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Attitude Stabilization of an Aircraft via Nonlinear Optimal Control Based on Aerodynamic Data

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Abstract: In this paper, a stabilization problem for an aircraft at a high angle of attack is considered. Though aircrafts obtain large lift force over the high angle of attack regime, nonlinearities of aerodynamics over the regime are considerably high. Using aerodynamic coeffients data given as a table, we design a nonlinear optimal control law to stabilize an existing aircraft at a high angle of attack by a new method based on stable manifold theory. As a result, the nonlinear optimal control law stabilizes attitudes of the aircraft in a larger region than the linear optimal control does.

Keywords: Nonlinear Control, Flight Control, High Angle of Attack

NUMENCLATURE

b = wing span
c = mean aerodynamic chord
g = acceleration of gravity
I = moment of inertia about body axis Y
m = aircraft mass
q = dynamic pressure
S = wing area
u, w = body-axis velocity components
V = aircraft speed
α = angle of attack
δ = elevator angle
θ = pitch angle
ρ = atmospheric density

1. INTRODUCTION

When an aircraft operates at high angle of attack, large aerodynamic forces exert on it. These forces make advantages such as large lift, enhancement of agility, et cetera. The dynamics of aircraft, however, is highly nonlinear over the regime where the nonlinearities of aerodynamics considerably high, so it is difficult to stabilize by a linear control. Therefore, we need to design nonlinear control laws to stabilize an aircraft over the regime.

In this paper, we design a nonlinear optimal control law to stabilize an aircraft at a high angle of attack. Models for design and evaluation are based on aerodynamic data of an existing aircraft. The control law will be developed using a new method derived from stable manifold theory to solve a Hamilton-Jacobi equation approximately[5]. It will be shown that the nonlinear optimal control law thus obtained stabilizes attitudes of the aircraft in a larger region than linear optimal control does.

2. MODELS

The aircraft to be controlled in this paper is the research vehicle used in ALFLEX(Automatic Landing FLight Experiment) conducted by NAL and NASDA to develope their automatic landing technologies. The vehicle is essentially unstable (in particular it is statically unstable in longitudinal motion), so it is necessary to stabilize it by feedback control. This characteristic makes it easy to compare how control laws effectively work. We construct a model for the ALFLEX veichle using aerodynamic data that is on the basis of secondary low-speed wind tunnel test. [7]

2.1 State Space Equation

In this paper, we will be concerned only with the longitudinal dynamics of the aircraft. In body fixed coordinates (Fig. 1), the equations of motion are described as

$$\begin{aligned} X &: m(\dot{u} + w\dot{\theta}) = -mg\sin\theta + F_x \\ Z &: m(\dot{w} - u\dot{\theta}) = mg\cos\theta + F_z \\ \text{Pitch} &: \quad I\ddot{\theta} = M \end{aligned} \tag{1}$$

Aerodynamic forces are described as

$$F_x = L\sin\alpha - D\cos\alpha\ ,\ F_z = -L\cos\alpha - D\sin\alpha$$
$$L = qSC_L\ ,\ D = qSC_D\ ,\ M = qScC_M$$

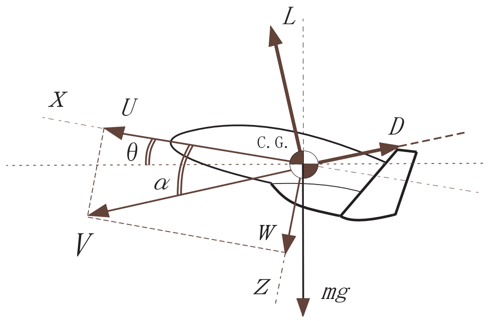

Fig. 1 body fixed coordinates

where $q = 0.5\rho V^2$. Let $u = U = \text{const}$ (**i.e.**, $\dot{u} = 0$) and substituting $w = U\tan\alpha$ into Eq. (1), we finally obtain

$$\dot{\alpha} = \frac{\cos^2\alpha}{U} \times \left\{ U\dot{\theta} + g\cos\theta - \frac{qS}{m}\left(C_L\cos\alpha + C_D\sin\alpha\right) \right\}$$

