Fast Torque Control System of PMSM based on Model Predictive Control

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***Abstract*—This paper describes a fast torque control system of permanent magnet synchronous motor (PMSM) based on model predictive control (MPC). This torque controller selects directly one of the voltage vector of voltage source PWM inverter considering voltage saturation explicitly. To obtain the fast torque response at the transient state and the stable current response at the steady state, the problem with selecting the voltage vector to output is formulated based on MPC. In this paper, real-time implementation method using a look-up table, which is designed beforehand, is discussed and the experimental results are shown.**

I. INTRODUCTION

Permanent Magnet Synchronous Motor (PMSM) has been used in a wide field because of high efficiency and high power per volume and weight. Recently, PMSM is applied to traction drive for electric vehicles and railway vehicles [1] [2].

In these applications, a fast torque response at high speed region is important. However, large back EMF is generated by permanent magnet at high speed region. And also, controller makes more higher voltage reference to obtain a fast torque response. On the other hand, inverter has a voltage limiter depending on a DC-link voltage. Even if controller generates a large voltage reference, inverter can't output the same voltage as the reference. In a word, voltage saturation occurs. As a result, current and torque response gets worse [3].

When the voltage saturation occurs, voltage limiter methods dominate a current response, which also does a torque response. In the case of interior permanent magnet synchronous motor (IPMSM), however, improvement of current response does not guarantee one of torque response. Therefore, proposed voltage limiters have restrictive performance [4], or need controller switching for compatibility.

To overcome these problems, we propose a new torque control system being able to obtain the fast torque response at the transient state and the stable current response at the steady state considering the various constraint on the inverter explicitly. Proposed controller directly selects a switching mode of the inverter by using Model Predictive Control (MPC) [5]. It may be similar to Direct Torque Control (DTC). To be precise, our proposed method can be a superset of DTC, which means that it is able to control not only torque but also currents and switching freqency explicitly, with prediction of future current and torque behavior by using the inverter switching mode and mathematical model of PMSM.

In this paper, the proposed MPC torque control system and its real-time implementation methods using a look-up table, which designed beforehand, are discussed and the experimental results are shown.

II. TORQUE CONTROL SYSTEM BASED ON MPC

Our proposed torque control system based on MPC is shown in Fig.1. Current reference generator consists of Maximum Torque per Ampere (MTPA) Control [6], Flux Weakening Control [7], and so on.

In this following section, the MPC controller in Fig.1 is described.

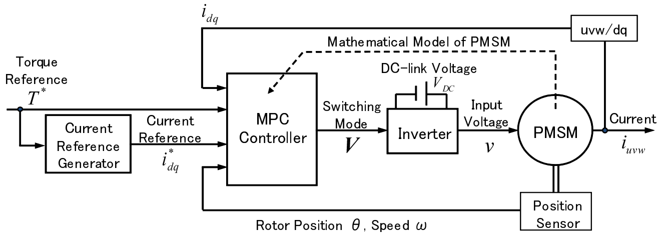


Fig. 1. Torque contorl system based on MPC

A. *Voltage limit and MPC controller*

Initially, voltage limit should be discussed. Voltage reference vector for inverter generated by the upper controller (e.g. PI current controller) must be limited in the voltage that inverter is able to output.

In this limitation of the voltage vector, there is a redundancy of not only the limitation of norm but also voltage phase control. In a general vector control system, some limitation methods, such as fixed voltage phase method and fixed d-axis voltage method, are proposed [4] [8] [9] [10].

However, it is difficult to design a voltage limiter as the limiter influences control performance designed by the upper controller.

On the other hand, MPC controller directly generates one of eight voltage vectors decide by limited DC-link voltage and combinations of switch of each phase (Fig.2, Fig.3). Then MPC controller searchs and selects a suitable voltage vector from among eight voltage vectors based on a reference of upper controller or control error. As a result, the voltage limiter is naturally achieved [11].

