left(\frac{e}{g^{(N)}}\right) B^{(N)}$$

とあらわされる。Z のほかにもう一つ新しくつけ加わった U(1) のために、Z′ があらわれる。

$$(W^{(3)}, B^{(Y)}, B^{(N)}) \Longrightarrow (A, Z, Z')$$

変換は 3×3 の matrix であらわすことができる。

◪ クォークの電荷の大きさは　+2/3　と −1/3
レプトンの電荷の大きさは　0　と　−1
故に前ページで示された coupling は　$I^{(3)} - 2\sin^2\theta\, Q^{\gamma}$　であり、
これはクォークとレプトンの入れかえによって違いをあたえることになる。