Theory of Kondo Effect in type II Superconductors

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Abstract

We study the equilibrium properties of type II superconductors with Kondo effect. The Kondo effect associated with the impurity spins is taken into account within the interpolation approximation, which was used previously in our calculations of the superconducting transition temperature[1)] and the specific heat jump at the transition temperature.[2)] Using this approximation, the general Ginzburg-Landau equations are derived for superconductors with Kondo effect and the Ginzburg-Landau parameter $\kappa_2(T)$ is calculated. One finds that the initial slope of the Ginzburg-Landau parameter shows a continuous change with T_k/T_{co} and approaches the BCS-value at the end of an infinite Kondo temperature $T_k/T_{co}=\infty$ in contrast to Maki's theory.[3)]

§1. Introduction

The equilibrium properties of type-II superconductors with Kondo effect was first explained theoretically by Maki.[3] He predicted the various interesting behaviors of the type-II superconductors in the presence of the magnetic field using the pole approximation employed by Müller-Hartmann and Zittartz.[4] Unfortunately, the pole approximation based on the Suhl-Nagaoka approximation is wrong at $T < T_k$. Therefore his predictions seem questionable, as far as the behaviors at $T < T_k$ are concerned. It is necessary to revise his calculations given in Ref.3).

The purpose of the present paper is to present a corrected version of the theory of type-II superconductors with Kondo effect. We confine ourselves to the properties in the vicinity of the upper critical field $H_{c2}(T)$, where the superconducting order parameter is small. We will take into account the Kondo effect associated with impurity spins based on the interpolation approximation which was used by Matsuura, Nagaoka and the present author to calculate the transition temperature[1] and the specifie heat jump.[2] We neglect both the effect of an applied magnetic field on the impurity spins and that of a Pauli paramagnetism on superconductors. We assume further that the electron mean free path in the superconductor is so short that the dirty-limit treatment would apply, for the mathematical simplicity. In this paper the p-wave scattering of the impurities is omitted, leading to the substitution of the transport lifetime τ_{tr} by the s-wave scattering lifetime τ in the formula.

This paper is arranged as follows: Section 2 is devoted to the generalization of the Ginzburg-Landau equations. In §3, we discuss the magnetic properties near the upper critical field, and in particular calculate the Ginzburg-Landau parameter $\kappa_2(T)$, which describes the magnetization in this region. In §4, we give summary.

§2. Generalized Ginzburg-Landau Equations

In the previous papers[1),2)] we assumed that the order parameter is constant in space. There are, however, other classes of interesting phenomena, which involve a spatially varing order parameter. To describe the spatial variation of the order parameter it is convenient to start with the generalized Ginzburg-Landau equations. In the following we will consider only the gapless region, where the order parameter $\Delta(r)$ is small. Then the self-consistent equation is given up to third order in $\Delta(r)$ by [5)]

$$\Delta^+(r) = |g|T\sum_\omega\int d^3r'Q_\omega(r,r')\Delta^+(r') + |g|T\sum_\omega\int\cdots\int d^3r_1\cdots d^3r_3 B_\omega(r,r_1,\cdots,r_3)\Delta^+(r_1)\Delta(r_2)\Delta^+(r_3) \quad , \tag{2.1}$$

where $Q_\omega(r,r')$ and $B_\omega(r,r_1,\cdots,r_3)$ are the two-particle and the four-particle Green's functions respectively.

1) the first term of Eq. (2.1).

Introducing the vertex correction $\gamma(\tilde{q}^2,\omega)$ and the bare two-particle Green's function $\tilde{Q}^0_\omega(\tilde{q})$ in the presence of magnetic fields, the first term of Eq. (2.1) can be written as

$$T\sum_\omega\int d^3r'Q_\omega(r,r')\Delta^+(r') = T\sum_\omega\gamma(\tilde{q}^2,\omega)\tilde{Q}^0_\omega(\tilde{q})\Delta^+(r) \quad . \tag{2.2}$$

Using the renormalized one-particle Green's function in the presence of magnetic field $G_k(\omega)$,[6)] the bare two-particle Green's function

$\tilde{Q}^0_\omega(\tilde{q})$ is given as

$$\tilde{Q}^0_\omega(\tilde{q}) = \sum_k G_k(\omega) G_{-k-q}(-\omega) = \frac{2\pi N\rho}{V_F|\tilde{q}|} \tan^{-1}\left(\frac{v_F|\tilde{q}|}{2|\tilde{\omega}|}\right) , \qquad (2.3)$$

where ρ is the density of states of conduction electrons per atom per spin, N is the number of atoms and $\tilde{\omega}$ is the renormalized frequency. $\tilde{q}$ is defined as $\vec{q}-2e\vec{A}$ or $\vec{q}+2e\vec{A}$ depending on whether it operates on $\Delta^+(r)$ or $\Delta(r)$ using the external momentum $\vec{q}$.

$$G_k(\omega) = [i\omega - e\vec{v}\cdot\vec{A} - \xi_k - \sum(\omega)]^{-1}$$

$$G_{-k}(-\omega) = [-i\omega + e\vec{v}\cdot\vec{A} - \xi_k - \sum(\omega)]^{-1} , \qquad (2.4)$$

where $\omega=(2n+1)\pi T$, ξ_k is the one-electron energy of conduction electrons and $\sum(\omega)$ is the self-energy correction due to both the magnetic and the non-magnetic impurities. $\vec{A}$ is the vector potential and $\vec{v}$ the Fermi velocity.

