

Cyclic Scheduling and Re-scheduling in Response to Change of Product Mix

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Abstract—This paper deals with a cyclic job shop scheduling problem requiring agile planning of an appropriate production schedule corresponding to a change of product mix. The product mix, which is defined as the total composite of products offered by a company, is generally changed to meet demand in a market. We propose the formulation of a cyclic scheduling problem in the form of mixed integer programming to plan optimum schedules under different product mix conditions. We also introduce a formulation of the re-scheduling problem that allows shifting to a new production schedule as soon as possible. Numerical simulations are carried out to verify the proposed formulations.

Index Terms—cyclic scheduling, re-entrant process, product mix, mixed integer programming, optimum solution

I. INTRODUCTION

In current industrial manufacturing, high-volume production is still mainstream, while remarkable techniques for low-volume and high-variety productions are being studied in order to meet the diverse preferences of customers. This paper focuses on the cyclical, high-volume processing of several types of products in using the same manufacturing resource. Especially, the use of one manufacturing resource such as a machine tool or robot more than twice in one production sequence is permitted in order to reduce the cost for the equipments in a factory.

In this paper, a formulation for a cyclic scheduling problem including re-entrant processes under a given product ratio is defined by mixed integer programming. Furthermore, we introduce another formulation for the re-scheduling problem that allows response a new production plan when the product ratio is changed to meet market demand.

II. DEFINITION OF CYCLIC SCHEDULING PROBLEM

A. Nomenclature

The following symbols are used for representations of the formulations defined here.

$j_{\alpha,i}$	i th job for product α .
$s_{\alpha,i}, f_{\alpha,i}$	relative start time and finish time of job $j_{\alpha,i}$ on cyclic Gantt chart.
$S_{\alpha,i}, F_{\alpha,i}$	absolute start time and finish time of job $j_{\alpha,i}$ on conventional Gantt chart.
$p_{\alpha,i}$	processing time of $j_{\alpha,i}$
$\mu_{\alpha,i}$	machine operating job $j_{\alpha,i}$
n_{α}	number of processes for product α
P	set of products α

TABLE I

AN EXAMPLE OF SCHEDULING PROBLEM
(4 PRODUCTS CONSISTING OF 8 PROCESSES BY 4 MACHINES)

No	Machine No. : Processing time							
1	1:1	2:2	3:2	4:1	1:3	2:4	3:2	4:1
2	1:2	3:1	4:2	2:1	3:4	4:2	1:2	2:1
3	1:3	4:2	3:1	2:1	4:3	3:2	1:2	2:1
4	2:2	1:1	3:3	4:1	3:2	1:1	2:1	4:2

R	set of machines $\mu_{\alpha,i}$
T	cycle time
M	large real constant number
$x_{\alpha,i,\beta,k}$	0-1 integer variable representing precedence of job $j_{\alpha,i}$ and $j_{\beta,k}$
$y_{\alpha,i}$	0-1 integer variable representing cyclic term of $j_{\alpha,i+1}$ toward $j_{\alpha,i}$

B. Job shop and cyclic scheduling problems including re-entrant processes

Stable and constant cyclic operations in industrial manufacturing must be achieved for high-volume production. Such cyclic operations are planned with use of a unit schedule. The unit schedule is obtained by a cyclic scheduling procedure, which is different from the well-known job shop scheduling procedure.

Table 1 shows an example of production procedures and operating times for four products with eight processes by 4 machines. Each machine is used twice for each product. For instance, the first and fifth processes for the No.1 product are operated by machine 1 as shown in Table 1. These processes are called re-entrant processes and appear in semiconductor production. In semiconductor processes, the machines and equipment are extremely expensive therefore, the machines are used repeatedly in the process. In addition to the special production required by expensive machines, multi-functional machine tools provided with milling and turning operations have been developing to make machine parts for common industrial products. The re-entrant process plans supported by the multi-functional machines can contribute to achieve the high volume productions in further low costs.

The cyclic production based on conventional techniques for a job shop scheduling procedure is shown in Fig.1 for an example.

