TDOA UWB Positioning with Three Receivers Using Known Indoor Features

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***Abstract*—Ultra-Wideband is an attractive technology for short range positioning, especially indoors. However, for normal 3D Time Difference of Arrival (TDoA) positioning, at least four receivers with unblocked direct path to the transmitter are required. A requirement that is not always met. In this work, a novel method for TDoA positioning using only three receivers is presented and tested using real-world measurements. Positioning with three receivers is possible by exploiting the knowledge of some of the indoor features, namely positions of big flat reflective surfaces, reflectors, for example ceiling and walls.**

***Index Terms*—UWB, positioning, TDoA, indoor**

I. Introduction

The main challenge for indoor positioning is the big number of reflections and diffractions, from walls, furniture, people, that introduce strong fading. The conventional way to combat fading is to increase the bandwidth of the signal, which not only limits fading but also increases time resolution and accuracy of time-based ranging. Ultra-Wideband (UWB) radio is then a natural choice for high precision indoor positioning. Such positioning has many possible applications, for example in-building goods tracking, workers monitoring or access control [1].

For standard Three Dimensional (3D) Time Difference of Arrival (TDoA) Positioning ranges to four or more receivers are needed [2]. This is because, in addition to 3D transmitter position, time sent also has to be estimated. When only three receivers are available, the conventional method is to reduce the problem to 2D by assuming height[3]. Method that, apart from not estimating height, can introduce a height-related 2D bias to the results.

In this paper we propose a novel method for three receiver positioning that uses the high time resolution of UWB signals and knowledge of big flat reflective surfaces in the environment, which we will call reflectors, to estimate the 3D position.

Because of the high time resolution of UWB signal, multipath components (MPCs) are distinguishable. Some MPCs are caused by reflections from known reflectors. We will use MPC delays together with knowledge of reflector positions for positioning.

Such usage of later arriving MPCs was not, to our knowledge, reported in the literature before. The proposed method was tested using signals obtained during a measurement campaign.

This paper builds upon our previous work on ToA positioning methods. In [4] we presented a MPC-based ToA method for two receivers using Least Squares approach, validated by simulation. In [5] we introduced a much improved Maximum Likelihood(ML)-based method, using measurement results for validation. Finally, in this paper we tackle TDoA positioning with three receivers and a new ML-based TDoA method.

The rest of the paper is organized as follows: We start with a general description of the problem in section II. Next in section III, we define the system elements. We then introduce our own method in section IV, dividing it into result curve calculation step in IV-A and ML on-curve position estimation step in IV-B. Next, in section V we present results of the proposed method when used with measurement data. Finally, we draw conclusions in section VI.

II. Problem Statement

In TDoA ranging transmitter T is unsynchronized with receivers $\mathbf{R} = [R_1, \cdots, R_N]$. Because signal sent time is not known, range from receiver R_n to transmitter T, d_n, cannot be directly determined from signal reception time t_n. Instead, range differences between different receivers R_n can be computed. Choosing R_1 as reference, for each other R_n, estimate of range difference, $\hat{d}_{n,1}$, can be calculated as:

$$\hat{d}_{n,1} = C(t_n - t_1), \qquad n = 1, 2, \ldots, N \tag{1}$$

where C is the speed of light.

Next, remembering that $d_{n,1} = d_n - d_1$, position of the transmitter T can be estimated by solving:

$$\begin{cases} \hat{d}_{2,1} = \sqrt{(x_t - x_2)^2 + (y_t - y_2)^2 + (z_t - z_2)^2} \\ \qquad - \sqrt{(x_t - x_1)^2 + (y_t - y_1)^2 + (z_t - z_1)^2} \\ \qquad \vdots \\ \hat{d}_{N,1} = \sqrt{(x_t - x_N)^2 + (y_t - y_N)^2 + (z_t - z_N)^2} \\ \qquad - \sqrt{(x_t - x_1)^2 + (y_t - y_1)^2 + (z_t - z_1)^2} \end{cases} \tag{2}$$

where $T := [x_t, y_t, z_t]$ is the position of transmitter and $R_n := [x_n, y_n, z_n]$ that of receiver n, $n = 1 \cdots N$. For $N > 4$, a data fusion method, such as Least Squares, can be used. For $N = 4$, the set can be solved directly.

