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Time-varying path following control for port-Hamiltonian systems

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Abstract— **This paper is devoted to path following control for port-Hamiltonian systems whose desired path is time-varying. Most of the existing results on path following can only take care of time invariant paths, hence they cannot be applied to control systems whose environments change, e.g., path following control with moving obstacle avoidance or with a path crossing itself. The proposed method solves this problem by employing decoupling control of three particular directions in the phase space which allows one to assign time-varying potential functions and vector fields.**

I. INTRODUCTION

Path following control which is to make the system track the desired path is an important task for control of mechanical systems. Most of the existing results for this problem use the distance between the current state and its desired path. However, it is difficult to measure the smallest distance (between the current state and the desired path) for complicated desired paths. Several methods not using the distance directly are proposed to overcome this problem. Salisbury [1] and Hogan [2] proposed a method employing virtual potential function which take its minimum value on the desired path. Li et al. [3], [4] proposed a method called passive velocity field control (PVFC) to design vector fields to track a desired path directly. This method employs a virtual potential energy like function but it does not have intuitive meaning to the control system. Inspired by the idea of PVFC, Duindam et al. [5], [6] proposed a method to design vector fields directly with a natural potential function which takes its minimum value on the desired path. The authors proposed a method [7] generalizing the results [5], [6]. We re-formulated the existing results to be applicable to port-Hamiltonian systems and derive a path following method applicable to a wider class of systems. Although the system tracks a fixed desired path in this method, it is not applicable for the desired path crossing itself, e.g., a figure of eight.

In the present paper, the result [7] is further developed to make the system track time-varying desired paths. In this method, the system can track a fixed path crossing itself, such as a figure of eight, by regarding it as a time-varying desired path identical to the fixed path near the current state. In order to construct time-varying potential functions and vector fields, we introduce a region moving according to the current state along the time-varying desired path. This region includes a part of the time-varying desired path. By controlling the system to be included in this region for all time, the system will track the time-varying desired path. Furthermore, if there exist moving obstacles around the desired path, we can avoid collisions between the system and them by choosing the potential energy large near them.

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II. PORT-HAMILTONIAN SYSTEMS

Let us consider the following port-Hamiltonian system [8], [9].

$$\begin{pmatrix}\dot q\\ \dot p\end{pmatrix}=\Bigg\{\underbrace{\begin{pmatrix}0 & J_{12}(q)\\ -J_{12}(q)^{\mathrm T} & J_{22}(q,p)\end{pmatrix}}_{J}-\underbrace{\begin{pmatrix}0 & R_{12}(q)\\ R_{12}(q)^{\mathrm T} & R_{22}(q,p)\end{pmatrix}}_{R}\Bigg\}\times\begin{pmatrix}\frac{\partial H}{\partial q}^{\mathrm T}\\ \frac{\partial H}{\partial p}^{\mathrm T}\end{pmatrix}+\begin{pmatrix}0\\ G(q)\end{pmatrix}u$$

$$H(q,p)=\frac{1}{2}p^{\mathrm T}M(q)^{-1}p \tag{1}$$

Here $x=(q,p)\in\mathbb{R}^l\times\mathbb{R}^m (l\geq m)$. The Hamiltonian function $H(q,p)\in\mathbb{R}$ describes the kinetic energy of the system. The symmetric semi-positive definite matrix $R(q,p)\in\mathbb{R}^{(l+m)\times(l+m)}$ describes the energy dissipation. It is supposed that the matrix $G(q)\in\mathbb{R}^{m\times m}$ is nonsingular, that the matrix $J_{12}(q)\in\mathbb{R}^{l\times m}$ is column full rank that the matrix $J_{22}(q,p)\in\mathbb{R}^{m\times m}$ is skew-symmetric, that the matrix $J(q,p)\in\mathbb{R}^{(l+m)\times(l+m)}$ is skew-symmetric, and that the matrices $J(q,p),R(q,p)\in\mathbb{R}^{m\times m}$ and $M(q)^{-1}\in\mathbb{R}^{m\times m}$ are continuous. This dynamics is a generalized version of conventional mechanical systems. It can describe a mechanical system with a class of nonholonomic constraints with respect to the velocity $\dot q$. In such a case, the matrices $J(q,p)$ is determined by the constraints.

In this paper, the inner product on the phase space is defined by

$$\langle p_u,p_v\rangle := p_u^{\mathrm T}M(q)^{-1}p_v \tag{2}$$

for $p_u,p_v\in\mathbb{R}^m$. Accordingly, the norm is defined by

$$\|p_u\| := \sqrt{\langle p_u,p_u\rangle}\geq 0. \tag{3}$$

A scalar valued function $\mathrm{sgn}(x)\in\mathbb{R}$ returns the sign of the argument $x\in\mathbb{R}$ as

$$\mathrm{sgn}(x) := \begin{cases}1 & (x\geq 0)\\ -1 & (x<0)\end{cases}. \tag{4}$$

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III. Main results

This section gives the main result of the present paper path following control of port-Hamiltonian system (1). Here we deal in the desired path characterized by the configuration state q and time t. Let us consider a potential function $U_1(q,t)$ which takes its minimum value on the desired path for all time t. More precisely, the potential function $U_1(q,t)$ is chosen in such a way that the following assumption holds.

Assumption 1: The scalar function $U_1 \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R})$ satisfies the following conditions.

- $U_1(q,t) \geq 0$.
- $U_1(q,t)$ takes its minimum value 0 if and only if q is on the desired path.

We call the value of this potential function $U_1(q,t)$ potential energy. Furthermore, let us define the desired vector $p_w(q,t)$ and the slope vector $p_{\hat{w}}(q,t)$ on the phase space.

Assumption 2: The vector valued function $p_w \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R}^m)$ satisfies the following conditions.

- $\langle (J_{12}(q)^{\mathrm{T}} - R_{12}(q)^{\mathrm{T}})(\partial U_1/\partial q)^{\mathrm{T}}, p_w(q,t)\rangle = 0$.

Assumption 3: The vector valued function $p_{\hat{w}} \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R}^m)$ satisfies the following conditions.