$$\ddot{\theta} = \frac{qScC_M}{I} \tag{2}$$

V is equal to $\dfrac{U}{\cos\alpha}$, and the parameters are given by

$$\begin{aligned} b &= 3.295 \quad [\mathrm{m}] \\ c &= 3.154 \quad [\mathrm{m}] \\ g &= 9.80665 \quad [\mathrm{kg/sec^2}] \\ I &= 1366 \quad [\mathrm{kgm^2}] \\ m &= 760 \quad [\mathrm{kg}] \\ S &= 9.45 \quad [\mathrm{m}] \\ \rho &= 1.225 \quad [\mathrm{kg/m^3}] \end{aligned}$$

By setting aerodynamic coefficients C_L, C_D, C_M as functions of $\alpha, \theta, \dot{\theta}$, the state space model is developed completely.

2.2 Aerodynamic Data

According to [7], each aerodynamic coefficient in Eq. (2) is described as

$$\begin{aligned} C_L(\alpha,\delta) &= C_{L(\text{basic})}(\alpha) + \Delta C_{L(\delta)}(\alpha,\delta) \\ &\quad + \Delta C_{L(\text{gear})}(\alpha) \\ C_D(\alpha,\delta) &= C_{D(\text{basic})}(\alpha) + \Delta C_{D(\delta)}(\alpha,\delta) \\ &\quad + \Delta C_{D(\text{gear})}(\alpha) \\ C_M(\alpha,\delta,\dot{\theta},V) &= C_{M(\text{basic})}(\alpha) + \Delta C_{M(\delta)}(\alpha,\delta) \\ &\quad + \Delta C_{M(\text{gear})}(\alpha) \\ &\quad + \Delta C_{M(\dot{\theta})}(\alpha)\cdot\frac{1}{2V}\dot{\theta}c. \end{aligned} \tag{3}$$

Each function is given as a table for several values of α or (α,δ) in the range of $\alpha \in [-10, 30]$ [deg], $\delta \in [-25, 35]$ [deg]. To write these functions as analytic functions, we approximate them by polynomial functions (degrees are at most five) using the method of least squares, shown as Figs. 2∼14. Note that two variable functions are assumed to be linear for δ (**e.g.**, $C_L(\alpha,\delta) \simeq C'_L(\alpha)\cdot\delta$).

2.3 Design Model and Evaluation Model

We design control laws for the system Eq. (2) where the aerodynamic coefficients are approximated as polynomial functions as mentioned in §2.2. Notice that $\Delta C_{D(\delta)}(\alpha,\delta)$ is treated as 0 in the model. To evaluate the effect of this neglection and the modeling error in the approximations of polynomial functions, we build another model where all functions in Eq. (3) are approximated as numerical functions generated by interpolation of the table. In this model, all aerodynamic coefficients, including $\Delta C_{D(\delta)}(\alpha,\delta)$, are taken account of. In the following, the former is called "design model", and the latter is "evaluation model".

