Voltage Limiter Calculation Method for Fast Torque Response of IPMSM in Overmodulation Range

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***Abstract*—In this paper, a voltage limiter calculation method for the fast torque response of an interior permanent magnet synchronous motor (IPMSM) when an inverter operates in the overmodulation range is considered. The relation between torque response of an IPMSM and voltage phase angle in the overmodulation range is analyzed, and based on this relation, the optimal voltage limiter calculation method that can be realized by real time calculation is proposed. By using the proposed method, torque response of an IPMSM can be improved as fast as twice the conventional methods. The effectiveness of the proposed method is confirmed by simulation and experimental results.**

I. Introduction

In transient condition of a motor drive system with current control loop (Fig.1), the reference voltage amplitude calculated from the current controller is usually larger than the limited voltage that an inverter can generate. In this voltage saturation, unless proper voltage limiter calculation method, current and torque response of a motor will deteriorate or eventually be unstable. By operating an inverter in the overmodulation range in transient condition (dynamic overmodulation [1]), the limited voltage can be increased, however voltage saturation still occurs in the case of high gain controller.

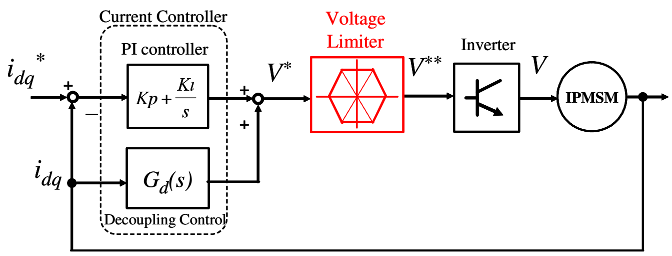

Fig. 1. Current control system of an IPMSM with voltage limiter

In this paper, we focus on the fast torque response of an interior permanent magnet synchronous motor (IPMSM) when an inverter operates in the overmodulation range in transient condition. There are two problems shown below that make this topic difficult.

(1) The nonlinear relation between torque and currents of an IPMSM

(2) The complicated characteristic of inverter output voltage amplitude and phase angle in the overmodulation range [2] (Fig.2)

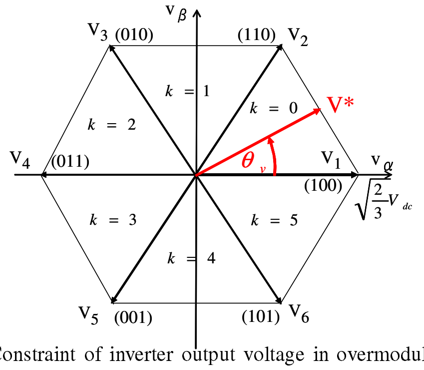

Fig. 2. Constraint of inverter output voltage in overmodulation range

Actually , many voltage limiter calculation methods in the overmodulation range have been proposed, which are minimum phase error method [3] (Fig.3), minimum amplitude error method [4] (Fig.4) and dynamic field weakening method [5] (Fig.5). However, in these conventional methods, fast torque response of an IPMSM is not the main objective.

The voltage limiter for fast torque response of an IPMSM in the linear range of an inverter has been already proposed by the authors [7]. In this paper, work is extended to the overmodulation range of an inverter. The relation between torque response of an IPMSM and inverter voltage phase angle in the overmodulation range is analyzed, and based on this relation, the optimal voltage limiter for fast torque response of an IPMSM is proposed. Moreover, by using the proposed method, an undershoot of current may occur, hence the current limiter calculation method is also proposed here too.

II. Proposed Voltage Limiter

Torque equation of an IPMSM based on the rotating frame (d-q) is shown in (1), when T is torque, P is pole pairs, i_d, i_q are d axis and q axis currents, k_e is back EMF. constant, L_d, L_q are d axis and q axis inductances respectively.

$$T = Pk_e i_q + P(L_d - L_q)i_d i_q \tag{1}$$

Because the constraint of inverter output voltage in the overmodulation range is determined in the stationary frame ($\alpha - \beta$) as shown in Fig.2, then it is easier to consider the torque response in the stationary frame rather than the rotating frame.

