TOA UWB Position Estimation with Two Receivers and a Set of Known Reflectors

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***Abstract*—Ultra-Wideband is an attractive technology for short range positioning, especially indoors. However, for normal Time of Arrival(ToA) positioning, at least 3 receivers with unblocked direct path to the transmitter are required. A requirement that is not always met. In this work, a novel algorithm for ToA positioning using only 2 receivers is presented. This is possible by exploiting reflections from a set of known flat reflectors, for example ceiling and walls. The proposed algorithm was tested using self-developed UWB propagation simulator testbed.**

I. Introduction

One of the reasons why ultra-wideband (UWB) radio is considered a viable solution for indoor localization is the great time resolution of the signal, which translates into good ranging accuracy and good ability to distinguish different arriving multi-path components (MPCs). Research until now concentrated mostly on detection and usage of the MPC corresponding to the direct path propagation between transmitter and receiver, which in a line-of-sight (LoS) situation would be the first arriving MPC [1]. All but a few approaches totally ignore later arriving MPCs. Exceptions include use of channel delay spread for non line-of-sight (NLoS) situation (blocked direct path) detection in [2] and [3], as well as tracking a few later MPCs using Kalman filters to be used for positioning when the direct path is temporarily blocked in [4].

In this paper, we introduce a novel way of using later arriving MPCs. Having the knowledge of a set of big flat reflectors ($\mathbf{FR}$), applying geometric principle, we will use the information contained in known reflector-reflected MPCs. We will demonstrate the strength of this new approach with a novel positioning system using only 2 beacon receivers.

The knowledge of the reflectors can be either given a priori, or gained using a calibration step, with moving known-position transmitter used for reflector position estimation. This will be a future step in our research.

UWB positioning with 2 receivers is also possible using angle-of-arrival (AoA) measurements, [5]. Unfortunately, for UWB AoA measurements, a receiver with an antenna array is needed, making the beacon receiver bigger and more expensive.

The rest of the paper is organized as follows: In section II, we define the system elements and we present standard ToA positioning algorithm for 3 receivers, as a starting point for our own. We then introduce our own algorithm in section III, starting with position circle calculation in III-A. We follow that with the description of an algorithm for finding the position on the circle using known reflectors in III-B, which is the core of our positioning algorithm. We finally present the full algorithm in III-C. Next, in section IV we present layout and parameters for our simulation, results of which are shown and discussed in section V. Finally, we draw conclusions and present our ideas for future work in section VI.

II. System model

A. Definitions

Consider a system with a mobile UWB transmitter sending impulses that are received by a set of N stationary beacon receivers $\mathbf{R} = [R_1 \cdots R_N]$. Each receiver R_n is described by its 3D position vector, $[x_n\ y_n\ z_n]$. Signal at R_n is often represented in the literature as:

$$r_n(t) = \sum_{k_n=1}^{K_n} \alpha_{k_n} s(t - \tau_{k_n}) + n(t), \qquad (1)$$

where K_n is the number of MPCs, α_k and τ_k are the fading coefficient and delay of k_{th} MPC, respectively, $n(t)$ is zero-mean additive white Gaussian noise (AWGN), and $s(t)$ is the transmitted impulse shape. Subscript n defines to which receiver's received signal the parameter applies to.

A set of M known big, flat reflectors, $\mathbf{FR} = [FR_1 \cdots FR_M]$, is assumed. Each reflector FR_m is described by roughness $\sigma^2_{FR_m}$ and a 3D surface equation:

$$\begin{aligned} A_m x + B_m y + C_m z + D_m = 0 \\ \sqrt{A_m^2 + B_m^2 + C_m^2} = 1 \end{aligned} \qquad (2)$$

where $[A_m\ B_m\ C_m\ D_m]$ are normalized surface coefficients of reflector FR_m and $[x\ y\ z]$ are coordinates in 3D space.

We define the normal vector of FR_m surface as $FR_m^{norm} = [A_m\ B_m\ C_m]$. Roughness $\sigma^2_{FR_m}$ is the added path length variance caused by reflection from FR_m.

Mirror image of receiver R_n through flat reflector FR_m will be designated as R_n^m.

$$R_n^m = R_n - 2(R_n \cdot FR_m^{norm} + D_m) FR_m^{norm} \qquad (3)$$

Where R_n^m stands for receiver position vector $[x_n^m\ y_n^m\ z_n^m]$, R_n for $[x_n\ y_n\ z_n]$. R_n^m will be of use because a FR_m-reflected path between transmitter and receiver R_n can be represented as a direct path between transmitter and R_n^m.