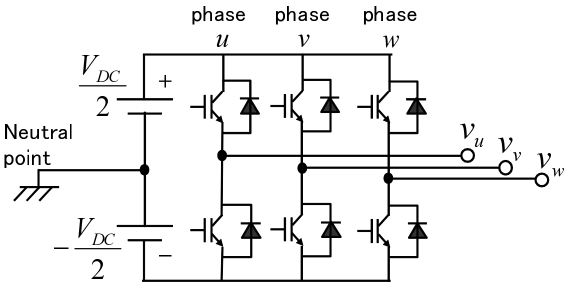


Fig. 2. Voltage source PWM inverter

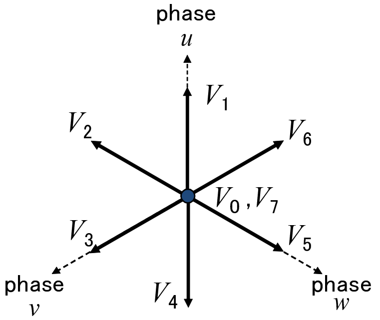


Fig. 3. Output voltage vector of the inverter

B. Selection of voltage vector based on MPC

MPC controller predicts current behavior and torque behavior by using a mathematical model of PMSM. And then, it selects a voltage vector that the inverter outputs based on prediction results.

The state equation of PMSM is given as follows:

$$\frac{d}{dt}\boldsymbol{i}_{dq} = \underbrace{\begin{bmatrix} -\frac{R}{L_d} & \frac{\omega_{re}L_q}{L_d} \\ -\frac{\omega_{re}L_d}{L_q} & -\frac{R}{L_q} \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{i}_{dq} + \underbrace{\begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix}}_{\boldsymbol{B}} \boldsymbol{v}_{dq} + \underbrace{\begin{bmatrix} 0 \\ -\frac{\omega_{re}K_E}{L_q} \end{bmatrix}}_{\boldsymbol{e}} \quad (1)$$

where $\boldsymbol{i}_{dq}\left(= [i_d \ i_q]^T\right)$ and $\boldsymbol{v}_{dq}\left(= [v_d \ v_q]^T\right)$ are the d-q axis vectors which denote the stator currents and the stator voltages. ω_{re} is the electric rotor speed. R is the resistance of a stator winding, L_d,L_q are the d-axis and q-axis inductances of a stator winding, and K_E is the permanent magnet flux.

The discrete-time state equation of (1) (with sampling time t_s) is given as follows:

$$\boldsymbol{i}_{dq}(n+1) = \underbrace{\exp(\boldsymbol{A}t_s)}_{\boldsymbol{A}_d} \boldsymbol{i}_{dq}(n) + \underbrace{\int_0^{t_s} \exp(\boldsymbol{A}\tau)d\tau \, \boldsymbol{B}}_{\boldsymbol{B}_d} \boldsymbol{v}_{dq}(n) + \underbrace{\int_0^{t_s} \exp(\boldsymbol{A}\tau)d\tau \, \boldsymbol{e}}_{\boldsymbol{e}_d} \quad (2)$$

The variable n is the discrete-time instant and $n = 0$ points the present time. MPC controller predicts the current response and the torque response by using the equation (2).

A concrete process of selection is shown as follows.

First, the finite-time period from $n = 0$ to $n = N_p$ (N_p is finite positive integer.) is defined as the prediction period as shown in Fig.4. Next, the finite sequence of the inverter voltage vectors $\left(\boldsymbol{V}_0 \text{ to } \boldsymbol{V}_7\right)$ in the prediction period $\boldsymbol{V}_p^{(k)}$ is shown as follows:

$$\boldsymbol{V}_p^{(k)} = \left[\boldsymbol{V}^{(k)}(0), \quad \boldsymbol{V}^{(k)}(1), \ \cdots \ , \boldsymbol{V}^{(k)}(N_p - 1)\right] \quad (3)$$