For $N = 3$, only three receivers, the result is not a point, but a second degree curve (hyperbola, ellipse or parabola).

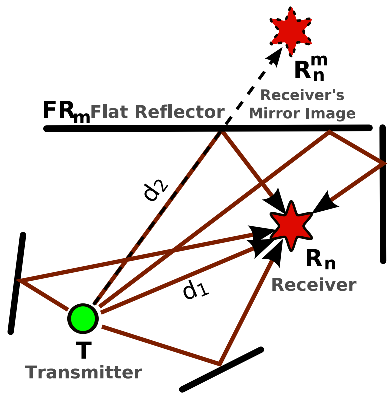

Fig. 1. Many reflected paths with similar path length - problem with finding MPC matching FR_m reflection. Example of receiver's mirror image concept.

More information is needed to find 3D position of the transmitter. The conventional method is to assume the height coordinate and reduce the problem to 2D. This method, if transmitter's height is not nearly constant, will produce a bias. Since this bias is height-related, it cannot be reduced by filtering (in case of a moving tag).

To solve this problem, to find the 3D transmitter position, information contained in received MPCs is used.

If for each receiver-reflector pair it was known which MPC in the signal detected by receiver matches the reflection from the reflector, using MPC delays for positioning would be trivial. Each reflector-MPC match found would be an extra range measurement to an imaginary receiver, mirror image of the real receiver.

Unfortunately, indoor environment is rich in reflections and diffractions that cause many MPCs. A simple example is presented in Fig. 1. Correct reflector-MPC matches are not known a priori. This paper concentrates on the problem of using information contained in MPCs, assuming only that for each reflector there is a matching MPC among detected MPCs.

III. System model

A mobile UWB transmitter T is sending pulses. Each pulse is received by a set of N stationary receivers $\mathbf{R} = [R_1, \cdots, R_N]$. Each receiver is described by its 3D position vector, $R_n := [x_n, y_n, z_n]$. Signal at R_n is often represented in the literature as:

$$r_n(t) = \sum_{k_n=1}^{K_n} \alpha_{k_n} s(t - \tau_{k_n}) + n(t), \tag{3}$$

where K_n is the number of MPCs, α_k and τ_k are the fading coefficient and delay of k_{th} MPC, respectively, $n(t)$ is zero-mean additive white Gaussian noise (AWGN), $s(t)$ is the transmitted pulse shape. Subscript n defines to which receiver's received signal the parameter applies to.

The transmitter $T := [x_t, y_t, z_t]$ is inside Service Area SA, defined by vector $[x_{\min}, y_{\min}, z_{\min}, x_{\max}, y_{\max}, z_{\max}]$.

$$T \in SA \Leftrightarrow \begin{array}{l} x_{\min} \le x_t \le x_{\max} \\ y_{\min} \le y_t \le y_{\max} \\ z_{\min} \le z_t \le z_{\max} \end{array} \tag{4}$$

No knowledge about transmitter's position in SA is assumed.

Knowledge of a set of M big, flat reflectors, for example ceiling and walls, is assumed. The set is represented as $\mathbf{FR} = [FR_1, \cdots, FR_M]$. Each reflector FR_m is described by roughness $\sigma^2_{FR_m}$ and a 3D surface equation:

$$\begin{array}{l} A_m x + B_m y + C_m z + D_m = 0 \\ \sqrt{A_m^2 + B_m^2 + C_m^2} = 1 \end{array} \tag{5}$$

where $[A_m, B_m, C_m, D_m]$ are normalized surface coefficients of reflector FR_m and $[x, y, z]$ are coordinates in 3D space.

Roughness $\sigma^2_{FR_m}$ is the assumed added path length variance caused by reflection from FR_m. $\sigma^2_{FR_m}$ models both error in assumed reflector position and irregularities of the reflector.

The knowledge of $\mathbf{FR}$ can be either given a priori, or gained using a calibration step. Mirror image of receiver R_n through reflector FR_m will be denoted as R_n^m.

$$R_n^m = R_n - 2(R_n \cdot [A_m\ B_m\ C_m] + D_m)[A_m\ B_m\ C_m] \tag{6}$$

where $R_n^m := [x_n^m, y_n^m, z_n^m]$.