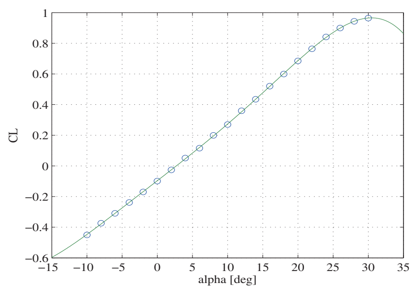

Fig. 2 approximation of $C_{L(\text{basic})}(\alpha)$

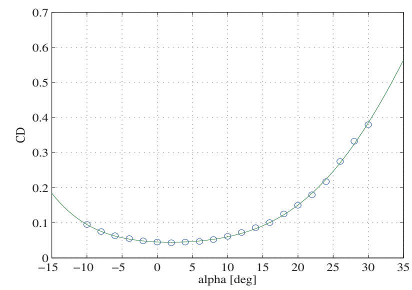

Fig. 3 approximation of $C_{D(\text{basic})}(\alpha)$

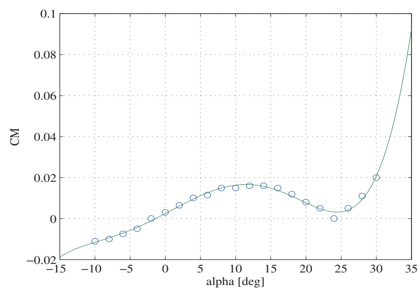

Fig. 4 approximation of $C_{M(\text{basic})}(\alpha)$

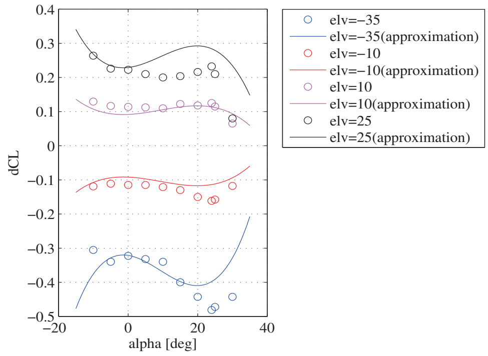

Fig. 5 approximation of $\Delta C_{L(\delta)}(\alpha,\delta)$ (δ is fixed)

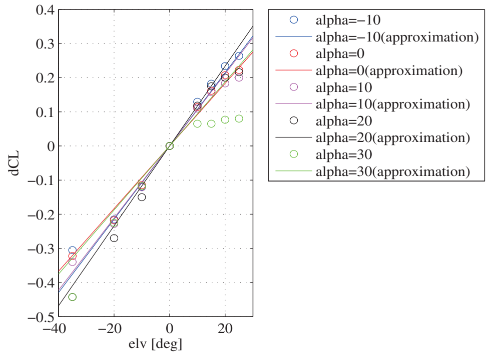

Fig. 6 approximation of $\Delta C_{L(\delta)}(\alpha,\delta)$ (α is fixed)

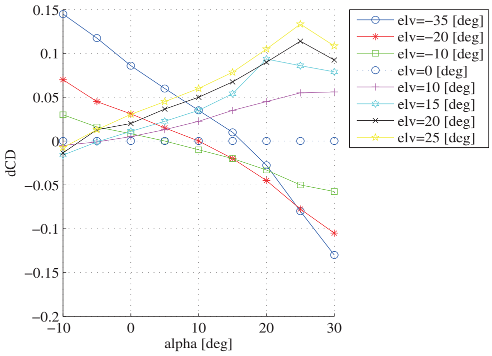

Fig. 7 approximation of $\Delta C_{D(\delta)}(\alpha,\delta)$ (δ is fixed)

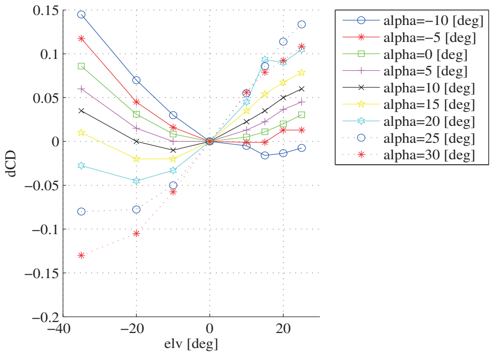

Fig. 8 approximation of $\Delta C_{D(\delta)}(\alpha,\delta)$ (α is fixed)

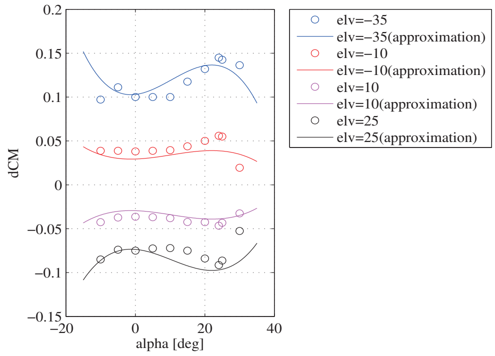

Fig. 9 approximation of $\Delta C_{M(\delta)}(\alpha,\delta)$ (δ fixed)

