Design of Man-Machine Cooperative Nonholonomic Two-Wheeled Vehicle Based on Impedance Control and Time-State Control

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Abstract—This paper presents a new control methodology for a nonholonomic electric two-wheeled vehicle wherein the autonomous and man-machine cooperative controls are synthesized. In the proposed control scheme, the 'autonomous control' and the 'man-machine cooperative control' are designed by synthesizing time-state control and impedance control. The time-state controller tries to reduce the machine's deviation from the guideline, the impedance controller, on the other hand, generates power to assist the operator's maneuver. Furthermore, experimental results are shown to demonstrate the usefulness of the proposed strategy.

I. INTRODUCTION

Power-assisting wheeled vehicle is a promising technology for aging society and care service. As typical examples, electric wheelchairs [1], [2], walking-support systems [3], and power assisting carts [4], [5] have been developed. Such power assisting systems are required not only to reduce the operator's burden but also to improve the operator's skill.

Hara [4] proposed power assisting control for holonomic carrying machines in which impedance control and servo control switch each other according to the state of the machine. This method is especially effective for accurate positioning of works. Switching of autonomous control and cooperative control is considered to be a significant method to improve the operator's skill and reduce the operator's burden simultaneously.

Peshkin and Colgate et al. [5] proposed an architecture of nonholonomic power assisting robot, called cobot. By the steering controller, the cobot can control the direction of the cobot's motion and switch two control modes, free mode and path mode. In the free mode, the operator can move the cobot to any direction that he/she intends and, in the path mode, the movement of the cobot is guided to a particular trajectory, called a virtual surface. This architecture also ensures the operator's safety because of the cobot's passive mechanism. Ensuring safety of the operator is also an important aspect of power assisting systems.

This paper presents a novel assisting system for an electric two-wheeled vehicle which has nonholonomic dynamical characteristics and commonly used as the wheelchairs. The proposed method realizes reduction of the operator's burden, improvement of the operator's skill, and securement of the operator's safety simultaneously. Figure 1 depicts the overview of the motivating environment wherein the vehicle can move in the space consisting of straight corridors with

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guidelines and free-movement spaces. The operator can move the vehicle freely in a free-motion space and can move the vehicle in a corridor safely by making it track on the guidelines with small effort. For this problem setting, we propose an assisting system consisting of impedance control and time-state control. The role of the impedance control is to amplify the operator's power, while the time-state control is designed to reduce the vehicle's deviation from the guideline. Furthermore, the synthesis of these two control strategies are addressed from the viewpoint of designing a variable gain.

This paper is organized as follows. In section II, we describe the man-machine cooperative two-wheeled vehicle which is used in our experiments. In section III and IV, impedance control and time-state control are introduced, respectively. Section V proposes the synthesis of the impedance controller and the time-state controller to realize the assisting system. Section VI shows the experimental result to verify the proposed assist system. Section VII concludes this paper.

II. MAN-MACHINE COOPERATIVE TWO-WHEELED VEHICLE

Figure 2 shows the developed electric two-wheeled vehicle which has nonholonomic dynamics. This vehicle has two motored wheels, a personal computer as the controller and a battery by which the vehicle can move autonomously. This vehicle is designed so that the center of mass coincides with the center of the two wheels. An operator operates the vehicle by pushing the handling bar on which a force sensor is attached to detect the operator's pushing force and the rotating torque. The driving force F (N) in the longitudinal direction of the vehicle and the rotational torque N (Nm) around the force sensor are calculated from the output of the force sensor. The vehicle is modelled as a vehicle as



Fig. 1. Overview of the environment of the cooperative two-wheeled vehicle

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Fig. 2. nonholonomic electric two-wheeled vehicle which is used in experiment



Fig. 3. Model of electric two-wheeled vehicle

depicted in Fig. 3. The global position of the center of mass of the vehicle is expressed by (x, y), and the rotational angle around the rotating center is expressed by θ in the global coordinate system. The velocity in the longitudinal direction of the vehicle and the rotational angular velocity around the rotating center are expressed by v and ω , respectively.

