

Identification of Probability Weighted Multiple ARX Models and Its Application to Behavior Analysis

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Abstract—This paper proposes a Probability weighted ARX (PrARX) model wherein the multiple ARX models are composed by the probabilistic weighting functions. As the probabilistic weighting function, a ‘softmax’ function is introduced. Then, the parameter estimation problem for the proposed model is formulated as a single optimization problem. Furthermore, the identified PrARX model can be easily transformed to the corresponding PWARX model with complete partitions between regions. Finally, the proposed model is applied to the modeling of the driving behavior, and the usefulness of the model is verified and discussed.

I. INTRODUCTION

Continuous/discrete hybrid system model is one of the promising mathematical representations which expresses some complex but important class of systems. In the hybrid system identification, a PieceWise affine AutoRegressive eXogeneous (PWARX) model is widely used as the basic mathematical model. Many identification algorithms for the PWARX model have been proposed [1][2][3][4][5]. The main concern in the hybrid system identification is how to identify the parameters in the ARX models and ones in the hyperplanes which specify the partition of the regressor space. Most of the identification algorithms developed so far often required huge computational cost because the estimation of parameters in each ARX model and in each hyperplane must be executed simultaneously.

In this paper, first of all, a Probability weighted ARX (PrARX) model wherein the multiple ARX models are composed by the probabilistic weighting function is proposed. This model is obtained by implementing the ‘softmax function’ in the expression of the partition instead of the deterministic partition used in the PWARX model. The partition is characterized by the parameters in the softmax function in the PrARX model. Then, the parameter estimation algorithm for the PrARX model is addressed. The estimation problem for the parameters in the ARX models and the softmax functions are formulated as a single optimization problem thanks to the continuity of the softmax function. Furthermore, the identified PrARX model can be easily transformed to the corresponding PWARX model wherein the complete partition is obtained automatically by the parameters in the softmax functions.

Finally, the PrARX model is applied to the modeling of the driving behavior. In hybrid system modeling of the driving behavior, the mode segmentation (partition) can be

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regarded as a kind of driver’s decision making. Therefore, the introduction of hybrid system model leads to understanding of the human behavior wherein the motion control and the decision making functions are synthesized. Furthermore, since the PrARX model can express the stochastic variance of the mode segmentation, i.e., the decision making, it can be used to quantify a ‘decision entropy’ in the human behavior.

II. PROBABILITY WEIGHTED ARX MODEL

A. Definition of the Model

We propose a Probability weighted ARX model wherein the multiple ARX models are composed by the probabilistic weighting functions. The PrARX model is defined by the form

$$y_k = f_{Pr}(r_k) + e_k \quad (1)$$

where $k \geq 0$ denotes the sampling index, $y \in \mathbb{R}^q$ is the output variable, e_k is an error term. r_k is a regressor vector defined by

$$r_k = [y_{k-1}^T \cdots y_{k-n_a}^T u_{k-1}^T \cdots u_{k-n_b}^T]^T \quad (2)$$

where $r_k \in \mathbb{R}^n$, $n = p \cdot n_a + q \cdot n_b$, and $u \in \mathbb{R}^p$ is the input variable. $f_{Pr}(r_k)$ is a function of the form

$$f_{Pr}(r_k) = \sum_{i=1}^s P_i \theta_i^T \varphi_k \quad (3)$$

where $\varphi_k = [r_k^T \ 1]^T \in \mathbb{R}^{n+1}$. $\theta_i \in \mathbb{R}^{(n+1) \times q}$ ($i = 1, \dots, s$) is an unknown parameter matrix of each mode. s is the number of modes. P_i denotes the probability that the corresponding regressor vector r_k belongs to the mode i , and is given by the softmax function as follows:

$$P_i = \frac{\exp(\eta_i^T \varphi_k)}{\sum_{j=1}^s \exp(\eta_j^T \varphi_k)}, \quad (4)$$

$$\eta_s = 0$$

where η_i ($i = 1, \dots, s-1$) is an unknown parameter that characterizes the probabilistic partition between regions.