Based on the transformations in (2) and (3) , when i_α, i_β are α axis and β axis currents, θ_{re} is rotor angle in electrical

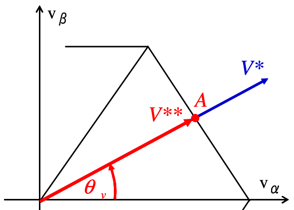

Fig. 3. Minimum phase error method

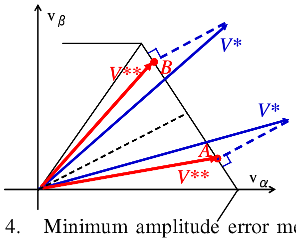

Fig. 4. Minimum amplitude error method

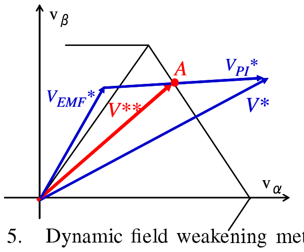

Fig. 5. Dynamic field weakening method

degree, torque equation of an IPMSM in the stationary frame can be written as shown in (4).

$$i_q = -i_\alpha sin\theta_{re} + i_\beta cos\theta_{re} \quad (2)$$

$$i_d = i_\alpha cos\theta_{re} + i_\beta sin\theta_{re} \quad (3)$$

$$T = Pk_e(-i_\alpha sin\theta_{re} + i_\beta cos\theta_{re}) + P(L_d - L_q) \quad (4)$$
$$((i_\beta^2 - i_\alpha^2)cos\theta_{re} sin\theta_{re} + i_\alpha i_\beta(cos^2\theta_{re} - sin^2\theta_{re}))$$

By differentiating (4) with time (t), torque response (dT/dt) equation can be achieved as shown in (5), when the definitions of constant A', B', K' are shown in (6)∼(8) respectively ($\omega_{re} = d\theta_{re}/dt$ is rotational speed in electrical degree).

$$\frac{dT}{dt} = A'\frac{di_\alpha}{dt} + B'\frac{di_\beta}{dt} + K' \quad (5)$$

$$A' = -Pk_e sin\theta_{re} + P(L_d - L_q) \quad (6)$$
$$\times(i_\beta(cos^2\theta_{re} - sin^2\theta_{re}) - 2i_\alpha cos\theta_{re} sin\theta_{re})$$
$$B' = -Pk_e cos\theta_{re} + P(L_d - L_q) \quad (7)$$
$$\times(i_\alpha(cos^2\theta_{re} - sin^2\theta_{re}) - 2i_\beta cos\theta_{re} sin\theta_{re})$$
$$K' = -Pk_e(i_\alpha cos\theta_{re}\omega_{re} + i_\beta sin\theta_{re}\omega_{re}) \quad (8)$$
$$+P(L_d - L_q)(i_\beta^2\omega_{re}(cos^2\theta_{re} - sin^2\theta_{re})$$
$$+i_\alpha^2\omega_{re}(sin^2\theta_{re} - cos^2\theta_{re}) - 4i_\alpha i_\beta cos\theta_{re} sin\theta_{re}\omega_{re})$$

Voltage equations of an IPMSM in the stationary frame can be written as shown in (9) and (10), when v_α, v_β are α axis and β axis voltages, R is resistance, $L_\alpha, L_\beta, L_{\alpha\beta}$ are α axis inductance, β axis inductance and mutual inductance between α axis and β axis respectively.

$$v_\alpha = Ri_\alpha + L_\alpha\frac{di_\alpha}{dt} + L_{\alpha\beta}\frac{di_\beta}{dt} - \omega_{re}k_e sin\theta_{re} \quad (9)$$

$$v_\beta = Ri_\beta + L_\beta\frac{di_\beta}{dt} + L_{\alpha\beta}\frac{di_\alpha}{dt} + \omega_{re}k_e cos\theta_{re} \quad (10)$$