$$\boldsymbol{V}^{(k)}(n) \in \boldsymbol{V}_0 \sim \boldsymbol{V}_7$$

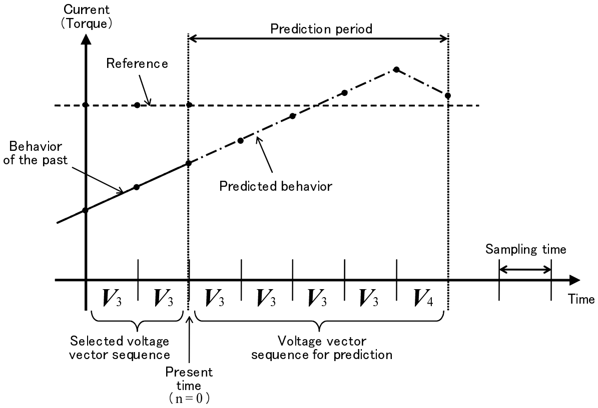


Fig. 4. Prediction of the current(torque) behavior

The superscript $\left(k\right)$ denotes that $\boldsymbol{V}_p^{(k)}$ is k-th sequence of all possible sequences. The coordinate transformation using the rotor position eventually yields the voltage vector sequence in d-q axis $\left([\boldsymbol{v}_{dq}^{(k)}(0), \ \boldsymbol{v}_{dq}^{(k)}(1), \ \cdots \ , \ \boldsymbol{v}_{dq}^{(k)}(N_p - 1)]\right)$.

Then, assuming that the rotor speed is constant in the prediction period, which indicates $\boldsymbol{A}_d$, $\boldsymbol{B}_d$ and $\boldsymbol{e}_d$ in (2) are constants, the equation (2) and the voltage sequence yield the sequence of the predicted current behavior in the prediction period $\boldsymbol{I}_p^{(k)}$ as follows:

$$\boldsymbol{I}_p^{(k)} \ = \ \left[\boldsymbol{i}_{dq}^{(k)}(1) \quad \boldsymbol{i}_{dq}^{(k)}(2) \ \cdots \ \boldsymbol{i}_{dq}^{(k)}(N_p)\right] \quad (4)$$

For the current limiter calculation, if the calculated $|\boldsymbol{i}_{dq}^{(k)}(n)|$ is larger than the maximum current, then $\boldsymbol{v}_{dq}^{(k)}(n)$ in that case will be rejected.

If the predicted current behavior can be obtained like (4), the torque behavior in the future can be predicted. In this paper, torque equation is as follows:

$$T = P_n i_q\{K_E + (L_d - L_q)i_d\} \quad (5)$$

where P_n is number of pole pairs.

Finally, the evaluation function (for example, the equation (6)) is calculated from the predicted current sequence $\boldsymbol{i}_{dq}(n)$ and torque sequence $T(n)$ and the current reference $\boldsymbol{i}_{dq}^*$ and torque reference T^*. In addition, W_T, W_{i_d} and W_{i_q} indicate the weights of each term.

$$J = \sum_{n=1}^{N_p} W_T\left|T^* - T(n)\right| + \sum_{n=1}^{N_p} W_{i_d}\left|i_d^* - i_d(n)\right| + \sum_{n=1}^{N_p} W_{i_q}\left|i_q^* - i_q(n)\right| \quad (6)$$

The prediction and the evaluation mentioned above are done to all possible voltage sequences. If k_{op}-th sequences, $\boldsymbol{V}_p^{(k_{op})}$ and $\boldsymbol{I}_p^{(k_{op})}$, minimize the value of the objective function, MPC controller selects the switching mode corresponding to $\boldsymbol{V}^{(k_{op})}(0)$ ($\in \boldsymbol{V}_0$ to $\boldsymbol{V}_7$) as optimal output. This process is repeated at every sampling time.

C. Design of weight in the evaluation function

An example of evaluation function which was proposed in the equation (6) includes three design parameters (W_T, W_{i_d}, W_{i_q}). These design parameters decide a characteristics of torque and current control.