The sample model is shown in Fig.1. This model is the single output PrARX model with 3 modes. The model parameters are given by

$$\begin{aligned} \theta_1 &= [0.5 \quad -5]^T, & \theta_2 &= [-0.1 \quad 3]^T, \\ \theta_3 &= [-0.4 \quad 15]^T, \\ \eta_1 &= [-3 \quad 45]^T, & \eta_2 &= [-1.5 \quad 30]^T, \\ \eta_3 &= [0 \quad 0]^T. \end{aligned} \quad (5)$$

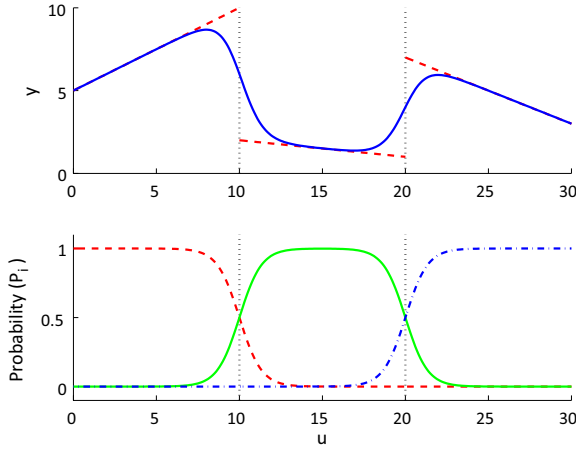


Fig. 1. Sample model of the single output PrARX model with 3 modes.

The upper figure shows the three ARX models (dashed line) and PrARX model (solid line) which is given by the weighted composition of three ARX models. The lower figure shows the three softmax functions used as the weighting probabilities in PrARX model.

It can be seen that the ARX model is smoothly connected around $u = 10$ and 20 . These connecting points, i.e., the partitions can be calculated from the η_1 and η_2 (See III).

B. Identification Algorithm

In order to identify the parameters in the PrARX model, the steepest descent method is used. The cost function is defined as the square norm of the output error as follows:

$$\varepsilon = \frac{1}{N} \sum_{k=1}^N \|e_k\|^2 = \frac{1}{T} \sum_k e_k^T e_k \quad (6)$$

where N is the number of the sampling data. Then, the partial differentiation of the objective function is given as follows:

$$\frac{\partial \varepsilon}{\partial \theta_i} = -\frac{1}{N} \sum_{k=1}^N 2P_i \varphi_k e_k^T, \quad (7)$$

$$\frac{\partial \varepsilon}{\partial \eta_i} = -\frac{1}{N} \sum_{k=1}^N 2P_i(1-P_i) \varphi_k (\theta_i \varphi_k)^T e_k. \quad (8)$$

The minimization of the cost function is obtained by updating the parameters in the steepest descent direction as follows:

$$\theta_i^{(t+1)} = \theta_i^{(t)} - \alpha \frac{\partial \varepsilon^{(t)}}{\partial \theta_i^{(t)}}, \quad (9)$$

$$\eta_i^{(t+1)} = \eta_i^{(t)} - \alpha \frac{\partial \varepsilon^{(t)}}{\partial \eta_i^{(t)}} \quad (10)$$

where $\theta_i^{(t)}$, $\eta_i^{(t)}$ and $\varepsilon^{(t)}$ are updated parameters and the cost function at t -th iteration. α is a small positive number.

Remark 2.1: The parameters θ_i in the ARX models and η_i in the partitions of the regions (softmax function) can be optimized simultaneously by the single optimization thanks to the continuity of the softmax function.