- There exists a scalar $k_1(q,t)$ satisfying $p_{\hat{w}}(q,t) = k_1(q,t)(J_{12}(q)^{\mathrm{T}} - R_{12}(q)^{\mathrm{T}})(\partial U_1/\partial q)^{\mathrm{T}}$.

Let $p_{we}(q,t)$ and $p_{\hat{w}e}(q,t)$ denote the normalized version of $p_w(q,t)$ and $p_{\hat{w}}(q,t)$, respectively, for $\|p_w(q,t)\| \neq 0$ and $p_{\hat{w}}(q,t) \neq 0$, namely,

$$
\begin{aligned}
p_{we}(q,t) :=& \frac{p_w(q,t)}{\|p_w(q,t)\|}\\
p_{\hat{w}e}(q,t) :=& \frac{p_{\hat{w}}(q,t)}{\|p_{\hat{w}}(q,t)\|}.
\end{aligned}
$$

Due to Assumptions 3, $\langle p_{\hat{w}e}, p_{we}\rangle = 0$.

Next let us decompose p into three elements: one is linearly dependent on $p_{we}(q,t)$, one is linearly dependent on $p_{\hat{w}e}(q,t)$ and the other is orthogonal to $p_{we}(q,t)$ and $p_{\hat{w}e}(q,t)$ which is denoted by $p_{\tilde{w}}(q,p,t)$. That is, p is decomposed as

$$
\begin{aligned}
&p = \alpha_w(q,p,t)\ p_{we}(q,t) + \alpha_{\hat{w}}(q,p,t)\ p_{\hat{w}e}(q,t) + p_{\tilde{w}}(q,p,t),\\
&\langle p_{we}(q,t), p_{\tilde{w}}(q,p,t)\rangle = \langle p_{\hat{w}e}(q,t), p_{\tilde{w}}(q,p,t)\rangle = 0 \qquad (5)
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_w(q,p,t) :=& \langle p_{we}(q,t), p\rangle\\
\alpha_{\hat{w}}(q,p,t) :=& \langle p_{\hat{w}e}(q,t), p\rangle\\
p_{\tilde{w}}(q,p,t) :=& p - \alpha_w(q,p,t)p_{we}(q,t) - \alpha_{\hat{w}}(q,p,t)p_{\hat{w}e}(q,t).
\end{aligned}
$$

According to the decomposition (5), we can decompose the Hamiltonian function as

$$
\begin{aligned}
H(q,p) &= H_{k,w}(q,p,t) + H_{k,\hat{w}}(q,p,t) + H_{k,\tilde{w}}(q,p,t)\\
H_{k,w}(q,p,t) &= \frac{1}{2}\alpha_w(q,p,t)^2\\
H_{k,\hat{w}}(q,p,t) &= \frac{1}{2}\alpha_{\hat{w}}(q,p,t)^2\\
H_{k,\tilde{w}}(q,p,t) &= \frac{1}{2}\langle p_{\tilde{w}}(q,p,t), p_{\tilde{w}}(q,p,t)\rangle.
\end{aligned}
$$

Here $H_{k,w}(q,p,t)$, $H_{k,\hat{w}}(q,p,t)$ and $H_{k,\tilde{w}}(q,p,t)$ denote the kinetic energy with respect to the desired direction $p_{we}(q,t)$, that with respect to the slope one $p_{\hat{w}e}(q,t)$ and that with respect to the undesired one $p_{\tilde{w}}(q,p,t)$.

Due to Assumption 1, the system tracks the desired path if and only if the condition $U_1(q,t) = 0$ holds. If $p_{\hat{w}}(q,t) \neq 0$, then the time derivative of $U_1(q,t)$ along the port-Hamiltonian system (1) is calculated as

$$
\begin{aligned}
\frac{\mathrm{d}U_1}{\mathrm{d}t} =& \frac{\partial U_1}{\partial q}\dot{q} + \frac{\partial U_1}{\partial t}\\
=& \frac{\partial U_1}{\partial q}(J_{12} - R_{12})M^{-1}p + \frac{\partial U_1}{\partial t}\\
=& \left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right)\frac{\partial U_1}{\partial q}^{\mathrm{T}}, p\right\rangle + \frac{\partial U_1}{\partial t}\\
=& \alpha_{\hat{w}}\left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right)\frac{\partial U_1}{\partial q}^{\mathrm{T}}, p_{\hat{w}e}\right\rangle + \frac{\partial U_1}{\partial t}.
\end{aligned}
\qquad (6)
$$

Therefore if $\alpha_{\hat{w}} = 0$ and $\partial U_1/\partial t = 0$, then the potential energy U_1 dose not change. Hence if $U_1 = 0$, $\alpha_{\hat{w}} = 0$ and $\partial U_1/\partial t = 0$, then the system tracks the desired path. We will design a controller to make U_1 and $\alpha_{\hat{w}}$ converge to 0 but there are some problems. The system cannot track the desired path when $\partial U_1/\partial t \neq 0$. Moreover p_{we} is not defined when $p_w = 0$ and so is $p_{\hat{w}e}$. Consequently, let us consider a time-varying region $A(t) \subset \mathbb{R}^l$ not including those points.

Assumption 4: The region $A(t) \subset \mathbb{R}^l$ satisfies the following conditions for all time t.

- There exists a $q \in A(t)$ such that $U_1(q,t) = 0$.
- $p_w(q,t) \neq 0, p_{\hat{w}}(q,t) \neq 0$, $\forall q \in A(t)$.
- $\|\left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right)(\partial U_1/\partial q)^{\mathrm{T}}\| \neq 0$, $\forall q \in A(t)$ except on the desired path.
- There exists a neighborhood of q, $B(t) \subset \mathbb{R}^l$, for all q on the desired path such that $(\partial U_1/\partial t)/\|\left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right)(\partial U_1/\partial q)^{\mathrm{T}}\| = 0$, $\forall q \in A(t) \cap B(t)$.
- The initial condition $q(0) \in A(t)$.
- $A(t)$ is connected.
- Measure of $A(t)$ is not 0.