Based on (9) and (10), current responses in the stationary frame ($di_\alpha/dt, di_\beta/dt$) can be written as shown in (11) and (12), when the definitions of constant L, K_1, K_2 are shown in (13),(14) and (15) respectively.

$$\frac{di_\alpha}{dt} = \frac{1}{L}(L_\beta v_\alpha - L_{\alpha\beta}v_\beta + K_1) \quad (11)$$

$$\frac{di_\beta}{dt} = \frac{1}{L}(L_\alpha v_\beta - L_{\alpha\beta}v_\alpha + K_2) \quad (12)$$

$$L = L_\alpha L_\beta - L_{\alpha\beta}^2 \quad (13)$$

$$K_1 = L_\beta(\omega_{re}k_e sin\theta_{re} - Ri_\alpha) + L_{\alpha\beta}(\omega_{re}k_e cos\theta_{re} + Ri_\beta) \quad (14)$$

$$K_2 = -L_\alpha(\omega_{re}k_e cos\theta_{re} + Ri_\beta) - L_{\alpha\beta}(\omega_{re}k_e sin\theta_{re} - Ri_\alpha) \quad (15)$$

By substitution (11) and (12) into (5), and with some arrangement, the relation between torque response and $\alpha - \beta$ voltages can be written as shown in (16), when the definitions of constants A, B, K are shown in (17) ∼ (19) respectively.

$$\frac{dT}{dt} = Av_\alpha + Bv_\beta + K \quad (16)$$

$$A = A'L_\beta/L - B'L_{\alpha\beta}/L \quad (17)$$
$$B = B'L_\alpha/L - A'L_{\alpha\beta}/L \quad (18)$$
$$K = A'K_1/L + B'K_2/L + K' \quad (19)$$

Next, from Fig.2., $\alpha - \beta$ voltages can be calculated as shown in (20) and (21), when θ_v is voltage phase angle in the stationary frame, V_{dc} is DC link voltage and k is an integer indicates inverter operating sector ($k = 0 \sim 5$).

$$v_\alpha = \frac{(V_{dc}/\sqrt{2})cos\theta_v}{sin((1-k)\frac{\pi}{3} + \theta_v)} \quad (20)$$

$$v_\beta = \frac{(V_{dc}/\sqrt{2})sin\theta_v}{sin((1-k)\frac{\pi}{3} + \theta_v)} \quad (21)$$

The voltage phase angle θ_v that can generate the fastest torque response of an IPMSM can be found by partial differentiating the torque response with voltage phase angle and set to be zero as shown in (22). By using (16), (20) and (21), the result of (22) can be achieved as shown in (23), and by some arrangement in trigonometric terms, the result in (23) can be written as shown in (24).

$$\frac{\partial(dT/dt)}{\partial\theta_v} = 0 \quad (22)$$

$$\frac{\partial(dT/dt)}{\partial\theta_v} = 0 = \quad (23)$$
$$\frac{AV_{dc}}{\sqrt{2}}\left(\frac{-sin\theta_v sin((1-k)\frac{\pi}{3} + \theta_v) - cos\theta_v cos((1-k)\frac{\pi}{3} + \theta_v)}{sin^2((1-k)\frac{\pi}{3} + \theta_v)}\right)$$
$$+\frac{BV_{dc}}{\sqrt{2}}\left(\frac{cos\theta_v sin((1-k)\frac{\pi}{3} + \theta_v) - sin\theta_v cos((1-k)\frac{\pi}{3} + \theta_v)}{sin^2((1-k)\frac{\pi}{3} + \theta_v)}\right)$$

$$Bsin((1-k)\frac{\pi}{3}) - Acos((1-k)\frac{\pi}{3}) = 0 \quad (24)$$

From (24), we can find that the result of (22) is in the form of "$constant = 0$" (no parameter θ_v in (24)), or we can say that the relation between torque response and voltage phase angle in each voltage sector is the linear relation, hence, the optimal point is on the boundary between each sector.