Remark 2.2: Since the steepest descent method may result in local minima, many initial parameters must be tested to find the global optimal parameters

III. TRANSFORMATION TO PWARX

A. PWARX Model

The PWARX model is defined by the form

$$y_k = f_{PW}(r_k) + e_k. \quad (11)$$

$f_{PW}(r_k)$ is a PWA function of the form

$$f_{PW}(r_k) = \begin{cases} \theta_1^T \varphi_k & \text{if } r_k \in \mathcal{R}_1 \\ \vdots & \vdots \\ \theta_s^T \varphi_k & \text{if } r_k \in \mathcal{R}_s \end{cases} \quad (12)$$

where $\{\mathcal{R}_i\}_{i=1}^s$ gives a complete partition of the regressor domain $\mathcal{R} \subseteq \mathbb{R}^n$. Each region \mathcal{R}_i is a convex polyhedron described by

$$\mathcal{R}_i = \{r \in \mathbb{R}^n : H_i \varphi \preceq_{[i]} 0\} \quad (13)$$

where H_i is a matrix which defines the partition $\{\mathcal{R}_i\}_{i=1}^s$. The symbol $\preceq_{[i]}$ denotes a vector whose elements can be the symbols \leq and $<$.

B. Transformation to PWARX from PrARX model

Consider the assignment of each r_k to the mode i using the following rule.

$$r_k \in \mathcal{R}_i \quad i = \arg \max_{i=1, \dots, s} P_i \quad (14)$$

This mode assignment implies that the $\{\mathcal{R}_i\}_{i=1}^s$ is represented by using η_i s as follows:

$$\mathcal{R}_i = \{r \in \mathbb{R}^n : H_i \varphi \preceq_{[i]} 0\} \quad (15)$$

$$H_i = [(\eta_1 - \eta_i) \cdots (\eta_s - \eta_i)]^T \quad (16)$$

Theorem 3.1: $\{\mathcal{R}_i\}_{i=1}^s$ given by (15) and (16) is a complete partition of \mathbb{R}^n , i.e.,

$$\mathcal{R}_1 \cup \cdots \cup \mathcal{R}_s = \mathbb{R}^n \quad (17)$$

$$\mathcal{R}_l \cap \mathcal{R}_m = \emptyset \quad \forall l, \forall m, l \neq m. \quad (18)$$

Proof: P_i can be calculated for any $r \in \mathbb{R}^n$ by (4). r always belongs to one of the area $\{\mathcal{R}_i\}_{i=1}^s$. Therefore, $\mathcal{R}_1 \cup \cdots \cup \mathcal{R}_s = \mathbb{R}^n$ is obvious.

Next, consider the intersection $\mathcal{R}_l \cap \mathcal{R}_m$ for any l and m .

$$H_l = [(\eta_1 - \eta_l) \cdots (\eta_m - \eta_l) \cdots (\eta_s - \eta_l)]^T, \quad (19)$$

$$H_m = [(\eta_1 - \eta_m) \cdots (\eta_l - \eta_m) \cdots (\eta_s - \eta_m)]^T. \quad (20)$$

For H_l and H_m ,

$$\{(\eta_m - \eta_l)^T \varphi \leq 0\} \cap \{(\eta_l - \eta_m)^T \varphi \leq 0\} = \emptyset. \quad (21)$$

always holds. Therefore, $\mathcal{R}_l \cap \mathcal{R}_m = \emptyset$. As the consequence, $\{\mathcal{R}_i\}_{i=1}^s$ is the complete partition of the \mathbb{R}^n . ■

Thanks to this theorem, the PrARX model can be directly transformed to the PWARX model by simply applying (16) to the identified PrARX model.