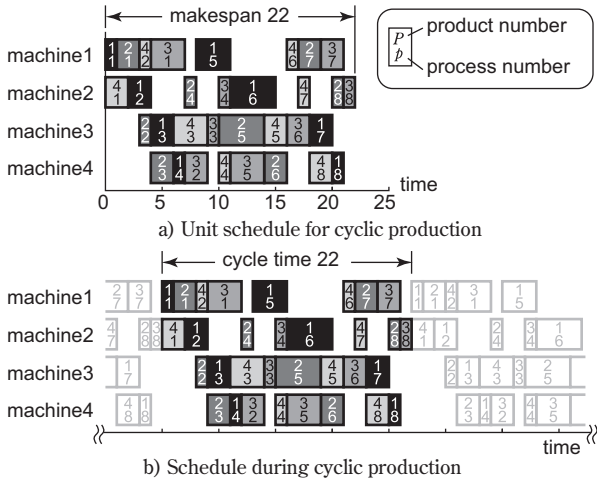


Fig. 1. Examples of unit schedule and cyclic schedule

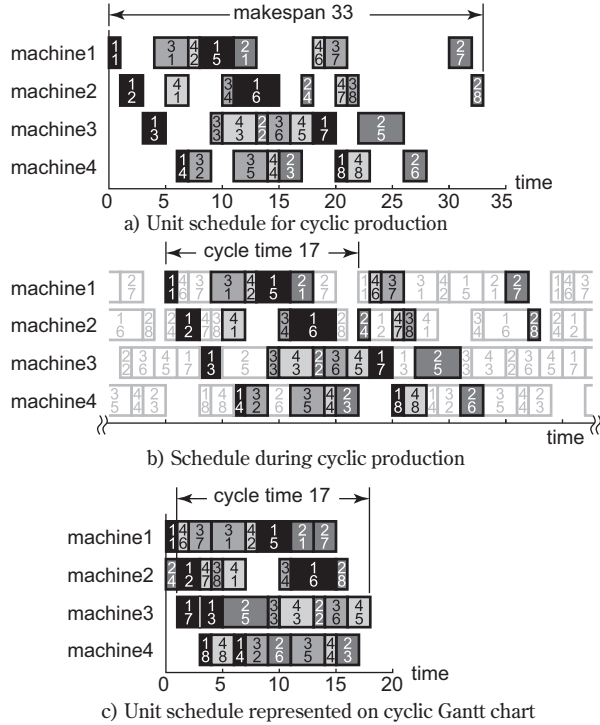


Fig. 2. Examples of unit schedule and cyclic schedule

Figure 1(a) shows a unit schedule for cyclic production planned to minimize the makespan corresponding to the production problem given by Table 1. The makespan is defined as the time from the beginning of the first operating job to the completion of the final operating job in the overall unit schedule. Figure 1(b) is the schedule during cyclic production derived from the basic schedule shown in Fig.1(a). The plan for every machines has idle times during production. This means that the maximum productivity is not achieved by this unit schedule.

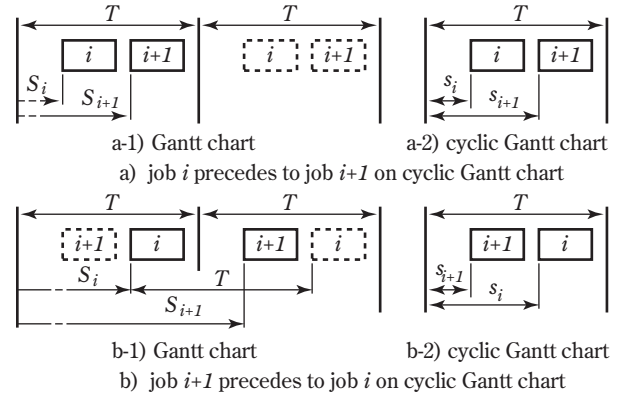


Fig. 3. Relationship between continuous jobs on Gantt chart and cyclic Gantt chart

Figure 2(a) shows an optimum unit schedule for the cyclic production planned by the formulations introduced here. Figure 2(b) is the schedule based on the unit schedule, and machine 3 has no idle time during the production. Therefore the schedule gives the maximum productivity of the factory for the product problem. Figure 2(c) is another unit schedule shown in cyclic Gantt chart. It also gives the optimum cyclic schedule shown in Figure 2(b). A formulation for cyclic scheduling problem can be derived in referring to Figure 2(a) and 2(c).