To confirm the above assumption, the calculation results of relation between torque response and voltage phase angle will be shown. When parameters of IPMSM shown in TABLE I are used, DC link voltage of an inverter is fixed to 70V , rotational speed is fixed to 1,600 rpm, rotor position θ_{re} is fixed to 0 °, the cases that amplitudes of current which are 1A,2A,3A are considered (phase angles of all currents are fixed to 120° measured from d-axis), the relations between torque response and voltage phase angle in these conditions can be calculated as described in the previous section and the results are shown together in Fig.6.

TABLE I
PARAMETERS OF IPMSM

pole pairs P	2
EMF constant k_e	0.104V/(rad/s)
Resistance R	0.45Ω
d-axis inductance L_d	4.15mH
q-axis inductance L_q	16.74mH

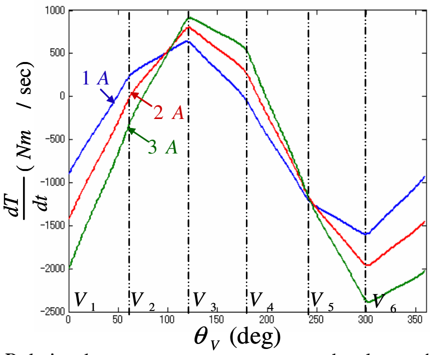

Fig. 6. Relation between torque response and voltage phase angle

From the results in Fig.6. , we can find that the relation between torque response and voltage phase angle in each voltage sector is the linear relation and the optimal point is one of the six voltage vectors of the hexagon vertices (in this case is vector V_3) which conforms with the above analysis.

Based on the above results, we can find that the voltage phase angle that will generate the fastest torque response of an IPMSM is one of the voltage vectors among the six vectors on hexagon vertices ($V_1 \sim V_6$) shown in Fig.2., hence for generating the fastest torque response in voltage saturation condition, we can just consider only these six voltage vectors and use (16) as the evaluation function.

However, such calculation consumes so much time and may not be used in the real time calculation. Hence, for reducing the calculation time, by observing the results in Fig.6., and based on the information of reference voltage vector (V^*) calculated from current controller, we can find the much less time consuming method as shown in Fig.7. From Fig.7., the voltage vector that can generate the fastest torque response is one of the six vectors of hexagon vertices, that lines nearest to the reference voltage vector V^* calculated from current controller in the direction rotates to the minus d axis vector (in this case V_4 vector is selected). The physical meaning of this voltage vector selection method is that, the fastest torque response of an IPMSM can be achieved when the d axis current is large enough in minus direction (dynamic field weakening effect) as described in [6][7].

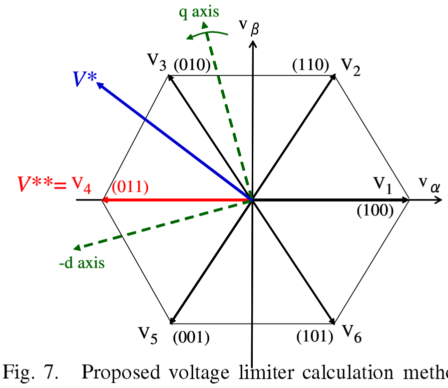

Fig. 7. Proposed voltage limiter calculation method

III. SIMULATION AND EXPERIMENTAL RESULTS

To confirm the effectiveness of the proposed methods, simulation and experimental results are shown. Parameters of an IPMSM are shown in TABLE I and setting conditions of current controller and operating conditions of an IPMSM for both simulation and experiment are shown in TABLE II and TABLE III respectively.

