# Urban-Specific Technological Progress in a Harris-Todaro Model with an Oligopolistic Urban Sector

NAKAMURA Takeho

This paper extends a version of Harris-Todaro model presented by Calvo (1978) to introduce a Cournot-oligopoly market structure in an urban sector. Following the Calvo (1978) model, the equilibrium urban wage rate is determined by the mechanism of bargaining process between a monopolistic urban trade union and urban firms. An exogenous shock of urban technological progress, through the bargaining mechanism, always increases the urban-rural wage gap and the urban uniemployment; while it always decreases the rural employment. However, its effect on the urban employment is shown to be generally ambiguous. Moreover, it is shown that the urban technological progress always increases the social welfare (defined by social utility function), although it exacerbates the social income inequality (measured by movements in Lorenz curve).

## I. Introduction

In a perfectly competitive market economy where homogeneous workers are employed in multi-sector, the equilibrium value of the marginal labor productivity is equated across all sectors. Then, full employment is achieved, and the intersectoral labor allocation becomes efficient. However, looking at the real world, we have been commonly and historically observed large intersectoral wage differentials accompanied by high urban unemployment — especially, it is typical in the developing countries.[1)]

What are the important differences in characteristics between the above-mentioned ideal perfect-competitive market and the reality of developing countries? The model presented by Harris and Todaro (1970), hereinafter HT model, has been the one of the most frequently applied model to tackle this problem.[2)3)] In the original HT (1970) model, the rural wage is determined at the market-clearing level, while the urban wage is *exogenously* fixed above the market-clearing level. The assumptions behind the fixed urban wage are minimum wage legislations, well-organized trade unions, pension schemes and so on. Calvo (1978) incorporates the mechanism of bargaining process between a monopolistic urban trade union and urban firms; thus, the *ex-post* fixed urban wage (above the market-clearing level) is *endogenously* determined in his model.

This paper is basically based on the modified HT model presented by Calvo (1978), but we assume *Cournot-type oligopoly* in the urban sector. The main purpose of this paper is to show that how this modification of Calvo's model affects urban-rural wage gap, unemployment, and social income inequality—we especially focus on the effects of urban technological progress.

The rapid, but sector-specific economic growth can be observed in several developing countries—such as, China and India (so-called new economic giants). In such countries, a dual structure between urban and rural sectors strongly persist.[4] From the traditional viewpoint of development economics, a number of dualities existing in the developing countries are the principal cause of persistent inequality. Then, a question arises as follows. How does a technological progress of urban (formal) sector, which is observed in the above-mentioned developing countries, affect the domestic economy as a whole? This paper examines this question using a modified HT dual economy model with an oligopolistic urban sector. The introduction of the urban oligopolistic structure can be partly justified by a high entry barrier caused by a large technological gap between urban and rural firms and/or government regulation policies. The former cause will suit for our model, because such a technological gap is assumed to be sufficiently large in our model.

Our closed economy model consists of two sectors: urban and rural. Each sector produces different type of goods: manufactured and agricultural. The manufactured good is produced by oligopolistic urban firms. An exogenous technological progress is assumed to occur in the urban area; it affects on the urban firms' outputs, profits, as well as the relative price of their products. These regards are omitted in the Calvo's original study, because he assumes a small open economy (where the prices of goods are given) with perfectly competitive urban firms.

The urban technological progress is shown to increase urban unemployment and decrease the amount of rural employment. These results are consistent with the empirical observations in several developing countries in the process of rapid but sector-specific economic growth (such as China and India). The urban technological progress effect on urban employment is generally ambiguous. However, regardless of how the urban employment changes, the urban technological progress unambiguously increases the social welfare (defined by social utility function) and social income inequality (measured by movements in Lorenz curve).

This paper is organized in the following manner. Section Ⅱ lays out the basic framework of our model and examines its equilibrium. Section Ⅲ analyzes comparative statics; that is, the effects of urban technological progress are investigated. Section Ⅳ concludes the paper.

## Ⅱ. The model

Consider a closed 'dual' economy with two sectors: rural and urban. The dual structure of the economy exists between the rural and urban sectors, whose basic framework is borrowed from the literature on HT model (as explained by the next paragraph). Labor is the only production factor that we consider. There are homogeneous domestic workers, whose (exogenously fixed) total number is denoted by $\bar{L}$. We assume that goods and labor can freely move between two sectors, while the location of firms is fixed.[5] Therefore, we label two types of firms as 'rural firms' and 'urban firms' according to their location.[6] In the rural sector, perfectly competitive rural firms produce a single agricultural good (hereinafter good $y$); whereas the good produced by urban firms is a single manufactural good (hereinafter good $x$). Good $y$ is assumed as the numéraire and its price is normalized to unity. Let $p$ denotes the relative price of good $x$. The equilibrium value of $p$ is endogenously determined in the model.