C. Formulation for schedule in cyclic production

The formulations for the cyclic schedule are defined as follows.

$$\text{minimize } C_{max} \quad (1)$$

subject to

$$C_{max} \geq S_{\alpha, n_{\alpha}} + p_{\alpha, n_{\alpha}} \quad (2)$$

$$S_{\alpha, 1} = 0 \quad \alpha \in P \quad (3)$$

$$s_{1,1} = 0 \quad (4)$$

$$s_{\alpha, i} \leq T \quad (5)$$

$$S_{\alpha, i+1} - S_{\alpha, i} \geq p_{\alpha, i} \quad (6)$$

$$S_{\alpha, i+1} - S_{\alpha, i} = s_{\alpha, i+1} - s_{\alpha, i} + T \cdot y_{\alpha, i} \quad (7)$$

$$s_{\beta, k} \geq s_{\alpha, i} + p_{\alpha, i} - T \cdot x_{\alpha, i, \beta, k} \quad (8)$$

$$s_{\alpha, i} \geq s_{\beta, k} + p_{\beta, k} - T \cdot (1 - x_{\alpha, i, \beta, k})$$

$$\{\mu_{\alpha, i}, \mu_{\beta, k} | \mu_{\alpha, i} = \mu_{\beta, k},$$

$$\mu_{\alpha, i}, \mu_{\beta, k} \in R\},$$

$$x_{\alpha, i, \beta, k}, y_{\alpha, i} \in \{0, 1\}, \alpha, \beta \in P,$$

$$i < k, i = 1, 2, \dots, n_{\alpha}, k = 1, 2, \dots, n_{\beta}$$

Equation (1) shows that the purpose of the cyclic scheduling is to minimize the maximum value of the flow times of the products. Each flow time of the product is defined by the difference between the absolute finish time of the last process and the absolute start time of the first job for every product.

The absolute start time of all the first processes $S_{\alpha, 1}$ can be fixed to zero temporarily for the optimization in order to

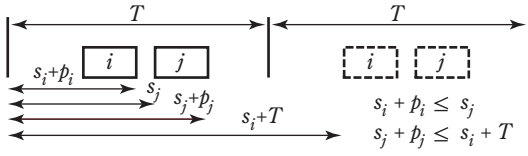


Fig. 4. Restriction between two processes

shorten the time required for the optimization in equation (3). Therefore, the maximum flow time is represented only by the finish time as shown in equation (2). The minimization of the flow time means reducing the amount of stock of the products in the factory. After the optimization is completed, the absolute start and finish times are modified according to the following equations (9).

$$S'_{\alpha,i} = S_{\alpha,i} + s_{\alpha,1} \quad \alpha \in P, \quad i = 1, 2, \dots, n_{\alpha} \quad (9)$$

One of the relative start times of the jobs must be fixed to a certain value to avoid circulation; thus, the start time of the first process of the first product $s_{1,1}$ is set to zero as shown in the equation (4). The relative start times are constrained within cycle time T as represented in the equations (5). The equations (6) give the processing order; namely, the succeeding job cannot be performed until the operation of the preceding job is completed. The equations (7) show the relationship between the relative and absolute start times, where $y_{\alpha,i}$ ($\alpha \in P, i = 1, 2, \dots, n_{\alpha}$) are 0-1 integer values. If $y_{\alpha,i}$ is zero then the job $j_{\alpha,i}$ precedes $j_{\alpha,i+1}$ on the cycle Gantt chart and gives a shorter flow time as shown in Fig.3.

Equations (8) define two important constraints to two different jobs operated by the same machine. One constraint is to prohibit two or more jobs being scheduled at the same time on one machine. This constraint is formulated in the same manner as for the conventional job shop scheduling problem. Another constraint is that all jobs assigned to one machine must be completed within cycle time T . In the equations (8), $x_{\alpha,i,\beta,k}$ ($\alpha \in P, i = 1, 2, \dots, n_{\alpha}$) are 0-1 integer values. If $x_{\alpha,i,\beta,k}$ is zero, then the machine operates the job $j_{\beta,k}$ after the operation of job $j_{\alpha,i}$ is completed. In contrast, if $x_{\alpha,i,\beta,k}$ is 1, then the job $j_{\alpha,i}$ is operated after the operation of job $j_{\beta,k}$ is completed. In addition to this familiar constraint, the equations (8) define another proper constraint for the re-entrant job shop scheduling problem. According to these two pairs of equations (8), every machine must complete every job between a pair of remaining jobs. Figure 4 shows that the job $j_{\beta,k}$ is operated between two jobs $j_{\alpha,i}$. By the combinations of equations (5) and (8), every job processed by the machines is assigned within the cycle time T .

