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Channel Estimation and Tracking Schemes for the Pulse-Shaping OFDM Systems

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***Abstract*—Robust channel estimation scheme is essential for pulse-shaping OFDM systems in the multipath mobile environment. This paper proposes three types of channel estimation schemes for the general class of pulse-shaping OFDM systems. The first two types are suboptimal low-complexity maximum likelihood estimators. The last type is adaptive Kalman filter channel estimator. We numerically evaluate the performance of each estimator using computer simulation.**

***Index Terms*—pulse-shaping OFDM, BFDM/OQAM, channel estimation and tracking, Kalman filter.**

I. INTRODUCTION

Recently, there has been an increased interest in using OFDM systems in a mobile environment. They are considered to be promising candidates for the next generation mobile communications. The mobile environment causes time-frequency dispersion. A signal distortion caused by the time-frequency dispersion depends crucially on the time-frequency localization of the pulse-shaping filter [1]. For instance, conventional OFDM systems that use rectangular-type pulse with guard interval can prevent intersymbol interference (ISI), but do not combat intercarrier interference (ICI). The design of time-frequency well-localized pulse-shaping OFDM filter is, therefore, an active area of research [1],[2].

By introducing well-localized transmitter pulse-shapes, it is possible to reduce the ISI/ICIs [3]. The performance of such pulse-shaping OFDM system in dispersive time-varying channels depends critically on the time-frequency localization (TFL) of the transmitter and receiver filters. Ideally, the system should use time-frequency well-localized transmitter pulse while keeping maximal spectral efficiency. For large number of subcarriers the spectral efficiency ρ of the OFDM system may be approximated by $\rho = 1/(TF)$ symbols per second per Hertz, here T is a symbol period and F is a subcarrier separation. From Balian-Low theorem [4], it is obvious that maximal spectral efficiency is $\rho_{max} = 1$. However, at the maximal spectral efficiency, it is hard to find a system with both of transmitter and receiver (dual) pulses have good TFL.

A number of interesting pulse-shaping OFDM systems are proposed recently to deal with the problem. Biorthogonal Frequency Division Multiplexing systems based on Offset QAM (BFDM/OQAM) allow the construction of well-localized dual pulses at the maximal spectral efficiency ,i.e. $TF = 1$, which is desirable in high data-rate applications [2], [5], and [6]. BFDM/OQAM relaxes orthogonality condition and uses linearly independent subcarriers. As a result, it allows a broader class of pulse-shapes, particularly the Gaussian transmitter pulse, which has the best TFL. Therefore, we can consider BFDM/OQAM as a generalization of pulse-shaping OFDM.

The use of time-frequency well-localized transmitter pulses for BFDM/OQAM systems results in a decrease in ISI/ICI, and the ISI/ICI may be neglected in some practical cases [3], [7]. However, in highly mobile environments ISI/ICIs increase and can no longer be neglected. A careful investigation of statistical properties of ISI/ICIs and proper channel estimation schemes thereof are needed for further improvement of the systems [8].

Optimal bayesian channel estimation scheme is considered in [8]. However, it appears to be computationally prohibitively complex. This paper proposes three suboptimal low-complexity estimation scheme. Firstly, we propose estimators that employ time-frequency separated impulses as a training sequence. However, the training sequence becomes too long deteriorating overall system efficiency. We can decrease the separation between impulses to increase the efficiency. However, with this type of estimation scheme, we pay large cost in computational complexity to achieve little increase in the efficiency. To deal with this problem we propose indirect estimator which uses only one training OFDM symbol. It estimates the actual physical channel and then calculates so-called channel parameters with analytic formulae. Lastly, we construct adaptive channel estimation and tracking schemes based on Kalman filter. The idea behind this scheme is similar with indirect estimator.

Formulation of the problem in our paper is quite general, thus, the proposed estimation schemes can be applied to any class of OFDM systems.

This paper is organized as follows. Section II introduces statistics of the typical mobile wireless channel. Next, Section III briefly reviews the for general class of pulse shaping OFDM system, defines channel parameters, Then, Sections IV and V present the proposed estimators. Lastly, Section VI discusses numerical study.

