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# Partial Coordination in Local Debt Policies 

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#### Abstract

The efficiency of decentralized local debt policy under partial and full coordination is examined. The earlier studies show that independent regional governments issue an excessive amount of local public debt, and call on the central government to place restrictions on local debt policies. This paper compares several rules of coordinated debt policies to show that the central government intervention is not required to carry out an efficient local debt policy. The main argument is that a form of partial coordination results in the efficient outcome rather than a full coordination in local debt policies.


Keywords. local public debt, migration, partial coordination
JEL classification. H41, H71, H74

## 1 Introduction

A paper by Bruce (1995) analyzes the efficiency of debt policies by regional governments, where individuals have a short-run attachment to a particular region but are free to migrate between regions in the long-run. The main argument of Bruce's paper is that regional governments issue an excessive amount of local public debt compared with a debt policy carried out by a central government. The purpose of our paper is to re-examine Bruce's argument under simple rules of coordinated local debt policies to show that the central government intervention is not required to carry out an efficient debt policy.

Economists are much concerned with the efficiency of local debt issuance. Bruce's view on the inefficient local debt issuance was established by Musgrave (1959) in relation with Ricardian equivalence; Ricardian equivalence fails in local public debt since individuals who migrate can avoid a future tax burden by moving to other regions when local public debts are repaid. The avoidance of debt burden by migration is the cause of bilking problem in local debt financing, i.e., if the transmigrants can shit the burden of debt onto the future residents, they under-evaluate the cost of local debt financing thereby over-issue the local debt. The advocacy of local debt limitation by central government is referred to this bilking problem. The bilking problem with the issues on debt neutrality is directly linked with normative properties arousing the interest of the economist for a long time. For instance, in 1970's, heated debates have been generated on Wagner (1970)'s analysis, which demonstrates that individuals who recognize the probability of relocating to a new jurisdiction demand an excessive amount of local public debt [Hand and Mitchell (1971), Aronson (1971), and Wagner (1971)] ${ }^{1}$.

[^0]With the background of market globalization, the studies from the 1990's have also been considered the interactions between local public debt, migration and taxation. Specifically, the political effort to create broad economic union is recognition of the importance of regional debt management. Integration such as the European Union must have promoted strong correlation among national governments' debt policies compared to the past less closed market. Similar to Bruce's view, the main argument in the literature is that since shifts in the intertemporal pattern of local taxes occur in diverse ways, the inefficiencies associated with the choice of local public debt are still present ${ }^{2}$.

One of the significant features that make our model different from previous local debt studies is that we consider a form of coordination in local debt policies. Much of the studies mentioned above has assumed that the regional governments completely independent in issuing and repaying local public debt. This is somewhat surprising since the regions often act harmoniously in local debt financing ${ }^{3}$. Specifically, since it is hard for small

[^1]regional governments that have less credence in the bond market to raise funds, these small regional governments frequently issue public bonds cooperatively by establishing the organizations for the joint issuance of public debt and/or by concluding policy agreement. These organizations and agreements have different schemes for raising and repaying public debt, that make us to examine the efficiency of local debt policies under different scheme of coordinated debt policies. In sum, Bruce (1995) discusses the efficiency of local debt policies in the non-cooperative framework, but we further consider several regimes of coordinated local debt policies. Specifically, this paper sheds light on the question of what type of coordination helps to internalize the externalities and eliminates the bilking problem in local debt policies.

Two specific forms of policy coordination are analyzed in a model where the economy is made up out of two symmetric jurisdictions. One form covers coordination either in the issuance or in the repayment of local debt. This type of policy accord is referred to partial coordination. The other form is a full coordination that covers coordination both in the issuance and repayment of local public debt. The results demonstrate that efficient outcome is obtained in the partial coordination where regional governments coordinate the issuance, but independently repay the local debt. Full policy coordination can solve the bilking problem, but it causes a new trouble, what one calls common pool problem. Partial coordination in debt repayment results in the worst outcome that involves both bilking and common pool problem.

The paper is organized as follows. Section 2 presents the basic model, following Bruce (1995). In Section 3, we first classify four regimes of decentralized debt policies. Then, the analysis is devoted to derive the equilibrium of each regime. The equilibrium properties are compared in section 4. Section 5 concludes the paper.

## 2 Model

### 2.1 Assumptions

To highlight the differences of the results derived in coordinate and noncoordinate debt polices, the model follows Bruce (1995), which develops the non-cooperative model of local debt policy. Throughout this paper we use the two regions $(i=w, e)$ and two-period $(t=1,2)$ model. The superscript $i$ denotes the region and the subscript $t$ indicates the time period, respectively.