In this paper, we use the conventional triangle intersection modulation method with feedforward amplitude compensation for the overmodulation range [8]. In transient condition, with this modulation method, the phase shift between inverter reference and output voltages will occur [1]. However, in this paper, this phase shift problem is already investigated and then compensated by using the offline table. Hence the phase angle of inverter output voltage can be exactly controlled same as the case of space-vector modulation method [2].

TABLE II
SETTING CONDITIONS OF CURRENT CONTROL SYSTEM

Bandwidth of PI current controllers	2000rad/s
Current control period	100μsec
Inverter carrier frequency f_c	10kHz
DC link voltage V_{dc}	70V
Antiwindup calculation method	Back calculation method

TABLE III
OPERATING CONDITIONS OF IPMSM

Rotational speed ω_{re}	1600rpm constant
Reference current amplitude	$0A \rightarrow 4A$ in step
Reference current phase angle	120° (from d-axis)

The simulation results of the conventional and proposed methods are shown together from Fig.8 ∼ Fig.11. From these results , we can find that the torque response of the proposed method (Fig.11(b)) can be improved as fast as twice the

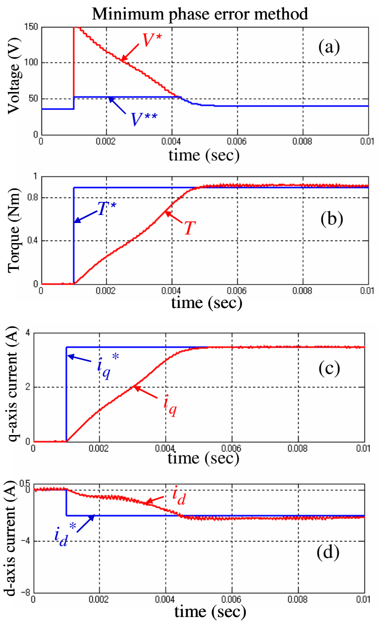

Fig. 8. Simulation results of minimum phase error method: (a) Voltage amplitude (b) Torque response (c) q-axis current response (d) d-axis current response

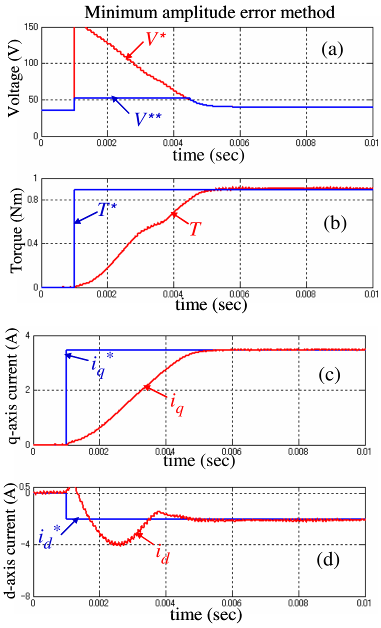

Fig. 9. Simulation results of minimum amplitude error method: (a) Voltage amplitude (b) Torque response (c) q-axis current response (d) d-axis current response

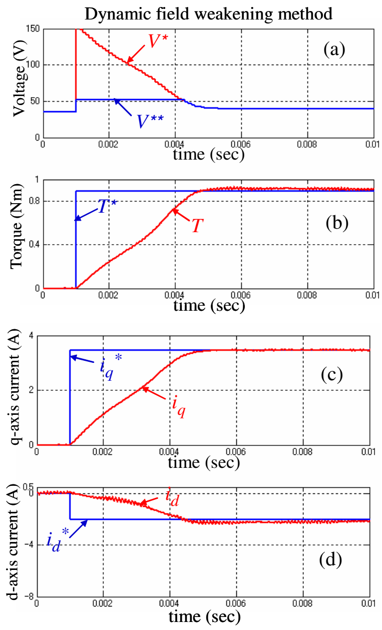

Fig. 10. Simulation results of dynamic field weakening method: (a) Voltage amplitude (b) Torque response (c) q-axis current response (d) d-axis current response

