Bit and Power Allocation for Power-Line Communications under Nonwhite and Cyclostationary Noise Environment

Naoya SAWADA*, Takaya YAMAZATO[‡] and Masaaki KATAYAMA[‡]
*Department of Electrical Engineering and Computer Science, Nagoya University. Nagoya, Japan.
Phone: +81-52-789-2729, Fax: +81-52-789-3173, E-mail: nsawada@katayama.nuee.nagoya-u.ac.jp

[‡]EcoTopia Science Institute, Nagoya University, Nagoya, Japan.

Phone: +81-52-789-2743, Fax: +81-52-789-3173, E-mail: {yamazato,katayama}@nuee.nagoya-u.ac.jp

Abstract—This manuscript discusses the adaptive OFDM system for narrow-band power-line communications with nonwhite and cyclostationary noise environment. The bit and power allocation algorithm is proposed with the considerations on the cyclostationarity of power-line noise. Numerical evaluation shows that the proposed method can improve the average BER performance under PLC noise environment.

Keywords—Power-line communication (PLC), bit allocation, power allocation, narrow-band, nonwhite, cyclostationary noise.

I. INTRODUCTION

Power-line communication (PLC) systems employ legacy power-lines as communication media. In Japan, the regulations allow both narrow-band (10–450kHz) and wide-band (2–30MHz) for PLC systems. Conventionally, narrow-band was used for communication at relatively low speeds up to 100kbps[1], [2]. This manuscript discusses the narrow-band PLC systems with higher data-rate by using multicarrier modulation with sophisticated bit and power allocation scheme.

The power-lines are not designed for communications, and their characteristics are quite different from those of other communication channels. It is known that power-line noise generated by electric appliances is nonwhite and nonstationary[3]. Especially for narrow band PLC systems, the cyclostationary noise model, where the instantaneous noise power changes periodically and the noise spectrum is decreasing exponentially with frequency, is proposed in [4]. In such noise environment, conventional communication schemes designed for stationary additive white Gaussian noise (AWGN) may not result in a good performance. In other words, the improvement of performance is possible with the considerations on the noise environment of power-line.

In wireless communication, for example, the scheme which allocates bit and power to subcarriers according to the propagation characteristics is proposed in [5], [6]. The basic idea of this scheme can be applied for time varying and nonwhite PLC noise environment.

In PLC, adaptive modulation for OFDM systems under impulsive power-line channel and adaptive interleaver under cyclostationary noise are proposed in [7], [8]. In [7], adaptive bit and power allocation for multipath PLC channels with Gaussian and Middleton's Class A noise is discussed. Since this work is for wide-band PLC systems, cyclostationary features, which are typical in narrow-band PLC systems, are not considered. In [8], the cyclostationary PLC noise is considered in codeword mapping, but the system uses a single modulation index with the uniform power allocation for all symbols and subcarriers.

In this paper, we study the adaptive OFDM system with power allocation together with bit allocation for nonwhite and cyclostationary noise environment. The performance of the proposed system is then evaluated under the measured powerline noise environment.

When power allocation is used, the constraints on the power spectral density (PSD) may present a problem because of the variation of the PSD of each subcarrier. In the case of wideband PLC systems, often regulations define a strict maximum PSD spectral masks. Under such restriction, power allocation is difficult to realize good performance. On the contrary, in the case of narrow-band PLC systems, the spectrum mask is often more relaxed. For example, in Japan, there is only total power limit and no constraint on PSD. This fact justifies our proposed method that employs not only bit allocation but also power allocation.

II. SYSTEM MODEL

The noise in narrowband PLC is modeled by cyclostationary Gaussian noise with a period $T_{\rm AC}/2$, where $T_{\rm AC}$ is cycle duration of the mains voltage[4]. With respect to time domain, we set frame length to be the same as this period. In frequency domain, the frequency band used for communication is from f_0 to f_0+W . With this set up, modulation can be understood as mapping of data to the time-frequency space of the area $(T_{\rm AC}/2)\cdot W$.