II. MOBILE MULTIPATH CHANNEL

In this paper we consider a Rayleigh mobile channel with exponentially decaying multipath intensity profile, with Jakes'

Doppler power spectrum. The maximum Doppler shift is $f_d = f_c v/c$, where f_c is carrier frequency, v is vehicle speed, and c is the speed of light. If transmitted low-pass signal has bandwidth $W \gg f_d$, then we can consider low-pass frequency response of the channel within the band $(-\frac{1}{2}W; \frac{1}{2}W)$. Then, it is easy to see that the time-varying impulse response $h(\tau;t)$ can be approximated as the tapped-delay-line:

$$h(\tau;t) = \sum_{m=1}^{M} h_m \xi_m(t) \delta\left(\tau - \frac{m}{W}\right) \tag{1}$$

where, h_ms are tap weights, $\xi_m(t)$ is a flat fading process at each tap, and $\delta(\tau)$ is the delta function. We make a standard assumption of wide-sense stationary uncorrelated scattering (WSSUS). Each tap weights are complex gaussian random variables with average energy:

$$\sum_{m:\tau_m \in (\tau, \tau+d\tau)} \mathbf{E}\{|h_m|^2\} = \frac{1}{\tau_0} e^{-\tau/\tau_0} d\tau \tag{2}$$

where $\mathbf{E}\{\cdot\}$ is the mathematical expectation, τ_0 is root mean square delay spread. Autocorrelation and crosscorrelation functions of the fading processes $\xi_m(t) = \xi_m^{\mathscr{R}}(t) + j\xi_m^{\mathscr{I}}(t)$ are all identical and given by [9]:

$$\begin{aligned} R_{\xi_m^{\mathscr{R}} \xi_m^{\mathscr{R}}}(\Delta t) &= \mathbf{E}\{\xi_m^{\mathscr{R}}(t+\Delta t)\xi_m^{\mathscr{R}}(t)\} = J_0(2\pi f_d \Delta t) \\ R_{\xi_m^{\mathscr{I}} \xi_m^{\mathscr{I}}}(\Delta t) &= \mathbf{E}\{\xi_m^{\mathscr{I}}(t+\Delta t)\xi_m^{\mathscr{I}}(t)\} = J_0(2\pi f_d \Delta t) \\ R_{\xi_m^{\mathscr{R}} \xi_m^{\mathscr{I}}}(\Delta t) &= R_{\xi_m^{\mathscr{I}} \xi_m^{\mathscr{R}}}(\Delta t) = 0 \end{aligned} \tag{3}$$

where, $J_0(\cdot)$ is 0-order Bessel function of the first kind.

III. BFDM/OQAM SYSTEM, ISI/ICI AND CHANNEL PARAMETERS

Let us define a set $\mathbf{G}$ that consists of the pairs of translations and modulations of a real-valued transmitter pulse $g(t)$:

$$\mathbf{G} = \begin{cases} g_{k,l}^{\mathscr{R}}(t) = g(t-lT)e^{j2\pi Fkt} \\ g_{k,l}^{\mathscr{I}}(t) = g(t-lT+T/2)e^{j2\pi Fkt} \end{cases}$$

where, T is the symbol period, F is subcarrier separation, $k \in \mathbb{Z}$ is subcarrier number, and $k \in \mathbb{Z}$ is symbol number. From the Gabor theory [4], if $TF \geq 1$ and $g(t) \in \mathbf{L}^2$ then there exist a set $\mathbf{W}$ (dual bases), constructed as $\mathbf{G}$, using pulse $w(t)$

$$\mathbf{W} = \begin{cases} w_{k,l}^{\mathscr{R}}(t) = w(t-lT)e^{-j2\pi Fkt} \\ w_{k,l}^{\mathscr{I}}(t) = w(t-lT+T/2)e^{-j2\pi Fkt} \end{cases}$$

that satisfies the biorthogonality conditions in [2]. For more detail discussions on this topic, please refer to [4], and [5].

The baseband BFDM/OQAM signal can be expressed as:

$$x(t) = \sum_{k=0}^{K-1} \sum_{l=-\infty}^{\infty} \left\{ c_{k,l}^{\mathscr{R}} g_{k,l}^{\mathscr{R}}(t) + j c_{k,l}^{\mathscr{I}} g_{k,l}^{\mathscr{I}}(t) \right\} \tag{4}$$

where, K is the number of subcarrier, and $c_{k,l}^{\mathscr{R}}, c_{k,l}^{\mathscr{I}}$ are real and imaginary parts of the transmitted symbols $c_{k,l}$, respectively. Transmitted signal propagates through mobile multipath channel described in Section II. Received noisy signal may be written as:

$$s(t) = \sqrt{E_s} \sum_m h_m x\left(t - \frac{m}{W}\right) \xi_m(t) + n(t) \tag{5}$$

where E_s is signal energy per channel use, $\xi_m(t)$ is flat fading process of the m-th path and $n(t)$ is additive white Gaussian noise (AWGN) within signal bandwidth with the variance $N_0/2$ per complex dimension.