The cycle time T is the maximum sum of time assigned to each machine. The machine giving the value of T has no idle time and the productivity of the factory is maximized.

According to the formulations, the optimum schedule is planned by a general optimization tool[4] which is available as open-source software.

TABLE II

AN EXAMPLE OF PROCESS PLANS

No.	Machine No. : Processing time	Total time
1	1:1 2:2 3:1 4:3 5:2	9
2	3:3 4:2 5:4 1:3 2:2	14

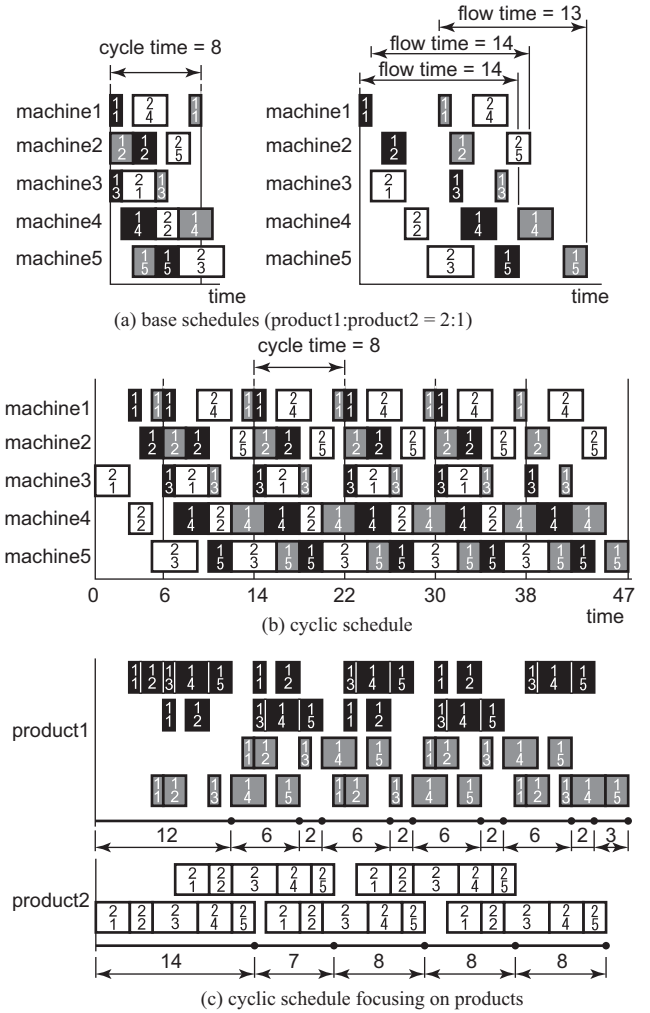


Fig. 5. Examples of cyclic schedule for product mix of two products

D. Scheduling for product mix

A management division in a company tries to decide the appropriate product mix given the demand in a market. The product mix is defined as the total composite of products manufactured by the company. The derived formulation here can be adopted to make an appropriate production schedule for any given product mix.

Table II shows production procedures and operating times for two products by five machines. As an example, an optimal schedule is shown for the given product mix of two units of product 1 and one of product 2. Figure 5 shows the optimal schedule derived from proposed formulations. As shown in Fig.5(a), a cyclic scheduling problem for three products is solved to make the schedule. The cycle time is eight because

the total operating time assigned to machine 4 and machine 5 are eight, the maximum time among the five machines. Figure 5(a) also shows both a cycle Gantt chart representing the relative start times of the jobs and a conventional Gantt chart representing absolute start times. Figure 5(b) represents the cyclic operating plan for high-volume production based on the unit schedule shown in Fig.5(a). There is no idle time on machines 4 and 5, and the schedule results in the maximum productivity for the factory. Figure 5(c) shows the same cyclic operating plan focusing on the products. It is confirmed that two unit of product 1 are produced while one unit of product 2 is produced.