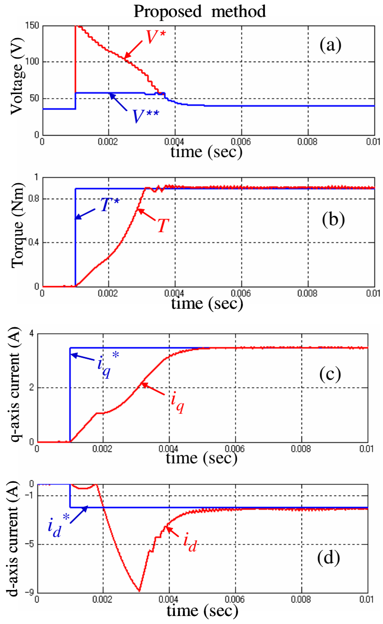

Fig. 11. Simulation results of proposed method: (a) Voltage amplitude (b) Torque response (c) q-axis current response (d) d-axis current response

conventional methods, however the large undershoot in d-axis current of the proposed method also occurs (Fig.11(d)). To solve this current undershoot problem, the current limiter calculation method will be proposed in the next section.

The experimental results of each methods are shown from Fig.12 $\sim$ Fig.15. In these experimental results, torque responses are calculated from currents and real parameter values as shown in (1). The experimental conditions are same as in the simulation, and the experimental results also match with the simulation results very well.

IV. Proposed Current Limiter

As stated in the previous section, by using the proposed voltage limiter, the large undershoot in d axis current will occur, and this may be a problem in some system. Hence, in this paper we also propose the current limiter calculation method too.

Fig.16 shows the relation between d axis current in control step k ($i_d[k]$) and the previous control step $k-1$ ($i_d[k-1]$). If we define the maximum allowance d axis current as i_{dmax}, then the maximum allowance d axis current response in control step k is $(di_d/dt)_{max}[k]$ which can be calculated as shown in (25), when t_{cc} is current control period.