The frameworks of our model basically follow the literature on HT model as follows.

1. The structures of the two sectors are different from each other: the rural wage is determined at perfectly competitive market-clearing level, while the urban wage is subject to an influential (monopolistic) trade union, as in Calvo (1978). The introduction of the Calvo-type urban union gives rise to equilibrium *urban-rural wage differentials* and *urban unemployment*.
2. All workers are risk-neutral. They have an equal likelihood of finding job in urban sector. Let $u \in (0, 1)$ denotes the probability of unemployment (as well as the urban unemployment rate), which will be discussed in detail later. Then, the well-known HT migration equilibrium condition can be expressed as

$$w_r = (1-u)w, \tag{1}$$

where $w_r$ and $w$ denote the rural wage and the urban wage in terms of good $y$, respectively.

The main departure of our model from standard HT models is the introduction of a *Cournot-oligopoly* market structure in the urban sector. Assume that there are $n$ identical urban firms operating in the urban sector; the number of them, $n$, is fixed throughout the paper.

### 1. Utility

The model economy consists of two types of individuals: workers and entrepreneurs. The total population of these individuals

is normalized to unity. As already mentioned, the (fixed) total number of workers is denoted by $\overline{L}$; therefore $1-\overline{L}$ is the total number of entrepreneurs. The entrepreneurs are assumed to own the urban firms; any positive profit of the urban firms goes to the owners (i.e., entrepreneurs) of these firms.[7] The profit of urban firms is the only source of the income of entrepreneurs; while the only income source for workers is wage income. All workers are innately homogeneous, but will be divided into three different income classes in equilibrium.
We assume that every individual—including both of workers and entrepreneurs—has an identical preference on how she consumes goods $x$ and $y$. Let the utility function of an arbitrary individual be represented by

$$U(d_x,\ d_y) = V(d_x)+d_y;\quad V' > 0,\ V'' < 0, \tag{2}$$

where $d_i$ $(i = x,\ y)$ denotes her consumption of good $i$.

Let $e$ denotes her nominal income measured in terms of good $y$. Maximizing (2) subject to her budget constraint: $e = pd_x+d_y$, we obtain the following first-order conditions.

$$V'(d_x) = \lambda p,\quad \lambda = 1, \tag{3}$$

where $\lambda$ is the Lagrange multiplier. From (3) and the above-mentioned budget constraint, we can obtain the following demand functions.

$$\begin{aligned} d_x &= d_x(p), \\ d_y &= d_y(p,\ e) = e-pd_x. \end{aligned} \tag{4}$$

The demand function of good $y$ depends on both her income and the relative price, while the demand function of good $x$ only depends on the relative price—that is, there is no income effect: $\partial d_x/\partial e = 0$.[8]

In order to obtain closed forms of solutions, we further assume that the function $V(d_x)$ takes the following form:

$$V(d_x) = \beta d_x-\frac{\alpha}{2}d_x^2;\quad \alpha,\ \beta > 0, \tag{5}$$

where $\alpha$ and $\beta$ are positive constants.[9] Using (5), the demand functions of (4) can be reduced to

$$\begin{aligned} d_x(p) &= \frac{\beta-p}{\alpha}, \\ d_y(p,\ e) &= e-\frac{(\beta-p)p}{\alpha}. \end{aligned} \tag{6}$$

Substituting these results into (2) and (5) yields the following indirect utility function:

$$\widetilde{U}(e,\ p) = e+\frac{(\beta-p)^2}{2\alpha}. \tag{7}$$

This is the maximum utility of an arbitrary individual who possesses nominal income $e$.

For future reference, we sum up the maximum utility of each individual and let it denoted by $\tilde{S}$:

$$\tilde{S} = \tilde{S}(E,\ p) = E + \frac{(\beta - p)^2}{2\alpha}. \tag{8}$$

where $E$ denotes the nominal national income. In equilibrium the nominal national income is perfectly divided into the consumption expenditures of goods $x$ and $y$:

$$E = D_x + D_y, \tag{9}$$

where $D_x$ and $D_y$ denote aggregate consumption of goods $x$ and $y$, respectively. Using (6) and (9), their equilibrium values can be given by

$$D_x = \frac{\beta - p}{\alpha},\quad D_y = E - \frac{(\beta - p)p}{\alpha}. \tag{10}$$

As one of the simplest way to assess changes in social welfare of the economy, we assume $\tilde{S}$ as the social welfare.