III. ASSISTANCE FOR OPERATOR'S POWER BY IMPEDANCE CONTROL

Impedance control generates power which assists the operator who carries the vehicle. Based on impedance control, the force which the operator exerted on the force sensor is amplified so that he/she may feel virtual impedance. In order to realize impedance control, the velocity control impedance control is adopted. The target velocity v_{imp} (m/s) in the longitudinal direction of the vehicle and the target rotational angular velocity ω_{imp} (rad/s) around the rotating center of the vehicle are calculated respectively as follows:

$$v_{imp} = \frac{1}{Ms + C_1}F,\tag{1}$$

$$\omega_{imp} = \frac{1}{Is + C_2} N,\tag{2}$$

where M, I, C_1 , C_2 are the virtual mass, the virtual moment of inertia, the virtual viscous coefficients in the longitudinal direction of the vehicle and the virtual viscous coefficients Impedance Controller



Fig. 4. Blockdiagram of impedance control of two-wheeled vehicle

around the rotating center of the vehicle, respectively. In our experiment, the values of virtual impedance parameters are derived empirically:

$$M = 20$$
 (kg), $I = 1.0$ (kgm²),
 $C_1 = 8.0$ (Ns/m), $C_2 = 4.0$ (Nms/rad)

Figure 4 shows the blockdiagram of the impedance controller. The motors of the vehicle are controlled by means of PID control in order to converge the velocity and the rotational angular velocity to the target values (1) and (2).

IV. AUTONOMOUS TRACKING BY TIME-STATE CONTROL

In this research, autonomous control is also installed on the vehicle in order to precise tracking the guideline. The autonomous control is usually achieved by feedback control to converge the vehicle to the guideline. However, since the vehicle has nonholonomic characteristic, the equilibrium points cannot be stabilized asymptotically by means of smooth time varying feedback control [6]. In order to solve this problem, Time-State Control [7] is introduced on the autonomous controller. Time-state control can stabilize some specific class of nonholonomic systems which are expressed as "Chained form".

A. Chained form

First, the state equation of the vehicle is expressed as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega.$$
(3)

Next, the following coordinate transformation and input transformation are applied to (3):

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} x \\ \tan \theta \\ y \end{bmatrix}, \quad \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{\nu_1}{\cos \theta} \\ \nu_2 \cos^2 \theta \end{bmatrix}, \quad (4)$$

where the rotational angle θ is considered to satisfy $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then, the transformed state equation is expressed as follows:

$$\begin{bmatrix} \dot{z}_1\\ \dot{z}_2\\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \nu_1\\ \nu_2\\ z_2\nu_1 \end{bmatrix}.$$
 (5)



Fig. 5. Blockdiagram of time-state control with stabilization feedback

Moreover, the following input transformation (6) is applied to (5):

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \frac{\nu_2}{\nu_1} \end{bmatrix}.$$
 (6)

Finally, the state equation is divided into two parts; one is time control part (7) and the other is state control part (8):

$$\frac{dz_1}{dt} = \mu_1,$$
(7)
$$\frac{d}{dz_1} \begin{bmatrix} z_3\\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1\\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_3\\ z_2 \end{bmatrix} + \begin{bmatrix} 0\\ 1 \end{bmatrix} \mu_2.$$
(8)

These equations are collectively called "time-state control form" [7].

B. Tracking on x-axis based on time-state control

Because the state control part (8) is controllable canonical form, it is easy to derive the stabilization feedback. For example, the candidate of feedback law is given by

$$\mu_2 = -k_2 z_2 - k_3 z_3 \tag{9}$$

where k_2 and $k_3 > 0$ are constant parameters but the sign of k_2 has to be changed according to the sign of μ_1 in order to stabilize the state control part:

$$\begin{cases} k_2 > 0 & \text{if } \mu_1 \ge 0\\ k_2 < 0 & \text{otherwise.} \end{cases}$$
(10)

Figure 5 is the blockdiagram of the time-state controller with stabilization feedback. When $\mu_1 \ge 0$ is satisfied, z_1 increases monotonically as well as usual time does. Therefore, z_2 and z_3 are stabilized by the feedback (9) when the constant parameters k_2 is positive. This implies that the vehicle converges to the *x*-axis with running in the direction of *x*-axis. On the other hand, when the vehicle runs in the opposite direction of *x*-axis, that is $\mu_1 < 0$, it also converges to the *x*-axis by changing the sign of constant parameters k_2 to negative.