The fourth item of Assumption 4 is to prevent the input defined later from diverging, and this condition is satisfied if $\partial U_1/\partial t(q,t) = 0$ for all $q \in A(t)$. We want to design a controller to make q stay in $A(t)$ for all time t. For this purpose let us define another potential function $U_2(q,t)$ which becomes large enough near the boundary of $A(t)$.

Assumption 5: The scalar function $U_2 \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R})$ satisfies the following conditions for $\forall q \in A(t)$.

- $U_2(q,t) \geq 0$.
- $U_2(q,t) \gg 1$ near the boundary of $A(t)$.
- There exists a constant $k_2 > 0$ such that if $U_2(q,t) \ll 1$ then $U_2^{k_2}/\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right)(\partial U_2/\partial q)^{\mathrm{T}}, p_{we}\rangle \ll 1$

Due to the second item of Assumption 5, if $U_2(q,t)$ dose not diverge, then q is in $A(t)$ for all time t. The time derivative of $U_2(q,t)$ along the port-Hamiltonian system (1)

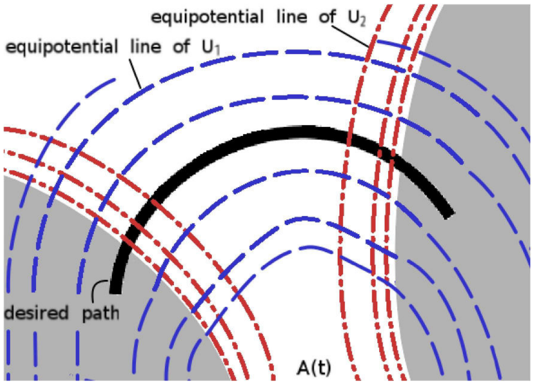

Fig. 1. Desired path and equipotential lines

is calculated as

$$\begin{aligned}\frac{\mathrm{d}U_2}{\mathrm{d}t} =& \frac{\partial U_2}{\partial q}\dot{q} + \frac{\partial U_2}{\partial t}\\ =& \frac{\partial U_2}{\partial q}\left(J_{12} - R_{12}\right) M^{-1} p + \frac{\partial U_2}{\partial t}\\ =& \left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial U_2}{\partial q}^{\mathrm{T}}, p \right\rangle + \frac{\partial U_2}{\partial t}\\ =& \alpha_w \left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial U_2}{\partial q}^{\mathrm{T}}, p_{we} \right\rangle\\ &+ \alpha_{\hat{w}} \left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial U_2}{\partial q}^{\mathrm{T}}, p_{\hat{w}e} \right\rangle\\ &+ \left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial U_2}{\partial q}^{\mathrm{T}}, p_{\tilde{w}} \right\rangle + \frac{\partial U_2}{\partial t}.\end{aligned} \tag{7}$$

Though we want to prevent $U_2(q,t)$ from diverging by controlling $\alpha_w(q,p,t)$, if $\langle (J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}) (\partial U_2/\partial q)^{\mathrm{T}}, p_{we}\rangle = 0$ then we cannot control $\mathrm{d}U_2/\mathrm{d}t$ by controlling $\alpha_w(q,p,t)$. In what follows, we consider a region close to the desired path which dose not include a region $\Big\{(q,t) | U_2(q,t) \neq 0, \langle (J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}) (\partial U_2/\partial q)^{\mathrm{T}}, p_{we}\rangle = 0\Big\}$ and semi-global stability of the desired path within this region is discussed.

Fig.1 shows the example of U_1, U_2 and A. The white region denotes the region A. The dashed (blue) line denotes the equipotential lines of U_1 which increases as the distance from the desired path increases. The dashed-dotted (red) one denotes that of U_2 which increases as the derivation from the boundary of A decreases.

In what follows, the following notations are used for the sake of simplicity: $d_q U_1(q,t) := \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) (\partial U_1/\partial q)^{\mathrm{T}}(q,t)$ and $d_q U_2(q,t) := \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) (\partial U_2/\partial q)^{\mathrm{T}}(q,t)$.

The path following controller is designed by four steps. In the first step, a controller called *nominal controller* is applied to keep $\alpha_w(q,p,t)$, $\alpha_{\hat{w}}(q,p,t)$ and $\|p_{\tilde{w}}(q,p,t)\|$ constant in order to decouple the behavior of them.

In the second step, another controller called *asymptotic controller* is added to make $p_{\tilde{w}}(q,p,t)$ converge to 0 while $\alpha_w(q,p,t)$ and $\alpha_{\hat{w}}(q,p,t)$ do not change.

In the third step, a controller called *gradient controller* is added to make $U_1(q,t)$ and $\alpha_{\hat{w}}(q,p,t)$ converge to 0 while $\alpha_w(q,t)$ and $\|p_{\tilde{w}}(q,p,t)\|$ do not change. Since the asymptotic controller reduces $H_{k,\tilde{w}}(q,p,t)$, $H_{k,\hat{w}}(q,p,t)$ and $U_1(q,t)$ down to 0, the condition $H(q,p) + U_1(q,t) = H_{k,w}(q,p,t)$ is achieved asymptotically while $H_{k,w}(q,p,t)$ is time invariant.

In the fourth step, a controller called *velocity controller* is added to make q stay in $A(t)$ for all time t. This controller reduces $U_2(q,t)$ by controlling α_w.

A. Nominal controller

This subsection gives the nominal controller. The objective of this controller is to make $\alpha_w(q,p,t)$, $\alpha_{\hat{w}}(q,p,t)$ and $\|p_{\tilde{w}}(q,p,t)\|$ constant in order to decouple their behavior.