All receivers R_n are assumed to be able to detect distinct MPCs, and produce a vector of measured MPC delays $[\hat{\tau}_{1_n} \dots \hat{\tau}_{J_n}]$, where $\hat{\tau}_{j_n} \sim N(\tau_{k_n}, \sigma^2_{\tau j_n})$. The number of detected MPCs, J_n, is smaller than the total number of MPCs, K_n.

N vectors of distance estimates $\mathbf{d_n} = [\hat{d}_{1_n} \dots \hat{d}_{J_n}]$ are calculated from $[\hat{\tau}_{1_n} \dots \hat{\tau}_{J_n}]$ using $\hat{d}_{j_n} = \hat{\tau}_{j_n} C$, where C is the speed of light, $\hat{d}_{j_n} \sim N(d_{j_n}, \sigma^2_{j_n})$. There will be a positive bias in $\hat{d}_{j_n}$ if the corresponding MPC path crossed any obstacle.

We assume that the MPCs corresponding to direct paths are detected, as well as most of MPCs corresponding to the $\mathbf{FR}$-reflected paths (ie. paths reflected by different FR_m). Under those assumptions, the distance of the first incoming MPC, $\hat{d}_{1_n}$ corresponds to the direct path distance between the transmitter and R_n. Also, the $\mathbf{d_n}$ vector will contain a subset corresponding to $\mathbf{FR}$-reflected path distances.

B. Standard ToA Positioning Algorithm

In the standard approach to positioning using ToA ranging, at least 3 receivers are needed. First, each receiver measures $\hat{d}_{1_n}$. Next, the distances $[\hat{d}_{1_1}\ \hat{d}_{1_2}\ \hat{d}_{1_3}]$ can be used to estimate the position of the transmitter, $[x_t\ \ y_t\ \ z_t]$ by solving the following set of quadratic equations:

$$\begin{cases} (x_t-x_1)^2+(y_t-y_1)^2+(z_t-z_1)^2 = \hat{d}^2_{1_1} \\ (x_t-x_2)^2+(y_t-y_2)^2+(z_t-z_2)^2 = \hat{d}^2_{1_2} \\ (x_t-x_3)^2+(y_t-y_3)^2+(z_t-z_3)^2 = \hat{d}^2_{1_3} \end{cases} \quad (4)$$

where $[x_n\ \ y_n\ \ z_n]$ is the position of the receiver R_n, $n = 1, 2, 3$.

There are usually two point solutions to this set of equations, corresponding to two transmitter position candidates (TPC). However, in practice, very often one of the candidates can be rejected basing either on the geometry of the building or the knowledge of previous positions of the transmitter. For example, if the receivers are mounted on the ceiling, one candidate will be over the ceiling.

III. Position Estimation with Two Receivers

A. Position Circle Calculation

If there are only two receivers available, the solution of (4) for 2 receivers:

$$\begin{cases} (x_t-x_1)^2+(y_t-y_1)^2+(z_t-z_1)^2 = \hat{d}^2_{1_1} \\ (x_t-x_2)^2+(y_t-y_2)^2+(z_t-z_2)^2 = \hat{d}^2_{1_2} \end{cases} \quad (5)$$

will usually be a circle with one degree of freedom. This result circle (RC) can be parametrically described as:

$$\begin{cases} x'_t = r_c \cos\alpha_t \\ y'_t = r_c \sin\alpha_t \\ z'_t = 0 \end{cases} \quad , \mathbf{x_t} = Q(\mathbf{x'_t}) \quad (6)$$

where $\mathbf{x'_t} = [x'_t\ y'_t\ z'_t]$ are possible transmitter positions in prime coordinates, on XY plane, r_c is result circle's radius, α_t is a free parameter, $\mathbf{x_t} = [x_t\ y_t\ z_t]$ are possible transmitter positions in real coordinates and $Q()$ is a translation plus rotation transform that moves the result circle from prime coordinates to natural coordinates.

Point's position on the RC circle is described by α_t.

To find the transmitter's position, more information about it is needed. One of the solutions, proposed in [6], is to assume the value of height coordinate. This method works as long as the assumption is reasonably correct. The assumption is usually based on previous position estimates or knowledge of the possible positions of transmitters.

In our approach, detailed in the next section, we start with the result circle (6), and find a point on the circle that would produce MPC patterns fitting best with measured MPC patterns $\mathbf{d_n}, n \in [1, N]$ and known reflectors $\mathbf{FR}$.