In each region, there exists a single regional/local government. The population of this economy is constant over time and is assumed to equal $\bar{n}$. We will henceforth follow the convention that a bar on top of a variable denotes a fixed value of variable. Denoting $n_{t}^{i}$ as the population in region $i$ at period $t$, we have $\bar{n}=n_{t}^{w}+n_{t}^{e}$. Following Bruce (1995)'s assumption that individuals have a short-run attachment to a particular region but are free to migrate between regions in the long run, we assume individuals choose there region of residence only in the second period of their life. Hence, the population in each region in the first period is constant and is given by $n_{1}^{i}$. All individuals, working for two-period, are alike in their initial endowments for labor. Each individual has one unit of homogeneous labor in each period and supply it in the individual's region of residence. All individuals have identical preferences described by the utility function, given by $U\left(c_{t}^{i}\right)$, where $c_{t}^{i}$ is the consumption of private goods $\left(U_{c}\left(c_{t}^{i}\right)>0, U_{c c}\left(c_{t}^{i}\right)<0\right)$.

The production of private goods in region $i$ requires the use of labor and land. The production function in region $i$ is $F^{i}\left(n_{t}^{i}\right)$, where $n_{t}^{i}$ is the amount of labor and land is omitted from the expression to simplify the notation ( $F_{n}^{i}>0, F_{n n}^{i}<0$ ). The individual budget constraint of individual residing in region $i$ at period $t$ is formally given by

$$
\begin{equation*}
c_{t}^{i}+\tau_{t}^{i}=w_{t}^{i}, \tag{1}
\end{equation*}
$$

where $\tau_{t}^{i}$ is the lump-sum tax/transfer, $w_{t}^{i}$ is the residents' income. By assuming that the return to the land is portioned equally amoung the current residents of the region, we assume $w_{t}^{i}=F^{i}\left(n_{t}^{i}\right) / n_{t}^{i}$.

Government collects a certain amount of revenue by issuing debt in the world market. The revenue made by debt issuance is distributed to the individuals in a lump-sum manner. In this paper, we disregard real public expenditure, so that tax revenues are used only for servicing the public debt. The new debt issued in period $t$ reaches maturity in period $t+1$, so that $\tau_{1}^{i}<0$ and $\tau_{2}^{i}>0$ in general.

### 2.2 Centrally Planed Economy

In this part, we first examine the coordinated debt policy carried out by a central planner.

Second Period. Individuals move freely between the two regions only in the second period to respond to differentials in utility level so that utility levels are equalized. Formally, the substitution of (1) into the utility function gives the migration equilibrium in the second period as follows.

$$
\begin{equation*}
U\left(\frac{F^{w}\left(n_{2}^{w}\right)}{n_{2}^{w}}-\tau_{2}^{w}\right)=U\left(\frac{F^{e}\left(\bar{n}-n_{2}^{w}\right)}{\bar{n}-n_{2}^{w}}-\tau_{2}^{e}\right) \tag{2}
\end{equation*}
$$

From (2), the regional population in the second period, $n_{2}^{i}$, can be obtained as a function of the tax rates; $n_{2}^{i}\left(\tau_{2}^{i}, \tau_{2}^{j}\right)$. The differentiation of (2) with respect to the tax rates yields

$$
\begin{aligned}
d n_{t}^{i} / d \tau_{t}^{i} & =U_{c}\left(c_{2}^{i}\right) / \Delta, \\
d n_{t}^{i} / d \tau_{t}^{j} & =-U_{c}\left(c_{2}^{j}\right) / \Delta,
\end{aligned}
$$

where $\Delta \equiv U_{c}\left(c_{2}^{w}\right) \hat{F_{n}^{w}}+U_{c}\left(c_{2}^{e}\right) \hat{F_{n}^{e}}$, and $\hat{F_{n}^{i}} \equiv\left[n_{2}^{i} F_{n}^{i}\left(n_{2}^{i}\right)-F^{i}\left(n_{2}^{i}\right)\right]\left(n_{2}^{i}\right)^{-2}$. To ensure stable migration equilibrium, we assume the case of 'excess popu-
lation', $\hat{F}_{n}^{i}<0(i=w, e)$, implying $\Delta<0$ [Atkinson and Stiglitz (1980, p.528-535)].

Substitution of $n_{2}^{i}\left(\tau_{2}^{i}, \tau_{2}^{j}\right)$ into the utility function in the second-period yields the common indirect utility value as $V\left(\tau_{2}^{i}, \tau_{2}^{j}\right)$.

First Period. Since individuals are fixed in the region of residence, the regional population is exogenously fixed in the first-period: $\overline{n_{1}^{\bar{w}}}$ and $\overline{n_{1}^{e}}=$ $\bar{n}-n_{1}^{\bar{w}}$. Then the life-time utility obtained in region $i$ is given by $U\left(w_{1}^{i}-\right.$ $\left.\tau_{1}^{i}\right)+\delta V\left(\tau_{2}^{i}, \tau_{2}^{j}\right)$, where $\delta$ is the discount factor and $\overline{w_{1}^{i}}$ is the (fixed) labor income obtained in the first period: $\overline{w_{1}^{i}}=F^{i}\left(\overline{n_{1}^{i}}\right) / \overline{n_{1}^{i}}$.