III. RE-SCHEDULING FOR CHANGE OF PRODUCT MIX

It is common for the product mix to change during the manufacturing to obtain the optimal benefits corresponding to unanticipated market demand. Figure 6(a) shows the change of the product mix schematically. Figure 6(b) shows the schedule for the product mix (product 1: product 2 = 2:1) again. Figure 6(c) shows a schedule for a new product mix (product 1: product 2 = 1:1) planned by the same procedure. The company desires to change the new manufacturing plan within a short term. Figure 6(d) shows the schedule adopted in the transition period between the two different product mixes, it is obtained by a simple combination of the two schedules shown in Fig.6(a) and (b). Idle time is identified on every machine, therefore, the schedule in the transition period must be modified to prevent the loss of production.

The following equations give the optimum schedule during the transition period when the product mix is changed.

$$\text{minimize } C_{max} \quad (10)$$

subject to

$$S_{\alpha,i} \geq \hat{S}_{\alpha,i} \quad (11)$$

$$C_{max} \geq F_{\alpha,i} + \hat{F}_{\alpha,i} \quad (12)$$

$$F_{\alpha,i} = S_{\alpha,i} + p_{\alpha,i} \quad (13)$$

$$S_{\alpha,i+1} \geq F_{\alpha,i} \quad (14)$$

$$S_{\beta,k} \geq F_{\alpha,i} - M \cdot x_{\alpha,i,\beta,k} \quad (15)$$

$$S_{\alpha,i} \geq F_{\beta,k} - M \cdot (1 - x_{\alpha,i,\beta,k})$$

$$\{\mu_{\alpha,i}, \mu_{\beta,k} | \mu_{\alpha,i} = \mu_{\beta,k}, \mu_{\alpha,i}, \mu_{\beta,k} \in R\},$$

$$x_{\alpha,i,\beta,k} \in \{0, 1\}, \alpha, \beta \in P,$$

$$\hat{j}_{\alpha,i}, \hat{j}_{\beta,k} \in \Omega_{1-2}$$

$$i < k, i = 1, 2, \dots, n_{\alpha}, k = 1, 2, \dots, n_{\beta}$$

The scheduling problem for the transition period is most immediately derived from a conventional job shop scheduling problem. Differences are found in the constraint for the start times of the operations and the objective value weighted by completion time by each machine.

The operations are categorized into three sets, or Ω_1 , Ω_2 and Ω_{1-2} as shown in Fig.7. Ω_1 and Ω_2 are the sets for the jobs before and after the transition respectively. Ω_{1-2} is the set for the jobs during the transition. A re-scheduling is executed for the jobs in the set Ω_{1-2} .

