Maximum Torque Control with Inductance Setting of Extended EMF Observer

Takumi Ohnuma, Shinji Doki and Shigeru Okuma
Department of Electrical Engineering and Computer Science
Nagoya University
Aichi, Japan
Email: ohnuma@nagoya-u.jp, doki@nagoya-u.jp, okuma@nagoya-u.jp

***Abstract*—This paper presents maximum torque control with inductance settings in extended EMF observers for sensorless controls. The maximum torque control is defined on maximum torque control frame (f–t axes), whose f-axis is parallel to tangential lines of the constant torque curves in the d–q current axes. The angle between f–t axes and d–q axes denoted by ϕ is defined as the tangential angles of the constant torque curves. As position estimation error caused by inductance error can be calculated, the inductance is set so as to make the estimated position error equal to ϕ.**

Since the maximum torque control is realized simply by making f-axis current component zero, complicated calculation is not required. Moreover, the proposed control is robust against magnetic saturation. The validity of the proposed system is confirmed by experiments.

I. Introduction

Salient-pole synchronous motors are widely used in variable speed drive applications due to their high efficiency. It is important not only to accurately detect rotor positions but also to control current at optimum phases for high efficiency and wide range operations such as maximum torque control and flux weakening control [1]-[3]. Although rotor positions can be detected precisely with position sensors, mechanical position sensors have several problems such as cost and low reliability. Therefore, many sensorless control methods have been proposed [4], [5]. Authors have also proposed sensorless control techniques with observers based on extended electromotive force (EEMF) model [5]-[7].

In position sensorless controls, it is known that position estimation errors are sensitive to q-axis inductance set in the observers. Such parameter errors affect both position estimation and current vector phase accuracy. Position estimation errors caused by inductance errors have been analyzed [6]-[8].

One of the typical trajectories in a current phase control is the maximum torque per ampere (MTPA) control, which is also called maximum torque control [1]-[3]. It is important for high efficiency drives, because the reluctance torque can be used effectively the most and the copper losses are minimized. The conventional methods are that d-axis current commands for the current control loop are set to negative values at maximum torque per ampere trajectory. It can be obtained by solving an extremal problem that the motor torque is maximized with respect to the current phase angle at a constant current amplitude [2].

By utilizing these relations, unified methods for position estimation and current phase control with inductance setting have been reported [8], [9]. The inductance setting which is set so as to associate the current vector with a quasi-optimal trajectory is presented in [8]. For example, the trajectory is located between unity power factor trajectory and minimum copper loss trajectory.

According to [9], the maximum torque control is realized with inductance setting in a modified EEMF model. The robustness against magnetic saturation has been experimentally pointed out. However, this approach is based on a particular EEMF model constructed on maximum torque control frame. Therefore considerable knowledge (ex. robustness) on EEMF model must be reconsidered.

Based on these concepts, this paper presents maximum torque control with inductance setting of normal EEMF observers. In case of normal EEMF observer, the relations between position estimation errors and parameter errors have been derived analytically in steady state [6], [7]. In addition, a phase angle of the maximum torque control frame which is equal to a current phase angle under the maximum torque control has also been derived [10]. In the proposed method, the inductance is set so as to make the estimated position error equal to the phase angle of the maximum torque control frame. Also the proposed method is simply constructed and robust against magnetic saturation. Based on this approach, the validity of the proposed method can be explained in the conventional frame work.

II. Principle of Maximum Torque Control

A. Position Estimation Errors by the Inductance Errors

An extended electromotive force (extended EMF) model [5] of synchronous motors on the stationary reference frame (α–β axes) is written as (1),(2).