The optimization problem for the central planner is now defined as follows:

$$
\begin{array}{cl}
\max _{\tau_{1}^{e}, \tau_{1}^{w}, \tau_{2}^{e}, \tau_{2}^{w}} & U\left(\overline{w_{1}^{w}}-\tau_{1}^{w}\right)+\delta V\left(\tau_{2}^{w}, \tau_{2}^{e}\right), \\
\text { s.t. } & U\left(\overline{w_{1}^{e}}-\tau_{1}^{e}\right)+\delta V\left(\tau_{2}^{w}, \tau_{2}^{e}\right) \geq \bar{U}, \\
& R\left[n_{1}^{\bar{w}} \tau_{1}^{w}+\left(\bar{n}-n_{1}^{\bar{w}}\right) \tau_{1}^{e}\right]+\tau_{2}^{w} n_{2}^{w}\left(\tau_{2}^{w}, \tau_{2}^{e}\right) \\
& +\left(\bar{n}-n_{2}^{w}\left(\tau_{2}^{w}, \tau_{2}^{e}\right)\right) \tau_{2}^{e}=0 . \tag{4}
\end{array}
$$

To determine the efficient tax rate under centrally planned economy, we maximize the utility of a representative household living in region $w$ subject to the constraints (3) and (4). (3) reflects that the utility of residents in region $e$ is fixed at a certain level. (4) is the intertemporal budget constraint, where $R$ denotes the gross rate of interest determined in the world debt market ${ }^{4}$. Denoting $\lambda$ as the Lagrangian multiplier associated with (3), and solving the problem, the first-order conditions become as

$$
\begin{equation*}
\frac{U_{c}\left(c_{1}^{w}\right)}{U_{c}\left(c_{1}^{e}\right)}=\frac{\lambda n_{1}^{\bar{w}}}{\bar{n}-n_{1}^{w}}, \tag{5}
\end{equation*}
$$

[^2]\[

$$
\begin{align*}
F_{n}^{w}\left(n_{2}^{w}\right)-c_{2}^{w} & =F_{n}^{e}\left(\bar{n}-n_{2}^{w}\right)-c_{2}^{e}  \tag{6}\\
\left(\frac{\bar{n}-n_{2}^{w}}{\bar{n}} \frac{1}{U_{c}\left(c_{2}^{e}\right)}+\frac{n_{2}^{w}}{\bar{n}} \frac{1}{U_{c}\left(c_{2}^{w}\right)}\right) & =\delta R\left(\frac{\bar{n}-n_{1}^{w}}{\bar{n}} \frac{1}{U_{c}\left(c_{1}^{e}\right)}+\frac{n_{1}^{\bar{w}}}{\bar{n}} \frac{1}{U_{c}\left(c_{1}^{w}\right)}\right)(.7)
\end{align*}
$$
\]

(5) is the condition for the efficient allocation of private goods between the two regions in the first-period. (6) is the efficient condition of population allocation and describes the optimal locational pattern for individuals in the second-period. The marginal social benefits of mobile individual must be equalized across regions. (7) is the condition for intertemporal allocation of private goods. If the two regions are symmetric $\left(n_{1}^{w}=\overline{n_{1}^{e}}=\bar{n} / 2\right)$, (5) becomes as

$$
\begin{equation*}
U_{c}\left(c_{1}^{w}\right)=U_{c}\left(c_{1}^{e}\right) \tag{8}
\end{equation*}
$$

which can be also derived in the different manner of the maximization of utilitarian social welfare function. Under (8), the migration constraint implies $U_{c}\left(c_{2}^{w}\right)=U_{c}\left(c_{2}^{e}\right)$. Hence (7) reduces to

$$
\begin{equation*}
U_{c}\left(c_{1}^{i}\right)=R \delta U_{c}\left(c_{2}^{i}\right) \tag{9}
\end{equation*}
$$

In the following section, (9) will be compared with decentralized equilibrium derived under the debt policies by local governments.

## 3 Decentralized Debt Policies

### 3.1 Classification of the Debt System

Decentralized local debt polices are characterized by a taxation in the secondperiod, $\tau_{2}^{i}$, and the amount of debt issuance in the first-period, $B^{i}$. We have no public goods in this economy, so that the governments' revenue raised by the bond issuance will hand out to the individuals as the lump-sum transfer, which is denoted by $\tau_{1}^{i}$.

In the decentralized debt market, local governments often establish the financial organizations (authorities) to provide debt services in co-operation
with other local governments. To establish joint authorities, individual governments enter into a mutual basic contract regarding on the debt issuance. It is often the case that small local governments in particular have participated to raise funds through cooperative debt issuance and repayment. Although these organizations have different schemes for raising and repaying public debt, we shall distinguish the following four regimes of local debt system.