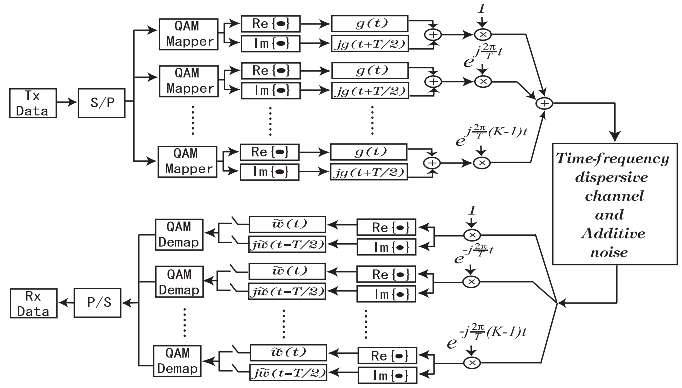

Fig. 1. BFDM/OQAM system, $\tilde{w}(t) = w(-t)$

Block diagram of the BFDM/OQAM system is shown in Figure 1. $\tilde{w}(t) = w(-t)$ can be seen as the matched filter to the transmitter pulse $g(t)$. If $(g(t)$ and $w(t))$ appear to be equal, then the system simply turns to be the conventional pulse-shaping OFDM/OQAM.

Within the *coherence time* of the channel, time variance of the channel parameters can be neglected. Then received symbols $d_{k,n} = d_{k,n}^{\mathscr{R}} + jd_{k,n}^{\mathscr{I}}$ can be rewritten as [8]:

$$d_{k,n}^{\mathscr{R}} = \sum_{i,j} \left\{ h_{i,j,\mathscr{R}}^{k,\mathscr{R}} c_{k\ominus i,n-j}^{\mathscr{R}} + h_{i,j,\mathscr{I}}^{k,\mathscr{R}} c_{k\ominus i,n-j}^{\mathscr{I}} \right\} + n_{k,n}^{\mathscr{R}} \tag{6}$$

$$d_{k,n}^{\mathscr{I}} = \sum_{i,j} \left\{ h_{i,j,\mathscr{R}}^{k,\mathscr{I}} c_{k\ominus i,n-j}^{\mathscr{R}} + h_{i,j,\mathscr{I}}^{k,\mathscr{I}} c_{k\ominus i,n-j}^{\mathscr{I}} \right\} + n_{k,n}^{\mathscr{I}} \tag{7}$$

where, $\ominus$ is a modulo-K subtraction, $h_{i,j,\mathscr{R}}^{k,\mathscr{R}}$, $h_{i,j,\mathscr{I}}^{k,\mathscr{R}}$, $h_{i,j,\mathscr{R}}^{k,\mathscr{I}}$, and $h_{i,j,\mathscr{I}}^{k,\mathscr{I}}$ are so-called *channel parameters*, $n_{k,n}^{\mathscr{R}}$ and $n_{k,n}^{\mathscr{I}}$ are the noise components. The summations in (6) and (7) are taken over some rectangular area [8]: $-\theta \leq i \leq \theta$ and $-\gamma \leq j \leq \gamma$. The channel parameters show amount of interference and are expressed [8]:

$$h_{i,j,\mathscr{R}}^{k,n,\mathscr{R}} = \int_{-\infty}^{\infty} dt \Re \left\{ \sum_m h_m \xi_m(t) e^{j2\pi F(k-i)(t-\frac{\alpha}{2}-\frac{m}{KF})} \right. \tag{8}$$

$$\left. e^{-j2\pi Fk(t-\frac{\alpha}{2})} \right\} g(t-(n-j)T - \frac{m}{KF}) w(t-nT) \tag{9}$$

The other channel parameters are expressed in the similar way.

IV. PROPOSED CHANNEL ESTIMATORS

We have derived the second-order statistics of the channel parameters in [8]. As long as the channel parameters are gaussian random variables, we can fully describe their prior joint distribution. Therefore, it is natural to consider the maximum a posteriori (MAP) method for the channel estimation. In [8] we have derived an optimal MAP estimator. However, due to large number of the channel parameters, its computational complexity appeared to be prohibitively complex. Specially, calculation of the correlation matrix of the channel parameters take huge computational resources. Therefore, in this section we propose two types of low complexity estimators.