IV. NUMERICAL EXPERIMENTS

A. Example 1

Let the data $\{(u_{k-1}, y_k)\}_{k=1}^{300}$ be generated by a system given by

$$y_k = f_{Pr}(u_{k-1}) + e_k,$$

$$\begin{aligned} \theta_1 &= [1 \quad -0.5]^T, & \theta_2 &= [-1.5 \quad 0.5]^T, \\ \theta_3 &= [1 \quad -0.5]^T, \\ \eta_1 &= [120 \quad 60]^T, & \eta_2 &= [-60 \quad 40]^T. \\ \eta_3 &= [0 \quad 0]^T. \end{aligned} \quad (22)$$

where e_k is a sequence of the normally distributed random numbers with zero mean and variance $\sigma_e^2 = 0.025$. In this example, although the modes 1 and 3 have the same ARX parameters θ_i , they are expressed as the different modes because they are located on different regions. The region of the mode 2 is located between the regions of the mode 1 and the mode 3. The true model, the identified model, and the data set used for identification are shown in Fig.2. From this figure, it can be confirmed that the mode 1 and the mode 3 can be identified as the different modes successfully. The estimated parameters are

$$\begin{aligned} \theta_1 &= [1.20 \quad -0.53]^T, & \theta_2 &= [-1.36 \quad 0.42]^T, \\ \theta_3 &= [1.18 \quad -0.66]^T, \\ \eta_1 &= [-133.6 \quad 74.9]^T, & \eta_2 &= [-78.8 \quad 52.8]^T, \\ \eta_3 &= [0 \quad 0]^T. \end{aligned} \quad (23)$$

In addition, the parameters in the hyperplane in the corresponding PWARX model are obtained from η_i by applying (16). The obtained hyperplane parameters are

$$H_1 = [[0 \ 0]^T \ [54.8 \ -22.1]^T \ [133.6 \ -74.9]^T]^T, \quad (24)$$

$$H_2 = [[-54.8 \ 22.1]^T \ [0 \ 0]^T \ [78.8 \ -52.8]^T]^T, \quad (25)$$

$$H_3 = [[-133.6 \ 74.9]^T \ [-78.8 \ 52.8]^T \ [0 \ 0]^T]^T. \quad (26)$$

Therefore, the deterministic partition between the modes 1 and 2 is given by

$$[54.8 \quad -22.1] \begin{bmatrix} u_{k-1} \\ 1 \end{bmatrix} = 0. \quad (27)$$

Similarly, the partition between the modes 2 and 3 is given by

$$[78.8 \quad -52.8] \begin{bmatrix} u_{k-1} \\ 1 \end{bmatrix} = 0. \quad (28)$$

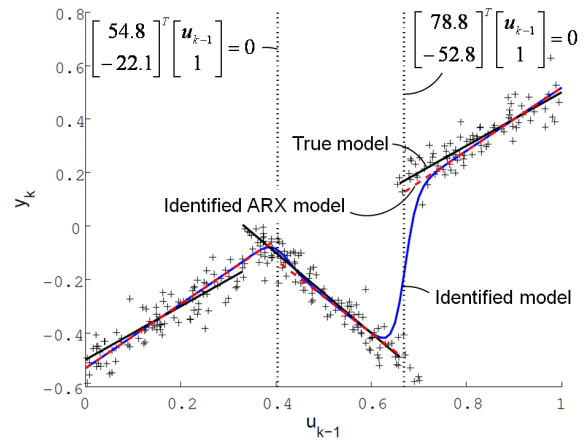


Fig. 2. Identification of PrARX model with 3 modes wherein 2 modes have same ARX parameters.

B. Example 2

Let the data $\{(u_{1,k-1}, u_{2,k-1}, y_k)\}_{k=1}^{500}$ be generated by a system given by

$$y_k = f_{Pr}([u_{1,k-1}, u_{2,k-1}]^T) + e_k \quad (29)$$

$$\begin{aligned} \theta_1 &= [5 \quad -4 \quad -3]^T & \theta_2 &= [6 \quad 3 \quad -6]^T \\ \theta_3 &= [-3 \quad 5 \quad 0]^T \\ \eta_1 &= [-30 \quad 0 \quad 15]^T & \eta_2 &= [-15 \quad -15 \quad 15]^T \\ \eta_3 &= [0 \quad 0 \quad 0]^T \end{aligned}$$