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \{(R+pL_d)\boldsymbol{I}+\omega_{re}(L_q-L_d)\boldsymbol{J}\}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}+\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = \{\omega_{re}K_E-(L_q-L_d)(\omega_{re}i_d-\dot{i}_q\}\begin{bmatrix} -\sin\theta_{re} \\ \cos\theta_{re} \end{bmatrix} \quad (2)$$

Where, R is stator resistance, L_d,L_q are inductances of d,q axes, K_E is EMF constant, p and "$\dot{}$" represent time derivative,

ω_{re} is rotor angular frequency, θ_{re} is rotor position, respectively. The extended EMF in Eq.(2) is a vector to the direction of q-axis that is synchronous with a rotor. Rotor positions can be estimated from the phase information of extended EMF with the following disturbance observer.

From this model, a linear state equation is derived as follows:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{e}\end{bmatrix}=\begin{bmatrix}\boldsymbol{A_{11}} & \boldsymbol{A_{12}}\\ \boldsymbol{O} & \boldsymbol{A_{22}}\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{e}\end{bmatrix}+\begin{bmatrix}\boldsymbol{B_1}\\ \boldsymbol{O}\end{bmatrix}\boldsymbol{v}+\begin{bmatrix}\boldsymbol{O}\\ \boldsymbol{W}\end{bmatrix} \tag{3}$$

$$\begin{aligned}
\boldsymbol{i} &= [i_\alpha i_\beta]^T\\
\boldsymbol{v} &= [v_\alpha v_\beta]^T\\
\boldsymbol{e} &= [e_\alpha e_\beta]^T\\
\boldsymbol{A_{11}} &= -(R/L_d)\boldsymbol{I}-\{\omega_{re}(L_q-L_d)/L_d\}\boldsymbol{J}\\
\boldsymbol{A_{12}} &= -(1/L_d)\boldsymbol{I}\\
\boldsymbol{A_{22}} &= \omega_{re}\boldsymbol{J}\\
\boldsymbol{B_1} &= (1/L_d)\boldsymbol{I}\\
\boldsymbol{W} &= -(L_q-L_d)(\omega_{re}\dot{i}_d-\ddot{i}_q)\begin{bmatrix}-\sin\theta_{re}\\ \cos\theta_{re}\end{bmatrix}
\end{aligned} \tag{4}$$

In Eq.(4), $\boldsymbol{W}$ is ignored as modeling error ($\boldsymbol{W}\approx\boldsymbol{0}$) because $\boldsymbol{W}$ is transient term and cannot be represented as a linear state equation. The minimal order observer to estimate the extended EMF can be constructed based on Eq.(3).

$$\begin{aligned}
\dot{\hat{\boldsymbol{i}}} &= \tilde{\boldsymbol{A}}_{\boldsymbol{11}}\boldsymbol{i}+\tilde{\boldsymbol{A}}_{\boldsymbol{12}}\hat{\boldsymbol{e}}+\tilde{\boldsymbol{B}}_{\boldsymbol{1}}\boldsymbol{v}\\
\dot{\hat{\boldsymbol{e}}} &= \boldsymbol{A_{22}}\hat{\boldsymbol{e}}+\tilde{\boldsymbol{G}}\left(\dot{\hat{\boldsymbol{i}}}-\dot{\boldsymbol{i}}\right)
\end{aligned} \tag{5}$$

$$\begin{aligned}
\tilde{\boldsymbol{A}}_{\boldsymbol{11}} &= -(\tilde{R}/\tilde{L}_d)\boldsymbol{I}-\{\omega_{re}(\tilde{L}_q-\tilde{L}_d)/\tilde{L}_d\}\boldsymbol{J}\\
\tilde{\boldsymbol{A}}_{\boldsymbol{12}} &= -(1/\tilde{L}_d)\boldsymbol{I}\\
\tilde{\boldsymbol{B}}_{\boldsymbol{1}} &= (1/\tilde{L}_d)\boldsymbol{I}\\
\tilde{\boldsymbol{G}} &= \alpha\tilde{L}_d\boldsymbol{I}+(\omega_{re}-\beta)\tilde{L}_d\boldsymbol{J}
\end{aligned} \tag{6}$$

Where "ˆ" and "˜" represent estimated values in the observer and nominal values of the parameter respectively, and $\boldsymbol{G}$,α,β are gain and poles of the observer. The rotor position θ_{re} can be calculated by Eq.(7) as the phase angle of extended EMF vector from the β-axis.