The vertex correction $\gamma(\tilde{q}^2,\omega)$ is given by solving the following equation:

$$\gamma(\tilde{q}^2,\omega) = 1 + T\sum_{\omega'}\Gamma_{\uparrow\downarrow}(\omega,\omega')\tilde{Q}^0_{\omega'}(\tilde{q})\gamma(\tilde{q}^2,\omega')$$

$$= 1 + \frac{2\pi N\rho}{v_F|\tilde{q}|} T\sum_{\omega'}\Gamma_{\uparrow\downarrow}(\omega,\omega')\gamma(\tilde{q}^2,\omega')\tan^{-1}\left(\frac{v_F|\tilde{q}|}{2|\tilde{\omega}'|}\right) , \qquad (2.5)$$

where $\Gamma_{\uparrow\downarrow}(\omega,\omega')$ is the irreducible vertex of the effective interaction between electrons due to both the magnetic and the non magnetic impurities. Introducing the electronic relaxation time due to nonmagnetic impurities alone τ_0 and the approximation used in Ref.1), we get an approximate expression for $\Gamma_{\uparrow\downarrow}(\omega,\omega')$ as

$$\Gamma_{\uparrow\downarrow}(\omega,\omega') = \frac{\delta_{\omega,\omega'}}{2\pi\tau_0 TN\rho} + \frac{n}{N}\left[\frac{1}{T}\Gamma_1(\omega)\delta_{\omega,\omega'} - \frac{f(\omega)f(\omega')}{4T_k\rho^2}\right], \tag{2.6}$$

where

$$f(\omega) = \left(1 + \frac{\pi|\omega|}{4T_k}\right)^{-2}, \tag{2.7}$$

and n is the magnetic impurity concentration. If we neglect the effect of an applied magnetic field on impurity spins, $\Gamma_1(\omega)$ is independent on the magnetic field and is given in Ref. 1) as

$$\Gamma_1(\omega) \simeq \begin{cases} \dfrac{f(\omega)}{(\pi\rho)^2} \text{ for } |\omega| \ll T_k \\ -\dfrac{3}{16\rho^2}\left(\ln\dfrac{|\omega|}{T_k}\right)^{-2} \text{ for } |\omega| \gg T_k . \end{cases} \tag{2.8}$$

Using the expansion

$$\frac{2|\tilde{\omega}|}{v_F|\tilde{q}|}\tan^{-1}\left(\frac{v_F|\tilde{q}|}{2|\tilde{\omega}|}\right) = 1 - \frac{1}{3}\left(\frac{v_F|\tilde{q}|}{2|\tilde{\omega}|}\right)^2 + \cdots, \tag{2.9}$$

the vertex correction $\gamma(\tilde{q}^2,\omega)$ is finally obtained as

$$\gamma(\tilde{q}^2,\omega) = \frac{|\omega| + |\Sigma(\omega)|}{|\omega| + n\alpha(\omega) + \frac{1}{2}D_1(\omega)\tilde{q}^2}\cdot\left[1 - \frac{n}{4T_k\rho}\cdot\frac{f(\omega)\tilde{\Phi}_1(\tilde{q}^2)}{1 + \frac{n}{4T_k\rho}\tilde{\Phi}_2(\tilde{q}^2)}\right], \tag{2.10}$$

where

$$\tilde{\Phi}_k(\tilde{q}^2) = \pi T\sum_{\omega}\frac{[f(\omega)]^k}{|\omega| + n\alpha(\omega) + \frac{1}{2}D_2(\omega)\tilde{q}^2}, \tag{2.11}$$

and $\alpha(\omega)$ is the pair-breaking parameter which appears in Ref. 1). Two kinds of diffusion constants $D_1(\omega)$ and $D_2(\omega)$ are written by the diffusion constant D_0 which is determined by nonmagnetic impurities alone: i.e.

$$D_1(\omega) = D_0\cdot\frac{1 + 2\pi\tau_0 n\rho\Gamma_1(\omega)}{[1 + \tau_0/\tau_1]^2} \tag{2.12}$$

$$D_2(\omega) = \frac{D_0}{1+\tau_0/\tau_1} \quad , \tag{2.13}$$

where $D_0 \equiv \frac{\tau_0 v_F^2}{3}$ and τ_1 is the electronic relaxation time due to magnetic impurities alone. When the electron mean free path is so short that it is hardly affected by a small amount of magnetic impurities, we can approximate them by D_0,

$$D_1(\omega) \simeq D_2(\omega) \simeq D_0 \quad \text{for} \quad \tau_0 / \tau_1 \ll 1. \tag{2.14}$$

Finally substituting Eq. (2.10) into Eq. (2.2), we arrive at

$$T\sum_{\omega}\int d^3r' Q_\omega(r,r')\Delta^+(r') = N\rho[\tilde{\Phi}_0(\tilde{q}^2) - \frac{n}{4T_k\rho}\cdot\frac{\{\tilde{\Phi}_1(\tilde{q}^2)\}^2}{1+\frac{n}{4T_k\rho}\tilde{\Phi}_2(\tilde{q}^2)}]\Delta^+(r) \quad . \tag{2.15}$$

2) the second term of Eq. (2.1).