Regime (a). The local governments choose the amount of local debt issuance and repay their debt independently.

$$
\tau_{1}^{i}=-B^{i} / \overline{n_{1}^{i}} \quad \text { and } \quad \tau_{2}^{i}=R B^{i} / n_{2}^{i}
$$

Regime (b). The local governments non-cooperatively choose the amount of local debt issuance, but they repay their debt cooperatively.

$$
\tau_{1}^{i}=-B^{i} / \overline{n_{1}^{i}} \quad \text { and } \quad \tau_{2}^{w}=\tau_{2}^{e}=R\left(B^{w}+B^{e}\right) / \bar{n}
$$

Regime (c). The local governments choose the amount of local debt issuance cooperatively, but they repay their debt independently.

$$
\tau_{1}^{w}=\tau_{1}^{e}=-\left(B^{w}+B^{e}\right) / \bar{n} \quad \text { and } \quad \tau_{2}^{i}=R B^{i} / n_{2}^{i}
$$

Regime (d). The local governments choose the amount of local debt issuance cooperatively. In addition, they repay their debt cooperatively.

$$
\tau_{1}^{w}=\tau_{1}^{e}=-\left(B^{w}+B^{e}\right) / \bar{n} \quad \text { and } \quad \tau_{2}^{w}=\tau_{2}^{e}=R\left(B^{w}+B^{e}\right) / \bar{n}
$$

Clearly, Regime (a) copies Bruce's analysis, where the debt is chosen non-cooperatively by each local government and each government is responsible for the debt its past residents took on. The partial coordination is
characterized by Regime (b) and (c). In Regime (b), the local governments repay collectively and choose debt independently, while they choose debt level cooperatively and repay independently in Regime (c). The Regime (d) describes the case of full coordination, in which the debt level is chosen collectively and repaid collectively.

In the following analysis, to focus on the efficiency of decentralized debt policies, we assume that the local governments voluntarily make interregional transfers so that the allocation of population is efficient in the secondperiod [Myers (1990) and Bruce (1995)]. This assumption allows us to isolate the intertemporal efficiency problem of local debt policies from the efficiency issues on horizontal allocation of population.

### 3.2 Regime (a)

In Regime (a), the private goods consumption in the second-period is expressed by

$$
\begin{equation*}
c_{2}^{i}=\frac{F\left(n_{2}^{i}\right)-Y^{i}+Y^{j}}{n_{2}^{i}}-\frac{R B^{i}}{n_{2}^{i}} \tag{10}
\end{equation*}
$$

where $Y^{i} \geq 0$ denotes the interregional transfers made by local government $i$.

The condition for migration equilibrium, thus, is given by

$$
U\left(\frac{F\left(n_{2}^{w}\right)-R B^{w}-Y^{w}+Y^{e}}{n_{2}^{w}}\right)=U\left(\frac{F\left(n_{2}^{e}\right)-R B^{e}-Y^{e}+Y^{w}}{n_{2}^{e}}\right)
$$

With $\bar{n}=n_{2}^{w}+n_{2}^{e}$, this equation gives us the second-period population in region $i$ as $n_{2}^{i}\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$. The comparative statics results yield

$$
\begin{align*}
\frac{\partial n_{2}^{i}}{\partial B^{i}} & =\frac{R U_{c}\left(c_{2}^{i}\right)}{n_{2}^{i} H}<0  \tag{11}\\
\frac{\partial n_{2}^{i}}{\partial B^{j}} & =-\frac{R U_{c}\left(c_{2}^{j}\right)}{n_{2}^{j} H}>0 \tag{12}
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial n_{2}^{i}}{\partial Y^{i}}=\frac{n_{2}^{j} U_{c}\left(c_{2}^{i}\right)+n_{2}^{i} U_{c}\left(c_{2}^{j}\right)}{n_{2}^{i} n_{2}^{j} H}<0 \tag{13}
\end{equation*}
$$

where

$$
H \equiv\left(\frac{\left[F_{n}\left(n_{2}^{w}\right)-c_{2}^{w}\right] U_{c}\left(c_{2}^{w}\right)}{n_{2}^{w}}\right)+\left(\frac{\left[F_{n}\left(n_{2}^{e}\right)-c_{2}^{e}\right] U_{c}\left(c_{2}^{e}\right)}{n_{2}^{e}}\right)
$$

and $H<0$ is assumed for the stability of the migration equilibrium. Substituting $n_{2}^{i}\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$ into the second-period utility function, we have $V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$. The comparative statics show

$$
\begin{align*}
\frac{\partial V}{\partial Y^{i}} & =-\frac{\partial V}{\partial Y^{j}}=\frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}}  \tag{14}\\
\frac{\partial V}{\partial B^{i}} & =-\frac{\partial V}{\partial B^{j}}=\frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)-R\right)}{n_{2}^{i}} \tag{15}
\end{align*}
$$

The local government $i$ wishes to maximize the lifetime utility of the resident, taking the tax rates and the amount of debt issuance of other regions as given. Noting that, in Regime (a), the lump-sum transfer received by the individuals in the first-period is given by $B^{i} / \overline{n_{1}^{i}}$, the maximization problem will be defined as

$$
\max _{B^{i}, Y^{i}} U\left(\frac{F\left(\overline{n_{1}^{i}}\right)}{\overline{n_{1}^{i}}}+\frac{B^{i}}{\overline{n_{1}^{i}}}\right)+\delta V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)
$$