$$\hat{\theta}_{re}=\tan^{-1}\left(\frac{-\hat{e}_\alpha}{\hat{e}_\beta}\right) \tag{7}$$

Here, we consider the case when q-axis inductance settings in the observer are different from the nominal values of the motor. Then steady state error occurs in the position estimation [6]. Assuming that motor inductance L_q varies due to magnetic saturation while nominal values of inductance $\tilde{L}_q$ set in the observer, motor inductance L_q is defined as Eq.(8)

$$L_q=\tilde{L}_q+\Delta L_q \tag{8}$$

That is, we suppose that the q-axis inductance of motor (L_q) varies ΔL_q from the nominal value ($\tilde{L}_q$). Then, the estimation error of extended EMF ($\varepsilon_{\Delta L}$) at steady state can be derived from Eq.(1) and Eq.(5) as follows:

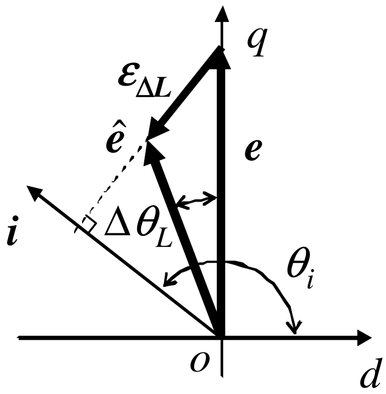

Fig. 1. Position estimation error caused by inductance errors.

$$\begin{aligned}
\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{\Delta L}} &= \dot{\hat{\boldsymbol{e}}}-\dot{\boldsymbol{e}}\\
&= (-\alpha\boldsymbol{I}+\beta\boldsymbol{J})\boldsymbol{\varepsilon}_{\boldsymbol{\Delta L}}+\{\alpha\boldsymbol{I}+(\omega_{re}-\beta)\boldsymbol{J}\}\omega_{re}\Delta L_q\boldsymbol{J}\boldsymbol{i}
\end{aligned} \tag{9}$$

In Eq.(9), if current is sinusoidal wave of fundamental component that angular frequency is ω_{re}, then the estimation error results Eq.(10).

$$\boldsymbol{\varepsilon}_{\boldsymbol{\Delta L}}=\omega_{re}\Delta L_q\boldsymbol{J}\boldsymbol{i} \tag{10}$$

From Fig.1, this extended EMF error vector generates the position estimation error $\Delta\theta_L$ shown in Eq.(11).

$$\Delta\theta_L=\tan^{-1}\left(\frac{\Delta L_q i_q}{K_E-(L_q-L_d)i_d+\Delta L_q i_d}\right) \tag{11}$$

Hence, the position estimation error by the inductance error can be evaluated by Eq.(11) with the motor parameters and d–q currents.

Note that, position estimation errors caused by d-axis inductance variation have been analyzed in the same way. According to the analysis, it has been shown that the steady state error does not depend on L_d [6], [8].

B. Maximum Torque Control Frame

The maximum torque control frame, which is denoted as f–t axes, is defined by a rotation angle ϕ from the d–q axes shown in Fig.2 [10]. The phase angle ϕ is derived from angles of tangential lines of the constant torque curves as follows.

The motor torque T is given by Eq.(12) as the cross product of armature current $\boldsymbol{i}=[i_d,i_q]^T$ and armature flux $\boldsymbol{\Psi_o}=[K_E+L_d i_d, L_q i_q]^T$.

$$T=P_n\{K_E-(L_q-L_d)\,i_d\}\,i_q \tag{12}$$

Where P_n is the number of pole pairs. Loci of the current vectors where the motor torque T is constant, called constant torque curves are given by the following hyperbola.