When the time-state controller is applied to the powerassist system, the operator is responsible for the motion in the direction of x-axis. This motion along x-axis is assisted by the impedance controller described in section III. Consequently, the motion in the direction of x-axis is controlled



Fig. 6. Tracking on x-axis in power-assist system



Fig. 7. Guideline and its local coordinate system

only by the impedance controller, and the motion of rotation and the motion in the direction of y-axis are done by both the impedance controller and the time-state controller. This scheme of power assisting system has great advantage from the viewpoint of safety because the vehicle closes to x-axis only when the operator pushes it in the direction of x-axis. In other words, the vehicle stops immediately when the operator releases the force sensor bar of the vehicle.

In actual operation of the power-assist system, the x-axis corresponds to a guideline embedded on the ground and the origin of the coordinate system is the beginning point of the guideline as shown in Fig. 6. Since it is considered that there are many guidelines in a building as shown in Fig. 1, a local coordinate system is allocated in each guideline. As shown in Fig. 7, the local coordinate system for *n*-th guideline is defined as $O_n - x_n y_n$ in which the state of vehicle is defined as (x_n, y_n, θ_n) . The vehicle is controlled to converge to the closest guideline by time-state control based on the local coordinate system.

When the operator tries to change the guideline to another one, the operator makes force so as to leave the vehicle from the guideline and the impedance controller amplifies his/her force. On the other hand, the time-state controller simultaneously tries to keep the vehicle to converge the guideline. As the result, the forces exerted by the two controllers conflict with each other. In order to solve this conflict, some balance adjustment is needed between the forces exerted by the two controllers. In the next section, the method to synthesize the impedance controller and the time-state controller is introduced, and the method to adjust the balance of the two controllers is also introduced.

V. COMBINATION OF TWO ASSISTING CONTROLS

A. Overview of power-assist system

Figure 8 depicts the blockdiagram of proposed powerassist system wherein the impedance controller and the timestate controller are synthesized. The parameters used in this blockdiagram are defined as follows:

- F: Force exerted by the operator
- N: Rotary torque exerted by the operator
- v_{imp} : Target velocity calculated in the impedance controller
- ω_{imp} : Target angular velocity calculated in the impedance controller
- ω_{tsc} : Target angular velocity calculated in the time-state controller
- G: Variable gain
- v^* : Target velocity of the two-wheeled vehicle
- ω^* : Target angular velocity of the two-wheeled vehicle
- Φ_L : Angle of left wheel of the two-wheeled vehicle
- Φ_R : Angle of right wheel of the two-wheeled vehicle
- $\dot{\Phi}_L^*$: Target angular velocity of the left wheel of twowheeled vehicle
- $\dot{\Phi}_R^*$: Target angular velocity of the right wheel of twowheeled vehicle

The impedance controller depicted in the upper left part of Fig. 8 calculates both the target values of velocity in the direction of the guideline v_{imp} and angular velocity around the rotating center of the vehicle ω_{imp} . The timestate controller depicted in the lower right part of Fig. 8 calculates the target value of angular velocity ω_{tsc} based on the position and angle relative to the guideline. In addition, the gain controller tunes the variable gain G based on the position and angle relative to the guideline and the force exerted by the operator. After that, the motor controller of the vehicle receives the target value of angular velocity ω^* which is the sum of ω_{imp} and $G\omega_{tsc}$, and the left and right motors are controlled based on PID control so as to follow the target values $\dot{\Phi}_L^*$ and $\dot{\Phi}_R^*$.