Solving the problem, we have ${ }^{5}$

$$
\begin{align*}
& \frac{U_{c}\left(c_{1}^{i}\right)}{n_{1}^{i}}+\frac{\delta U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)-R\right)}{n_{2}^{i}}=0,  \tag{16}\\
& \frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}} \leq 0, Y^{i} \geq 0, \\
& Y^{i}\left(\frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}}\right)=0 . \tag{17}
\end{align*}
$$

[^3]Note that (14) indicates that non-negativity constraint on the inter-regional transfer cannot be binding in both regions. Hence, assuming that it binds only in one of the two regions, we have

$$
\begin{equation*}
F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}=F_{n}\left(n_{2}^{j}\right)-c_{2}^{j} \tag{18}
\end{equation*}
$$

which is identical to (6). This means that the efficient allocation of population is achieved through interregional transfer made by local governments.

Inserting (11) and (18) into (16), we have

$$
\begin{equation*}
U_{c}\left(c_{1}^{i}\right)=\frac{\overline{n_{1}^{i}}}{\bar{n}} R \delta U_{c}\left(c_{2}^{i}\right) \tag{19}
\end{equation*}
$$

Comparing (9) with (19), we have $U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$ since $\overline{n_{1}^{i}} / \bar{n}<1$. This stands for the excessive consumption in the first-period caused by the excessive debt flotation. Summarizing above discussion, we confirm the following result.

Proposition 1 (Bruce (1995)). When the local governments choose the amount of local debt issuance and repay their debt independently, they overissue the local debt in the first-period; $U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$.

The reason behind this result is provided in Bruce (1995) as follows. While the debt issuance made by local government $i$ gives benefits only on residents in region $i$ since there is no migration in the first-period, the future cost of this debt issuance is diluted by migration in the second-period. Hence, the local governments under-evaluate the cost of debt issuance, so that they demand an excessive amount of public debt. In addition to Bruce's interpretation, we can provide an alternative expression to put the explanation in more detail. Since both regions are overpopulated, they have incentives to reduce the regional population by raising the second-period tax
rate, implying an increase in the first-period debt issuance. While the population outflow induced by an increase in the tax rate increases the utility of its own residents, it externally reduces the utility of residents residing in other region. This negative externality is ignored in the local governments' decision-making so that the local governments issue public debt excessively.

### 3.3 Regime (b)

In Regime (b), local governments repay their debt cooperatively, while they choose the amount of local debt issuance independently. Then, the private goods consumption in the second-period is given by

$$
\begin{equation*}
c_{2}^{i}=\frac{F\left(n_{2}^{i}\right)-Y^{i}+Y^{j}}{n_{2}^{i}}-\frac{R\left(B^{i}+B^{j}\right)}{\bar{n}} . \tag{20}
\end{equation*}
$$

The condition for migration equilibrium is now given by
$U\left(\frac{F\left(n_{2}^{w}\right)-Y^{w}+Y^{e}}{n_{2}^{w}}-\frac{R\left(B^{w}+B^{e}\right)}{\bar{n}}\right)=U\left(\frac{F\left(n_{2}^{e}\right)-Y^{e}+Y^{w}}{n_{2}^{e}}-\frac{R\left(B^{w}+B^{e}\right)}{\bar{n}}\right)$.
With $\bar{n}=n_{2}^{w}+n_{2}^{e}$, this equation gives us the second-period population in region $i$ as $n_{2}^{i}\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$. The comparative statics results show

$$
\begin{align*}
& \frac{\partial n_{2}^{i}}{\partial B^{i}}=\frac{R U_{c}\left(c_{2}^{i}\right)}{\bar{n} \Gamma}<0  \tag{21}\\
& \frac{\partial n_{2}^{i}}{\partial B^{j}}=-\frac{R U_{c}\left(c_{2}^{j}\right)}{\bar{n} \Gamma}>0  \tag{22}\\
& \frac{\partial n_{2}^{i}}{\partial Y^{i}}=\frac{n_{2}^{j} U_{c}\left(c_{2}^{i}\right)+n_{2}^{i} U_{c}\left(c_{2}^{j}\right)}{n_{2}^{i} n_{2}^{j} \Gamma}<0, \tag{23}
\end{align*}
$$

where
$\Gamma \equiv\left(\frac{\left[F_{n}\left(n_{2}^{w}\right)-c_{2}^{w}-\frac{R\left(B^{w}+B^{e}\right)}{\bar{n}}\right] U_{c}\left(c_{2}^{w}\right)}{n_{2}^{w}}\right)+\left(\frac{\left[F_{n}\left(n_{2}^{e}\right)-c_{2}^{e}-\frac{R\left(B^{w}+B^{e}\right)}{\bar{n}}\right] U_{c}\left(c_{2}^{e}\right)}{n_{2}^{e}}\right)$,
and $\Gamma<0$ is assumed. Substituting $n_{2}^{i}\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$ into the secondperiod utility function, we have $V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)$. The comparative statics results show