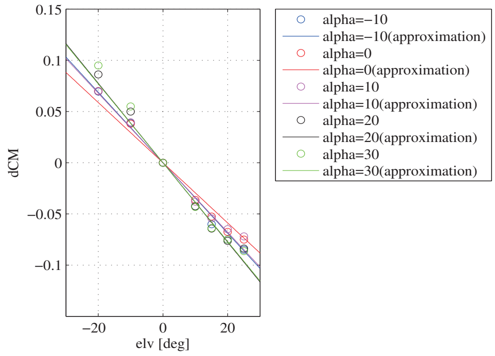

Fig. 10 approximation of $\Delta C_{M(\delta)}(\alpha,\delta)$ (α is fixed)

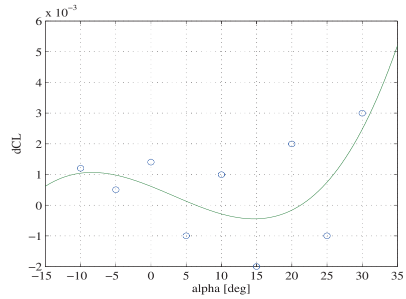

Fig. 11 approximation of $\Delta C_{L(\mathrm{gear})}(\alpha)$

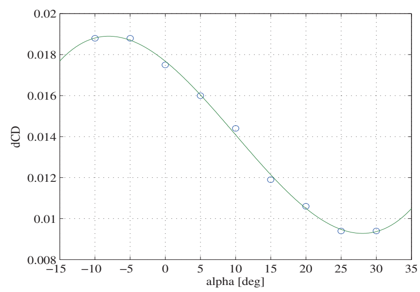

Fig. 12 approximation of $\Delta C_{D(\mathrm{gear})}(\alpha)$

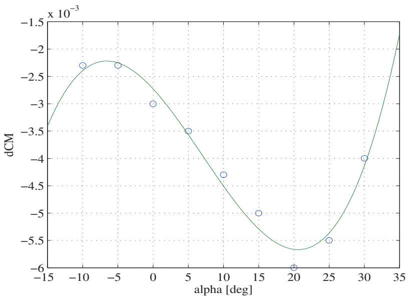

Fig. 13 approximation of $\Delta C_{M(\mathrm{gear})}(\alpha)$

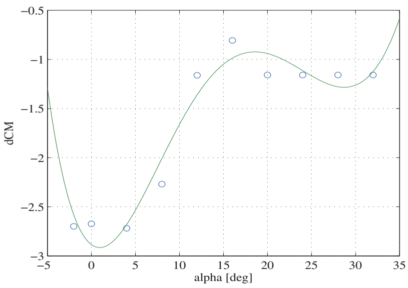

Fig. 14 approximation of $\Delta C_{M(\dot{\theta})}(\alpha)$

3. OPTIMIZATION PROBLEM

3.1 Problem Settings

Let us consider the control problem for landing of an aircraft. For a gentle landing, the speed of the aircraft must be sufficiently slow, and also the aircraft needs sufficient lift force. Unfortunately, the lower the aircraft speed is, the weaker aerodynamic forces are. Thus, the aircraft needs to hold at high angle of attack to obtain large lift if it try to land at low speed.

Here, we design a nonlinear optimal control law to stabilize the aircraft at a high angle of attack and low speed. The trim condition are $U = 35$ [m/sec], $\alpha = 23$ [deg], $\theta = 23.3$ [deg], $\dot{\theta} = 0$ [deg/sec], $\delta = -0.45$ [deg]. Eq. (2) is transformed into

$$\dot{x} = f(x) + g(x)v, \;\; f(0) = 0 \qquad (4)$$

by setting the states and the input as $x^T = [\, x_1 \; x_2 \; x_3 \,] = [\, \alpha - 23 \quad \theta - 23.3 \quad \dot{\theta} \,]$, $v = \delta - (-0.45)$. The cost function to be minimaized is defined as

$$J = \int_0^\infty x^T Q x + r v^2 dt \qquad Q = I_{(3\times 3)} \;, \; r = 1.$$

3.2 Control Law Design

To design control laws, some simplifications are made. Trigonometric functions (**i.e.**, $\sin, \cos, \tan$) are approximated as 2nd degree polynomial function by the 2nd degree Taylor expansion. As a result, each component of right hand side of Eq. (4) is a polynomial function. The terms whose degree is larger than 5 are neglected. In addition, input coefficient function $g(x)$ is treated as constant matrix $B = g(0)$.