A. Low Complexity Direct Estimators

In this subsection, we derive low complexity estimators that employ time-frequency well separated impulses. With this kind of training sequence, MAP estimation algorithm requires only a few number of elements of the correlation matrix $\mathbf{R}$ of channel parameters, defined in Subsection 4.2 of [8]. We choose separations of the training impulses in the time-frequency lattice, so that every single received symbol during training period will be interfered by only one impulse.

Firstly, we derive *double-term* estimator. Let us assume that a pulse with amplitude $1+j$ is transmitted at node α,β on the time-frequency lattice, We assume that any received symbol is interfered by only one transmitted symbol. That is, all of the adjacent symbols $c_{\alpha+k,\beta+l}$ are equal to zero, at least within the square $-2\psi \le k \le 2\psi$ and $-2\phi \le l \le 2\phi$. Then, real part of the received symbol (6) is expressed as:

$$d^{\mathscr{R}}_{\alpha+k,\beta+l} = h^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{R}} + h^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{I}} + n^{\mathscr{R}}_{\alpha+k,\beta+l} \tag{10}$$

and, MAP estimates can be checked:

$$\begin{aligned} \hat{h}^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{R}} &= \frac{2d^{\mathscr{R}}_{\alpha+k,\beta+l}(\sigma_1+\rho\sqrt{\sigma_1\sigma_2})}{N_0+2(\sigma_1+\sigma_2+\rho\sqrt{\sigma_1\sigma_2})} \\ \hat{h}^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{I}} &= \frac{2d^{\mathscr{R}}_{\alpha+k,\beta+l}(\sigma_2+\rho\sqrt{\sigma_1\sigma_2})}{N_0+2(\sigma_1+\sigma_2+\rho\sqrt{\sigma_1\sigma_2})} \end{aligned} \tag{11}$$

where, $\sigma_1 = \mathbf{E}\{h^{\alpha-k,\mathscr{R}\,2}_{k,l,\mathscr{R}}\}$, $\sigma_2 = \mathbf{E}\{h^{\alpha-k,\mathscr{R}\,2}_{k,l,\mathscr{I}}\}$, and $\rho = \mathbf{E}\{h^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{R}} h^{\alpha-k,\mathscr{R}}_{k,l,\mathscr{I}}\}/\sqrt{\sigma_1\sigma_2}$. Other channel parameter estimates can be derived in the same way. We observe that estimation requires $6(2\gamma+1)(2\theta+1)K$ elements of $\mathbf{R}$.

We may also derive so-called *single-term* (ST) estimator. It sends impulses 1 and j separately (DT estimator sends 1 and j simultaneously as $1+j$). ST estimator are more accurate and simpler. However, it uses twice longer training sequence. To save space we omit derivation of the ST estimator.

Due to structure of employed training sequence, operational complexities of ST and DT estimators (11) are considerably low compared with the optimal estimator derived in [8]. However, the use of time-frequency separated impulses results in a long training sequence, degrading the overall efficiency of a system. This is the main shortcoming in the estimators. Making the separation shorter the received symbol will be interfered with more than one transmitted impulses, and estimation requires off-diagonal elements of $\mathbf{R}$. If we still use the same estimator for this case, the estimation error increases inevitably. The optimal point should be carefully investigated. In Section VI we numerically evaluate how the separation of the impulses affects the estimation error.

B. Low Complexity Indirect Estimator

A proposed estimator in this subsection estimates the channel parameter indirectly. Firstly, we estimate a weight vector $\mathbf{h}_{tap} = [h_1\xi_1(0),\cdots,h_M\xi_M(0)]^T$ of the tapped-delay-line model, assuming the estimation is conducted at $t=0$. Then the channel parameters are calculated by (8) using the estimate of $\mathbf{h}_{tap}$. As training signal we use a conventional OFDM signal (i.e. $g(t)$ is rectangular) with cyclic prefix greater than M/W. We transmit one OFDM symbol $\mathbf{c}_{tr} = [c_0,\cdots,c_{N-1}]^T$ with the cyclic prefix. It is easy to check that, the received symbol at carrier k can be expressed:

$$d_k = \sum_{m=1}^{M} h_m\xi_m(0)c_k e^{-j\frac{2\pi}{N}mk} + n_k = H_k c_k + n_k \tag{12}$$

where, H_k is the k-th element of the N-point FFT of $\mathbf{h}_{tap}$, n_k is a additive noise component with the variance $N_0/2$ per complex dimension.