$$i_q=\frac{T}{P_n\{K_E-(L_q-L_d)\,i_d\}} \tag{13}$$

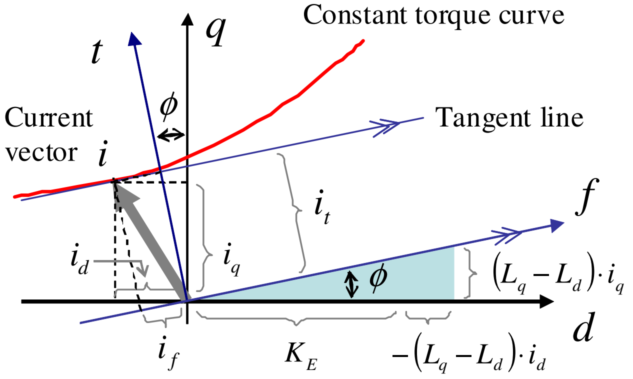

Fig. 2. Definition of maximum torque control frame.

The slopes of tangential lines along the constant torque curves are given by Eq.(14).

$$\begin{aligned}\frac{di_q}{di_d} &= \frac{T}{P_n}\frac{L_q - L_d}{\{K_E - (L_q - L_d)i_d\}^2} \\ &= \frac{(L_q - L_d)i_q}{K_E - (L_q - L_d)i_d} \end{aligned} \quad (14)$$

And the slopes can be denoted by the rotation angle from d–axis as Eq.(15).

$$\phi = \tan^{-1}\frac{(L_q - L_d)i_q}{K_E - (L_q - L_d)i_d} \quad (15)$$

Since the maximum torque control is realized simply by making f-axis current component zero, f–t axes are called maximum torque control frame [10].

C. Transformation of the Estimated Reference Frame with Inductance Settings

We now regard the inductance error as a setting value in order to assign the position estimation error to a desirable value. It has been discussed about the position estimation error $\Delta\theta_L$ caused by the inductance error in section II-A, and it also has been derived the phase angle of the maximum torque control frame ϕ in section II-B. In this section, the inductance is set so as to make the position estimation error equal to ϕ.

We suppose that the position estimation error $\Delta\theta_L$ in Eq.(11) is equal to the phase angle of the maximum torque control frame ϕ in Eq.(15).

$$\Delta\theta_L = \phi \quad (16)$$

This equation can be solved for the inductance error ΔL_q as Eq.(17).

$$\Delta L_q = \frac{(L_q - L_d)\{K_E - (L_q - L_d)i_d\}}{K_E - 2(L_q - L_d)i_d} \quad (17)$$

Here, we regard L_q in the observer as inductance settings $L_q^\dagger$, and substitute $\tilde{L}_q$ in Eq.(8) for $L_q^\dagger$. Then the inductance settings $L_q^\dagger = L_q - \Delta L_q$, which synchronize the estimated reference frame and the maximum torque control frame, is derived as Eq.(18).

$$L_q^\dagger = \frac{L_d K_E - (L_q^2 - L_d^2)i_d}{K_E - 2(L_q - L_d)i_d} \quad (18)$$

D. Inductance Settings for Maximum Torque Control

Equation (18) is given for i_d, besides under the maximum torque control the relation between i_d and i_q is given by Eq.(19)[2].

$$i_d = \frac{K_E - \sqrt{K_E^2 + 4(L_q - L_d)^2 i_q^2}}{2(L_q - L_d)} \quad (19)$$

Therefore, this is applied as the constrained condition and putting Eq.(19) into Eq.(18) gives inductance settings for maximum torque control as Eq.(20).

$$L_q^\dagger = L_0 + \frac{L_1 K_E}{\sqrt{K_E^2 + (4L_1 i_q)^2}} \quad (20)$$

The inductance L_0, L_1 is given in the following definition.

$$L_0 = \frac{L_d + L_q}{2} \quad (21)$$

$$L_1 = \frac{L_d - L_q}{2} \quad (22)$$

For the reasons mentioned above, the maximum torque control is realized simply by setting f-axis current component to zero and setting $L_q^\dagger$ determined based on Eq.(20) instead of L_q in the normal extended EMF observer.