Theorem 1: (i) Consider a port-Hamiltonian system (1) *with vectors $p_w(q,t)$ and $p_{\hat{w}}(q,t)$. Suppose that Assumptions 2 and 3 hold. Then $\alpha_w(q,p,t)$, $\alpha_{\hat{w}}(q,p,t)$ and $\|p_{\tilde{w}}(q,p,t)\|$ are constant along the state trajectory of the closed loop system derived by the nominal controller defined by*

$$\begin{aligned}u_n =& \langle p_{we}, p\rangle G^{-1}\left(\eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}\right)\\ &+ \langle p_{\hat{w}e}, p\rangle G^{-1}\left(\eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}\right)\\ &- \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\tilde{w}}\right\rangle G^{-1} p_{we}\\ &- \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{\tilde{w}}\right\rangle G^{-1} p_{\hat{w}e} + G^{-1}\bar{R}M^{-1}p.\end{aligned} \tag{8}$$

Here $\eta_{Jw}(q,p,t)$ and $\eta_{J\hat{w}}(q,p,t) \in \mathbb{R}^m$ are defined by

$$\begin{aligned}\eta_{Jw}(q,p,t) :=& \xi_{Jw}(q,p,t) - \bar{J}(q,p)M^{-1}p_{we}\\ \eta_{J\hat{w}}(q,p,t) :=& \xi_{J\hat{w}}(q,p,t) - \bar{J}(q,p)M^{-1}p_{\hat{w}e},\end{aligned}$$

skew-symmetric matrix $\bar{J}(q,p) \in \mathbb{R}^{(l+m)\times(l+m)}$ and symmetric matrix $\bar{R}(q,p) \in \mathbb{R}^{(l+m)\times(l+m)}$ are defined by

$$\bar{J} := \left(\frac{1}{2}M\begin{bmatrix}\frac{\partial M^{-1}}{\partial q_1}p & \cdots & \frac{\partial M^{-1}}{\partial q_l}p\end{bmatrix} \quad I\right) J \begin{pmatrix}\frac{1}{2}\begin{bmatrix}p^{\mathrm{T}}\frac{\partial M^{-1}}{\partial q_1}\\ \cdots\\ p^{\mathrm{T}}\frac{\partial M^{-1}}{\partial q_l}\end{bmatrix} M\\ I\end{pmatrix} \tag{9}$$

$$\bar{R} := \left(\frac{1}{2}M\begin{bmatrix}\frac{\partial M^{-1}}{\partial q_1}p & \cdots & \frac{\partial M^{-1}}{\partial q_l}p\end{bmatrix} \quad I\right) R \begin{pmatrix}\frac{1}{2}\begin{bmatrix}p^{\mathrm{T}}\frac{\partial M^{-1}}{\partial q_1}\\ \cdots\\ p^{\mathrm{T}}\frac{\partial M^{-1}}{\partial q_l}\end{bmatrix} M\\ I\end{pmatrix}, \tag{10}$$

and $\xi_{Jw}(q,p,t), \xi_{Rw}(q,p,t), \xi_{J\hat{w}}(q,p,t)$ and $\xi_{R\hat{w}}(q,p,t) \in$

$\mathbb{R}^m$ *are vectors defined by*

$$\xi_{Jw} := \left(\frac{\partial p_{we}}{\partial q} + \frac{1}{2}M\left[\tfrac{\partial M^{-1}}{\partial q_1}p_{we}\cdots\tfrac{\partial M^{-1}}{\partial q_l}p_{we}\right]\right)J_{12}M^{-1}p$$
$$\xi_{Rw} := -\left(\frac{\partial p_{we}}{\partial q} + \frac{1}{2}M\left[\tfrac{\partial M^{-1}}{\partial q_1}p_{we}\cdots\tfrac{\partial M^{-1}}{\partial q_l}p_{we}\right]\right)R_{12}M^{-1}p$$
$$\xi_{J\hat{w}} := \left(\frac{\partial p_{\hat{w}e}}{\partial q} + \frac{1}{2}M\left[\tfrac{\partial M^{-1}}{\partial q_1}p_{\hat{w}e}\cdots\tfrac{\partial M^{-1}}{\partial q_l}p_{\hat{w}e}\right]\right)J_{12}M^{-1}p$$
$$\xi_{R\hat{w}} := -\left(\frac{\partial p_{\hat{w}e}}{\partial q} + \frac{1}{2}M\left[\tfrac{\partial M^{-1}}{\partial q_1}p_{\hat{w}e}\cdots\tfrac{\partial M^{-1}}{\partial q_l}p_{\hat{w}e}\right]\right)R_{12}M^{-1}p.$$

(ii) $\xi_{Jw}, \xi_{Rw}, \xi_{J\hat{w}}, \xi_{R\hat{w}}, \eta_{Jw}$ *and* $\eta_{J\hat{w}}$ *satisfy the following equation*

$$\langle \xi_{Jw}, p_{we}\rangle = \langle \xi_{Rw}, p_{we}\rangle = \langle \xi_{J\hat{w}}, p_{\hat{w}e}\rangle = \langle \xi_{R\hat{w}}, p_{\hat{w}e}\rangle = 0$$
$$\langle \eta_{Jw}, p_{we}\rangle = \langle \eta_{J\hat{w}}, p_{\hat{w}e}\rangle = 0. \quad (11)$$

Proof: We prove only (i) due to space limitation. Let us consider a feedback system with the port-Hamiltonian system (1) with the feedback $u = u_n$. First, we prove that α_w is constant. The time derivative of α_w is calculated as

$$\begin{aligned}\frac{\mathrm{d}\alpha_w}{\mathrm{d}t} &= \left\langle Gu - \bar{R}M^{-1}p, p_{we}\right\rangle + \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\tilde{w}}\right\rangle \\ &\quad - \langle p_{\hat{w}e}, p\rangle \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{we}\right\rangle \\ &= \langle p_{we}, p\rangle \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{we}\right\rangle \\ &= 0. \end{aligned} \quad (12)$$

This proves that α_w is constant.

First, we prove that $\alpha_{\hat{w}}$ is constant. The time derivative of $\alpha_{\hat{w}}$ is calculated as

$$\begin{aligned}\frac{\mathrm{d}\alpha_{\hat{w}}}{\mathrm{d}t} &= \left\langle Gu - \bar{R}M^{-1}p, p_{we}\right\rangle + \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{\tilde{w}}\right\rangle \\ &\quad - \langle p_{we}, p\rangle \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\hat{w}e}\right\rangle \\ &= \langle p_{\hat{w}e}, p\rangle \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{\hat{w}e}\right\rangle \\ &= 0 \end{aligned} \quad (13)$$

This proves that $\alpha_{\hat{w}}$ is constant.