B. Proposed Position on Circle Calculation Algorithm

We assume that the transmitter position lies on (or near to) the result circle (6). This is a reasonable assumption because the direct path length measurements, used for calculating the result circle, are the most reliable measurements available.

For each receiver and reflector pair $(R_n, FR_m) \in \mathbf{R} \times \mathbf{FR}$, we assume that an MPC corresponding to a FR_m-reflected path from transmitter to R_n is present in the received signal. FR_m-reflected path to R_n can be represented as a direct path to R_n^m, as discussed in Section II-A. If $\hat{d}_{j_n}$ was the distance associated with that MPC, then the transmitter position would be a solution of:

$$\begin{cases} x'_t = r_c \cos\alpha_t \quad , y'_t = r_c \sin\alpha_t \quad , z'_t = 0 \\ (x'_t - x_n^{m\prime})^2 + (y'_t - y_n^{m\prime})^2 + (z'_t - z_n^{m\prime})^2 = \hat{d}^2_{j_n} \\ R_n^{m\prime} = Q^{-1}(R_n^m) \quad , \mathbf{x_t} = Q(\mathbf{x'_t}) \end{cases} \quad (7)$$

where the first three equations are result circle (6) equations in prime coordinates and the fourth equation is a sphere representing distance of d_{j_n} from $R_n^{m\prime}$, R_n^m translated into prime coordinates. The resulting $\mathbf{x'_t}$ has to be translated back into natural coordinates. In a non-degenerate case, this set of equations will have 0,1 or 2 solutions, corresponding to the same number of transmitter position candidates (TPCs).

It is not known which MPC corresponds to FR_m reflection, but it is possible to create a set of all possible TPCs, by assuming each MPC in turn to be the FR_m-reflected MPC. We therefore solve (7) with $j_n \in [1, J_n]$ and compile a set of all solutions for all j_n, $\mathbf{TPC^{(n,m)}}$. $\mathbf{TPC^{(n,m)}}$ will contain P TPCs, $TPC_p^{(n,m)}$. $TPC_p^{(n,m)}$ can be equivalently described either by its position in real space $[x_p^{(n,m)}\, y_p^{(n,m)}\, z_p^{(n,m)}]$, or by result circle RC and angle $\alpha_p^{(n,m)}$. Later, mostly the second description will be used. If the borders of the service area of the system are known, $TPC_p^{(n,m)}$ that are outside can be removed from the list.

In order to account for the error of $\hat{d}_{j_n}$, the variance of

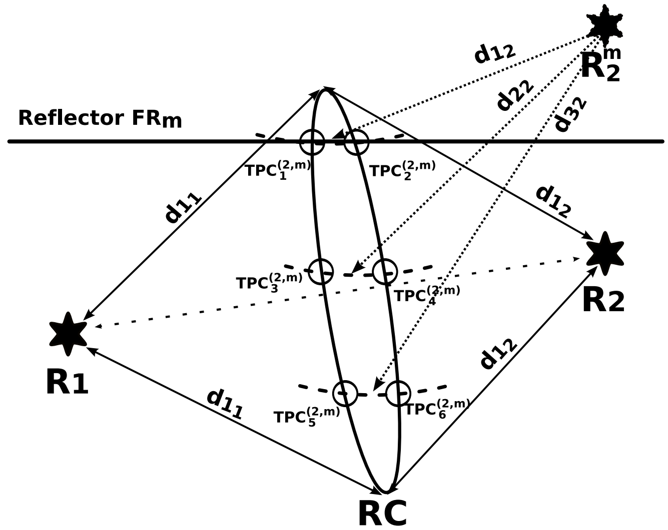

Fig. 1. A drawing illustrating (7) and creation of $\mathbf{TPC^{(n,m)}}$ vector

$\alpha_p^{(n,m)}$ for each TPC is calculated:

$$\begin{aligned} {\sigma_p^{(n,m)}}^2 &= \left(\frac{\delta \alpha_p^{(n,m)}}{\mathrm{d} d_{j_n}}\right)^2 (\sigma_{d_j}^2 + \sigma_{FR_m}^2) = \frac{\sigma_{d_j}^2 + \sigma_{FR_m}^2}{r_c^2 \cos\beta^2} \\ \beta &= \angle(|R, TPC_p|, \perp|RC, TPC_p|) \end{aligned} \quad (8)$$

where $\sigma_{FR_m}^2$ is the roughness of FR_m, β is the angle between receiver to position candidate path $|R, TPC_p|$ and tangent line of result circle RC at point $TPC_p^{(n,m)}$, $\perp|RC, TPC_p|$. To avoid infinite variance when $cos\beta = 0$, an upper bound for ${\sigma_p^{(n,m)}}^2$ is set.