In ranging, each receiver R_n can detect not only first, but all distinct MPCs. Result of ranging is a vector of measured MPC arrival times $[\hat{t}_n^1, \ldots, \hat{t}_n^J]$, where $\hat{t}_n^j$ is assumed to have a Gaussian error distribution: $\hat{t}_n^j = t_n^j + e_n^j \sim N(0, \sigma_n^{j2})$.

Without knowing t_0, signal sent time, ranges cannot be directly calculated. Range differences to a chosen reference are calculated instead. $\mathbf{R}$ is reordered so that $\hat{t}_1^1 = \min_{n\in(1,N)}(t_n^1)$, ie. R_1 becomes the closest receiver to T. R_1 and t_1^1 are chosen as reference.

N vectors of range difference estimates $\mathbf{d_n} = [\hat{d}_{n,1}^1, \cdots, \hat{d}_{n,1}^J]$ are calculated as:

$$\hat{d}_{n,1}^j = (\hat{t}_n^j - \hat{t}_1^1)C \tag{7}$$

$$\hat{d}_{n,1}^j = d_{n,1}^j + e_{n,1}^j \sim N(0, C^2(\sigma_n^{j2} + \sigma_1^{j2})) \tag{8}$$

where C is the speed of light.

In each received signal, the MPC corresponding to the direct path is assumed to be detected, as well as most MPCs matching reflections from $\mathbf{FR}$. The first MPC range of $\mathbf{d_n}$, $\hat{d}_{n,1}^1$ will then correspond to the direct path between T and R_n. $\mathbf{d_n}$ will also contain a subset corresponding to $\mathbf{FR}$-reflected paths. This subset is not known a priori.

IV. Position Estimation with Three Receivers

A. Result Curve(RC)

The first step of the proposed method is to use direct path range differences to calculate Result Curve(RC). RC should contain / be near to real transmitter position. RC is calculated as the solution of (2) for three receivers. In a non-degenrate case it will be a second degree curve, hyperbola, ellipse or parabola. An example RC is shown on Fig. 2. Paper

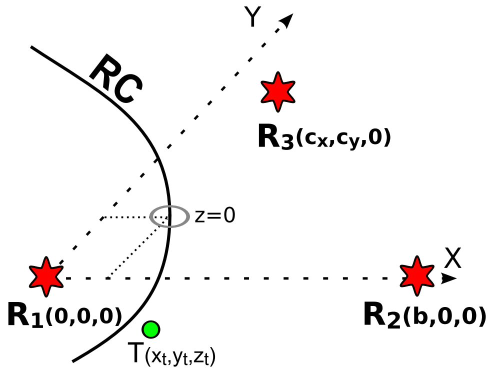

Fig. 2. Result Curve (RC) - Second degree curve, solution of eq. (2) for three receivers. Real transmitter position should be near RC. Presented in prime base.

[6] presents a simple way to calculate and represent RC. Following it, RC is defined with a $[g, h, d, e, f]$ parameter vector and a 3D rotation + translation transformation $Q(\cdot)$ from real base to prime base, defined by receiver positions: $R'_1 = [0, 0, 0]$, $R'_2 = [b, 0, 0]$, $R'_3 = [c_x, c_y, 0]$.

$$\begin{cases} y'_t = g x'_t + h \\ z'_t = \pm\sqrt{d x'^2_t + e x'_t + f} \end{cases} ,T = Q^{-1}(T') \quad (9)$$

where $T' := [x'_t, y'_t, z'_t]$ are possible transmitter positions in prime base, $T := [x_t, y_t, z_t]$ are possible transmitter positions in real base. Transmitter's position on the RC can be described with z'_t as $T(z'_t)$. For each z'_t there are two possible x'_t. x'_t corresponding to lower $|T'|$ (ie. closer to R_1) is chosen. This can cause error if RC is an ellipse, both x'_t being correct solutions. Since $T \in SA$, only $z'_t : T(z'_t) \in SA$ are considered.

B. Position on RC Calculation Algorithm

The second step of the proposed method is finding real transmitter position $T(z'_t)$ on RC. Reflector positions and MPC delays are used. Since reflector-MPC matches are not known, a statistical approach is used. Each detected MPC is assumed to be a possible match for reflector.