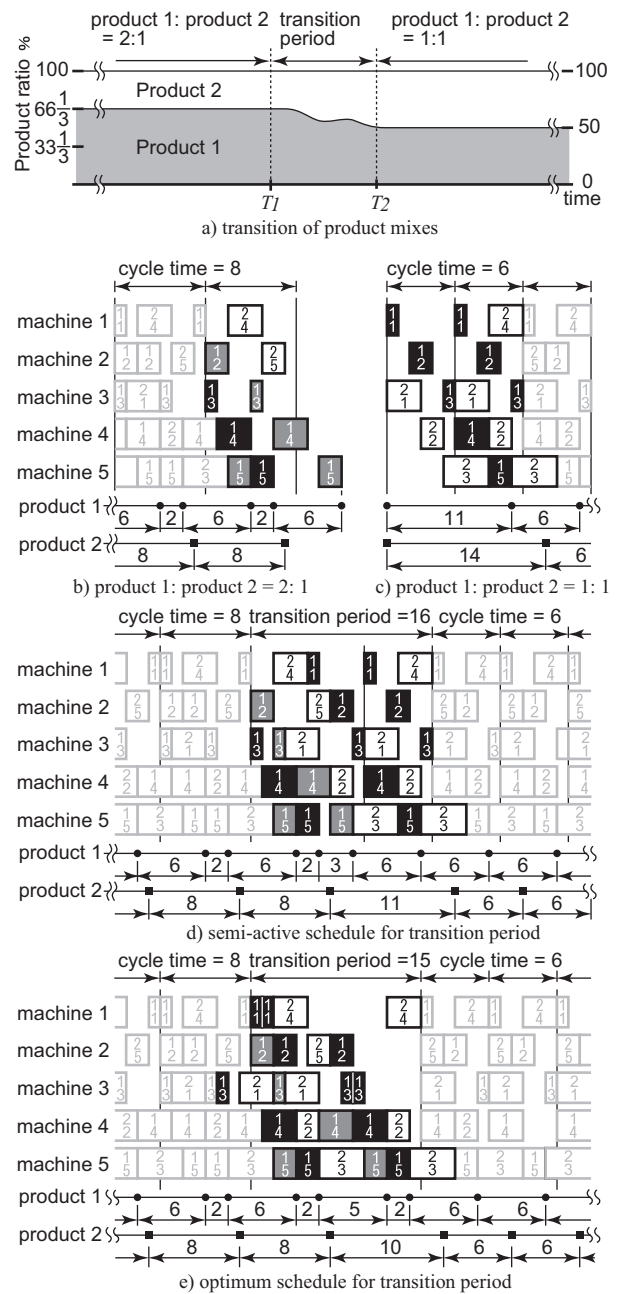


Fig. 6. Examples of transition of product mixes and re-scheduling

During the stable cyclic period in the manufacturing, all jobs in the unit schedule of product mix appear in the cyclic period as in the cyclic Gantt chart given by the optimization. In other words, if any cyclic period does not satisfy the one perfect set of the product mix, the jobs in the imperfect cyclic periods are assigned to Ω_{1-2} .

The transition period should be minimized for the optimal change to the new product mix. Therefore, the objective function is defined to the completion time as shown in equations (10) and (12). Equation (11) shows the constraint to the starting time of each job. Each machine has a different time $\hat{S}_{\alpha,i}$ to begin the first job in the transition period. The

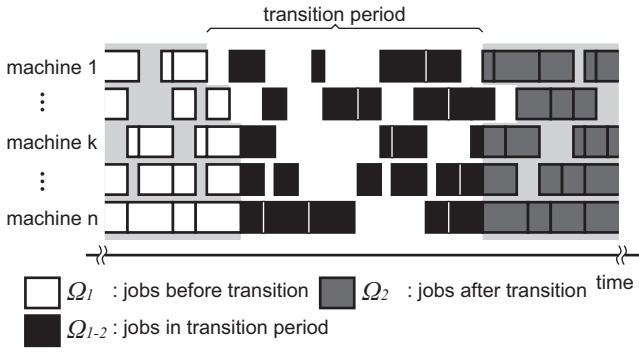


Fig. 7. Definition of jobs to be rescheduled

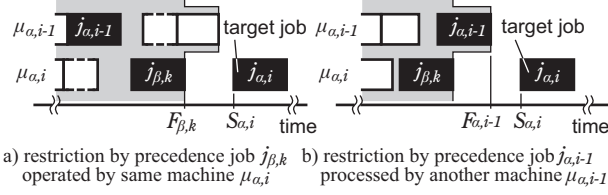


Fig. 8. Restriction of start time of job re-scheduled for transition period

$\hat{S}_{\alpha,i}$ is determined by the last job in the set Ω_1 as shown in Equation (16) referring to Fig.8.

$$\hat{S}_{\alpha,i} = \max\{F_{\beta,k}, F_{\alpha,i-1}\} \quad (16)$$

$$j_{\beta,k}, j_{\alpha,i-1} \in \Omega_1$$

Equation (12) shows a constraint to the objective value. The objective value is larger than the completion time weighted by a time $\hat{F}_{\alpha,i}$. The time $\hat{F}_{\alpha,i}$ is also different on each machine and is defined as shown in equation (17) referring to Fig.9.

Equation (14) and (15) are common constraints that appear in the conventional job shop scheduling problem. The value M is a large positive number to switch the valid constraints as shown in equation (15). The minimum value of number M is larger than the term in the transition period obtained by a simple combination of two different schedules for product mixes.

The optimization according to the described formulation gives an agile transition to the new plan for the given product mix as shown in Fig.6(e).

$$\hat{F}_{\alpha,i} = M - \min\{S_{\beta,k}, S_{\alpha,i+1}\} \quad (17)$$

$$j_{\beta,k}, j_{\alpha,i+1} \in \Omega_2$$

IV. NUMERICAL SIMULATIONS

The formulations for the scheduling problems for the given product mix and the transition to the new product mix are simply described in the form of mixed integer programming.