Again, we employ the MAP method to estimate $\mathbf{H} = [H_0,\cdots,H_{N-1}]$. Then, the estimate can be expressed as:

$$\hat{\mathbf{H}} = \arg\min_{\mathbf{H}} \left(\mathbf{H}^*\mathbf{Q}^{-1}\mathbf{H} + \frac{|\mathbf{d}-\mathbf{H}\mathbf{c}_{tr}|^2}{N_0} \right) \tag{13}$$

where, $\mathbf{d} = [d_0,\cdots,d_{N-1}]^T$ and $\mathbf{Q} = \mathbf{E}\{\mathbf{H}\mathbf{H}^H\}$ is the correlation matrix of $\mathbf{H}$ with diagonal elements $q_{ii}, i = 0,\cdots,N-1$. The solution of (13) verifies:

$$\hat{\mathbf{H}} = \mathbf{d}\left(\frac{N_0}{2}\mathbf{Q}^{-1} + diag(\bar{q}_1,\cdots,\bar{q}_{N-1}) \right)^{-1} \tag{14}$$

where, $\bar{q}_i = q_{ii}N_0/2 + c_i$.

V. KALMAN FILTER CHANNEL ESTIMATION AND TRACKING

In this section we propose a channel estimation scheme for the BFDM/OQAM system based on a Kalman filter. The idea behind our estimation scheme is to track the time-variance, i.e. fading process $\xi_m(t)$, of each tap of the channel and recalculate channel parameters with analytic equations (8) as for the case of indirect estimators. For the state vector of the Kalman filter we take samples of the fading process on each tap of the channel within a certain time window of fixed length.

A. Process Equation

We consider the following forward linear predictor on the fading process $h_m\xi_m(n)$:

$$\begin{aligned} \hat{\xi}_m(n+1) &= h_m \sum_{i=0}^{L-1} \omega_i \xi_m(n-i) = \vec{\omega}^T \bar{\mathbf{x}}_m(n), \qquad (15) \\ \vec{\omega} &= [\omega_0, \omega_1, \cdots, \omega_{L-1}]^T, \\ \bar{\mathbf{x}}_m(n) &= [h_m\xi_m(n),\cdots,h_m\xi_m(n+1-L)]^T, \end{aligned}$$

Optimal values (minimum mean-square error) of the tap weights can be calculated using Wiener-Hopf equation $\mathbf{S}\vec{\omega} = \mathbf{s}$, where

$$\mathbf{S} = \mathbf{E}\{\bar{\mathbf{x}}_m(n)\bar{\mathbf{x}}_m(n)^H\},\ \mathbf{s} = \mathbf{E}\{\bar{\mathbf{x}}_m(n)\xi_m(n+1)^*\}, \tag{16}$$

It is obvious that, as long as $\xi_m(n)$s (for all m) have the same autocorrelation function, the tap weights are identical for all m, no matter of the value h_m. If we denote the forward predictor error with $e_m(n+1) = h_m\hat{\xi}_m(n+1) - h_m\xi_m(n+1)$, then

$$\bar{\mathbf{x}}_m(n+1) = \bar{\mathbf{F}}_m\bar{\mathbf{x}}_m(n) + \bar{v}_m(n+1), \tag{17}$$

$$\bar{\mathbf{F}}_m = \begin{pmatrix} \omega_0 & \omega_1 & \cdots & \omega_{L-2} & \omega_{L-1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix},$$

$$\bar{v}_m(n+1) = [e_m(n+1), 0, \cdots, 0]^T \tag{18}$$

If we denote, $\bar{\mathbf{x}}(n+1) = [\bar{\mathbf{x}}_1(n+1)^T, \bar{\mathbf{x}}_2(n+1)^T, \cdots, \bar{\mathbf{x}}_M(n+1)^T]^T$, $\bar{v}(n+1) = [\bar{v}_1(n+1)^T, \bar{v}_2(n+1)^T, \cdots, \bar{v}_M(n+1)^T]^T$ and $\bar{\mathbf{F}} = diag\{\bar{\mathbf{F}}_1, \bar{\mathbf{F}}_2, \cdots, \bar{\mathbf{F}}_M\}$ then it can be checked that,

$$\bar{\mathbf{x}}(n+1) = \bar{\mathbf{F}}\bar{\mathbf{x}}(n) + \bar{v}(n+1), \tag{19}$$