Fig. 1 shows the narrowband PLC system discussed in this manuscript. At the transmitter, the OFDM modulator with symbol duration T_s and N subcarriers of the bandwidth Δ is employed. For convenience, we assume that both the number of symbol slots $(K = T_{\rm AC}/2T_s)$ in a frame and the number of subcarriers $(N = W/\Delta)$ are integers. Then, the time-frequency space of $(T_{\rm AC}/2) \cdot W$ is divided into $K \cdot N$ cells as shown in Fig. 2, where $T_s\Delta = 1$. For the ease of description, let us denote the cell corresponding to the k-th symbol $(k = 0 \dots K-1)$ and n-th subcarrier $(n = 0 \dots N-1)$ as "the cell at (k, n)".

Input data is denoted as $a_d = \pm 1 (d = 0, 1, \dots, B-1)$ where B is the amount of data bits transmitted in a frame. Mapping of these data and allocation of power to each cell are based on the statistical information of noise obtained from the receiver. Detailed algorithm of data mapping and power allocation are described in the following Section III.

The signal at the input of the receiver in Fig. 1 is represented as

$$r(t) = s(t) + n(t), \tag{1}$$

where n(t) is cyclostationary noise shown in [4]. The signal component, s(t), is expressed as

$$s(t) = \Re \left[\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \sqrt{2E_{k,n}} b_{k,n} g(t - kT_s) e^{j2\pi(f_0 + n\Delta)t} \right], \quad (2)$$

where $E_{k,n}$ is the power and $b_{k,n}$ is the data symbol of the cell at (k,n). In this manuscript M-ary PSK and M-ary QAM are employed for modulation of $b_{k,n}$, and thus $b_{k,n}$ is written as $b_{k,n} = b_{k,n}^I + j b_{k,n}^Q = \exp(j2\pi m/M)(m=0,\ldots,M-1)$ in the case of PSK, and if QAM is used, $b_{k,n}^I$ and $b_{k,n}^Q$ are represented as

$$b_{k,n}^{I} = \Re[b_{k,n}] = iq - \frac{Q-1}{2}q, \quad i = 0, \dots, Q-1,$$
 (3)

$$b_{k,n}^Q = \Im[b_{k,n}] = jq - \frac{Q-1}{2}q, \quad j = 0, \dots, Q-1,$$
 (4)

respectively. For simplicity, rectangular pulse is used for g(t), as follows:

$$g(t) = \begin{cases} 1, & 0 \le t \le T_s \\ 0, & \text{otherwise.} \end{cases}$$
 (5)

At the output of the OFDM demodulator, the received symbol corresponding to the cell (k, n) is given by

$$r_{k,n} = s_{k,n} + z_{k,n},$$
 (6)

where

$$s_{k,n} = \sqrt{2E_{k,n}}b_{k,n}T_s. \tag{7}$$

The noise component $z_{k,n}$ is Gaussian with zero mean and variance $\sigma_{k,n}^2$, which is a function of time and frequency, as follows:

$$\sigma_{k,n}^2 = \sigma_k^2 \cdot \alpha_n \tag{8}$$

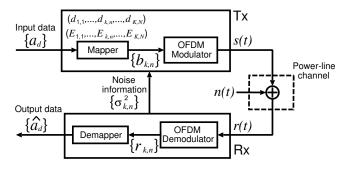


Fig. 1. System Model

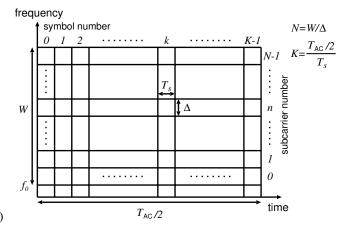


Fig. 2. Time-frequency space in a frame

where

$$\sigma_k^2 = \int_{kT_s}^{(k+1)T_s} \sum_{l=0}^{L-1} A_l |\sin(2\pi t/T_{AC} + \theta_l)|^{n_l} dt, (9)$$

$$\alpha_n = \int_{-\infty}^{(n+1)\Delta} \frac{a}{2} \exp(-a|f|) df.$$
(10)

In the above equations, Eq.(9) denotes time fluctuation of noise variance. The amplitude, peak position in time, and impulsiveness of the noise are represented by A_l , θ_l , and n_l , respectively. In Eq.(10), a is the noise attenuation factor of frequency domain.