For instance, setting weight for d, q axis current error W_{i_d}, W_{i_q} to zero gives priority to torque control and abandons current control. Therefore fast torque resoponse at the transient state is achieved and steady state errors of d, q axis current get worse because they are not controlled. It makes copper loss increase at the steady state greatly. On the other hand, setting weight for torque error W_T to zero gives priority to d, q axis current control. As a result, torque response depends on d, q axis current reference. It is, however, difficult to calculate current reference to attain a fastest torque response analytically, because it is necessary to consider various nonlinear factors caused by currents and voltages limitation and overmodulated operation of inverter [8].

In this paper, to obtain a fast torque response at the transient state and stable current response at steady state for the salient pole permanent magnet synchronous motor, the weights are proposed as follows:

$$\begin{cases} W_T = 1 \\ W_{i_d} = 0 \\ W_{i_q} = P_n\{k_e + (L_d - L_q)i_d^*\} \end{cases} \tag{7}$$

First, the weight of d-axis current W_{i_d} is discussed. The reference literature describes that the fast torque response is obtained when a negative d-axis current is applied greatly to use the reluctance torque effectively [4]. In fact, at the torque transient state, the d-axis current should be quite different from reference current in steady state, such as the MTPA condition, for fast torque response. And, in the steady state, if the torque and the q-axis current follow in the reference, the torque equation (5) makes the d-axis current follow the reference automatically. According to the above-mentioned two facts, it is not necessary to control explicitly d-axis current whether at the transient state or at the steady state. As a result, the weight of d-axis current W_{i_d} is set as zero.

Next, the weight of torque W_T and the weights of q-axis current W_{i_q} are discussed. Weights should be set as equation (7) by considering the steady state. where W_{i_q} is a coefficient that converts the q-axis current into the torque. As a result, the torque and q-axis current can be evenly controlled in the dimension of torque.

In addition, the weight of the torque W_T is made to increase when the difference between the predicted torque and the torque reference is large, to obtain a fast torque response. Finally, the weights are proposed as equation (8).

$$\begin{cases} W_T = 1 + \left|T^* - T(n)\right| \\ W_{i_d} = 0 \\ W_{i_q} = \frac{1}{1+\left|T^* - T(n)\right|} P_n\{k_e + (L_d - L_q)i_d^*\} \end{cases} \tag{8}$$

The reference literature deals with another approach of the MPC [12]. This method uses the approximation and parameters obtained empirically, therefore, it can't apply to other PMSMs because it is designed for a particular PMSM. On the other hand, our proposed method can apply to all of salient pole permanent magnet synchronous motors.

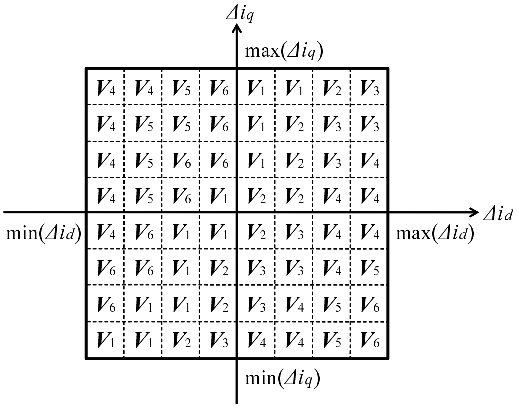
III. IMPLEMENTATION OF MPC CONTROLLER

MPC controller selects voltage vector as the result of predicting current and torque behavior and evaluating by evaluation function at every sampling time. However, it is hard to predict current and torque behavior and calculate evaluation function in such a short time with a current embedded processor performance.

In order to solve the problem, the voltage vectors are stored in the look-up table beforehand, and selected by referring it in every control period. The look-up table is designed beforehand by using simulation results at all the assumed operating points. This table is configured on the multidimensional state space that consists of rotor speed, rotor position, d-q axis currents, d-q axis current references, and so on. The process of designing this table is shown as follows [13].