Lastly, rewriting the first function of (10) makes the inverse demand function for good $x$ market:

$$p = \beta - \alpha D_x. \tag{11}$$

We use this function for the study of the oligopolistic urban firms—the producers of good $x$.

### 2. The rural sector

There are perfectly competitive firms in the rural sector. They produce good $y$ by means of labor. We employ the simplest form of aggregate production function of this sector as follows:

$$Q_y = L_y,$$

where $Q_y$ and $L_y$ denote the total output of good $y$ and the total labor employment of this sector, respectively. Recall that good $y$ is the numéraire good; and its price is given as one. Therefore, the rural wage rate, $w_r$, is determined as follows:

$$w_r = 1. \tag{12}$$

### 3. The urban sector

As mentioned above, $n$ oligopolistic firms operate in the urban sector. For convenience, we use $i(i = 1, 2, \ldots, n)$ to index them. They produce good $x$ and compete in the Cournot fashion. We assume that all of the urban firms are symmetric and their production technology takes the following form.

$$q_i = A \times l_i,\quad \forall i \in \{1, \ldots, n\}, \tag{13}$$

where $q_i$ is the output of a firm $i$; $l_i$ is that firm's employment of labor; and $A(>0)$ is a technological parameter, which has an important role in this paper. In the following section, we will examine the effects of an exogenous increase in $A$.

Some notices must be mentioned here. Since we assumed that all workers are *identical*, the main differences between the urban and rural sectors are (i) the type of produced goods, and (ii) the type of firms. If we pay particular notice to the second difference, the parameter $A$ can be interpreted as the technological gap between urban and rural firms.[10) ] As it will be specified later, we assume that $A$ takes a sufficiently large value.

From (13), we obtain the revenue of firm $i$ in terms of good $y$ as $pAl_i$. Recall that $w$ is measured in terms of good $y$. Thus, for each firm $i$, the total cost (i.e., total wage payment) is $wl_i$; the constant marginal cost, denoted by $c(=wl_i/q_i)$, equals to $w/A$; and the firm's profit is $(pA-w)l_i$. (All these variables are measured in terms of good $y$.)

As discussed in detail later, $w$ is determined by the interaction between the urban firms and a monopolistic urban trade union. We employ one of the two scenarios—regarding such interaction—presented by Calvo (1978) in order to determine the equilibrium value of $w$ as follows:[11)]

- A monopolistic urban trade union moves first and determines $w$ so as to maximize its object function. Then, each of the urban firms passively react as price-taking profit maximizers. Each of them maximizes its profit, taking the level of $w$ as given; the amount of total employment is determined by this profit-maximizing behavior.
- Note, however, that the union's objective function consists of the number of union members employed by the urban firms, and the urban-rural wage differential. Thus, the union must take the backward induction procedure to determine the level of $w$.

This is a brief outline of the interaction between the urban firms and union that we consider. Hence, we first look at the profit maximization problem of the firms taking $w$ as given, and then examine the optimizing behavior of the union.

## 4. Profit-maximizing behaviors of the urban firms

Since we consider a closed economy, the equilibrium condition of good $x$ can be simply represented by

$$D_x = Q_x, \tag{14}$$

where $Q_x(=\sum_{i=1}^{n} q_i)$ denotes the total output of good $x$. The profit-maximization problem of each urban firm $i$ can be expressed as follows (recall (11)):

$$\max_{q_i}(\pi_i=)pq_i-cq_i,$$
$$\text{s.t. } p=\beta-\alpha Q_x, \tag{15}$$

where $\pi_i$ denotes the profit of firm $i$. The first-order condition (FOC) of each firm $i$ can be readily obtained as

$$\beta - \alpha Q_x - \alpha q_i - c = 0, \quad \forall i \in \{1, \ldots, n\}. \tag{16}$$

Recall that all urban firms are symmetric. Using (16), we can add all firms' FOCs together in order to obtain the following:

$$n\beta - n\alpha Q_x - \alpha Q_x - nc = 0;$$

which determines the total and each firm's output level, respectively; that is,

$$Q_x = \frac{n(\beta - c)}{(1+n)\alpha}\ , \tag{17}$$

$$q_i = \frac{\beta - c}{(1+n)\alpha} \equiv q, \quad \forall i \in \{1, \ldots, n\}. \tag{18}$$

where $\beta > c = w/A$ is the necessary condition for $q$ and $Q_x$ to be positive—note that $w$ is an endogenous variable, while $\beta$ and $A$ are exogenous ones. We assume the following assumption throughout the paper in order to eliminate uninteresting equilibria.