If the variation of the values α_n and σ_k^2 within the range $T_s\Delta$ is small, they are approximated to

$$\sigma_k^2 \approx T_s \sum_{l=0}^{L-1} A_l |\sin(2\pi t/T_{AC} + \theta_l)|^{n_l} |_{t=kT_s}, (11)$$

$$\alpha_n \approx \Delta_{\frac{a}{2}} \exp(-a|f|), (12)$$

respectively. The demapper in the Fig. 1 outputs $\{\hat{a}_d\}$, estimates of $\{a_d\}$, based on the values $\{r_{k,n}\}$.

III. BIT AND POWER ALLOCATION ALGORITHM A. Problem Definition

In this manuscript, noise variance $\sigma_{k,n}^2$ of each cell, measured at the receiver are assumed to be perfectly known at

the transmitter. The transmitter chooses modulation schemes and allocates power for KN cells according to the noise information. The number of bits allocated to a cell is chosen from M+1 possible integers, i.e., $\mathcal{D} = \{d_0, d_1, d_2, \dots, d_M\}$ where $d_m < d_{m+1}$. Note that $d_0 = 0$, which means no bit is assigned to the cell. Let the total number of information bits transmitted in a frame be B. The mapper of the transmitter allocates these B bits to each of KN cells, and the modulator assigns power for then. The bits and power allocated to the cell at (k, n) are expressed as $d_{k,n} \in \mathcal{D}$ and $E_{k,n}$ respectively. Thus, the power ratio of signal bit and noise at (k, n) is written

$$\gamma_{k,n} = \frac{E_{k,n}/d_{k,n}}{\sigma_{k,n}^2}. (13)$$

Let the BER (bit-error-rate) of the cell at (k, n) be $P_{k,n}$ and the total number of cells with allocated bits be S_{use} . Then the problem is to find a set of all $d_{k,n}$ and $E_{k,n}$ which minimizes the average BER of the system. This can be formulated as

$$\min_{d_{k,n}, E_{k,n}} \left[P_{\text{ave}} = \frac{1}{S_{\text{use}}} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} P_{k,n} \right], \tag{14}$$

subject to the constraints:

$$\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} E_{k,n} \leq E_{\text{max}}, \qquad (15)$$

$$d_{k,n} \in \mathscr{D}, \qquad (16)$$

$$\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} d_{k,n} = B, \qquad (17)$$

$$d_{k,n} \in \mathscr{D}, \tag{16}$$

$$\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} d_{k,n} = B, \tag{17}$$

where $E_{\rm max}$ is the maximum available total transmission power for one frame. Note that $P_{k,n} = 0$ if $d_{k,n} = 0$.

B. Algorithm Design

The bit and power allocation formulated above by Eqs.(14) -(17) can be solved by discrete water-filling algorithm based on the greedy principle. We start bit allocation under constant transmission power on each cell. After the bit allocation, power is allocated to each cell aiming to make BER of all the cells the same, or in other words, to minimize the worst BER among all the cells. This scheme, based on min-max theorem, is employed as BER can be exponentially decreased by adding power, and the worst cell may dominate total BER. The detail of this algorithm can be written as follows:

- 1) Sort all cells according to the noise power $\sigma_{k,n}^2$.
- 2) For all $k \in \{0, \dots, K-1\}, n \in \{0, \dots, N-1\},$ set $d_{k,n} = d_M$ and $E_{k,n} = E_{\text{max}}/KN$.
- 3) Find the cell which has the worst BER among all cells with $d_{k,n} > d_0$. Then decrease the modulation level of the cell on step. If $d_{k,n} = d_m$ then the next $d_{k,n}$ is
- 4) Continue step 3 while $\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} d_{k,n} > B$. When the total number of bits allocated to all cells is equal to the number of bits to be transmitted, namely

TABLE I PARAMETERS FOR NUMERICAL EVALUATION

Mains Frequency	$1/T_{\rm AC}$	60 [Hz]	
Frame Length	$T_{ m AC}/2$	8.33 [ms]	
Bandwidth	W	432 [kHz]	
Lowest Subcarrier Frequency	f_0	62 [kHz]	
Signal Bandwidth	F	96 [kHz]	
Number of Subcarrier	N	8	
Number of OFDM symbols in a frame	K	450	
Total Data in a frame	В	3600, 7200 [bits]	
Information Rate	$B/(T_{ m AC}/2)$	432, 864 [kbps]	