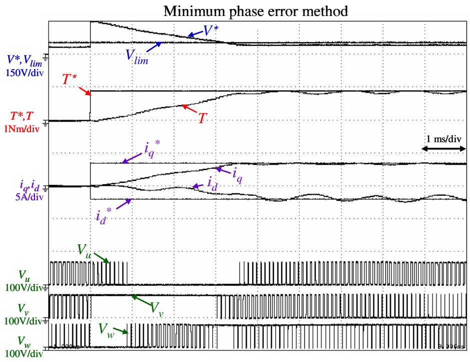

Fig. 12. Experimental results of minimum phase error method

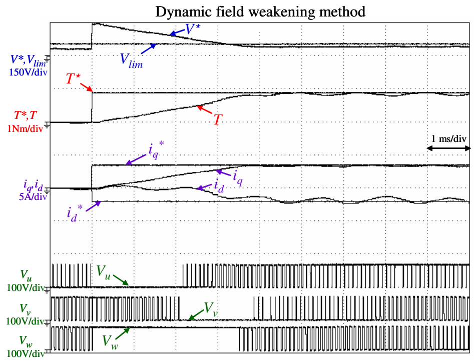

Fig. 14. Experimental results of dynamic field weakening method

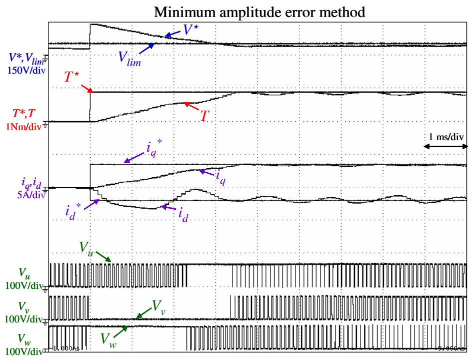

Fig. 13. Experimental results of minimum amplitude error method

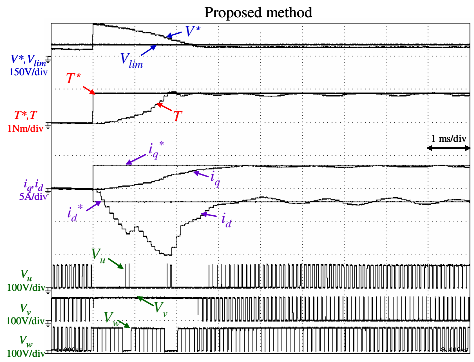

Fig. 15. Experimental results of proposed method

$$(\frac{di_d}{dt})_{max}[k] = \frac{i_{dmax} - i_d[k-1]}{t_{cc}} \tag{25}$$

From d axis voltage equation of an IPMSM, we can find that the d axis current response relates directly to d axis voltage, hence the maximum allowance d axis voltage in control step k, $v_{dmax}[k]$ can be calculated as shown in (26).

$$v_{dmax}[k] = L_d(\frac{di_d}{dt})_{max}[k] + Ri_d[k-1] - \omega_{re}L_q i_q[k-1] \tag{26}$$

For limiting d axis current in each control step k to be not larger than i_{dmax}, d axis voltage in each control step k must not exceed v_{dmax} calculated from (26). The voltage vector that should be selected in this case can be found by projecting the v_{dmax} from d axis to the circumference of hexagon as shown in Fig.17. (new reference voltage vector is V_x^{**} or V_y^{**} depends on the sign of q axis current, V_x^{**} when i_q is plus, V_y^{**} when i_q is minus).

By using the proposed voltage limiter and current limiter together, the calculation algorithm of the proposed method can be summarized as the flowchart shown in Fig.18.

The simulation results when using our proposed method with current limiter are shown in Fig.19. From these results, we can find that the maximum of d axis current in each case can be correctly controlled. Of course, the torque response in these cases are also slower than the case without current limiter, however torque responses are still faster than the conventional method responses, except the case that the maximum d axis current is same as the d axis reference current, the

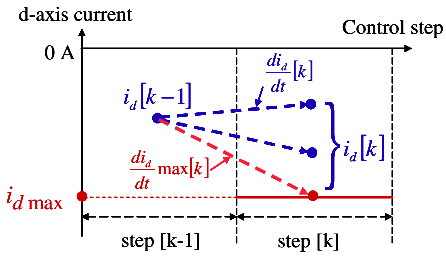

Fig. 16. d axis current in each control step

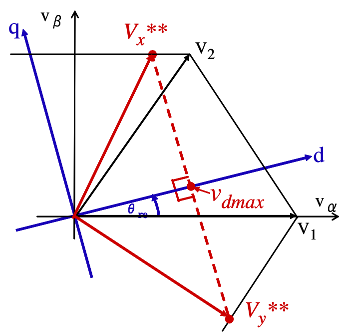

Fig. 17. Proposed current limiter calculation method

torque response of the proposed method becomes same as the conventional methods.

The experimental results of proposed method with current limiter when $i_{dmax} = -8A, -6A, -5A$ are shown together in Fig.20. which also conform with the previous simulation results.

V. Conclusion

In this paper, the voltage limiter calculation method in the overmodulation range of an inverter for generating the fastest