$$
\begin{align*}
\frac{\partial V}{\partial Y^{i}} & =-\frac{\partial V}{\partial Y^{j}}=\frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{R\left(B^{i}+B^{j}\right)}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}} \\
\frac{\partial V}{\partial B^{i}} & =-\frac{\partial V}{\partial B^{j}}=U_{c}\left(c_{2}^{i}\right)\left(\frac{\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{R\left(B^{i}+B^{j}\right)}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)}{n_{2}^{i}}-\frac{R}{\bar{n}}\right) \tag{24}
\end{align*}
$$

Noting $\tau_{1}^{i}=-B^{i} / \overline{n_{1}^{i}}$ in Regime (b), the maximization problem of the local governments can be defined as follows.

$$
\max _{B^{i}, Y^{i}} U\left(\frac{F\left(\overline{n_{1}^{i}}\right)}{\overline{n_{1}^{i}}}+\frac{B^{i}}{\overline{n_{1}^{i}}}\right)+\delta V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right) .
$$

Solving the problem, we have

$$
\begin{align*}
& \frac{U_{c}\left(c_{1}^{i}\right)}{n_{1}^{i}}+\delta U_{c}\left(c_{2}^{i}\right)\left(\frac{\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{R\left(B^{i}+B^{j}\right)}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)}{n_{2}^{i}}-\frac{R}{\bar{n}}\right)=0, \\
& \frac{U_{c}\left(c_{2}^{i}\right)}{n_{2}^{i}}\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{R\left(B^{i}+B^{j}\right)}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right) \leq 0, Y^{i} \geq 0,  \tag{26}\\
& Y^{i}\left(\frac{U_{c}\left(c_{2}^{i}\right)}{n_{2}^{i}}\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\tau_{2}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)=0 . \tag{27}
\end{align*}
$$

By assuming that the non-negativity constraint on the inter-regional transfer binds only in one of the two regions, from (27), we have the identical equation with (6), indicating the population allocation is efficient.

With the same procedure in Regime (a), using (6) and (21), (26) reduces to

$$
\begin{equation*}
U_{c}\left(c_{1}^{i}\right)=\frac{\overline{n_{1}^{i}} n_{2}^{i}}{(\bar{n})^{2}} R \delta U_{c}\left(c_{2}^{i}\right) . \tag{28}
\end{equation*}
$$

Since $(\bar{n})^{2}>\overline{n_{1}^{i}} n_{2}^{i}, U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$. Therefore, we have our second result as follows.

Proposition 2. When the local governments repay their debt cooperatively, while they independently choose the amount of local debt issuance, they over-issue the local debt in the first-period; $U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$.

### 3.4 Regime (c)

In this regime, the second-period equilibrium is as same as Regime (a). Hence, equations (10)-(15) are valid. In the first-period, the local governments choose the level of debt issuance cooperatively, so that the maximization problem of local government is given by

$$
\max _{B^{i}, Y^{i}} U\left(\frac{F\left(\overline{n_{1}^{i}}\right)}{\overline{n_{1}^{i}}}+\frac{B^{i}+B^{j}}{\bar{n}}\right)+\delta V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)
$$

Solving the problem, we have

$$
\begin{align*}
& \frac{U_{c}\left(c_{1}^{i}\right)}{\bar{n}}+\frac{\delta U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)-R\right)}{n_{2}^{i}}=0,  \tag{29}\\
& \frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}} \leq 0, Y^{i} \geq 0, \\
& Y^{i}\left(\frac{U_{c}\left(c_{2}^{i}\right)\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)}{n_{2}^{i}}\right)=0 . \tag{30}
\end{align*}
$$

As before, since the non-negativity constraint on the inter-regional transfer binds only in one of the two regions, from (30), we have (6). With the same procedure in Regime (a) and (b), substituting (6) and (11)into (29), we have

$$
\begin{equation*}
U_{c}\left(c_{1}^{i}\right)=R \delta U_{c}\left(c_{2}^{i}\right) . \tag{31}
\end{equation*}
$$

(31) is identical to (9), showing that the decentralized debt policy is efficient. Thus, we have third result of this paper as follows.

Proposition 3. When the local governments choose the amount of local debt issuance cooperatively, but they repay their debt independently, they choose efficient level of debt issuance; $U_{c}\left(c_{1}^{i}\right)=R \delta U_{c}\left(c_{2}^{i}\right)$.