First, operating range of PMSM is decided from the specifications, for example maximum rotor speed and maximum current. Next, rotor speed and d-q axis current of operating range are quantized from continuous quantities to discrete quantities. The domain of definition is given by the specifications mentioned above, and the width of division are decided by the pilot simulation beforehand. Finally, by predicting the states (current and torque) and evaluating the function at each operating point, the inverter output voltage vector is selected and stored in the multidimensional look-up table. For example, two demensional subspace of the look-up table is shown in Fig.5. Where Δi_d and Δi_q are d-axis and q-axis current errors.

Then, the voltage vector is selected by referring to the table with sensed states in every control period. As a result, the cost of the calculation can be greatly decreased, and real-time implementation of MPC controller becomes possible.



V_4	V_4	V_5	V_6	V_1	V_1	V_2	V_3
V_4	V_5	V_5	V_6	V_1	V_2	V_3	V_3
V_4	V_5	V_6	V_6	V_1	V_2	V_3	V_4
V_4	V_5	V_6	V_1	V_2	V_2	V_4	V_4
V_4	V_6	V_1	V_1	V_2	V_3	V_4	V_4
V_6	V_6	V_1	V_2	V_3	V_3	V_4	V_5
V_6	V_1	V_1	V_2	V_3	V_4	V_5	V_6
V_1	V_1	V_2	V_3	V_4	V_4	V_5	V_6

Fig. 5. Two dimensional subspace of a look-up table
(One example of a look-up table)

IV. EXPERIMENTAL VERIFICATION

In this section, we show the performance of proposed torque control system in experiment. Experiment system is shown in Fig.6. DSP is TMS320C6713-225 made by the Texas Instruments, Inc.

A. Experimental condition

This experiment verifies torque response and *d*-*q* axis current response under the condition that a demanded operating point of PMSM transfers from 0[Nm] to 1[Nm]. Rotor speed is set to be 1000[rpm] constant, because we want to compare the torque response. The block diagram of the experiment is shown in Fig.1. The MTPA control is employed in the current reference generator. To prevent the degauss, the *d*-axis current limiter is set to -6[A].

The parameters of PMSM and the inverter are shown in TABLE I. The configuration parameters of proposed controller are shown in TABLE II. And parameters of look-up table for the MPC controller are shown in TABLE III.

To evaluate our MPC controller, the performance is compared with one of conventional torque control system which consists of MTPA and PI current controller with fixed *d*-axis voltage method as voltage limiter and integration stop method as antiwindup [14]. The configuration parameters of PI current controller are shown in TABLE IV.

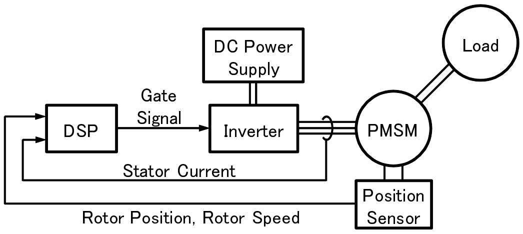


Fig. 6. Block diagram of experimental instrument

TABLE I
PARAMETERS OF PMSM AND INVERTER

Rated power	0.5 [kW]
Rated current	5.0 [A]
Rated speed	2500 [rpm]
Resistance (R)	0.45 [Ω]
Inductance (*d*-axis) (L_d)	4.15 [mH]
Inductance (*q*-axis) (L_q)	16.74 [mH]
EMF constant (k_e)	0.104 [V/(rad/s)]
Rotor inertia	1.5×10^{-3} [kg·m^2]
Number of pole pairs (p)	2
DC-link voltage (V_{DC})	50 [V]

TABLE II
PARAMETERS OF PROPOSED CONTROLLER

Control period	20 [μs]
Prediction time	100 [μs]

B. Experimental result

The experimental results of torque response and *d*-*q* axis current response in proposed torque control system based on MPC are shown in Fig.7, Fig.8. And the results in the conventional torque control system with PI current controller are shown in Fig.9, Fig.10.