After these simplifications, we calculate an approximate solution of canonical equation (*cf.* Appendix Eq. (6)) $x_1(t,\xi), p_1(t,\xi)$ using the stable manifold theorem, and obtain an approximate soltion V of corresponding Hamilton-Jacobi equation as a 10 degree polynomial function. Finally, a nonlinear optimal control law is obtained as $v = -\dfrac{1}{2} r^{-1} B^T \left(\dfrac{\partial V}{\partial x}\right)^T$. As for the calculation argorithm in detail, see [5].

4. RESULTS

To compare stabilizability of the obtained nonlinear optimal controller with a linear optimal controller that is developed for the linear system at the trim condition (the cost function is the same), simulation for "design model" is carried out. (Note that evaluation model is not appropriate for the purpose because it needs table-look-up that is impossible when value of α or δ get out of the table.)

Table. 1 shows the simulation results for several initial conditions [α θ 0] ($\dot{\theta} = 0$ is fixed), comparing the nonlinear optimal control with the linear optimal control.

Table. 1 comparison of stabilizable initial conditions (O : stabilizable, X : unstabilizable)

LINEAR CONTROL													
	θ [deg]												
		−15	−10	−5	0	5	10	15	20	25	30	35	40
α [deg]	0	X	O	O	O	O	O	O	O	O	O	O	O
	5	X	O	O	O	O	O	O	O	O	O	O	O
	10	X	X	O	O	O	O	O	O	O	O	O	O
	15	X	X	X	O	O	O	O	O	O	O	O	O
	20	X	X	X	X	O	O	O	O	O	O	O	O
	25	X	X	X	X	X	O	O	O	O	O	O	O
	30	X	X	X	X	X	X	O	O	O	O	O	O
	35	X	X	X	X	X	X	X	X	X	O	O	O
	40	X	X	X	X	X	X	X	X	X	X	X	X
	45	X	X	X	X	X	X	X	X	X	X	X	X
NONLINEAR CONTROL													
	θ [deg]												
		−15	−10	−5	0	5	10	15	20	25	30	35	40
α [deg]	0	O	O	O	O	O	O	O	O	O	O	O	O
	5	O	O	O	O	O	O	O	O	O	O	O	O
	10	O	O	O	O	O	O	O	O	O	O	O	O
	15	O	O	O	O	O	O	O	O	O	O	O	O
	20	O	O	O	O	O	O	O	O	O	O	O	O
	25	O	O	O	O	O	O	O	O	O	O	O	O
	30	O	O	O	O	O	O	O	O	O	O	O	O
	35	O	O	O	O	O	O	O	X	X	X	X	X
	40	O	O	O	O	O	O	X	X	X	X	X	X
	45	X	X	X	X	X	X	X	X	X	X	X	X

The colored regions in the table indicates the conditions that only linear/nonlinear controller can stabilize. We see from Table. 1 that the region of initial conditions where the nonlinear control stabilizes are considerably larger than the linear control does though there are several conditions for which the nonlinear control is inferior to the linear one.

The simulation for "evaluation model" is also carried out for the intial conditions and similar results are obtained, which suggests that the control law developed for design model is also valid for the real system. The values of cost function for several cases in the simulation for evaluation model are shown below. The initial conditions are

$$[\, \alpha \; \theta \; \dot{\theta} \,] = \begin{cases} \text{(a)} & [10[\mathrm{deg}] \quad 0[\mathrm{deg}] \quad 0[\mathrm{deg/sec}]] \\ \text{(b)} & [20[\mathrm{deg}] \quad 10[\mathrm{deg}] \quad 0[\mathrm{deg/sec}]] \\ \text{(c)} & [30[\mathrm{deg}] \quad 20[\mathrm{deg}] \quad 0[\mathrm{deg/sec}]] \end{cases}$$