As discuss in section III, although the exact value of $L_q^\dagger$ increases with i_q, decrease of L_q caused by magnetic saturation is canceled. Therefore the proposed control is robust against magnetic saturation.

III. Inductance Setting

A. Characteristics of Inductance Setting $L_q^\dagger$

Fig.3 shows the characteristics of the inductance setting $L_q^\dagger$ with respect to i_q for the test motor of TABLE I. As illustrated in Fig.3, $L_q^\dagger$ is equal to L_d at no load where i_q is equal to zero in Eq.(20). And then, $L_q^\dagger$ increases up to a limit of $L_0 = (L_d + L_q)/2$ with the load increasing. To sum up, $L_q^\dagger$ has the characteristics as follows:

$$L_q^\dagger = \begin{cases} L_d & ; i_q \to 0 \\ L_0 = \frac{L_d + L_q}{2} & ; i_q \to \infty \end{cases} \quad (23)$$

Therefore $L_q^\dagger$ varies greatly with respect to q-axis current for high salient-pole machines. On the other hand $L_q^\dagger$ is nearly constant for low salient-pole machines.

TABLE I
PARAMETERS OF MOTOR

d-axis inductance	4.15 [mH]
q-axis inductance	16.74 [mH]
EMF constant	0.104 [V·s/rad]
Rated current	5 [A]
Rated speed	2500[r/min]

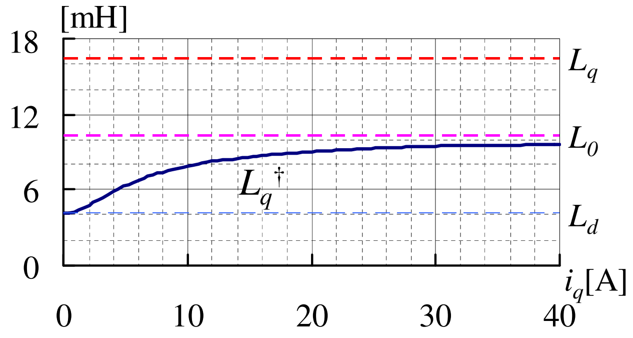

Fig. 3. Inductance – current ($L_q^\dagger$–i_q) characteristics.

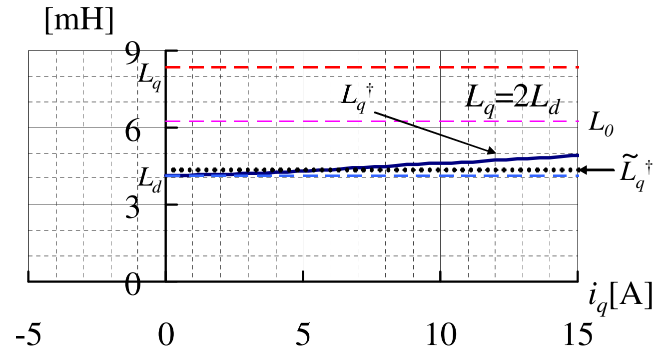

Fig. 4. Inductance setting for low saliency without magnetic saturation.

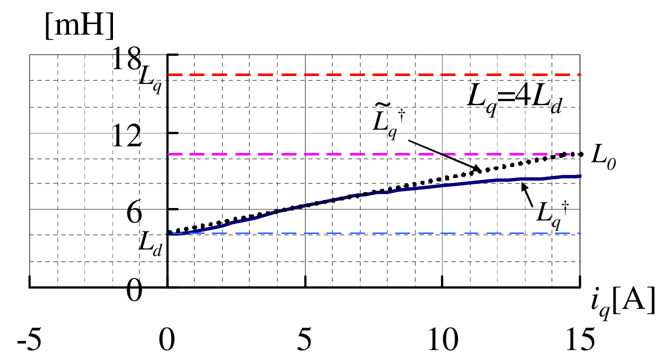

Fig. 5. Inductance setting for high saliency without magnetic saturation.