Next we prove that $\|p_{\tilde{w}}\|$ is constant. In order to prove that $p_{\tilde{w}}$ is constant, it is proven that $H_{k,\tilde{w}}$ is constant. The time derivative of $H_{k,\tilde{w}}$ along the closed loop system is calculated as

$$\begin{aligned}\frac{\mathrm{d}H_{k,\tilde{w}}}{\mathrm{d}t} &= \langle Gu - \bar{R}M^{-1}p, p_{\tilde{w}}\rangle \\ &\quad - \langle p_{we}, p\rangle \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\tilde{w}}\right\rangle \\ &\quad - \langle p_{\hat{w}e}, p\rangle \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{\tilde{w}}\right\rangle \\ &= 0 \end{aligned} \quad (14)$$

which implies that $H_{k,\tilde{w}}$ is constant. This also suggests that $\|p_{\tilde{w}}\| = \sqrt{2H_{k,\tilde{w}}}$ is constant as well. This completes the proof. ■

Let us consider a feedback system with the port-Hamiltonian system (1) with the feedback $u = u_n + \bar{u}$. The time derivative of $\alpha_w, \alpha_{\hat{w}}$ and $H_{k,\tilde{w}}$ along the closed loop system are calculated as

$$\frac{\mathrm{d}\alpha_w}{\mathrm{d}t} = \langle G\bar{u}, p_{we}\rangle \quad (15)$$
$$\frac{\mathrm{d}\alpha_{\hat{w}}}{\mathrm{d}t} = \langle G\bar{u}, p_{\hat{w}e}\rangle \quad (16)$$
$$\frac{\mathrm{d}H_{k,\tilde{w}}}{\mathrm{d}t} = \langle G\bar{u}, p_{\tilde{w}}\rangle\,. \quad (17)$$

If we choose $\bar{u} = k_3 G^{-1} p_{we}$ with a scalar function $k_3(q,p,t)$, then $\bar{u}$ changes α_w without changing $\alpha_{\hat{w}}$ and $H_{k,\tilde{w}}$ since $\langle p_{we}, p_{\hat{w}e}\rangle = \langle p_{we}, p_{\tilde{w}}\rangle = 0$. In the same way, we can change $\alpha_{\hat{w}}$ and $H_{k,\tilde{w}}$ without changing the others.

B. Path following

Let us introduce the asymptotic controller. The objective of this controller is to make $p_{\tilde{w}}(q,p,t)$ converge to 0. Let us define the asymptotic controller by

$$u_a = -\beta_a G^{-1} p_{\tilde{w}} \quad (18)$$

where a continuous function $\beta_a(q,p,t) > 0 \in \mathbb{R}$ is a design parameter.

Next let us introduce the gradient controller. The objective of this controller is to reduce the potential function U_1 to achieve the path following control. If $q \in A(t)$ then the time derivative of $U_1(q,t)$ along the port-Hamiltonian system (1) is calculated as

$$\frac{\mathrm{d}U_1}{\mathrm{d}t} = \langle d_q U_1, p\rangle + \frac{\partial U_1}{\partial t} = \alpha_{\hat{w}}\langle d_q U_1, p_{\hat{w}e}\rangle + \frac{\partial U_1}{\partial t}. \quad (19)$$

The gradient controller controls $\alpha_{\hat{w}}$ to reduce the potential function U_1. Let us define the desired value $\alpha_{\hat{w}r}(q,t) \in \mathbb{R}$ of $\alpha_{\hat{w}}$ that if $\alpha_{\hat{w}} = \alpha_{\hat{w}r}$ then U_1 decreases. The gradient controller makes $\alpha_{\hat{w}}$ converge to $\alpha_{\hat{w}r}$. More precisely, the desired value $\alpha_{\hat{w}r}(q,t)$ is chosen in such a way that the following assumption holds.

Assumption 6: The desired value $\alpha_{\hat{w}r} \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R})$ satisfies the following conditions for $(q,t) \in A(t) \times \mathbb{R}$.

- $\alpha_{\hat{w}r}(q,t) = 0$ if $U_1(q,t) = 0$.
- $\alpha_{\hat{w}r}(q,t)\mathrm{sgn}\langle d_q U_1, p_{\hat{w}e}\rangle < -(\partial U_1/\partial t)/\|d_q U_1\|$ if $U_1(q,t) > 0$.

For example, the following $\alpha_{\hat{w}r}(q,t)$ satisfies this assumption.

$$\alpha_{\hat{w}r} = \begin{cases} 0 & \text{when } U_1 = 0 \\ -\left(\gamma_p + \frac{\frac{\partial U_1}{\partial t}}{\|d_q U_1\|}\right)\mathrm{sgn}\langle d_q U_1, p_{\hat{w}e}\rangle & \text{when } U_1 > 0 \end{cases} \quad (20)$$

Here a continuous scalar function $\gamma_p(q,t) \geq 0 \in C^1$ is a design parameter which takes its minimum value 0 if and only if $U_1(q,t) = 0$.

Let us define the gradient controller by

$$u_p = -\beta_p\left(\alpha_{\hat{w}} - \alpha_{\hat{w}r}\right)G^{-1}p_{\hat{w}e} + \frac{\mathrm{d}\alpha_{\hat{w}r}}{\mathrm{d}t}G^{-1}p_{\hat{w}e} - d_q U_1 \quad (21)$$

where a continuous function $\beta_p(q,p,t) > 0 \in \mathbb{R}$ is a design parameter. The time derivative $\mathrm{d}\alpha_{\hat{w}r}/\mathrm{d}t(q,p,t)$ in

Equation (21) is evaluated along the port-Hamiltonian system (1) calculated as

$$\frac{\mathrm{d}\alpha_{\hat{w}r}}{\mathrm{d}t} = -\left\{\left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial \alpha_{\hat{w}r}}{\partial q}^{\mathrm{T}}, p\right\rangle + \frac{\partial \alpha_{\hat{w}r}}{\partial t}\right\}. \quad (22)$$

The first and second terms of u_p are to let $\alpha_{\hat{w}}$ converge to its desired value $\alpha_{\hat{w}r}$ and the third term is to reduce the potential function U_1.