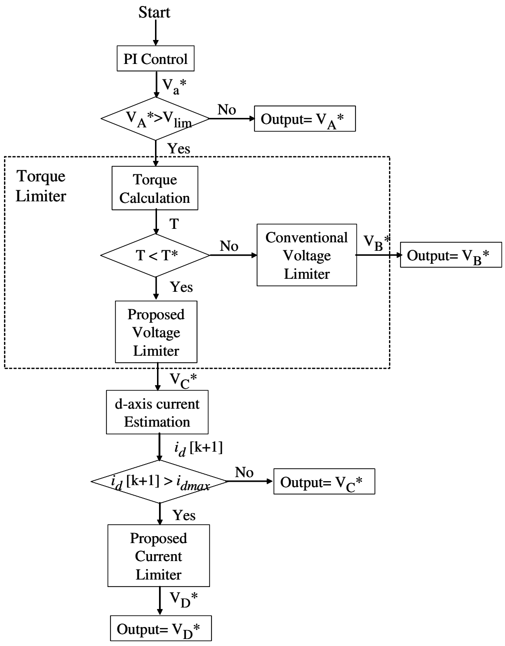

Fig. 18. Calculation flowchart of proposed method

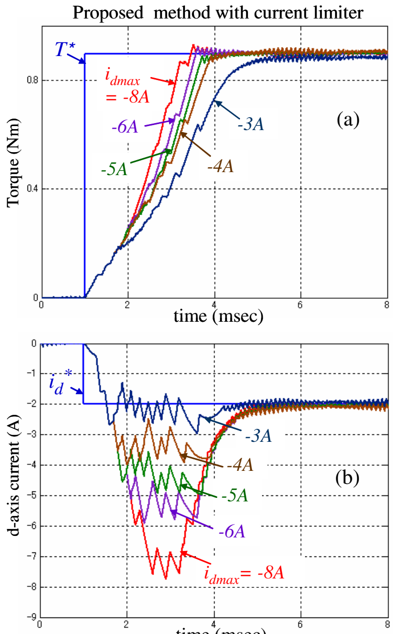

Fig. 19. Simulation results of proposed method with current limiter (a) Torque response (b) d-axis current response

torque response of an IPMSM is proposed. By using the proposed method, the torque response of an IPMSM can be improved as fast as twice the conventional methods. However, the large undershoot in d axis current also occurs when using the proposed method. For solving this problem, the current limiter calculation method is also proposed. By using the both proposed limiter calculation methods, the fastest torque response of an IPMSM can be achieved under the constraints of both voltage and current. The effectiveness of the proposed methods is confirmed by simulation and experimental results.

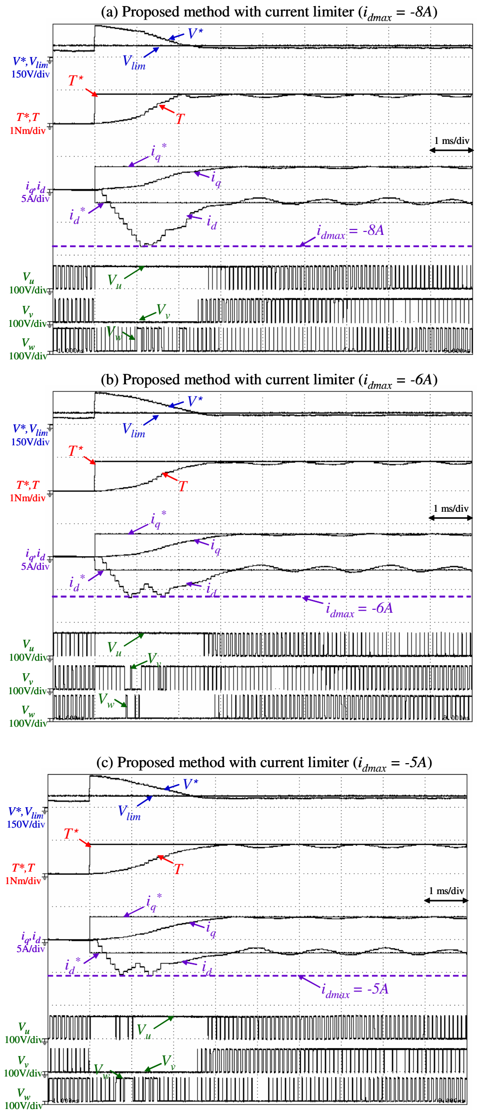

Fig. 20. Experimental results of proposed method with current limiter (a) $i_{dmax} = -8A$ (b) $i_{dmax} = -6A$ (c) $i_{dmax} = -5A$

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