Having done the above-mentioned calculations for all $(R_n, FR_m) \in \mathbf{R} \times \mathbf{FR}$, the next step is to combine the results, to find the most probable transmitter position candidate. We are looking for α_t that would minimize the variance-weighted sum of squared errors to the closest calculated TPC, or, in other words, we want to minimize the following expression:

$$\hat{\alpha}_t = \min_{\alpha_t} \left(\sum_{(R_n, FR_m) \in \mathbf{R} \times \mathbf{FR}} \min_{p \in [1,P]} \frac{(\alpha_t - \alpha_p^{(n,m)})^2}{{\sigma_p^{(n,m)}}^2} \right) \quad (9)$$

The position corresponding to the calculated α_t and result circle RC is the result of the algorithm.

The expression (9) may have more than one minimum, or some local minima with values very close to minimum. Their number is dependent on environment's geometry and delay measurement errors. These minima can be sometimes resolved using knowledge about the environment or previous positions of the transmitter, but in general it is best to choose the positions of beacon receivers so that to reduce the chance of false minima appearing.

A proposed technique is to place receivers asymmetrically, at different distances from reflectors FR_m , to avoid ambiguities when there is a second reflector present, parallel to FR_m (for example, if FR_m is a wall, the opposite wall).

C. Proposed Two Receivers Positioning

We assume that:

1) Two beacon receivers R_1 and R_2 with known positions $[x_1\ y_1\ z_1]$ and $[x_2\ y_2\ z_2]$ are given.
2) Receivers are capable of ToA ranging of not only the first arriving MPC, but also some of the later arriving MPCs.
3) Service area, 3D space in which the transmitter is supposed to be, is defined.
4) A set of big flat reflectors $\mathbf{FR}$ in the service area is known. Their surfaces are described by (2) and their roughness by $\sigma_{FR_m}^2$.
5) For measurement points inside the service area, MPCs corresponding to the direct paths, as well as most of MPCs corresponding to the $\mathbf{FR}$-reflected paths, are detectable by the receivers.
6) There is no knowledge about previous transmitter positions or position probability density function.

Under those assumptions, we propose the following algorithm for finding the position of transmitter:

1) Perform ToA ranging to the transmitter at both receivers, obtaining two MPC delay vectors $[\hat{\tau}_{1_1} \dots \hat{\tau}_{J_1}]$ and $[\hat{\tau}_{1_2} \dots \hat{\tau}_{J_2}]$. Convert those vectors into distance vectors $[\hat{d}_{1_1} \dots \hat{d}_{J_1}]$ and $[\hat{d}_{1_2} \dots \hat{d}_{J_2}]$.
2) As described in Section III-A, solve (7) using $\hat{d}_{1_1}$ and $\hat{d}_{1_2}$. In the case of:
   - No solution – return:
   $$\mathbf{x_t} = \frac{R_1/\hat{d}_{1_1} + R_2/\hat{d}_{1_2}}{1/\hat{d}_{1_1} + 1/\hat{d}_{1_2}} \quad (10)$$
   - Point solution – return the solution as result, end.
   - Circle solution – Save solution as result circle RC, continue.
3) Using algorithm described in Section III-B, find the best transmitter position candidate. When constructing $\mathbf{TPC^{(n,m)}}$ disregard TPCs that are outside of the service area.
4) Return the best transmitter position candidate.

IV. Simulation Layout

In order to prove usability of the proposed algorithm, simulations were performed using our self-developed UWB propagation simulator. The service area was a sparsely furnished room, as presented on Fig. 2. The size of the room was 8 m by 6 m, with ceiling at 3 m. Ceiling, floor and the wall with the door were chosen as the set of reflectors $\mathbf{FR}$. Two receivers were attached to the back wall. All antennas were assumed to be ideal isotropic with 0dB gain.

We used 1ns Gaussian modulated sinusoidal pulse, as presented in section 1.3.2. of [7], for the test signal:

$$\begin{aligned} s(t) &= a e^{-(kt)^2} \cos(2\pi f_c t) \\ a &= 1, k = \frac{\pi\sqrt{2}}{1ns}, f_c = 4GHz \end{aligned} \quad (11)$$

where a is signal's amplitude, k is envelope shape coefficient, $k = 1/(\sqrt{2}\sigma)$, and f_c is sinusoidal pulse's central frequency.