For each receiver-reflector pair $(R_n, FR_m) \in \mathbf{R} \times \mathbf{FR}$, $d^j_{n,1}$ matching to FR_m-reflected path to R_n will, with big probability, be among detected MPC range differences, $\mathbf{d_n}$. FR_m-reflected path to R_n can be represented as a direct path to R^m_n, as discussed in Section III. The error of MPC detection is assumed to be Gaussian with $\sigma^{m2}_n = \sigma^2_n + \sigma^2_{FR_m}$, σ^2_n being range measurement variance (assumed constant for all MPCs), $\sigma^2_{FR_m}$ being reflector's roughness. Then, likelihood function for possible $T(z'_t)$, $L(\mathbf{d_n}; z'_t)$ is constructed as follows:

$$L(\mathbf{d_n}; z'_t) = N_{\text{rm}} P_{\text{ndet}} + \\ + N_{\text{rm}} \sum_{j=1}^{J_n} P_j \exp\left(\tfrac{-1}{2} \left(\tfrac{d(T(z'_t), R^m_n) - (d^j_{n,1} + d(T(z'_t), R_1))}{\sigma^m_n} \right)^2 \right) \quad (10)$$

where N_{rm} is a normalization constant, P_{ndet} represents the chance that the matching MPC was not detected, $d(T(z'_t), R^m_n)$ is the range between $T(z'_t)$ and R^m_n, $d^j_{n,1}$ is the range difference for j_{th} MPC in the R_n received signal. P_j is a penalty

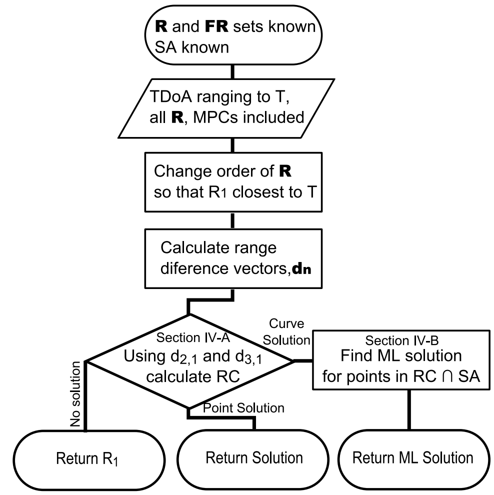

Fig. 3. Flowchart of the proposed algorithm

lowering contributions from early MPCs to offset the tendency of the algorithm to assign high probability near reflector FR_m.

$$P_j = \min\left(\exp\left(- P_{\text{fst}} \left(1 - \frac{d^j_{n,1} - d^1_{n,1}}{d_{max}}\right)\right), 1\right) \quad (11)$$

where P_{fst}, first MPC penalty, and d_{max}, maximum penalty length, are algorithm parameters.

If the matching $d^j_{n,1}$ is detected, the value of $L^m_n(z'_t)$ for $T(z'_t)$ near to the real position of T will be high. Because of other, non-matching MPCs the value of $L^m_n(z'_t)$ will also be high for other z'_t. However, for each (R_n, FR_m) pair, $L^m_n(z'_t)$ near T will be high but high value regions caused by non-matching MPCs will be different. If all $L^m_n(z'_t)$ for $(R_n, FR_m) \in \mathbf{R} \times \mathbf{FR}$ are combined, the result likelihood function $L(z'_t)$ should still be high near T, while semirandom and lower for other z'_t. The total likelihood function $L(z'_t)$ is calculated as follows:

$$L(z'_t) = \prod_{n \in [1,N], m \in [1,M]} L^m_n(z'_t) \quad (12)$$

This calculation assumes the independence between L^m_n which is not strictly correct, but the introduced error is small. Finally, z'_t estimate is found by maximizing likelihood:

$$\hat{z}'_t = \arg\max_{z'_t} \left(\sum_{n \in [1,N], m \in [1,M]} \ln L^m_n(z'_t) \right) \quad (13)$$

The result transmitter position is $T(\hat{z}'_t)$.

A flowchart of the complete algorithm is presented in Fig. 3

V. Measurement results

In order to verify the proposed method, we performed measurements at Warsaw University of Technology(PW), Department of Electronics and Information Theory(EiTI), in cooperation with Dr. Jerzy Kolakowski.

Fig. 4 presents measurement setup. The transmitted signal was designed to roughly correspond to the 3.4-4.8 GHz band.