- $\sum_{k=0}^{K-1}\sum_{n=0}^{N-1}d_{k,n}=B,$ then terminate the bit allocation and go to the next step 5.
- 5) After the bit allocation, distribute the power of the cells (with $d_{k,n} = d_0$) equally to all other cells to make $E_{k,n} = E_{\text{max}}/S_{\text{use}}$. After the distribution, compute P_{ave} that is the BER averaged over all the cells in a frame.
- 6) Change the power of each cell so that $P_{k,n} = P_{\text{ave}}$ for
- all cells with allocated bits. 7) Calculate $E=\sum_{k=0}^{K-1}\sum_{n=0}^{N-1}E_{k,n}.$
 - a) If $E = E_{\text{max}}$, algorithm stops.
 - b) If $E > E_{\text{max}}$, set $E_{k,n} = E_{\text{max}}/S_{use}$ for all the cells with allocated bits, and finish the procedure.
 - c) If $E < E_{\text{max}}$, distribute the surplus power E_{max} E to all used cells so that the increment of SNR on each cell becomes the same. Then newly allocated power to each cell is $(E_{\text{max}} - E)E_{k,n}/E$. This strategy improves BER of each cell equally when relation between SNR and BER is linear.

IV. PERFORMANCE EVALUATION

In this section, we numerically evaluate the performance of the proposed algorithm given in Section IV under the nonwhite and cyclostationary noise environment.

Parameters used for evaluation are shown in Table I. For the modulation schemes, BPSK, QPSK, 16QAM and 64QAM, that is, $\mathcal{D} = \{0, 1, 2, 4, 6\}$ with Gray mapping are employed.

The parameters to calculate the noise variance $\sigma_{k,n}^2$ of (8) are shown in Table II. These parameter sets, which are based on the results of experimental measurements[10], consider only the first three terms (L = 2). The first (l = 0) term describes stationary noise component, while the second and the third (l = 1 and 2) are for cyclostationary and impulse noise components, respectively. Figs. 3 and 4 show examples of the noise waveforms generated by these parameter sets. From these figures we can observe that the noise A represents cyclic and impulsive features of power-line noise, while the noise B is quasi-stationary case.

Since the power of noise and signal is time-variant, average SNR $\overline{E_{k,n}}/\sigma_{k,n}^2$ is defined as follows:

$$\frac{\overline{E_{k,n}}}{\sigma_{k,n}^2} = \frac{E_{\text{max}}}{KN\sum_{k=0}^{K-1}\sum_{n=0}^{N-1}\sigma_{k,n}^2}.$$
 (18)

TABLE II
POWER-LINE NOISE PARAMETERS

parameter	A_0	A_1, θ_1, n_1	A_2, θ_2, n_2	a
noise A	0.230	1.38,6,1.92	$7.13,-35,1.57\times10^5$	1.21×10^{-5}
noise B	0.819	0.160,86,5.00	$0.569,-21,6.90\times10^7$	6.92×10^{-6}

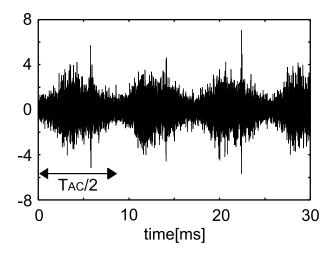


Fig. 3. Computer simulated power-line noise(noise A) [10]

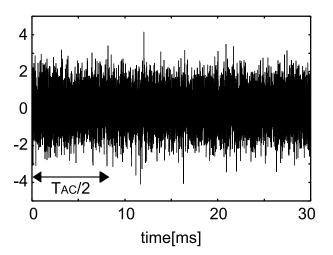


Fig. 4. Computer simulated power-line noise(noise B)[10]

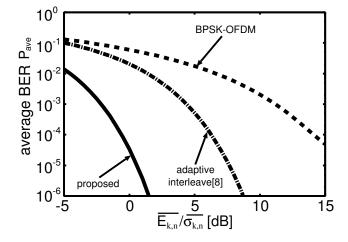


Fig. 5. Average bit error rate performance with lower data-rate (noise A)

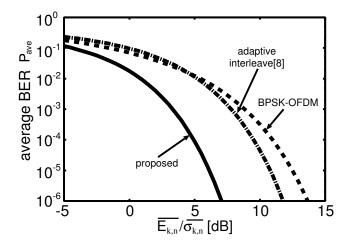


Fig. 6. Average bit error rate performance with lower data-rate (noise B)

For comparison, we also consider conventional BPSK-OFDM (QPSK-OFDM) with adaptive interleaver but without bit and power allocation [8]. This adaptive interleave scheme uses block coding to assign the coded bit to each cell so as to homogenize noise power per each information bit at the transmitter, and employs a maximum ratio combining receiver based on the noise level of each cell. For this scheme, we use QPSK (16QAM) modulation and repetition code whose rate is (1/2).