### 3.5 Regime (d)

The second-period equilibrium is as same as Regime (b). Hence, equations (20)-(25) could be applied in this regime. Since $\tau_{1}^{i}=-\left(B^{i}+B^{j}\right) / \bar{n}$, the maximization problem of local government is defined as

$$
\max _{B^{i}, Y^{i}} U\left(\frac{F\left(\overline{n_{1}^{i}}\right)}{\overline{n_{1}^{i}}}+\frac{B^{i}+B^{j}}{\bar{n}}\right)+\delta V\left(Y^{i}, Y^{j}, B^{i}, B^{j}\right)
$$

Solving the problem, we have

$$
\begin{align*}
& \frac{U_{c}\left(c_{1}^{i}\right)}{\bar{n}}+\delta U_{c}\left(c_{2}^{i}\right)\left(\frac{\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{B^{i}+B^{j}}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial B^{i}\right)}{n_{2}^{i}}-\frac{R}{\bar{n}}\right)=0,(3  \tag{32}\\
& \frac{U_{c}\left(c_{2}^{i}\right)}{n_{2}^{i}}\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{B^{i}+B^{j}}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right) \leq 0, Y^{i} \geq 0 \\
& \quad Y^{i}\left[\frac{U_{c}\left(c_{2}^{i}\right)}{n_{2}^{i}}\left(\left[F_{n}\left(n_{2}^{i}\right)-c_{2}^{i}-\frac{B^{i}+B^{j}}{\bar{n}}\right]\left(\partial n_{2}^{i} / \partial Y^{i}\right)-1\right)\right]=0 \tag{33}
\end{align*}
$$

As before, since the non-negativity constraint on the inter-regional transfer binds only in one of the two regions, from (33), we have (6). Substituting (6) and (21)into (32), we have

$$
\begin{equation*}
U_{c}\left(c_{1}^{i}\right)=\frac{n_{2}^{i}}{\bar{n}} R \delta U_{c}\left(c_{2}^{i}\right) \tag{34}
\end{equation*}
$$

Since $n_{2}^{i} / \bar{n}<1$, we have $U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$. Summarizing, we have the final result.

Proposition 4. When the local governments choose the amount of local debt issuance and repay their debt cooperatively, they over-issue the local debt in the first-period; $U_{c}\left(c_{1}^{i}\right)<R \delta U_{c}\left(c_{2}^{i}\right)$.

## 4 Comparison and Interpretations

We compare the equilibrium properties in each regime with offering the intuitive interpretation of the results. If we compare (19) with (28), we find that the debt level in Regime (b) is greater than the one in Regime (a) since $(\bar{n})^{2} /\left(n_{1}^{i} n_{2}^{i}\right)>\bar{n} / \overline{n_{1}^{i}}$. Since both Regime (a) and (b) results in overissuance of public debt, the local debt policy in Regime (b) increases the excessive debt accumulation further. This is simply because the cooperative repayment of debt potentially involves an additional source of over-issuance of debt, along with the source of excessive debt accumulation originated from bilking problem; Under Regime (b), local governments are possible to charge the burden of debt repayment not only on the residents in its own region but also individuals residing in other regions. This causes the familiar common pool problem: The resident fully internalizes the benefit of its debt issuance in the first-period, but (as repayment is shared) it internalizes only a fraction of $n_{2}^{i} / \bar{n}$ of the social marginal cost of local debt issuance in the second-period. This factor acts to reduce cost of debt issuance so that it makes local governments issue bonds further.

By contrast, in Regime (c), the benefit of debt financing lump-sum subsidies in the first-period does not fasten within the region. It spreads to the individuals of other regions, and that local government does not underevaluate the benefit of debt issuance. Furthermore, local governments have no ways to charge the burden of debt repayment on the individuals living in other region, since the repayment of debt is conducted independently. These lead local governments evaluate their cost and benefit of debt issuance cor-
rectly. In Regime (d), although the cooperative debt issuance contributes to buffer against over-issuance of debt and succeeds in avoiding the bilking problem, the common pool problem is generated by the cooperative repayment of debt, that leads local government issues excessive debt.

For further illustration, consider the familiar utility function: $U\left(c_{t}^{i}\right)=$ $\ln c_{t}^{i}$. In the symmetric equilibrium, we have ${ }^{6}$

$$
\begin{equation*}
B_{a}=B_{d}=\frac{F(2-R \delta)}{R(2+\delta)}, B_{b}=\frac{F(4-R \delta)}{R(4+\delta)}, B_{c}=\frac{F(1-R \delta)}{R(1+\delta)}, \tag{35}
\end{equation*}
$$

where $B_{i}$ is the amount of local debt in regime $i$ and $F \equiv F(\bar{n} / 2)$. (35) shows that $B_{c}<B_{a}=B_{d}<B_{b}$ for all $\delta>0$. To provide some insight of welfare ranking, we obtain the indirect utility level as follows;

$$
\begin{aligned}
U_{a} & =U_{d}=\ln \frac{4 F(R+1)}{R n(\delta+2)}+\delta \ln \frac{2 F \delta(R+1)}{n(\delta+2)} \\
U_{b} & =\ln \frac{8 F(R+1)}{R n(\delta+4)}+\delta \ln \frac{2 F \delta(R+1)}{n(\delta+4)} \\
U_{c} & =\ln \frac{2 F(R+1)}{R n(\delta+1))}+\delta \ln \frac{2 F \delta(R+1)}{n(\delta+1)}
\end{aligned}
$$

which show that $U_{c}>U_{a}=U_{d}>U_{b}$ for all $\delta>0$.
In sum, when local governments in Regime (c) choose debt level cooperatively they internalize the externality associated with migration- which is not internalized in Bruce's original set up (Regime (a))- and the resulting debt level is efficient. Neither Regime (c) nor Regime (d) leads to a first best level. Specifically, the inefficiency is aggravated, and that the welfare is the lowest when the repayment is collective- this reflects the well known common pool problem explained above.