**Assumption 1.** *We assume that $\beta > c$ (or equivalently, $w < A\beta$) in order to assure positive outputs of urban firms.*

From (11), (14) and (17), the price of good $x$ can be obtained as

$$p = \frac{\beta + nc}{1+n}. \tag{19}$$

Thus far, we take $c(= w/A)$ as given. We next examine the endogenous determination of $w$—by the union's optimizing behavior subject to (17).

## 5. Optimizing behavior of the urban trade union

According to Calvo (1978), the urban trade union prefers to increase both the number of workers employed by the urban firms, and the urban-rural wage differential. Formally, we assume the union's objective function is given by

$$G(L_x,\ w - w_r) = L_x \times (w - w_r), \tag{20}$$

where $L_x (\equiv \sum_{i=1}^{n} l_i)$ denotes the total employment in the urban sector.

The union determines the level of $w$ subject to the profit-maximizing behaviors of firms, which is already shown in the previous subsection. In other words, the union first decides the level of $w$, and then the firms maximizes their profits given the level of $w$. The firms' choice variable is their employment (or output). Since the union also desires to increase firms' employment level, the backward induction procedure must be taken to determine the level of $w$.

From (17), $L_x = Q_x/A$ and $c = w/A$, we obtain the optimal employment level determined by firms as follows:

$$L_x = \frac{n\left(\beta - \frac{w}{A}\right)}{A(1+n)\alpha} \quad , \tag{21}$$

or (rearranging this equation makes)

$$w = A\beta - \frac{\alpha A^2(1+n)}{n} L_x,$$

where a negative relation between $w$ and $L_x$ is implied. Thus, the union faces a trade-off between its two goals (i.e., increasing the levels of $w$ and $L_x$).

The union chooses the optimal level of $w$ so as to maximize (20) subject to (21); that is,

$$\max_w \quad L_x \times (w - w_r),$$
$$\text{s.t. } L_x = \frac{n\left(\beta - \frac{w}{A}\right)}{A(1+n)\alpha}.$$

We can readily obtain the optimal level of $w$ as follows:

$$w = \frac{A\beta + w_r}{2}. \tag{22}$$

This equation implies the following relationship:

$$w \gtreqless w_r \Leftrightarrow A\beta \gtreqless w.$$

Needless to say, what we want to examine is a type of equilibria where the urban wage is determined so as to be higher than the rural wage (i.e., $w > w_r$) —as the common assumption of the literature on HT model. Since we already assume that $A\beta > w$ by Assumption 1, $w > w_r$ always holds throughout the paper. (Otherwise, the production of good $x$ becomes zero, and the urban sector must disappear.)

## 6. Equilibrium

This subsection derives the equilibrium solutions of our small-scale general equilibrium model. First, the following equation stands for the supply and demand of domestic labor.

$$\bar{L} = L_x + L_y + L_u. \tag{23}$$

Here, the total supply of domestic labor ($\bar{L}$) is shown to be divided into the employed in the urban sector ($L_x$), that in the rural sector ($L_y$), and unemployed in the urban sector ($L_u$). The probability of unemployment, $u$, can be expressed as

$$u = \frac{L_u}{L_x + L_u}; \tag{24}$$

therefore (1) can be rewritten as

$$w_r = \frac{L_x}{L_x + L_u} w. \tag{25}$$

Solving equations (12), (21), (22), (23) and (25) simaltaneously, we obtain the equilibrium values of $L_x$, $L_y$, $L_u$, $w_r$ and $w$. Firstly, since the optimal level of the urban wage is determined by the trade union so as to satisfy (22), we readily obtain

$$w_r^* = 1, \quad w^* = \frac{A\beta+1}{2}, \tag{26}$$

from (12) and (22). (The superscript '*' is used to represent *equilibrium* values.) Substituting (22) into (21) yields the equilibrium total employment of labor as follows:

$$L_x^* = \frac{n(A\beta-1)}{2\alpha(1+n)A^2}, \tag{27}$$

where we use the result: $w_r = 1$ from (12). Next, using (12) and (25), we have

$$L_u^* = \frac{n(A\beta-1)^2}{4\alpha(1+n)A^2}, \tag{28}$$

where the second equality comes from (27). Lastly, substituting (27) and (28) into (23), we obtain

$$L_y^* = \bar{L} - \frac{n(A\beta-1)(A\beta+1)}{4\alpha(1+n)A^2}. \tag{29}$$

Meanwhile, the equilibrium condition of good $y$ can be expressed as

$$Q_y^* = D_y^*, \tag{30}$$

where

$$\pi^* = \left(p^* - \frac{w^*}{A}\right) q^*. \tag{31}$$

The national income of the economy (in terms of good $y$) is

$$E^* = w^* L_x^* + L_y^* + n\pi^*, \tag{32}$$

Using (4), (14), (17), (18), (19) and (26)-(32), we can obtain closed-form solutions for $E^*$, $D_x^*$, $D_y^*$, $Q_x^*$, $Q_y^*$, $\pi^*$, $p^*$ and $q^*$. (These solutions are explicitly derived in the next section.) Now, we are ready to examine comparative statics.

## Ⅲ. Comparative statics: Examining the technological progress effects of urban firms

This section demonstrates comparative statics, which show that how an exogenous increase in $A$ affects the equilibrium of our modified HT model. First, from (26), the urban-rural wage gap is given as

$$w^* - w_r^* = \frac{A\beta-1}{2}; \tag{33}$$

and the effect of an increase in $A$ on it is apparently positive:

$$\frac{\mathrm{d}(w^* - w_r^*)}{\mathrm{d}A} = \frac{\beta}{2} > 0. \tag{34}$$

**Proposition 1. (the urban-rural wage gap):** *An increase in $A$ increases the urban-rural wage gap.*

Next, partially differentiating (27), (28) and (29) with respect to $A$, respectively, we obtain

$$\frac{\mathrm{d}L_x^*}{\mathrm{d}A}=\frac{n(2-A\beta)}{2\alpha(1+n)A^3}\begin{cases}>0 \text{ if } A\beta<2,\\<0 \text{ if } A\beta>2,\end{cases} \tag{35}$$

$$\frac{\mathrm{d}L_u^*}{\mathrm{d}A}=\frac{n(A\beta-1)}{2\alpha(1+n)A^3}>0, \tag{36}$$

$$\frac{\mathrm{d}L_y^*}{\mathrm{d}A}=\frac{-n}{2\alpha(1+n)A^3}<0. \tag{37}$$

Note that (26), combined with Assumption 1, implies the following relationship:

$$1=w_r^*<w^*<A\beta;$$

however, it does not exclude both the cases where $A\beta<2$ and where $A\beta>2$. For the sake of convenience, we hereinafter call the former case as Case 1, and the latter case as Case 2; that is,

$$A\beta<2, \qquad \text{(Case 1.)}$$

$$A\beta>2, \qquad \text{(Case 2.)}$$

The results of (35)-(37) can be summarized as follows:

**Proposition 2. (the labor distribution):** *An increase in $A$ always increases the urban unemployment and decreases the rural employment. The urban employment increases under Case 1, while it decreases under Case 2.*

The effect of urban technological progress on urban unemployment rate, $u\equiv L_u/(L_u+L_x)$, can be obtained as

$$\frac{\mathrm{d}u^*}{\mathrm{d}A}=\frac{2\beta}{n(A\beta+1)^2}>0. \tag{38}$$

**Proposition 3. (the urban unemployment rate):** *An increase in $A$ always increases the urban unemployment rate.*

We next look at how the aggregate wage income of all workers changes in response to the urban technological progress. Let $\Omega$ denotes the aggregate wage income; that is,

$$\Omega\equiv wL_x+L_y. \tag{39}$$

Totally differentiating $\Omega$ with respect to $A$, we obtain

$$\frac{\mathrm{d}\Omega}{\mathrm{dA}}=L_x^*\frac{\mathrm{d}w^*}{\mathrm{d}A}+w^*\frac{\mathrm{d}L_x^*}{\mathrm{d}A}+\frac{\mathrm{d}L_y^*}{\mathrm{d}A}. \tag{40}$$