Figs. 5 and 6 show the average BER performance with lower data-rate (432kbps) under noise A and B, respectively. According to Fig. 5, in the case of noise A, which has large time variant features, the adaptive interleave scheme has 8dB

better than conventional BPSK-OFDM at the BER value of 10^{-4} , and the proposed scheme gives additional 7dB gain than the adaptive interleave scheme. As shown in Fig. 6, in the case of noise B, which has small time variation, the adaptive interleave scheme still has better performance than the conventional BPSK-OFDM, but the improvement is about 1.5dB at the BER value of 10^{-4} , which is smaller than in the case of noise A. In such a case, the proposed scheme still overperforms the conventional BPSK-OFDM and the adaptive interleave scheme by about 6dB and 4.5dB, respectively.

Figs. 7 and 8 show the average BER performance with higher data-rate (864kbps) under noise A and B, respectively. According to Fig. 7, the coding with adaptive interleaver

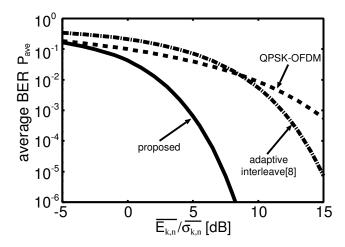


Fig. 7. Average bit error rate performance with higher data-rate (noise A)

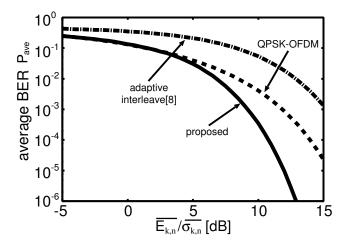


Fig. 8. Average bit error rate performance with higher data-rate (noise B)

realizes performance improvement only when signal is strong, and the scheme has worse performance than the conventional QPSK-OFDM under low SNR. In addition, when noise has smaller time variation, as in Fig. 8, the adaptive interleave scheme underperforms the conventional QPSK-OFDM for all range of SNR. On the contrary to these results, in these higher data-rate environments, it still can be confirmed that the proposed scheme achieves performance improvement.

In addition, Fig. 9 shows the difference of effect between bit allocation and power allocation on the proposed scheme when noise has large time variant features and data rate is 432kbps. Here the same power is allocated to all the cells with data in the "bit allocation only" case. In the "power allocation only", we allocate different power to the cells. Note that the "power allocation only" is the conventional water-filling. According to Fig. 9, bit allocation is more effective than power allocation, and the proposed scheme, which is the conbination of them, is superior to them.

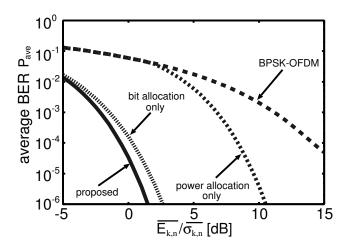


Fig. 9. Comparison of effect on performance improvement of bit allocation and power allocation

V. CONCLUSION

In this paper, we discuss the performance of adaptive OFDM system employing bit and power allocation algorithm for non-white and cyclostationary noise. Because of cyclostationary features of power-line noise, we can repeat the same allocation pattern for each frame with the duration of half cycle of mains voltage. As the result of performance evaluation, we confirm that the proposed method can greatly improve average BER performance when the instantaneous noise power variation is large. In addition, we show that the proposed method is effective even if the noise is almost quasi-stationary but nonwhite.

Propagation of narrow-band PLC signal is relatively flat and stationary compared with that of wide-band PLC[9]. Thus, this paper considers non-white and cyclostationary features of the noise but not of the propagation. If the transmitter knows these features of propagation, the same procedure shown in the proceding section can be used by using noise to signal level instead of noise level.

In the proposed algorithm, the allocation of power is done regardless of uniformity of power in time and frequency. Thus the instantaneous signal power may have large variation, and there may be some subcarriers with larger power than other subcarriers. The allocation with the considerations on the uniformity of power, it necessary, is one of future works.

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