[^4]
## 5 Final Remarks

In this paper we follow Bruce (1995) and have formulated a simple two region and two period model to examine the efficiency of local bond financing in a regional economy. With a simple coordination regarding on the amount of debt issuance between the two regions, the decentralized debt policies can achieve the first best outcome. More specifically, if the local governments partially coordinate to choose the amount of local debt issuance cooperatively, but they repay their debt independently, the decentralized local debt polices results in the efficient outcome. In one sense, this is hardly a surprising result since the benefit of debt issuance is weekend by the cooperative debt flotation since the benefits of debt issuance spread to other region. However, this result is still a striking result, and it should be apparent that only the cooperation on debt issuance is needed, meaning that local governments must not cooperate repaying their debt. Rather, if they cooperate both in debt issuance and repayment, they are caught up in the common pool problem and would over-issue the bond, as is the case with non-cooperative decentralized debt policies studied in Bruce (1995).

The basic model does at least suggest that a harmonizing public debt policy is needed, but the full coordination is not required. The results show that partial coordination may well be an effective mechanism by which local governments are forced to internalize the externalities of their own local public debt issuance. Our analysis contributes to clarify which kind of policy coordination is efficiency enhancing and that too much coordination may even harm efficiency. We believe that this result contributes to the discussion on harmonizing debt policies among the local governments in a country. In addition, political effort to create broad economic union such as the European Union is another recognition of the importance of our result. The analysis provides a framework for rationalizing the debt policy of the

European Union as the EU tries hard to coordinate national debt issuance and the Union leaves it to its members to repay accumulated debt ${ }^{7}$.

We believe the result derived in this paper is contributory to develop an efficient system for decentralized debt policies. In parallel, however, it should also be noted that some of the assumptions, such as the absence of household savings, the constant interest factor, and the 'excess population' could be modified. Specifically, the assumption on the constant interest factor should be relaxed because it is natural to consider some kind of coordination in debt financing aims lowering the borrowing cost. By relaxing these assumptions, we can offer more general examination in the future research.

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[^0]:    ${ }^{1}$ On the back of these studies, Tsuneki (1985), Akai (1994), and Ogawa and Yano (2007) provide the examination on Ricardian equivalence for local public debt, having been centered on the role of land taxation. The main difference of our paper with those is

[^1]:    that, to achieve the first-best outcome, we set a simple coordination rule for debt issuance and do not rely on the taxation on land. The role of land in achieving the efficient outcome in the local debt financing has been indicated by Daly (1969) and Oates (1972) in the early stage.
    ${ }^{2}$ Under two-period and two regions economy with migration, Schultz and Sjostrom (2001) show that regional governments undersupply public goods while it over-accumulates local public debt. Wellisch and Richter (1995) show that neutrality of local public debt is not guaranteed since shifts in the intertemporal pattern of local taxes change the net wealth of local property owners if distortionary residence-based taxes are imposed to service the local debt. Weichenrieder (1999) propose the 'pay as you finance' scheme to overcome the inefficiencies in local debt finance. Schultz and Sjostrom (2004) incorporate the voting process into the model to show that local public debt would be over-accumulated. Furthermore, under an alternative analytical framework, Homburg and Richter (1993) provide a negative argument on the efficiency of local public debt issuance when individuals freely migrate between regions.
    ${ }^{3}$ Many countries apply some form of local debt coordination as well as the restrictions to local government budgeting and borrowing, but in various forms and to varying degrees. For instance, it has been widely recognized that the Public Pfandbriefea and the Credit Locale de France in effect assume the role of coordinate local debt in Germany and France, respectively, but the forms of system differ to some tune. See Dafflon (2002) for the comprehensively review of European local debt management.

[^2]:    ${ }^{4}$ The interest rate is, thus, $R-1$. We can replace zero with a constant parameter in (4) since a tax revenue of zero is not crucial.

[^3]:    ${ }^{5}$ We assume $B^{i}>0$ and disregard why local governments issue public bonds. Jensen and Toma (1991) is one of the studies examining the reason local governments issue bonds in a framework of interregional competition.

[^4]:    ${ }^{6}$ The proof is available upon request.

[^5]:    ${ }^{7}$ The agreement of the coordinate issue of local debts started in 2003 in Japan has also intended to have the aspect that each jurisdiction repays its debt independently while it issues the debt cooperatively.