The values of cost function J for each cases are

$$\begin{cases} \text{linear control} & \text{(a) } 0.239 \quad \text{(b) } 0.0775 \quad \text{(c) } 0.0284 \\ \text{nonlinear control} & \text{(a) } 0.234 \quad \text{(b) } 0.0764 \quad \text{(c) } 0.0219 \end{cases}$$

Consequently, implovements for each cases are

$$\frac{J(\text{nonlinear})}{J(\text{linear})} = \begin{cases} \text{(a)} & \dfrac{0.234}{0.239} = 97.5\% \\ \text{(b)} & \dfrac{0.0764}{0.0775} = 98.6\% \\ \text{(c)} & \dfrac{0.0219}{0.0284} = 77.1\% \end{cases}$$

The time histories of states and input for each cases are shown as below.

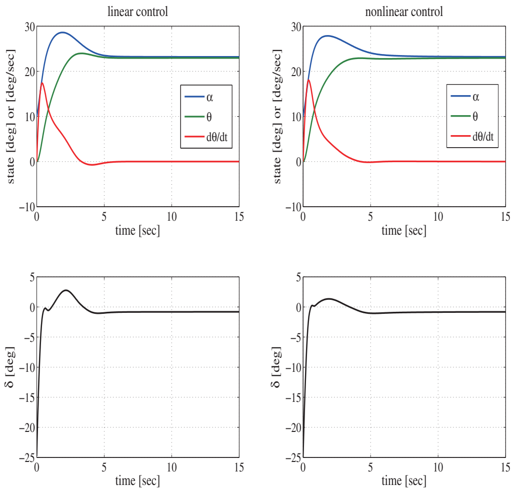

Fig. 15 Case (a) : time history of states and input

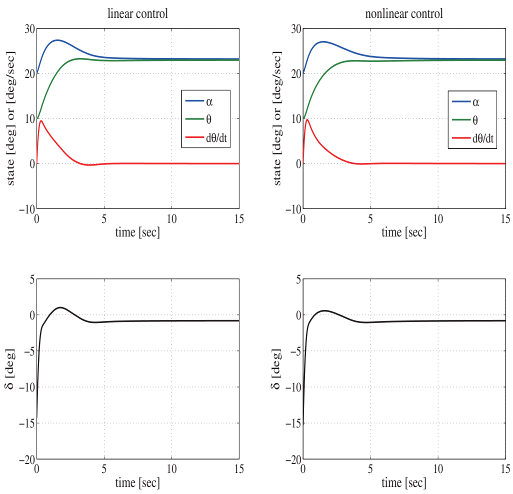

Fig. 16 Case (b) : time history of states and input

5. CONCLUSIONS

In this paper, we have considered an attitude stabilization problem to hold an aircraft at a high angle of attack.

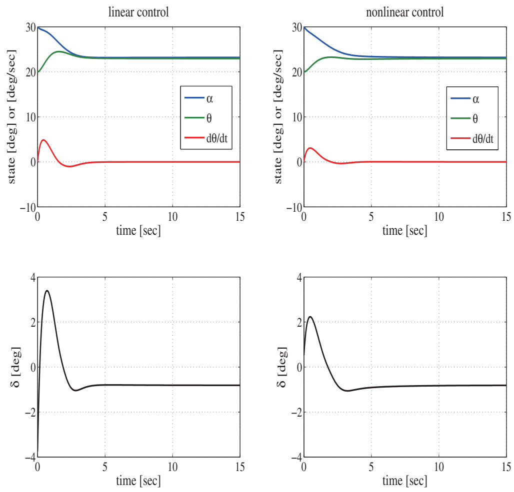

Fig. 17 Case (c) : time history of states and input

The models for control law design and performance evaluation have been developed based on aerodynamic data for an existing aircraft. A nonlinear optimal control law has been designed using a new method derived from stable manifold theory.

It has been found from the simulation results that the obtained nonlinear optimal control law stabilizes in a considerably larger region than linear control does, and is effective not only for the analytic design model but also the numerical model constructed by interpolation of aerodynamic data table. In the latter case, it has been confirmed that value of the cost function can be decreased more than 20% compared with the linear optimal control.