B. Inductance Setting Method with Low Magnetic Saturation

Since the exact value of $L_q^\dagger$ varies with current, the maximum torque control is realized without approximation by setting the inductance table measured for each current. It is as troublesome as usual maximum torque control system in the meaning of necessity to have some table with some current in the system.

In the proposed method, the maximum torque control can be realized with an approximate value $\tilde{L}_q^\dagger$ because of the properties discussed in section III-A. In the case of low salient-pole machines, for example, the approximate value $\tilde{L}_q^\dagger$ can be set to the constant value of $L_q^\dagger$ at full load (i_q=5A) shown in Fig.4. In the case of high salient-pole machines, the inductance table can be reduced by using approximate value $\tilde{L}_q^\dagger$ which is set to L_d at no load and interpolated between the values at no load and full load, and then limit to L_0 as shown in Fig.5.

C. Inductance Setting Method with High Magnetic Saturation

It is known that the inductance of salient-pole synchronous motors varies due to the magnetic saturation as follows:

- L_q saturates stronger than L_d.
- L_q decreases with the current increasing.
- The relation between d and q inductance keeps negative saliency.($L_q \geq L_d$)

These assumptions may not always be valid, however, these are general properties for salient-pole machines. The robustness of the proposed method against magnetic saturation is demonstrated with these properties of motor inductance L_q and inductance setting $L_q^\dagger$ discussed in section III-A.

The inductance setting $L_q^\dagger$ is equal to L_d at no load and increases up to a limit of L_0 with the load increasing. On the contrary, q-axis inductance L_q decreases due to the magnetic saturation. Therefore, the convergent value L_0 decreases as results of the saturation of L_q. As a result, the parameter $L_q^\dagger$ in the maximum torque control is nearly constant in heavy load. It is not always true that L_q decreases so as to make $L_q^\dagger$ equal to constant, however, it is always true that the lower and upper limits are so close as to put $L_q^\dagger$ between them.

From the relation above, L_d and L_q are measured in order to take the magnetic saturation into account at first, and then $L_q^\dagger$ can be set to an approximate value $\tilde{L}_q^\dagger$ which is constant calculated at full load.

IV. EXPERIMENTAL RESULT

A. Inductance Measurement

The q-axis inductance of the test motor is measured with the inductance measurement method that makes sensorless position error zero [6]. In this method, estimated position is used to adjust L_q in extended EMF observers while a motor is controlled at certain speed with sensor.

The motor speed is controlled at 1000[r/min] here so as not to affect with errors of resistance and output voltage. The current references are set to the operating points of maximum torque control calculated by Eq.(19) with the parameters of nominal values shown in Table I.

Figure 6 shows measurement results of q-axis inductance L_q and inductance setting $L_q^\dagger$ calculated by Eq.(20). The results illustrates that the variation of L_q due to magnetic saturation is not so strong in spite of high saliency. As a result, it is just hard condition for our proposed methods, that $L_q^\dagger$ varies with i_q quite much.

B. Maximum Torque Control with Inductance Setting

The sensorless maximum torque control is performed to confirm the effectiveness of the proposed method. The motor

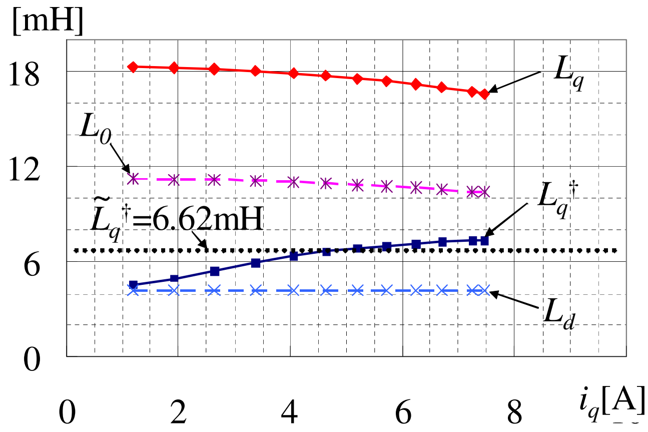

Fig. 6. Measurement results of L_q and $L_q^{\dagger}$.