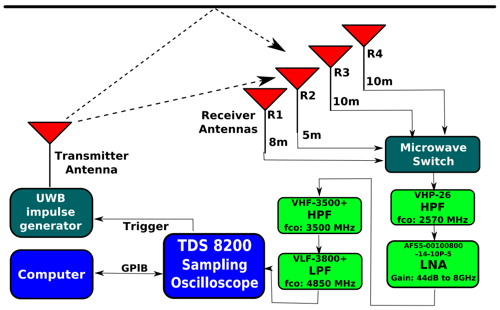

Fig. 4. Measurement Setup, schematic

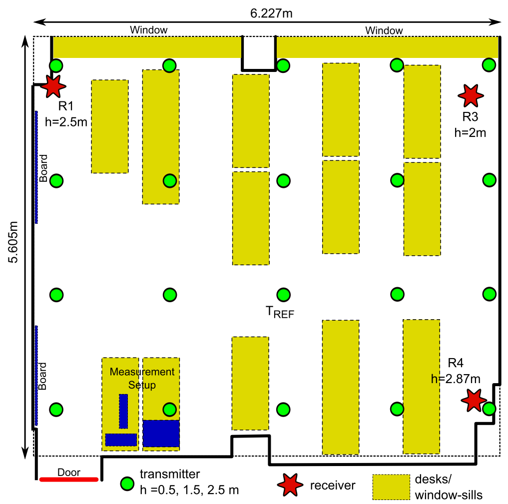

Fig. 5. Experiment Layout. Lecture Room S36, EiTI, PW

The service area was a classroom, as presented on Fig. 5. Receivers R1, R3 and R4, were used. The considered reflectors were: ceiling, floor, left, right and door walls, in that order. Measurements were performed for transmitters at heights of 0.5, 1.5 and 2.5m placed at points shown in Fig. 5, for a total of 60 positions. Modified CLEAN algorithm was used for MPC detection [7]. Table I presents method parameters used. 3D Mean Square Errors (MSE) for 60 transmitter positions using different 3-receiver methods are presented in Fig. 6.

Best results are achieved with Time of Arrival positioning. If available, ToA should be used. The standard three receivers TDoA algorithm, Assumed Height has MSE of 89 cm. The

TABLE I
PARAMETERS USED IN THE PROPOSED METHOD

P_{ndet}	0.05
P_{fst}	1.25
$d_{\max}[m]$	1.5
σ_n	$0.10m$
σ_{FR_1}	$0.12m$
σ_{FR_2}	$0.18m$
σ_{FR_3}	$0.20m$
σ_{FR_4}	$0.20m$
σ_{FR_5}	$0.30m$

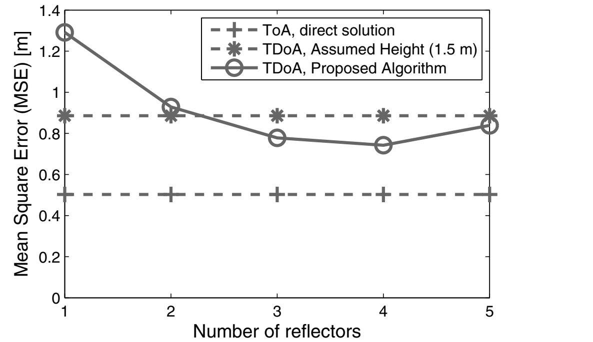

Fig. 6. Average 3D positioning error for different number of known reflectors

proposed method is better if 3 or more reflectors are used. Best result, achieved for 4 reflectors, lowers the MSE by 12% comparing to Assumed Height method.

Results of the proposed method generally improve with the number of reflectors used. However, this is not the case for the 5th reflector, the door wall. As can be seen on Fig. 5, door wall is uneven, with sections at different depths, making a flat reflector a poor approximation of its shape and introducing error defeating the gain.

VI. Conclusions

In this paper we presented a TDoA positioning method using three receivers and knowledge of indoor reflective surfaces. Measurement results show that it can determine the transmitter's position with better accuracy than the conventional Assumed Height method. A more random error distribution, which is useful in conjunction with position-tracking algorithms, is also a benefit. The proposed method is best to be used as a backup scheme in a bigger localization system, for cases when only three receivers are reliably in the range of the transmitter.

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