B. Variable gain

The gain controller calculates the variable gain G which can adjust the effect of the time-state controller. The variable gain G is considered to be $0 \le G \le 1$, and then the target value of angular velocity ω^* of the two-wheeled vehicle is calculated as follows:

$$\omega^* = \omega_{imp} + G\omega_{tsc}.\tag{11}$$

When G = 0, the vehicle moves based only on the impedance control. On the other hand, when G = 1, the vehicle is controlled by both the impedance controller and the time-state controller. The case of G = 0 corresponds to the case that the operator wants to move the vehicle freely, and the case of G = 1 corresponds to the case that the operator wants the vehicle to track on the guideline. Since the switching of G should obey the operator's intention, the variable gain G is determined according to the position and

angle relative to the guideline and also the force exerted by the operator. We empirically derived the conditions to change the variable gain G through many experiments. The conditions assumed that the vehicle is closest to the guideline n are described as follows:

(i) Condition specified by distance to guideline

The vehicle is consider to track on the guideline only when the vehicle is close to the guideline n. On the other hand, when the vehicle is far from the guideline n, the vehicle should move according to the operator intention. The condition specified the distance between the vehicle and the guideline n is expressed as follows:

$$\begin{cases} |y_n| \leq 0.6 \text{ (m)} \Rightarrow G = 1\\ |y_n| > 0.6 \text{ (m)} \Rightarrow G = 0. \end{cases}$$
(12)

(ii) Condition specified by angle to guideline

When the operator tries to pass over the guideline n, the vehicle should not try to track on the guideline. Also, when the operator makes the vehicle go away from the guideline, the time-state controller should halt as immediately as possible. Therefore, the condition specified by the angle to the guideline is represented as follows:

$$\begin{cases} y_n \times \dot{y}_n \le 0 \text{ and } 60^\circ < |\theta_n| < 120^\circ \Rightarrow G = 0\\ y_n \times \dot{y}_n > 0 \text{ and } 30^\circ < |\theta_n| < 150^\circ \Rightarrow G = 0. \end{cases}$$
(13)

The upper condition of (13) is for passing through the guideline and the lower one is for leaving the guideline. The range of $|\theta_n|$ of the lower condition is larger than that of the upper one so that the vehicle easily leaves from the guideline.

(iii) Condition specified by operator's force

The operator exerts large rotational torque on the force sensor when he makes the two-wheeled vehicle go away from the guideline. In this case, since the time-state controller works against the operator's intention, the effect of the time-state controller should be weakened. However, since the sudden halt of time-state controller is likely to cause a negative effect to the operational feeling, the variable gain Gshould be changed gradually. The condition is, consequently, described as follows:

$$\begin{cases} 0.0 \text{ (Nm)} \leq |N| < 1.8 \text{ (Nm)} \Rightarrow G = 1\\ 1.8 \text{ (Nm)} \leq |N| < 3.6 \text{ (Nm)} \Rightarrow G = \frac{3.6 - |N|}{1.8}\\ 3.6 \text{ (Nm)} \leq |N| \Rightarrow G = 0. \end{cases}$$
(14)

Meanwhile, when the above conditions are satisfied simultaneously, the least value of G is selected.

VI. EXPERIMENTAL RESULTS

A. Experimental condition

We executed some experiments by using the nonholonomic electric two-wheeled vehicle as shown in Fig. 2. An operator moves the vehicle in the field depicted in Fig. 9. This field has two guidelines which are orthogonal each other. The operator starts from the start point in the edge

Impedance Controller



Fig. 8. Blockdiagram of power-assist system for electric two-wheeled vehicle

of guideline 1 and arrive at the goal point in the edge of guideline 2. Therefore, the operator has to change the guideline once to reach the goal point. The gray area in Fig. 9 is a converging area of each guideline, in which the condition (i) in section V is satisfied. This examination was executed by three examinees, A, B and C under two conditions that only the impedance controller was applied and that both the impedance controller and the time-state controller were applied. The examinees were instructed to execute the examination as follows:

- Start from the origin of the global coordinate and track on the guideline 1.
- Change the guideline from 1 to 2 whenever the operator wants to do.
- Track on the guideline 2 as far as possible after change the guideline.
- Finish the task when $y \ge 4$ is satisfied.