where e_k is a sequence of the normally distributed random numbers with zero mean and variance $\sigma_e^2 = 0.025$. The estimated parameters are

$$\begin{aligned} \theta_1 &= [3.98 \quad -4.23 \quad -2.729]^T, \\ \theta_2 &= [6.18 \quad 1.78 \quad -6.00]^T, \\ \theta_3 &= [-3.46 \quad 4.84 \quad 0.50]^T, \\ \eta_1 &= [-23.95 \quad 0.42 \quad 11.42]^T, \\ \eta_2 &= [-11.48 \quad -10.56 \quad 11.24]^T, \\ \eta_3 &= [0 \quad 0 \quad 0]^T. \end{aligned} \quad (30)$$

In addition, the parameters in the hyperplane in the corresponding PWARX model are obtained from η_i by applying (16). The obtained hyperplane parameters are

$$H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 12.47 & -10.98 & -0.18 \\ 23.95 & -0.42 & -11.42 \end{bmatrix}, \quad (31)$$

$$H_2 = \begin{bmatrix} -12.47 & 10.98 & 0.18 \\ 0 & 0 & 0 \\ 11.48 & 10.56 & -11.24 \end{bmatrix}, \quad (32)$$

$$H_3 = \begin{bmatrix} -23.95 & 0.42 & 11.42 \\ -11.48 & -10.56 & 11.24 \\ 0 & 0 & 0 \end{bmatrix}. \quad (33)$$

$$H_3 = \begin{bmatrix} -23.95 & 0.42 & 11.42 \\ -11.48 & -10.56 & 11.24 \\ 0 & 0 & 0 \end{bmatrix}. \quad (34)$$

The data set used for identification and the partitions calculated from estimated η_i s are shown in Fig.3. In Fig.3, the \circ , \times and \triangle represent the mode 1, 2 and 3 assigned to data by applying (14), respectively. From this figure, it can be confirmed that the complete partition is realized.

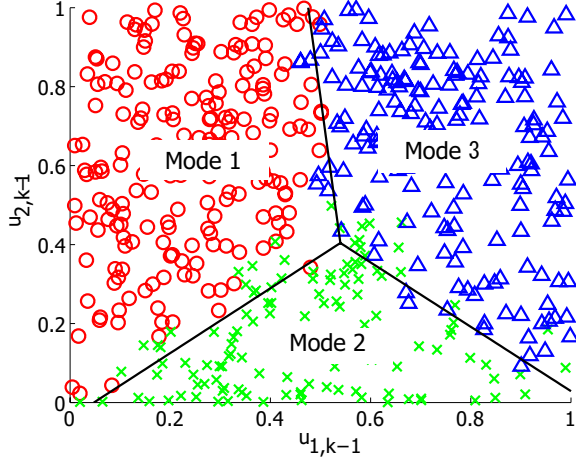


Fig. 3. Identified partitions and distribution of data.

V. APPLICATION TO DRIVING BEHAVIOR MODELING

The human behavior can be considered to consist of the function of the decision making and the motion control. The former can be characterized by the logical switching, whereas the latter can be described by the continuous dynamics. Therefore, by applying the hybrid system identification to the human behavioral data, it is expected to extract the decision making and the motion control aspects simultaneously from the observed behavioral data. In this section, the proposed PrARX model is applied to the analysis of the driving behavioral data, and the usefulness is verified.

A. Acquisition of Driving Data

The driving data is acquired through a driving simulator which provides a stereoscopic immersive environment. In this paper, we focus on the drivers' vehicle following task on the expressway. The velocity of the preceding vehicle varies from 0 to 130 [km/h]. All drivers were provided with the instruction 'Follow the preceding vehicle so as not to make collision'. Since this instruction is 'loose' instruction, the drivers do not concern much about the range and range rate. As the result, each driver can drive as his/her usual manner. The view from the driver is shown in Fig.4.