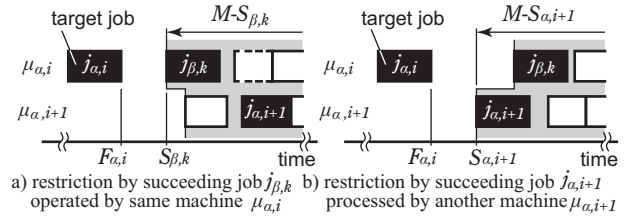


Fig. 9. Restriction of completion time of job re-scheduled for transition period

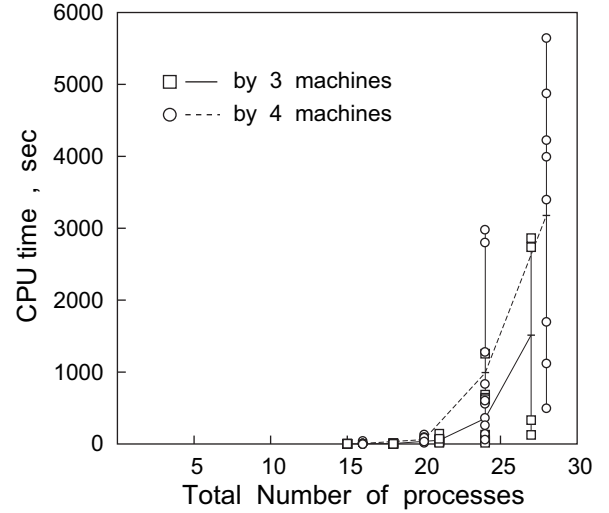


Fig. 10. Limitation to optimize plans by proposed formulation

The optimized solutions are obtained by commercially available optimization solvers directly. Open source software is also available for optimizations such as GLPK. However, the scheduling problem is a typical combination problem, and large-scale problems cannot be solved within a feasible time.

Figure 10 shows the result of simulation to know a determine the limitations of these approaches by the formulations defined here. The problems are prepared for the job shop scheduling for three products including re-entrant processes on three or four machines. The processing times are generated using random numbers in a range from 1 to 7. As shown in Fig.10, a problem comprised of less than 30 processes can be solved within two hours by the formulations described in the present paper. This is not a sufficient condition to adopt in actual industrial production, however, the method is a starting point for developing new solutions in future works.

Figure 11 shows the advantages of re-scheduling proposed here when the product mix is changed during the manufacturing. As a supposition, production leveling, which is to adjust the average load assigned to each machine, is achieved in the initial stage of the plan in manufacturing. Therefore, the machine operating rates before or after the transition period show high scores. As shown in Fig.11, the machine operating rates of the schedules after re-scheduling are higher than in

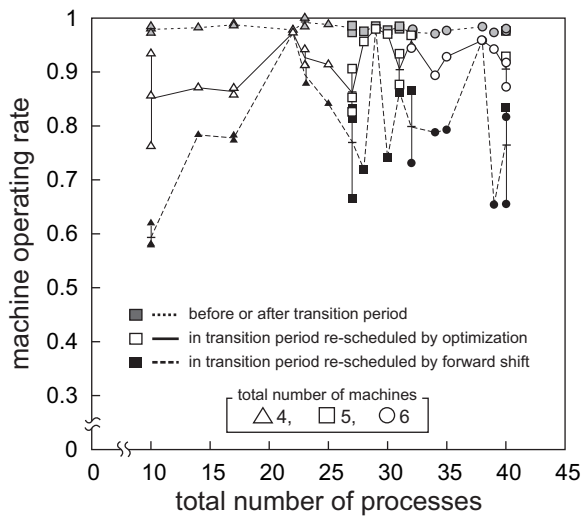


Fig. 11. Re-scheduling results for small size productions in transition period from product mix 1:1 to 2:1

the simple combination of the two schedules obtained by forward shift. Therefore, the proposed re-scheduling procedure contributes to the agile manufacturing that is able to respond quickly to changes in the markets.

V. CONCLUSION

Two scheduling problems are studied for high-volume production in the current industrial manufacturing setting. First, the formulation of a cyclic scheduling problem including re-entrant processes is derived in the form of mixed integer programming and its application in realizing a given product mix according to market demands is explained. Second, a change of the product mix is illustrated with re-scheduling problems and their solutions. Numerical simulations are executed to confirm the validity of the equations.

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