Finally let us introduce the velocity controller. The objective of this controller is to make $q \in A(t)$ for $\forall t$. Due to Assumption 5, if U_2 dose not diverge, then $q \in A(t)$ for $\forall t$. The time derivative of $U_2(q,t)$ along the port-Hamiltonian system (1) is calculated as

$$\frac{\mathrm{d}U_2}{\mathrm{d}t} = \alpha_w \langle d_q U_2, p_{we}\rangle + \alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle + \langle d_q U_2, p_{\tilde{w}}\rangle + \frac{\partial U_2}{\partial t}.$$

The velocity controller can reduced U_2 by assigning α_w appropriately since $\langle d_q U_2, p_{we}\rangle \neq 0$ holds in $A(t)$. Let us define the desired value $\alpha_{wr}(q,t) \in \mathbb{R}$ of α_w so that U_2 decreases if $\alpha_w = \alpha_{wr}$ and if U_2 is large enough. The velocity controller makes α_w converge to α_{wr}. More precisely, the desired value $\alpha_{wr}(q,t)$ is chosen in such a way that the following assumption holds.

Assumption 7: !! The desired value $\alpha_{wr} \in C^1(\mathbb{R}^l \times \mathbb{R}; \mathbb{R})$ satisfies the following conditions for $(q,t) \in A(t) \times \mathbb{R}$.

- There exists a constant $k_4 > 0 \in \mathbb{R}$ such that $\alpha_{wr}(q,t) < -(\alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle + \langle d_q U_2, p_{\tilde{w}}\rangle + \partial U_2/\partial t)/\langle d_q U_2, p_{we}\rangle$ if $U_2(q,t) > k_4$.
- There exists a constant $k_5 > 0 \in \mathbb{R}$ such that $\alpha_{wr} \langle d_q U_2, p_{we}\rangle + \alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle + \langle d_q U_2, p_{\tilde{w}}\rangle + \partial U_2/\partial t) \leq k_5$.

Though $p_{\tilde{w}}(q,p,t)$ and $\alpha_{\hat{w}}(q,p,t)$ depend on p in the first item of Assumption 7, we can always choose α_{wr} satisfying Assumption 7 as a function of q and t. This is because there always exists a function of q and t which is larger than the maximum values of $\|p_{\tilde{w}}\|$ and $\alpha_{\hat{w}}$ in the port-Hamiltonian system (1) with the input $u = u_n + u_a + u_p + u_r$ where u_r will be defined later.

For example, the following $\alpha_{wr}(q,t)$ satisfies Assumption 7.

$$\alpha_{wr} = \begin{cases} \gamma_{r1} & \text{when } U_2 = 0 \\ \gamma_{r1} - \gamma_{r2} U_2^{k_2} \frac{(\gamma_{r3} + p_{\tilde{w}0})\|d_q U_2\| + \frac{\partial U_2}{\partial t}}{\langle d_q U_2, p_{we}\rangle} & \text{when } U_2 > 0 \end{cases} \quad (23)$$

Here $\gamma_{r1}(q,t), \gamma_{r2}, \gamma_{r3}(t) \in \mathbb{R}$ are design parameters satisfying following conditions. $\gamma_{r1}(q,t)\mathrm{sgn}\langle d_q U_2, p_{we}\rangle \leq 0$ if $U_2 \geq 1/\sqrt[k_2]{\gamma_{r2}}$ for the constant $k_2 > 0$ satisfying Assumption 5. $\gamma_{r2} > 0$ is a constant. $\gamma_{r3}(t) \geq \sup_q(|\alpha_{\hat{w}r}| + \sqrt{2U_{10} + |\alpha_{\hat{w}0} - \alpha_{\hat{w}r0}|})$ where U_{10}, $\alpha_{\hat{w}0}$ and $\alpha_{\hat{w}r0}$ are the initial value of U_1, $\alpha_{\hat{w}}$ and $\alpha_{\hat{w}r}$ respectively. $\partial\gamma_{r1}/\partial q, \partial\gamma_{r1}/\partial t$ and $\mathrm{d}\gamma_{r3}/\mathrm{d}t$ are continuous. $p_{\tilde{w}0}$ is the initial value of $p_{\tilde{w}}$.

Let us define the velocity controller by

$$u_r = -\beta_r(\alpha_w - \alpha_{wr}) G^{-1} p_{we} + \frac{\mathrm{d}\alpha_{wr}}{\mathrm{d}t} G^{-1} p_{we} - \langle d_q U_2, p_{we}\rangle G^{-1} p_{we} \quad (24)$$

where a continuous function $\beta_r(q,p,t) \in \mathbb{R}$ is a design parameter that there exists a constant $k_6 > 0 \in \mathbb{R}$ such that $\beta_r(q,p,t) \geq 1/k_6$. The time derivative $\mathrm{d}\alpha_{wr}/\mathrm{d}t(q,p,t)$ in Equation (24) is evaluated along the port-Hamiltonian system (1) calculated as

$$\frac{\mathrm{d}\alpha_{wr}}{\mathrm{d}t} = -\left\{\left\langle \left(J_{12}^{\mathrm{T}} - R_{12}^{\mathrm{T}}\right) \frac{\partial \alpha_{wr}}{\partial q}^{\mathrm{T}}, p\right\rangle + \frac{\partial \alpha_{wr}}{\partial t}\right\}. \quad (25)$$

The first and second terms of u_r are to let α_w converge to its desired value α_{wr} and the third term is to reduce the potential function U_2.

We can prove the following theorem for the stability of the resulting feedback system.