Substituing (26), (27), (35) and (37) into the above equation, we obtain the following result:

$$\frac{\mathrm{d}\Omega}{\mathrm{d}A}=0, \tag{41}$$

that is, all effects in (40) cancel out each other, and then $\mathrm{d}\Omega/\mathrm{d}A$ becomes zero. This is a straightforward result, because we assume the standard HT migration equilibrium condition (1) —that is, in equilibrium the aggregate wage income, $\Omega$, equals to the (fixed) total endowment of labor, $\overline{L}$.[12)]

**Proposition 4. (the aggregate wage income):** *An increase in $A$ does not change the aggregate wage income of all domestic workers.*

The nominal national income, $E$, is the sum of the aggregate wage income and the aggregate profit of firms:

$$E = \Omega + n\pi^*.$$

Since $\mathrm{d}\Omega/\mathrm{d}A = 0$, an increase in $A$ affects the nominal national income only through the change in the profit of urban firms, $\pi^*$; that is,

$$\frac{\mathrm{d}E}{\mathrm{d}A} = n\frac{\mathrm{d}\pi^*}{\mathrm{d}A}.$$

The equilibrium value of each urban firm can be expressed as

$$\pi^* = (p^* - c^*)q^*, \tag{42}$$

where $c^* = w^*/A$. Totally differentiating this equation with respect to $A$, we obtain

$$\begin{aligned}\frac{\mathrm{d}\pi^*}{\mathrm{d}A} &= (p^* - c^*)\frac{\mathrm{d}q^*}{\mathrm{d}A} + q^*\frac{\mathrm{d}p^*}{\mathrm{d}A} - q^*\frac{\mathrm{d}c^*}{\mathrm{d}A} \\ &= \frac{\beta - c^*}{(1+n)^2\alpha A^2} > 0,\end{aligned} \tag{43}$$

where, we use the following results to get the above equation.

$$\frac{\mathrm{d}q^*}{\mathrm{d}A} = \frac{1}{2\alpha A^2(1+n)} > 0, \tag{44}$$

$$\frac{\mathrm{d}p^*}{\mathrm{d}A} = \frac{-n}{2(1+n)A^2} < 0, \tag{45}$$

$$\frac{\mathrm{d}c^*}{\mathrm{d}A} = \frac{-1}{2A^2} < 0. \tag{46}$$

From (45) and (46) we obtain

$$\frac{\mathrm{d}(p^* - c^*)}{\mathrm{d}A} = \frac{1}{2(1+n)A^2} > 0.$$

Hence, the drop in the marginal cost is greater than the drop in the relative price in response to the urban technological progress; and it contributes to increase the urban firms' profit.

**Proposition 5. (the nominal national income):** *An increase in $A$ increases the total profit of urban firms. Thus, the nominal national income increases as the same amount as the total profit of urban firms increases.*

Lastly, we examine how the urban technological progress affects the utility of each economic agent (i.e., workers and entrepreneurs). Each of their utility is decreasing in the relative price, $p$, and increasing in their own income. The decline in $p$ shown by (45) equally contributes to increase the utility of every economic agent. But, the directions and amounts of changes in each worker's income are different from each other. An exogenous occurrence of the urban technological progress provokes labor redistribution between the rural, urban-formal, and urban-informal (i.e., unemployed) sectors. Some of the new (rural-to-urban) immigrants will suffer from job loss and unambiguous deterioration in utility, while the others will enjoy the increase in their income and utility.

In the view of measuring the social welfare of this model economy, apparently, a Pareto-improving change will never happen in response to the urban technological progress. Then, as the simplest way of measuring the social welfare, we here examine how the maximum social utility, $\widetilde{S}$, given by (8) will be affected by the urban technological progress. Using (8)–(10), $\widetilde{S}$ also can be represented as

$$\begin{aligned}\widetilde{S}(E,\ p) &= \widetilde{S}(D_x^*(p),\ D_y^*(p,\ E)) \\ &= \beta D_x^* - \frac{\alpha}{2} D_x^{*\,2} + D_y^* \\ &= V(D_x^*) + D_y^*. \qquad (47)\end{aligned}$$

Thus, the welfare analysis presented here is equivalent to assuming the following social utility function; and let its value as a measure of social welfare.[13)]

$$S(D_x,\ D_y) = V(D_x) + D_y.$$

Totally differentiating (47) with respect to $A$, we obtain

$$\frac{\mathrm{d}\widetilde{S}(E,\ p)}{\mathrm{d}A} = -D_x^* \frac{\mathrm{d}p^*}{\mathrm{d}A} + n\frac{\mathrm{d}\pi^*}{\mathrm{d}A} > 0, \qquad (48)$$

and this result yields the following proposition.