B. Definition of Driver Input and Output

We have analyzed the driving data of 11 male drivers (20s years old). The input and output data are sampled and collected every 240[msec].



Fig. 4. Overview of driver and screen.

The driver is regarded as a kind of controller to follow the preceding vehicle. In this subsection, the driver input and output are defined. The driver input $u = [u_1 \ u_2 \ u_3]^T$, i.e., the sensory information of the driver is defined as follows:

- Index for approaching (KdB) : u_1
- Range between cars : u_2
- Range rate between cars : u_3

Here, KdB is an index which represents the logarithm of a time derivative of the area of the back of the preceding vehicle projected on the driver's retina [6]. In [6], it is verified that this index plays an important role in the vehicle following behavior from viewpoint of the cognitive science. The KdB can be expressed by using u_2 and u_3 as follows:

$$KdB = \begin{cases} -10 \times \log \left| -2 \times \frac{u_3}{u_2} \times \frac{1}{5 \times 10^{-8}} \right| & \text{if } u_3 > 0 \\ 10 \times \log \left| -2 \times \frac{u_3}{u_2} \times \frac{1}{5 \times 10^{-8}} \right| & \text{if } u_3 < 0 \end{cases} \quad (35)$$

Intuitively speaking, the large KdB implies that the driver is facing dangerous situation. Also, the driver output is defined as follows:

- Pedal operation : y

Here, the acceleration and braking operations are considered to take positive and negative values in y , respectively. Since we consider a first-order dynamics as the controller model, the regressor vector r_k is defined as follows:

$$r_k = [y_{k-1} \ u_{1,k-1} \ u_{2,k-1} \ u_{3,k-1}]^T. \quad (36)$$

All observed data are normalized before identification.

C. Modeling Results

1) *Mode Segmentation Results:* First, the driver model is identified by using 2434 points of data. The mode segmentation results in the case of the two-mode modeling of the drivers A and B are shown in Figs.5 and 6. In these figures, the red and blue marker shows the two modes, respectively in the Range rate -Range space (The color is changed gradually based on the probability). The dangerous region, where the range is small and the range rate is negative large, is indicated by the red mode. Figs. 7 and 8 show the mode segmentation results in the KdB - Pedal space. In these figures, we can see that the braking operation is activated many times in the red mode, and the red mode appears on the region where the KdB is large. This implies that the KdB strongly affects

on the mode segmentation, i.e., the decision making of the driver. This point can also be verified by investigating the magnitude of the parameters η_i , which is addressed in the section V-C.3.

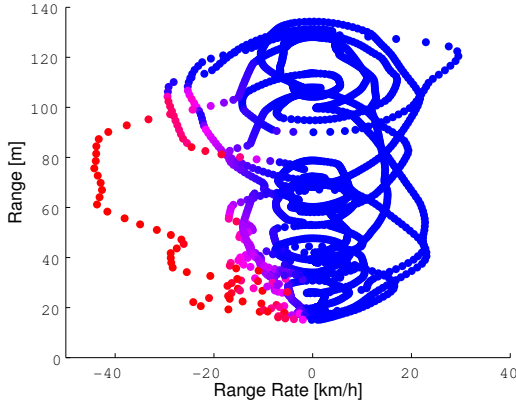


Fig. 5. Mode segmentation result of vehicle following task (Driver A, two mode model, range rate-range).

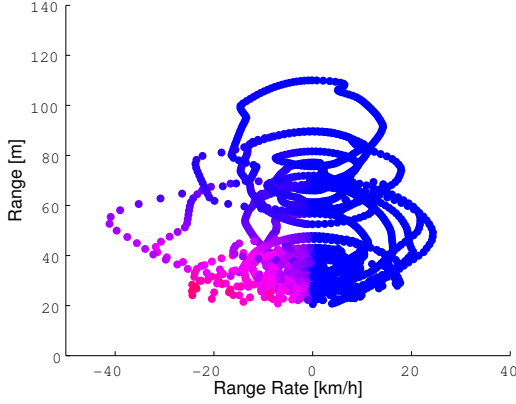


Fig. 6. Mode segmentation result of vehicle following task (Driver B, two mode model, range rate-range).