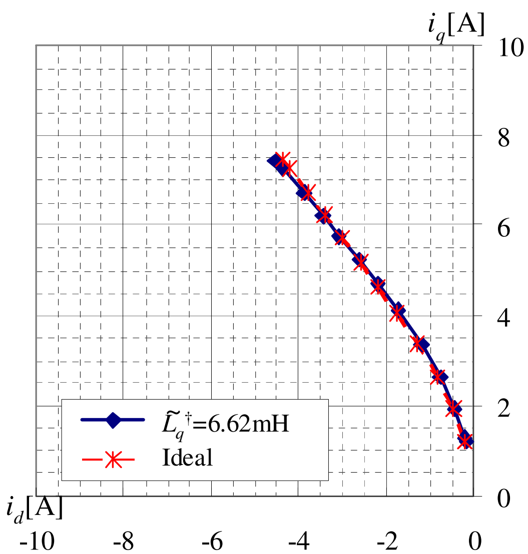

Fig. 7. Loci of the current vector on d–q axes.

is controlled with the estimated position from the extended EMF observer instead of the detected position from the sensor. The inductance setting $\tilde{L}_q^{\dagger}$ in the observer is set to the constant value of 6.62mH determined based on the inductance measurement at rated current in section IV-A. The current reference of a d-axis component is set to zero (i_d^*=0) for the maximum torque control. The d-q currents are calculated with the position sensor not to control the motor but to evaluate the errors. Figure 7 shows the vector loci of d-q currents. The ideal values are calculated with Eq.(19), which shows the relation between i_d and i_q under the maximum torque condition. In Eq.(19), the nominal values in Table I are used for L_d, K_E and the measurement values in Fig.6 are used for L_q. The result indicates that the current vector locus with the proposed method is equal to the ideal one.

Next, Fig.8 shows the phase errors between the position

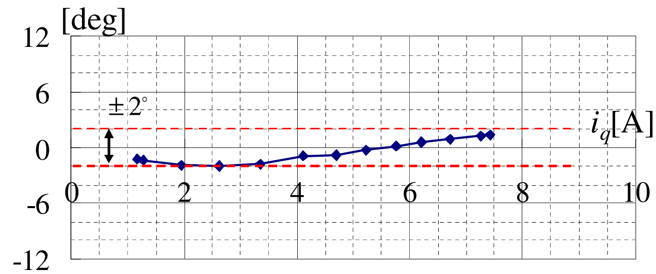

Fig. 8. Difference between $\Delta\theta_L$ and ϕ.

estimation error $\Delta\theta_{re}$ and the phase angle of the maximum torque control frame ϕ. These phase angles are supposed to be equal to each other in the proposed method. The phase angle of the maximum torque control frame ϕ is calculated from Eq.(15) where the nominal values are used for L_d, K_E and the measurement values in Fig.6 are used for L_q as with the calculations of the vector locus. The phase errors are within ±2 degrees. This errors are caused by the approximation of $L_q^{\dagger}$ which is treated as constant. Because the phase errors at light load have no serious effect in terms of the efficiency, it is reasonable to set $L_q^{\dagger}$ at the rated point for the approximate value $L_q^{\dagger}$ as used here. In addition, the approximation errors are not magnified so as to cancel the decreases of L_q with the increases of $L_q^{\dagger}$ even if the current goes up to overload region.

V. Conclusion

This paper presented maximum torque control only with extended EMF observers and general current control systems. In the proposed method, the q-axis inductances in the extended EMF observer for the sensorless control are set to the different values from the motor one so as to make the estimated reference frame equal to maximum torque control frame. The proposed system dose not explicitly have maximum torque controller, so the complicated calculation is not required. Moreover, the proposed control is robust against magnetic saturation. The effectiveness of the proposed method has been confirmed by experimental results.

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