Theorem 2: Consider a port-Hamiltonian system (1) *with scalar functions $U_1(q,t)$ and $U_2(q,t)$, vectors $p_w(q,t)$ and $p_{\hat{w}}(q,t)$, and a region $A(t)$. Suppose that Assumptions 1, 2, 3, 4 and 5 hold. Then the following properties hold along the state trajectory of the closed loop system derived by the feedback $u = u_n + u_a + u_p + u_r$: The state q stays in $A(t)$ for $\forall t$. The state p converges to $\alpha_w p_{we}$ as $t \to \infty$. The state q converges to the desired path as $t \to \infty$ for all initial conditions.*

Proof: Consider a feedback system with the port-Hamiltonian system (1) with the input $u = u_n + u_a + u_p + u_r$. First of all, it will be proven that $q \in A(t)$ for $\forall t$. In order to prove that $q \in A(t)$ for $\forall t$, it is proven that U_2 dose not diverge. Let us define a function

$$V_1(q,p,t) = U_2(q,t) + \frac{1}{2}\left(\alpha_w(q,p,t) - \alpha_{wr}(q,t)\right)^2.$$

The time derivative of V_1 is calculated as

$$\begin{aligned}
\frac{\mathrm{d}V_1}{\mathrm{d}t} =& \frac{\mathrm{d}U_2}{\mathrm{d}t} + (\alpha_w - \alpha_{wr})\left(\frac{\mathrm{d}\alpha_w}{\mathrm{d}t} - \frac{\mathrm{d}\alpha_{wr}}{\mathrm{d}t}\right) \\
=& \alpha_w \langle d_q U_2, p_{we}\rangle + \alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle + \langle d_q U_2, p_{\tilde{w}}\rangle \\
&+ \frac{\partial U_2}{\partial t} + (\alpha_w - \alpha_{wr})\Big(\langle Gu - \bar{R}M^{-1}p, p_{we}\rangle \\
&+ \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\tilde{w}}\right\rangle \\
&- \langle p_{\hat{w}e}, p\rangle \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{we}\right\rangle - \frac{\mathrm{d}\alpha_{wr}}{\mathrm{d}t}\Big) \\
=& \alpha_w \langle d_q U_2, p_{we}\rangle + \alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle + \langle d_q U_2, p_{\tilde{w}}\rangle + \frac{\partial U_2}{\partial t} \\
&+ (\alpha_w - \alpha_{wr})\left(\langle Gu_a + Gu_p + Gu_r, p_{we}\rangle - \frac{\mathrm{d}\alpha_{wr}}{\mathrm{d}t}\right) \\
=& -\beta_r(\alpha_w - \alpha_{wr})^2 + \alpha_{wr} \langle d_q U_2, p_{we}\rangle + \alpha_{\hat{w}} \langle d_q U_2, p_{\hat{w}e}\rangle \\
&+ \langle d_q U_2, p_{\tilde{w}}\rangle + \frac{\partial U_2}{\partial t}.
\end{aligned} \quad (26)$$

Consider constants k_4, k_5 and k_6. Suppose that Assumption 7 holds with $\beta_r(q,p,t) \geq 1/k_6$. Then $\mathrm{d}V_1/\mathrm{d}t = 0$ holds if $(\alpha_w - \alpha_{wr})^2 \geq k_5 k_6$ or $U_2 > k_4$ holds. This means $U_2 \leq V_1 < k_4 + k_5 k_6/2$ or $U_2 \leq V_1 \leq V_{10}$ holds where V_{10} denotes the initial value of V_1. Due to the fifth item of Assumption 4 and the second item of Assumption 5, this implies $q \in A(t)$ for $\forall t$.

Next it will be proven that the convergence of the state to the desired path. To this end, it is proven that U_1 and $\alpha_{\hat{w}}$ converge to 0. Let us define a Lyapunov like function

$$V_2(q,p,t) = U_1(q,t) + \frac{1}{2}\left(\alpha_{\hat{w}}(q,p,t) - \alpha_{\hat{w}r}(q,t)\right)^2 \geq 0.$$

Since $q \in A(t)$ for $\forall t$, the time derivative of V_2 is calculated as

$$\begin{aligned}
\frac{\mathrm{d}V_2}{\mathrm{d}t} =& \frac{\mathrm{d}U_1}{\mathrm{d}t} + (\alpha_{\hat{w}} - \alpha_{\hat{w}r})\left(\frac{\mathrm{d}\alpha_{\hat{w}}}{\mathrm{d}t} - \frac{\mathrm{d}\alpha_{\hat{w}r}}{\mathrm{d}t}\right)\\
=& \alpha_{\hat{w}}\left\langle d_q U_1, p_{\hat{w}e}\right\rangle + \frac{\partial U_1}{\partial t}\\
&+ (\alpha_{\hat{w}} - \alpha_{\hat{w}r})\Big(\left\langle Gu - \bar{R}M^{-1}p, p_{\hat{w}e}\right\rangle\\
&+ \left\langle \eta_{J\hat{w}} + \xi_{R\hat{w}} + \frac{\partial p_{\hat{w}e}}{\partial t}, p_{\tilde{w}}\right\rangle\\
&- \langle p_{we}, p\rangle \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\hat{w}e}\right\rangle - \frac{\mathrm{d}\alpha_{\hat{w}r}}{\mathrm{d}t}\Big)\\
=& \alpha_{\hat{w}}\left\langle d_q U_1, p_{\hat{w}e}\right\rangle + \frac{\partial U_1}{\partial t}\\
&+ (\alpha_{\hat{w}} - \alpha_{\hat{w}r})\left(\left\langle Gu_a + Gu_p + Gu_r, p_{\hat{w}e}\right\rangle - \frac{\mathrm{d}\alpha_{\hat{w}r}}{\mathrm{d}t}\right)\\
=& -\beta_p\left(\alpha_{\hat{w}} - \alpha_{\hat{w}r}\right)^2 + \alpha_{\hat{w}r}\left\langle d_q U_1, p_{\hat{w}e}\right\rangle + \frac{\partial U_1}{\partial t}\\
\leq& 0
\end{aligned} \tag{27}$$

This suggests that the scalar function $V_2(q,p,t)$ satisfies the following conditions.

- $V_2(q,p,t)$ is lower bounded.
- $\mathrm{d}V_2/\mathrm{d}t(q,p,t)$ is negative semi-definite.
- $\mathrm{d}V_2/\mathrm{d}t(q,p,t)$ is uniformly continuous in time.