2) *Identified Parameters θ_i* : The identified θ_i s in the PrARX model are shown in Table I. In this table, the mode 1 and 2 means the red and blue mode in Figs.5 and 6, respectively. The coefficient of the autoregressive term θ_{i1} takes smaller value in the mode 1 than the mode 2. This implies that the driver shows faster dynamics in the mode 1 than the mode 2. In addition, the other parameters θ_{i2} , θ_{i3} and θ_{i4} tend to be larger in the mode 1 than the mode 2. This implies that the driver uses higher gain feedback control in the mode 1.

3) *Identified Parameters η_i* : The identified η_i s in the PrARX model are shown in Table II. The probability which the r_k belongs to the corresponding mode can be calculated by these parameters. The matrix H_i which specifies the region of each mode are calculated by using (16), and is given by

$$H_1 = \begin{bmatrix} (\eta_1 - \eta_1) \\ (\eta_2 - \eta_1) \end{bmatrix} = \begin{bmatrix} 0 \\ -\eta_1 \end{bmatrix}, \quad (37)$$

$$H_2 = \begin{bmatrix} (\eta_1 - \eta_2) \\ (\eta_2 - \eta_2) \end{bmatrix} = \begin{bmatrix} \eta_1 \\ 0 \end{bmatrix}. \quad (38)$$

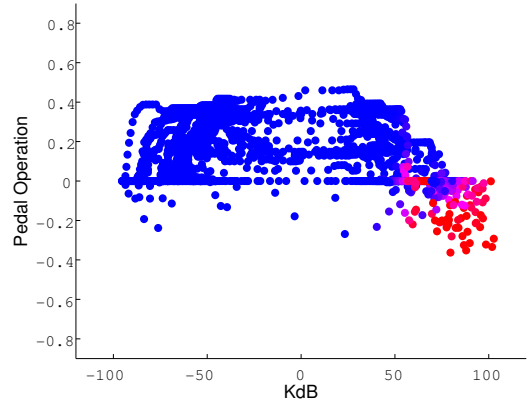


Fig. 7. Mode segmentation result of vehicle following task (Driver A, two mode model, KdB - Pedal).

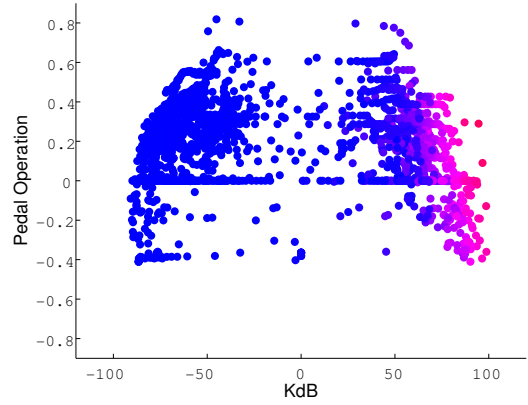


Fig. 8. Mode segmentation result of vehicle following task (Driver B two mode model, KdB - Pedal).

Then, the partition between the modes 1 and 2 is given by

$$\eta_1^T \varphi = 0. \quad (39)$$

Therefore, the large element in the η represents that it strongly affects on the partition between modes, i.e., the driver's decision making. In Table II, it can be seen that the KdB and the range rate have strong influence on the decision making in the driver A. Similarly, the KdB and the range have strong influence on the decision making in the driver B. Generally speaking, the KdB has the strong influence on the decision making. The parameter η_i can be an important feature value to design the assisting system which accommodates with each driver's personality.

4) *Modeling accuracy*: The output error of the PrARX model is compared with that of the PWARX model which is identified by using the clustering based approach [4]. The mean square errors are shown in Table III. In this table, the PrARX model shows better modeling accuracy than the PWARX model.