First, comparison of Fig.7 and Fig.9 shows that both transient response and steady state error of torque are improved by using MPC controller.

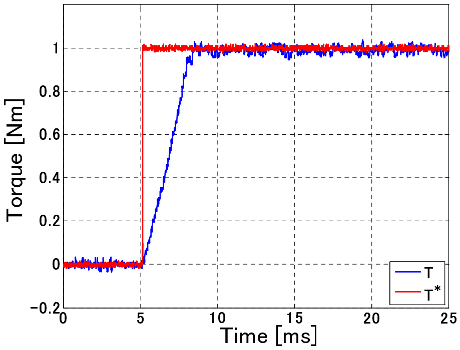


Fig. 7. Experimental Result :
Torque response by proposed MPC controller

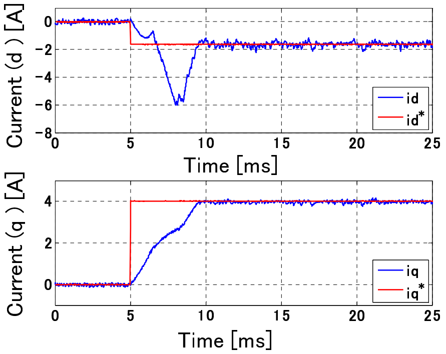


Fig. 8. Experimental Result :
d-*q* axis current response by MPC controller

TABLE III
PARAMETERS OF LOOK-UP TABLE

state	min	max	number of divisions
Δi_d	-6.0[A]	6.0[A]	50
Δi_q	-3.0[A]	3.0[A]	50
θ_{re}	0[deg]	360[deg]	18

TABLE IV
PARAMETERS OF PI CURRENT CONTROLLER

Control period	100 [μs]
PI-Gain of current controller	2000 [rad/s]
Inverter carrier frequency	5 [kHz]

Next, Fig.8 and Fig.10 show d-axis and q-axis current response of each controller. Proposed MPC controller generates negative large d-axis current at transient of torque. It makes proposed MPC torque control system possible to increase reluctance torque to get fast torque response. In addition, it is understood that the current limiter functions properly from d-axis current response. And steady state current shown in Fig.8 is kept stable, though d-axis current is not controlled directly in our proposed system.

These results show that proposed MPC torque controller has faster torque response and more stable steady current response compossible without any switch of control strategy than conventional torque controller.

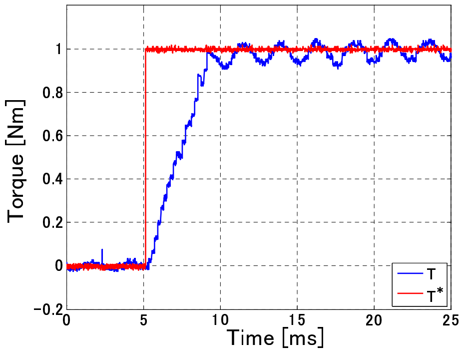


Fig. 9. Experimental Result :
Torque response by PI current controller

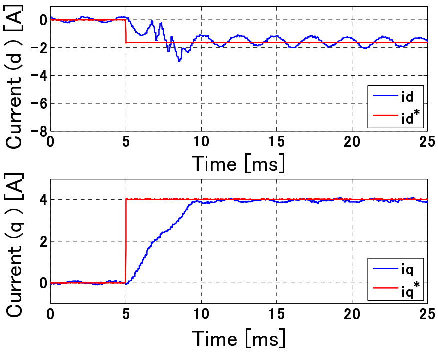


Fig. 10. Experimental Result :
d-q axis current response by PI current controller

V. CONCLUSION

This paper proposed the fast torque control system based on Model Predictive Control. And the evaluation function for MPC controller, which can realize a fast torque response at the transient state and a stable current response at the steady state without any switch of control strategy was discussed. Moreover, its real-time implementation methods using a look-up table, which designed beforehand, was shown.

Good performance of proposed MPC torque control system was shown by experimental results compared with conventional torque control system.

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