One can easily see that (19) gives us that process equation for a Kalman filter with state vector $\bar{\mathbf{x}}(n)$. Similar construction is proposed in [10]. However, it uses additional RLS filter to estimate $\vec{\omega}$. Our approach is to employ linear forward predictor. For convenience which will be clear in the next subsection, we define the *state vector* of the Kalman filter by merging the vector $\bar{\mathbf{x}}(n+1)$ with its conjugate:

$$\mathbf{x}(n) = [\bar{\mathbf{x}}(n)^T, \bar{\mathbf{x}}(n)^H]^T \tag{20}$$

Then, from (19) the *process equation* for the Kalman filter can be expressed as:

$$\mathbf{x}(n+1) = \mathbf{F}\mathbf{x}(n) + v(n+1), \tag{21}$$

where, $\mathbf{F} = [\bar{\mathbf{F}}, \bar{\mathbf{F}}^*]$ is the transition matrix, and $v(n+1) = [\bar{v}(n+1)^T, \bar{v}(n+1)^H]^T$ is a noise component.

B. Measurement Equation

We rewrite channel parameters in (8) as:

$$h^{k,\mathscr{R}}_{i,j,\mathscr{R}} = \Re\left\{\sum_m h_m\xi_m(n)e^{-j\frac{2\pi}{K}(k-i)m}gw(m,i,j)\right\} \tag{22}$$

$$gw(m,i,j) = e^{j\pi Fi\alpha}\int_{-\infty}^{\infty} e^{-j2\pi Fit}\cdot\cdot g(t+jT-\frac{m}{KF})w(t)dt$$

The channel parameters $h^{k,\mathscr{I}}_{i,j,\mathscr{R}}$, $h^{k,\mathscr{R}}_{i,j,\mathscr{I}}$ and $h^{k,\mathscr{I}}_{i,j,\mathscr{I}}$ can be expressed in the same way using similar notion of $gw_{T/2}(m,i,j)$, $g_{T/2}w(m,i,j)$, and $g_{T/2}w_{T/2}(m,i,j)$, respectively.

Then, from (6) and (7) it can be checked that:

$$d_{k,n} - n_{k,n} = \sum_m h_m\xi_m(n)\tilde{a}_{k,m}(n) + \sum_m h_m^*\xi_m(n)^*\tilde{b}_{k,m}(n) \tag{23}$$

where, $d_{k,n} = d^{\mathscr{R}}_{k,n} + jd^{\mathscr{I}}_{k,n}$, $n_{k,n} = n^{\mathscr{R}}_{k,n} + jn^{\mathscr{I}}_{k,n}$, and

$$\tilde{a}_{k,m}(n) = \frac{\sqrt{E_s}}{2}\sum_{i,j} e^{j\frac{2\pi}{K}(k-i)m}\Big\{c^{\mathscr{R}}_{k\ominus i,n-j}[gw(m,i,j) + jgw_{T/2}(m,i,j)] - c^{\mathscr{I}}_{k\ominus i,n-j}[g_{T/2}w_{T/2}(m,i,j) - jg_{T/2}w(m,i,j)]\Big\}$$

$$\tilde{b}_{k,m}(n) = \frac{\sqrt{E_s}}{2}\sum_{i,j} e^{-j\frac{2\pi}{K}(k-i)m}\Big\{c^{\mathscr{R}}_{k\ominus i,n-j}[gw(m,i,j)^* + jgw_{T/2}(m,i,j)^*] + c^{\mathscr{I}}_{k\ominus i,n-j}[g_{T/2}w_{T/2}(m,i,j)^* - jg_{T/2}w(m,i,j)^*]\Big\}$$