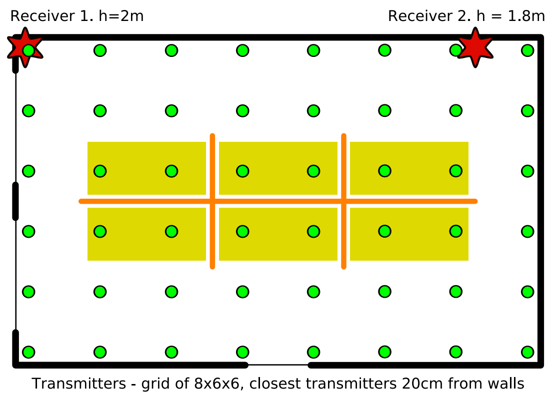

Fig. 2. Simulation Layout

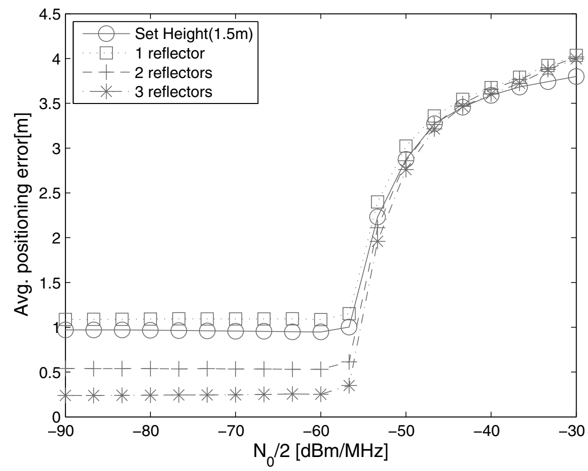

Fig. 3. Average localization error assuming height of 1.5m, or knowing position of 1,2 and 3 reflectors

The signal's power was set to $0dBm$, 100ns repetition interval was assumed.

CLEAN algorithm was used for MPC detection in received signals at each receiver R_n [8]. The algorithm can be summarized as (after [7]):

1) Calculate the autocorrelation of the template waveform $r_{ss}(t)$ and the cross-correlation of the template with the received waveform $r_{sr}(t)$.

$$r_{ss}(t) = \int_{-\infty}^{\infty} s(\tau)s(\tau - t)d\tau \quad (12)$$

$$r_{sr}(t) = \int_{-\infty}^{\infty} s(\tau)r(\tau - t)d\tau \quad (13)$$

2) Find the largest correlation peak in $r_{sr}(t)$, record the normalized amplitudes α_j and the relative time delay τ_j of the correlation peak.

$$\alpha_j = max(r_{sr}^j(t)), \qquad \tau_j = argmax(r_{sr}^j(t)) \quad (14)$$

where $r_{sr}^j(t)$ is the modified cross-corelation function used at j_{th} iteration of the algorithm. $r_{sr}^0(t) = r_{sr}(t)$.

3) Subtract $r_{ss}(t)$ scaled by α_j from $r_{sr}(t)$ at the time delay τ_j.

$$r_{sr}^{j+1}(t) = r_{sr}^j(t) - \alpha_j r_{ss}(t - \tau_j) \quad (15)$$

4) If a stopping criterion $(max(r_{sr}^{j+1}(t)) < threshold$, threshold set to 0.8875mV, which is 2.5% of expected direct path MPC signal amplitude at a distance of 1m) is not met, go to step 2. Otherwise, stop.

Vector of measured delays, $[\hat{\tau}_{1_n} \ldots \hat{\tau}_{J_n}]$, is the result of the algorithm, for each receiver R_n.

Simulation was performed for transmitter positions in a regular 8 by 6 by 6 grid, closest points being 20cm from each of the walls/ceiling/floor, as marked on Fig. 2. Simulations were repeated 100 times for each point and noise spectral density value.

V. Simulation Results

In Fig. 3, dependence of average localization error from noise spectral density $\frac{N_0}{2}$ is presented. The average is taken over all simulation points and localization error is counted as euclidean distance between detected and real position. Results are shown for the proposed algorithm with 1,2 and 3 reflectors known (ceiling, door wall, floor), as well as a simple set height algorithm, as presented in [6].