The examinees practiced the above task sufficiently before the experiment and executed the task ten times in the experiment.

B. Experimental result

The trajectory of the vehicle of the examinee A is shown in Fig. 10, in which the square and the center dot imply the posture and the position of the vehicle, respectively. The trajectory by the impedance controller (a) has larger fluctuation in the guideline 2 than that by the impedance and time-state controller (b).

Figure 11 shows the force and torque exerted by the examinee A. As for the time-state controller, the variable gain G is also shown in (e). The vertical lines in the graphs mean the time when the vehicle changes the guideline. When the vehicle was tracking on the guideline 1, the characteristics of the force and torque were similar between the impedance



Fig. 9. Field of experiment

controller (a) (c) and the impedance and time-state controller (b) (d), but the impedance and time-state controller needed some larger torque of the operator in order to leave the guideline 1. On the other hand, when the vehicle was tracking on the guideline 2, the operator needed large force and torque in the case of the impedance controller in order to adjust the position of the carrier machine.

Figure 12 shows the statistic result from each examinee's ten-times trials; (a) the completion time to carry the vehicle form the start point to the goal point, (b) the average power of the operator to push the vehicle forward, (c) the average power of the operator to turn the vehicle, (d) the total error of distance between the guideline 2 and the the position of the vehicle while it was tracking on the guideline 2. As already shown in Fig. 10, there is a significant difference in (d). Although there are not significant differences in both (a) and (b), the examinees needed much power to turn the two-wheeled vehicle in the case of the impedance controller as shown in (c) because they had to adjust the position of



Fig. 10. Trajectory of two-wheeled vehicle by examinee A



Fig. 11. Force and torque exerted by examinee A

the two-wheeled vehicle to the guideline 2.

VII. CONCLUSION

This paper proposed a new control methodology for a nonholonomic electric two-wheeled vehicle wherein the autonomous and man-machine cooperative controls are synthesized. In the proposed control scheme, the 'autonomous control' and the 'man-machine cooperative control' are designed by the time-state control and impedance control, respectively. From the experimental results, the usefulness of the proposed method was verified. Deriving the conditions to change the variable gain so as to follow the operator's intention is the future work.



С

C

Fig. 12. Statistical result of each examinee

REFERENCES

- [1] J. Miyata, Y. Kaida, T. Murakami, v-mathdotphi-Coordinate-Based Power-Assist Control of Electric Wheelchair for a Caregiver, Industrial Electronics, IEEE Transactions on, vol. 55, issue 6, pp. 2517-2524, 2008.
- S. Katsura, K. Ohnishi, Human cooperative wheelchair for haptic [2] interaction based on dual compliance control, Industrial Electronics, IEEE Transactions on, vol. 51, issue 1, pp. 221-228, 2004.
- O. Chuy, Y. Hirata, K. Kosuge, A New Control Approach for a Robotic [3] Walking Support System in Adapting User Characteristics, Systems, man, and cybernetics, IEEE Transactions on, part C, vol. 36, no. 6, pp. 725-733, 2006.
- Susumu Hara, A Smooth Switching From Power-Assist Control to [4] Automatic Transfer Control and Its Application to a Transfer Machine, Industrial Electronics, IEEE Transactions on, vol. 54, no. 1, pp. 638-650. 2007
- [5] Peshkin, M.A., Colgate, J.E., Wannasuphoprasit, W., Moore, C.A., Gillespie, R.B., Akella, P., Cobot architecture, Robotics and Automation, IEEE Transactions on, vol. 17, no. 4, pp. 377-390, 2001.
- R.W. Brockett, Asymptotic stability and feedback stabilization, Dif-[6] ferential Geometric Control Theory, vol. 27, pp. 181-191, 1983.
- Mitsuji Sampei, Feedback Control of nonholonomic Systems, Journal [7] of the Society of Instrument and Control Engineers, vol. 36, no. 6, pp. 396-403, 1997.