We can rewrite (23) in matrix form,

$$\mathbf{d}(n) - \mathbf{n}(n) = \tilde{\mathbf{A}}(n)\vec{\xi}(n) + \tilde{\mathbf{B}}(n)\vec{\xi}^*(n) \tag{24}$$

where,
$\mathbf{d}(n) = [d_{1,n}, d_{2,n}, \cdots, d_{K,n}]^T$, $\mathbf{n}(n) = [n_{1,n}, n_{2,n}, \cdots, n_{K,n}]^T$,

$$\tilde{\mathbf{A}}(n) = \{\tilde{a}_{k,m}(n)\}_{\substack{1\le k\le K,\\ 1\le m\le M}}, \ \tilde{\mathbf{B}}(n) = \{\tilde{b(n)}_{k,m}\}_{\substack{1\le k\le K,\\ 1\le m\le M}},$$

$$\vec{\xi}(n) = [h_1\xi_1(n), h_2\xi_2(n), \cdots, h_M\xi_2(M)]^T,$$

Define $K \times ML$ matrices $\mathbf{A}(n) = \tilde{\mathbf{A}}(n) \otimes [1, 0, ..., 0]$ and $\mathbf{B}(n) = \tilde{\mathbf{B}}(n) \otimes [1, 0, ..., 0]$, where $\otimes$ is the Kronecker product. Then, using (24), we derive a *measurement equation*:

$$\mathbf{d}(n) = \mathbf{A}(n)\bar{\mathbf{x}}(n) + \mathbf{B}(n)\bar{\mathbf{x}}^*(n) + \mathbf{n}(n) = \mathbf{C}(n)\mathbf{x}(n) + \mathbf{n}(n)$$

where, $\mathbf{C}(n) = [\mathbf{A}(n), \mathbf{B}(n)]$ and $\mathbf{x}(n)$ is the state vector defined in (20).

Since we have defined the state vector, and derived the process and measurement equations, we are ready to construct a Kalman filter to estimate the state vector $\mathbf{x}(n)$.

VI. NUMERICAL SIMULATION

As a measure for the mismatch of the proposed estimator we choose a total mean square error(MSE) defined as:

$$MSE = \sum_{k=0}^{K-1}\sum_{i,j}\mathbf{E}\{(\hat{h}^{k,\mathscr{R}}_{i,j,\mathscr{R}} - h^{k,\mathscr{R}}_{i,j,\mathscr{R}})^2 + \tag{25}$$
$$+(\hat{h}^{k,\mathscr{I}}_{i,j,\mathscr{R}} - h^{k,\mathscr{I}}_{i,j,\mathscr{R}})^2 + (\hat{h}^{k,\mathscr{R}}_{i,j,\mathscr{I}} - h^{k,\mathscr{R}}_{i,j,\mathscr{I}})^2 + (\hat{h}^{k,\mathscr{I}}_{i,j,\mathscr{I}} - h^{k,\mathscr{I}}_{i,j,\mathscr{I}})^2\}$$

where, $\hat{h}^{k,\mathscr{I}}_{i,j,\mathscr{R}}$, $\hat{h}^{k,\mathscr{R}}_{i,j,\mathscr{R}}$, $\hat{h}^{k,\mathscr{R}}_{i,j,\mathscr{I}}$, and $\hat{h}^{k,\mathscr{I}}_{i,j,\mathscr{I}}$ are the estimates of the channel parameters.

Transmitter pulse is the Gaussian pulse. The symbol period is $T = 3.2$ s and $TF = 1$ (maximal spectral efficiency). The number of subcarriers is $K = 64$. 200 simulation runs are averaged to plot the total MSE. The length of the training sequence for the single-term and double-term estimators with separation between impulses $[\psi, \phi]$=[3,2] is 80 OFDM symbols and 40 OFDM symbols, respectively. On the other hand, the length of the training sequence for indirect estimator is 1 OFDM symbol plus cyclic prefix.

Fig. 2 shows the total MSE of the channel estimators versus the speed of the vehicle, when SNR=30dB, the rms delay spread is $\tau_0 = 0.5$ s. The time-frequency separations $[\psi, \phi]$ between impulses in the training sequence are $[3,3]$ (curves $-*-$), $[2,2]$ (curves $-\diamond-$), and $[2,1]$ (curves $-\square-$). It can be observed that for the indirect estimator, the total MSE at speed 300km/h is 7.1dB higher than the one achieved at the stationary channel. For the single-term and double-term estimators the difference is 2.2dB and 4.2dB respectively.