D. Decision Entropy

By using the PrARX model, the 'Decision entropy' can be defined, which is a quantitative measure to evaluate

TABLE I
IDENTIFIED PARAMETERS θ_i

Driver	mode (<i>i</i>)	θ_{i1}	θ_{i2}	θ_{i3}	θ_{i4}	θ_{i5}
A	1	0.620	-0.101	0.198	0.353	0.027
	2	0.970	-0.002	0.007	0.105	0.001
B	1	0.739	-0.084	0.651	0.829	-0.174
	2	0.964	-0.008	0.010	0.141	0.003
C	1	0.784	-0.008	0.092	0.242	-0.012
	2	0.986	-0.006	0.021	0.112	-0.007
D	1	0.699	-0.314	0.695	0.694	0.162
	2	0.960	-0.006	0.016	0.050	0.000
E	1	0.894	-0.461	0.714	0.488	0.223
	2	0.941	-0.003	0.107	0.229	0.002

TABLE II
IDENTIFIED PARAMETERS η_i

Driver	mode (<i>i</i>)	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}
A	1	-9.944	13.072	3.903	-15.601	-13.290
B	1	1.543	4.006	-6.598	-1.493	-1.916
C	1	-17.014	13.734	-4.234	-3.839	2.837
D	1	-6.242	13.527	-8.316	-7.594	-9.805
E	1	9.358	23.673	-14.132	-8.314	-19.108

the vagueness of the decision making. Decision entropy is defined as follows:

$$H(P_i) = \int_{r \in D_H} \sum_{i=1}^s P_i \log(P_i) dr \quad (40)$$

The larger the decision entropy is, the more the vagueness in the decision making is. The calculated decision entropy of the drivers A and B are shown in Table IV. In this table, the decision entropy of the driver A is less than that of the driver B. This implies that the decision making of the driver B is more unclear than that of the driver A. This can also be verified by comparing Figs.5 and 6. The unclear region (between the red mode and the blue mode) of the driver B is larger than that of the driver A.

VI. DISCUSSION

The useful features of the proposed PrARX model are summarized as follows:

- In the understanding of the complex physical phenomena, such as the human behavior or the biological

TABLE III
COMPARISON OF MEAN SQUARE ERROR

Driver	PrARX	PWARX
A	0.0007	0.0009
B	0.0019	0.0024
C	0.0020	0.0025
D	0.0018	0.0022
E	0.0027	0.0049
ALL	0.0019	0.0024

TABLE IV
DECISION ENTROPY

Driver	A	B
Entropy	0.8273	2.2440

system, the probabilistic partition may fit well due to the continuity underlying the original phenomena. In these application fields, the modeling error tends to be smaller than the PWARX model which has the deterministic partition since the PrARX model can represent the composition of several modes. Furthermore, the stochastic characteristics of the partition may represent some important factor in the original phenomena like the decision entropy.

- The identification scheme of the PrARX model can be exploited as the identification scheme of the PWARX model by applying simple transformation rule. From viewpoint of the identification strategy of the PWARX model, the proposed identification scheme can realize the simultaneous estimation of the parameters in the ARX models and in the partitions by a single optimization. Furthermore, the obtained parameters give a complete partition.

VII. CONCLUSION

In this paper, we have proposed a Probability weighted ARX (PrARX) model wherein the multiple ARX models are composed by the probabilistic weighting functions. As the probabilistic weighting function, the ‘softmax’ function was introduced. Then, the parameter estimation problem for the proposed model was formulated as a single optimization problem, and the estimation algorithm was derived. Since the PrARX model can be easily transformed to the corresponding PWARX model with complete partition between regions, the proposed scheme can be exploited as the identification of the PWARX model. Finally, the proposed model was applied to the modeling of the driving behavior, and the usefulness of the model was verified and discussed.

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