Lyapunov-Like Lemma ([10], pp.127) implies that $\mathrm{d}V_2/\mathrm{d}t(q,p,t) \to 0$ as $t \to \infty$. This says $\alpha_{\hat{w}} \to \alpha_{\hat{w}r}$ and $U_1 \to 0$ since $\alpha_{\hat{w}r}\left\langle d_q U_1, p_{\hat{w}e}\right\rangle + \partial U_1/\partial t = 0$ holds if and only if $U_1 = 0$. Furthermore $\alpha_{\hat{w}} \to 0$ since $\alpha_{\hat{w}r} = 0$ when $U_1 = 0$. Hence the state converges to the desired path as $t \to \infty$.

Finally in order to prove that p converges to $\alpha_w p_{we}$, it is proven that $p_{\tilde{w}}$ converges to 0. Let us calculate the time derivative of $H_{k,\tilde{w}}$ along the state trajectory of the closed loop system as

$$\begin{aligned}
\frac{\mathrm{d}H_{k,\tilde{w}}}{\mathrm{d}t} =& \left\langle Gu - \bar{R}M^{-1}p, p_{\tilde{w}}\right\rangle\\
&- \langle p_{we}, p\rangle \left\langle \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{\tilde{w}}\right\rangle \qquad (28)\\
=& \left\langle Gu_p + Gu_r, p_{\tilde{w}}\right\rangle - \left\langle p_{\tilde{w}}, p_{\tilde{w}}\right\rangle\\
=& -\left\langle p_{\tilde{w}}, p_{\tilde{w}}\right\rangle.
\end{aligned}$$

Therefore $H_{k,\tilde{w}}$ converges to the set $p_{\tilde{w}} = 0$. Hence $p = \alpha_w p_{we} + \alpha_{\hat{w}} + p_{\hat{w}e} + p_{\tilde{w}}$ converges to $\alpha_w p_{we}$. This completes the proof. ■

Consider a feedback system with the port-Hamiltonian system (1) with the input $u = u_n + u_a + u_p + u_r$. From Equation (27), we obtain

$$U_1 \leq U_1 + \frac{1}{2}\left(\alpha_{\hat{w}} - \alpha_{\hat{w}r}\right)^2 \leq U_{10} + \frac{1}{2}\left(\alpha_{\hat{w}0} - \alpha_{\hat{w}r0}\right)^2. \tag{29}$$

Therefore if $U_1(q,t) > U_{10} + \left(\alpha_{\hat{w}0} - \alpha_{\hat{w}r0}\right)^2/2$ or $q \notin A(t)$ in neighborhoods of obstacles then the system avoids collisions with moving obstacles.

If U_2 and $\partial U_2/\partial t$ converge to 0 by choosing α_{wr} appropriately, then from (27) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}\left(\alpha_w - \alpha_{wr}\right)^2\right\} = -\beta_r\left(\alpha_w - \alpha_{wr}\right)^2. \tag{30}$$

Therefore α_w converges to its desired value α_{wr}. Then due to Theorem 2, p converges to $\alpha_{wr} p_{we}$. Hence the velocity of the system moving on the desired path can be assigned by choosing α_{wr}.

Thus, p is decomposed into three elements $\alpha_w p_{we}$, $\alpha_{\hat{w}} p_{\hat{w}e}$ and $p_{\tilde{w}}$ related the potential function U_1, then these elements are controlled independently. The state of the system converges to the desired path and avoids collisions with moving obstacles by controlling the slope element $\alpha_{\hat{w}} p_{\hat{w}e}$ of p. Furthermore q stays in the region $A(t)$ for all time t by controlling the desired element $\alpha_w p_{we}$ of p.

Since q stays in the region $A(t)$ for all time t, Assumptions 1, 2 and 3 do not need to hold for $q \notin A(t)$.

IV. Conclusions

This paper is devoted to a new path following method whose desired path is time-varying for a class of port-Hamiltonian systems. In the proposed method systems can avoid collisions with moving obstacles. We design potential functions and vector fields as time-varying functions. By controlling the desired, the slope and the undesired phase states independently, we can derive a control method to make systems converge to the time-varying desired path. Furthermore by making the potential energy large near the obstacles, we can avoid collisions between the systems and the obstacles.

The final version of this paper will exhibit numerical examples to control the behavior of a rolling coin.

References

[1] J. K. Salisbury, "Active stiffness control of a manipulator in Cartesian coordinates," in *Proc. 19th IEEE Conf. on Decision and Control*, 1980.

[2] N. Hogan, "Impedance contro: An approach to manipulation," *Trans. ASME, J. Dyn. Syst., Meas., Control*, vol. 107, no. 1, pp. 1–24, 1985.

[3] P. Y. Li and R. Horowitz, "Passive velocity field control of mechanical manipulators," *IEEE Trans. Robotics and Automation*, vol. 15, no. 4, pp. 751–763, 1999.

[4] ——, "Passive velocity field control (PVFC)," *IEEE Trans. Autom. Contr.*, vol. 46, no. 9, pp. 1346–1371, 2001.

[5] V. Duindam and S. Stramigioli, "Passive asymptotic curve tracking," in *Proc. IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control*, 2003, pp. 229–234.

[6] ——, "Passive compensation of nonlinear robot dynamics," *IEEE Trans. Robotics and Automation*, vol. 20, no. 3, pp. 480–487, 2004.

[7] K. Fujimoto and M. Taniguchi, "Passive path following control for port-hamiltonian systems," in *Proc. 47th IEEE Conf. on Decision and Control*, 2008.

[8] A. J. van der Schaft, *L_2-Gain and Passivity Techniques in Nonlinear Control*. London: Springer-Verlag, 2000.

[9] B. M. J. Maschke and A. J. van der Schaft, "A Hamiltonian approach to stabilization of nonholonomic mechanical systems," in *Proc. 33rd IEEE Conf. on Decision and Control*, 1994, pp. 2950–2954.

[10] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control.* Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1991.