**Proposition 6. (the social welfare):** *An increase in A increases the social welfare, defined by a social utility function.*

In (48), the first term in the right-hand side stands for the 'price-drop effect' and the second term represents the 'pro fit-increase effect'; both of them contribute to increase the social welfare.

### 1. Discussion: Change in social inequality

The central concern of this paper is the linkage between urban technological progress and income distribution among workers.[14)] In order to assess the social inequality of our model economy, we employ a graphical analysis of the Lorenz curve.[15)]

Figure 1 draws the initial Lorenz curve before an exogenous urban technological progress occurs. Since our model economy consists of three different income classes: the urban workers (i.e., the richest class), the rural workers (i.e., the middle class), and the unemployed people in the urban sector (i.e., the poorest class), the Lorenz curve is piecewise linear with two kinks. Start from the origin, the first (horizontal) segment (named Segment 1) is based on the income (zero) of the unemployed; the second segment (named Segment 2) on the income of the rural workers; and the third segment (named Segment 3) on the income of the urban workers. The slope of the each segment of this Lorenz curve is given by the ratio of the wage-income share of corresponding income class to the population share of that class. Therefore, the slope of Segment 2 is

$$\frac{w_r L_y/\Omega}{L_y/\overline{L}}, \tag{49}$$

and it can be rewritten as *unity*. (Recall that $\overline{L} = \Omega$ holds in equilibrium.) Thus, Segment 2 is parallel to the perfect equality line (the diagonal line in Figure 1). Similarly, the slope of Segment 3 is

$$\frac{wL_x/\Omega}{L_x/\overline{L}}, \tag{50}$$

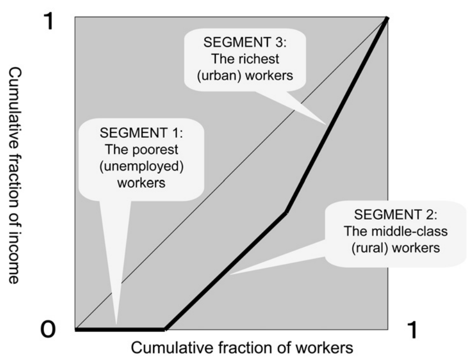


Figure 1. Initial Lorenz curve

and it can be rewritten as $w$ $(>1)$.

Utilizing the results of the comparative statics presented in (34)–(41), Figures 2 and 3 show alternative shifts in the Lorenz curve due to an urban technological progress. The main difference between those two figures is the directions of the changes in the urban employment; Figure 2 corresponds to Case 1 in (35), while Figure 3 corresponds to Case 2 in (35). In both of Case 1 and 2, new Lorenz curve (after change) must meet all of the following conditions. The slope of Segment 3 must increase, whereas that of Segment 2 remains unchanged;

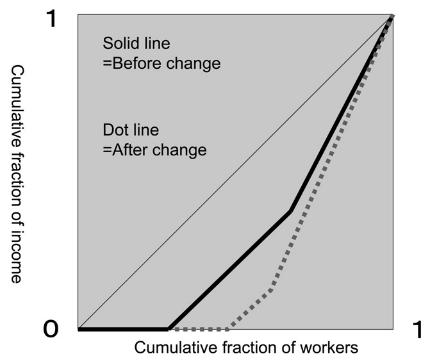


Figure 2. A shift under Case 1

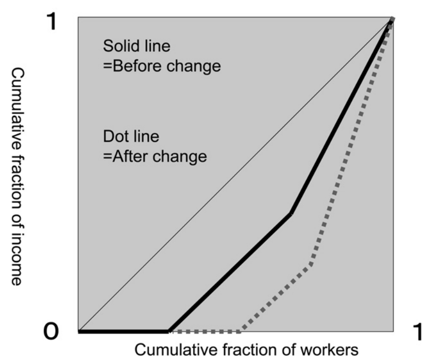


Figure 3. A shift under Case 2

the population share of unemployed workers must increase, while that of the rural workers must decrease; the wage-income share of the urban workers must increase, whereas that of the rural workers must decrease. The dotted lines depicted in Figures 2 and 3 are representative (though rather tentative) lines that meet all of these conditions.

The most important feature shown in Figures 2 and 3 is that the Lorenz curve shifts outwards in its entire range in the both figures. Thus, we can immediately state the following proposition.