Fig. 3 demonstrates the performance of Kalman filter channel estimation and tracking scheme. The channel is estimated

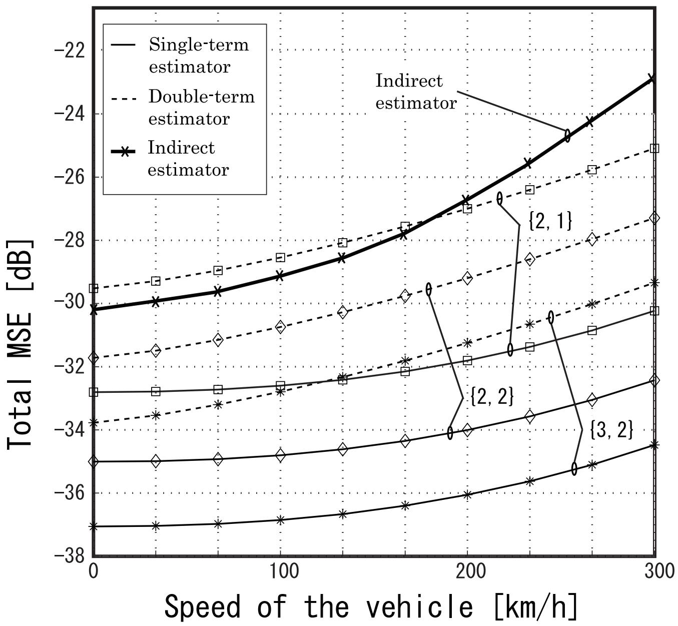

Fig. 2. The total MSE of channel estimators versus the speed of the vehicle, when SNR=30dB, $\tau_0 = 0.5$ s, time-frequency separation between impulses $[\psi, \phi] = [3,2], [2,2]$, and $[2,1]$.

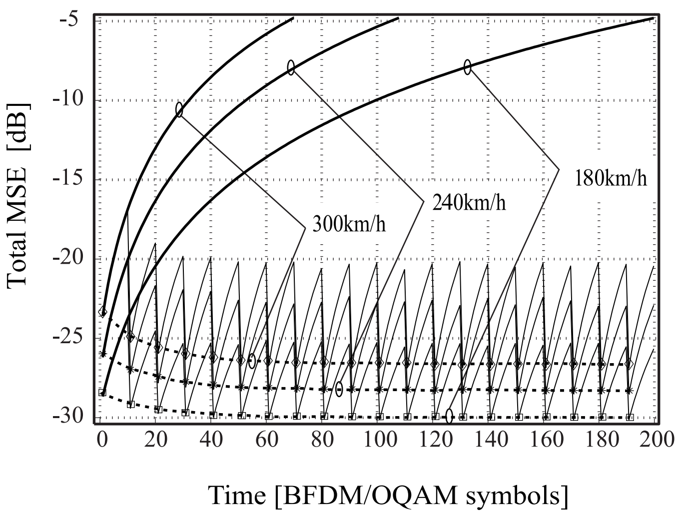

Fig. 3. The total MSE of channel estimators versus the time, when SNR=30dB, $\tau_0 = 0.5$ s, $L = 6$.

every 10OFDM symbols. We set the depth of the linear predictor $L = 6$. Dotted curves shows how the channel parameters deviates from their estimated values if no tracking is implemented. Solid curves shows the total MSE performance of the Kalman filter. In order to get faster convergence, we firstly estimate the channel with indirect estimator. Then, as an initial value for the state vector, we set all components of $\bar{\mathbf{x}}_m(n)$ in (15) to the estimated values of $h_m \xi_m(n)$. Thus, the total MSE at time 0 is the same with indirect estimator case. If converged, Kalman filter performs better than the indirect estimator. One can observe, for instance, at $v = 300$km/h the difference is 3dB. We believe, this is an effect of time averaging (with some weights in Kalman gain vector) nature the proposed Kalman filter.

VII. Conclusions

Optimal bayesian estimator for pulse-shaping OFDM system, which we have studied previously appears to be computationally prohibitively complex. To reduce complexity we have proposed three estimators. Direct estimators that employ time-frequency separated impulses as a training sequence. However, these training sequences become long, deteriorating system efficiency. In the direct estimators we can make the training sequence somewhat shorter with considerable cost in estimation accuracy and complexity. To tackle this problem we have proposed an indirect estimator which requires only one training symbol. Lastly, we have proposed adaptive estimation and tracking scheme based on Kalman filter.

We have numerically analyzed the total MSE performance of the estimators. As expected the single-term estimator is most accurate. Surprisingly, the indirect estimators perform well. Specially, in a high SNR region it performs better than double-term estimator with shortened training sequence. In high speed region the performance of the indirect estimator deteriorates faster. Kalman filter channel tracking appears to be reasonable alternative for the indirect estimator and if converges, it performs better.

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