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APPENDIX

A OPTIMAL CONTROL THEORY

Consider a system Σ and a cost function J as below. Assuming that the system has at least one equilibrium point. Without loss of generality, it is assumed to be $x = 0$.

$$\begin{cases} \Sigma : \dot{x} = f(x) + g(x)v \\ J = \int_0^\infty x^T Q x + v^T R v \, dt \end{cases}$$

The solution for the optimal control problem is given by

$$v = -\frac{1}{2} R^{-1} g(x)^T \left(\frac{\partial V}{\partial x} \right)^T$$

where $V(x)$ is the solution of nonlinear partial equation – so called Hamilton-Jacobi equation – described as

$$\left(\frac{\partial V}{\partial x} \right) f(x) - \frac{1}{4} \left(\frac{\partial V}{\partial x} \right) g(x) R^{-1} g(x)^T \left(\frac{\partial V}{\partial x} \right)^T + x^T Q x = 0$$

Furthermore, the following holds.

$$V(x_0) = \min_u J(x_0) \quad \text{for} \quad x(0) = x_0 \tag{5}$$

Now, let us consider the linear system

$$\Sigma' : \ \dot{x} = Ax + Bv$$
$$\text{where} \quad A = \left[\frac{\partial f}{\partial x} \right] \Bigg|_{x=0}, \ B = g(0)$$

Assuming that the stabilizing solution P exists for Riccati equation

(RIC) $PA + A^T P - PBR^{-1}B^T P + Q = O$

and W is the solution to the Lyapunov equation

(LYAP) $(A - SP)W + W(A - SP)^T = S$

where $S = BR^{-1}B^T$

and set

$$T = \begin{bmatrix} I & \frac{1}{2} W \\ 2P & PW + I \end{bmatrix}$$

The solution of Hamilton-Jacobi equation can be obtained from the stable manifold of the system of differential equations such as

$$\begin{cases} \dot{x} = \dfrac{\partial H}{\partial p} \\ \dot{p} = -\dfrac{\partial H}{\partial x} \end{cases} \tag{6}$$

where $H(x, p) = p^T f(x) - \frac{1}{4} p^T g(x) R^{-1} g(x)^T p + x^T Q x$

Transforming $[x, p]^T$ to $[x', p']$ by $[x, p]^T = T[x', p']^T$, then we obtain

$$\begin{bmatrix} x' \\ p' \end{bmatrix} = \begin{bmatrix} A - SP & O \\ O & -(A - SP)^T \end{bmatrix} \begin{bmatrix} x' \\ p' \end{bmatrix} + \begin{bmatrix} \phi(t, x', p') \\ \psi(t, x', p') \end{bmatrix}$$

where ϕ, ψ are smooth nonlinear functions. The stable manifold of this system of equations can be obtained approximately by theorem below.

Theorem 1 (Sakamoto, N., van der Schaft, A.J.) Consider a system of equations given by

$$\begin{cases} \dot{x} = \ \ Fx + \phi(x, y) \\ \dot{y} = -F^T y + \psi(x, y) \end{cases} \tag{7}$$

where F is a stable matrix, ϕ, ψ are smooth nonlinear functions. Then, consider recurrence relations;

$$\begin{cases} x_{k+1}(t, \xi) \\ \quad = e^{Ft}\xi + \int_0^t e^{F(t-s)} \phi(s, x_k(s, \xi), y_k(s, \xi)) ds \\ y_{k+1}(t, \xi) \\ \quad = -\int_t^\infty e^{-F^T(t-s)} \psi(s, x_k(s, \xi), y_k(s, \xi)) ds \end{cases}$$

$$\begin{cases} x_0(t, \xi) = e^{Ft}\xi \\ y_0(t, \xi) = \ \ 0 \end{cases}.$$

if $||\xi||$ is sufficiently small, $x_k(t, \xi), y_k(t, \xi)$ converge uniformly to the solution on the stable manifold of Eq. (7) as $k \to \infty$. □