As can be seen from the graph, all positioning algorithms suffer a breakdown near -55dBm/MHz. This is because the same impulse detection algorithm is used and at -55dBm/MHz noise level, the noise starts causing false positive impulse detections. It has to be noted that, if the right minimum on the position circle is found, the positioning error is small, so rising average accuracy here reflects not refining of the position estimates, but elimination of gross positioning errors.

The algorithm works much better with bigger number of reflectors. Comparing to 1 reflector case, 2 reflectors and 3 reflectors give 2 and 4 times better average accuracy, respectively. The reason is similar to the noise averaging phenomena - in profiles contributed to (9) by each reflector, the minimum connected with the real position will always be there, but other minima will be, roughly, randomly distributed. That rises the chance that the minimum of the sum of all profiles will correspond to the true transmitter position.

It can be noticed that a simple assumption of height to be 1.5m gives similar accuracy with the proposed algorithm, 1 reflector case. This is caused by the tendency of the proposed algorithm to produce additional minimum in the profile near the reflector, corresponding with a case when direct and reflected MPCs overlap and are indistinguishable. This is a much lesser problem with more than one reflector.

Fig. 4 shows the dependence of average localization error on the transmitter's height, in the low noise case. As could be predicted, the set height algorithm works best near the assumed height, here 1.5m. The proposed algorithm has better accuracy

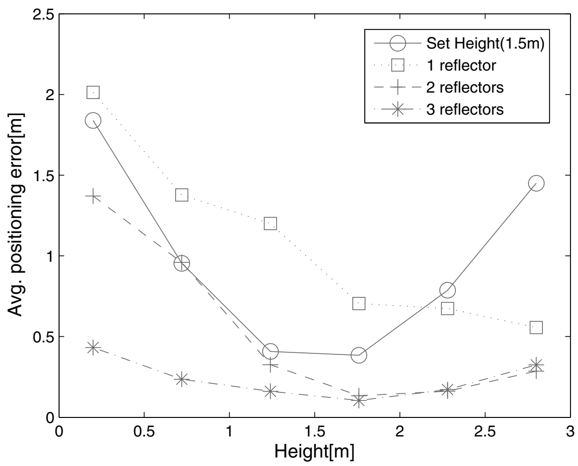

Fig. 4. Localization error dependence on transmitter's height

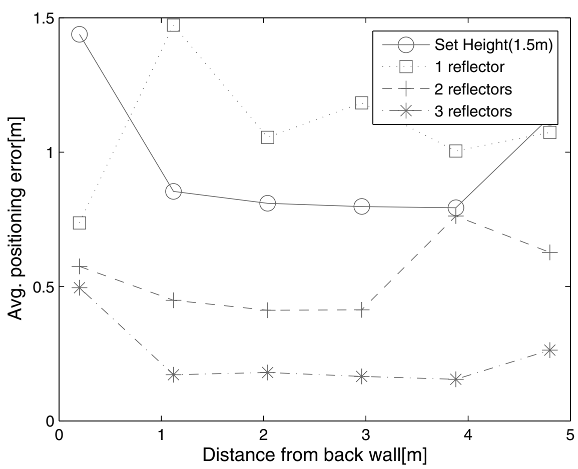

Fig. 5. Localization error dependence on distance to the back wall(with receivers)

for upper part of the room. This is caused by the absence of intervening furniture and closer distance to the receivers, which are mounted at heights of 1.8m and 2m.

Fig. 5 shows the dependence of average localization error on the distance from receiver's wall, in the low noise case. No clear dependence is observed. It is worth noting that, for the proposed algorithm with 3 reflectors, the biggest positioning error is for points very close to receivers, directly over or under.

It is caused in big part by MPCs reflected from the back wall arriving soon after the more direct MPCs, creating broader minima.

VI. Conclusion

In this paper we presented a TOA position estimation algorithm using 2 receivers and a set of known reflectors. Preliminary simulation results show that in most cases it can determine the transmitter's position with good accuracy. However, its dependence on detection of most of $\mathbf{FR}$-reflected MPCs in the received signal and minimization of a term that can have many local minima makes it often unreliable. It is best to be used as a backup scheme in a bigger localization system, for cases when only 2 receivers are reliably in the range of transmitter.

Our main goal for future is to improve upon our algorithm and extend it to bigger number of receivers in order to detect and allow positioning in NLoS propagation cases. We also want to make it more robust to $\mathbf{FR}$-reflected MPC non-detection. Also, by further developing our propagation simulator, we want to take into account impulse distortion caused by UWB antennas. Last but not least, in we plan a measurement campaign in August to provide real-world data for our positioning algorithm.

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