**Proposition 7. (the social inequality):** *An increase in $A$ always increases the social inequality, measured by shifts in the Lorenz curve.*

## Ⅳ. Concluding remarks

In the recent globalized world economy, several newly-industrialized countries, such as China and India, experienced the rapid pace of economic growth. But, technological progress in such countries tends to occur in sector-specific manner. This paper examines the effects of sector-specific technological progress using a HT model with an oligopolistic urban sector.

Our main results of comparative statics are as follows. As the results on employment, an urban technological progress unambiguously increases urban unemployment and decreases the amount of rural employment. Meanwhile, the technological progress effect on urban employment is generally ambiguous. It may increase or decrease depending on the values of parameters, $A$ and $\beta$. As the results on urban firms, the urban technological progress increases their profit and decreases the price of their products. These results contribute to increase social welfare defined by a social utility function. Lastly, the technological progress unambiguously increases the social inequality.

However, we cannot overlook the fact that our clear results highly depend on our assumptions, which make our analysis very simple. First, we assume a quasi-liner social utility function. Thus, there is no income effect on the consumption of good $x$, which is produced by the oligopolistic urban firms. Introducing more general form of social utility function may bring further complex interdependences into the welfare analysis. Secondly, the simple linear forms are used for the inverse demand function of good $x$ and the production technologies of goods $x$ and $y$. These assumptions may be influential on our clear results. Lastly, the objective function of urban trade union is not the only formulation that we can consider. If we employ more general framework of the behavior of trade unions, we could gain more insight on our results.

## Notes

1 ) See, for example, International Labour Office (2007) for the recent global trends on unemployment. An increasing number of empirical studies suggest the rising wage inequality in the developing countries: See Feenstra and Hanson (1997), Robbins (1994, 1996), Wood (1994, 1999), Beyer et al. (1999), Robbins and Gindling (1999), and Arbache et al. (2004) among others.

2 ) The brief but thoughtful explanations of HT model can be found in well-established textbooks written by Ray (1998), and Bardhan and Udry (1999).

3 ) The basic assumptions of HT model are explained and used in the following section.

4 ) Note that there also exists a dual structure between formal and informal sectors within an urban area. The formal sector is characterized by advanced technology and high fringe benefit. In contrast, the informal sector is characterized by low productivities, and poor and hazardous environment. According to traditional concept of HT model, in this paper, the formal sector is represented by the employed urban workers, while the informal sector is represented by the unemployed people in the urban area.

5 ) For simplicity, the transportation costs of goods and labor are assumed as nil.

6 ) According to the literature on HT model, we call two sectors 'rural' and 'urban', respectively. However, as pointed out by Calvo (1978; p.66), more appropriate names given to the two sectors would be 'traditional' and 'modern', respectively—because we focus on the characteristic differences rather than geographic distance between these two sectors. Thus, the names 'rural' and 'urban' need not always be interpreted literally.

7 ) We do not explicitly consider the owners of the rural firms, because their profits are zero.

8 ) This is a well-known result caused by assuming a quasi-linear utility function, and it highly simplifies our model with two sectors and two goods.

9 ) We assume that $\beta$ is sufficiently large so as to assure $V'(d_x) = \beta - \alpha d_x > 0$ holds in the equilibrium we consider.

10) Note, however, that even if $A = 1$, it does not necessary mean that the technological gap disappears, because the produced goods are different.

11) Calvo (1978) construct two different scenarios regarding the interaction between urban trade union and urban firms. One is the 'monopolistic trade union' scenario, which is employed in our present analysis. And the other one is the 'Nash arbitrator' case, where an arbitrator intermediates between the union and firms.

12) Strictly speaking, $\Omega = w_r \overline{L}$ holds under the HT migration equilibrium in our model. (Note, however, that $w_r = 1$ always holds in the current model). If we modify the current model to allow $w_r$ to vary, the equilibrium value of $\Omega$ will also vary according to any change in $w_r$.

13) Assuming a social utility function is a very standard way in the literature on applied

economic analyses, such as international trade theory. See Wong (1995, Chapter 8) for further discussion on social utility function.

14) The analysis presented in this subsection does not consider the entrepreneurs.

15) Using the Lorenz curve, Bourguignon (1990) examines the link between growth and income distribution in a dual economy model. See also Temple (2005) in the same vein.

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(Graduate School of Economics, Nagoya University)