Now we turn to the analysis on the second term of Eq. (2.1). It is convenient to rewrite it as follows:

$$T\sum_{\omega}\int\cdots\int d^3r_1\cdots d^3r_3 B_\omega(r,r_1,\cdots,r_3)\Delta^+(r_1)\Delta(r_2)\Delta^+(r_3)$$
$$= \lim_{\substack{r_1 \\ r_2 \\ r_3}\} \to r} T\sum_{\omega}\tilde{B}_\omega(\tilde{q}_i)\Delta^+(r_1)\Delta(r_2)\Delta^+(r_3) \quad . \tag{2.16}$$

The quantity $\tilde{B}_\omega(\tilde{q}_i)$ can be calculated by means of the same perturbation method that was used previously by the present author in the calculation of the specific heat jump at the transition temperature,[2)] though in this case the dependence of the external momentum $\tilde{q}$ is introduced into the s-electron Green's function $G_k(\omega)$ and vertex corrections $\gamma(\tilde{q}^2,\omega)$. Since it is tedious to write down various contributions of diagrams for $\tilde{B}_\omega(\tilde{q}_i)$, here we show only the results.

$$\tilde{B}_\omega(\tilde{q}_i) = -\frac{\pi N\rho}{2}\cdot\frac{|\omega|\eta_3(\omega)+\frac{1}{8}D_2(\omega)[(\tilde{q}_1-\tilde{q}_3)^2+(\tilde{q}_2-\tilde{q}_4)^2]}{|\omega|^4\{\prod_{j=1}^{4}\eta_2(\tilde{q}_j^2,\omega)\}}, \quad (2.17)$$

where $\sum_{i=1}^{4}\tilde{q}_i=0$ and $\tilde{q}_i$ operates only on $\Delta(r_i)$. Here use has been made of the fact that in the dirty limit, the term associated with the non-commutative nature of the operators $\tilde{q}_i$ only gives rise to a higher order correction in $1/\xi_0$ even at low temperature,[6] where l is the electronic mean free path and ξ_0 is the BCS coherence length. Thus we arrive at a quasi-local equation even at low temperature. Here $|\omega|\eta_3(\omega)$ is given in Ref. 2) as

$$|\omega|\eta_3(\omega) = |\omega| + \frac{n}{2\pi\rho}\chi(\omega) \quad (2.18)$$

$$\chi(\omega) = \begin{cases} \frac{\pi|\omega|}{2T_k} - \frac{2}{3}\left(\frac{\pi|\omega|}{4T_k}\right)^2 - \frac{2}{9}\left(\frac{\pi|\omega|}{4T_k}\right)^3 & \frac{\pi|\omega|}{4T_k} < 1 \\ \frac{10}{9}\cdot\left[\frac{\frac{3}{4}\pi^2}{(\ln\frac{\pi|\omega|}{4T_k})^2+\frac{3}{4}\pi^2}\right]^2 & \frac{\pi|\omega|}{4T_k} > 1 \end{cases}, \quad (2.19)$$

where the magnitude of the impurity spin is assumed $\frac{1}{2}$. The renormalization factor $\eta_2(\tilde{q}_i^2,\omega)$ is the extension of $\eta_2(\omega)$, which appears in Ref. 2), into a magnetic field dependent case and is given by

$$|\omega|\eta_2(\tilde{q}_i^2,\omega) = [|\omega|+n\alpha(\omega)+\frac{1}{2}D_1(\omega)\tilde{q}_i^2]\cdot\frac{1+\frac{n}{4T_k\rho}\tilde{\Phi}_2(\tilde{q}_i^2)}{1+\frac{n}{4T_k\rho}[\tilde{\Phi}_2(\tilde{q}_i^2)-\tilde{\Phi}_1(\tilde{q}_i^2)f(\omega)]} \quad (2.20)$$

Thus we arrive at the following equation:[6]

$$X(\tilde{q}^2)\Delta^+(r) + Y(\tilde{q}_i)\Delta^+(r_1)\Delta(r_2)\Delta^+(r_3)\Big|_{1=2=3=r} = 0, \quad (2.21)$$

where

$$X(\tilde{q}^2) = \frac{1}{|g|N\rho} - \tilde{\Phi}_0(\tilde{q}^2) + \frac{n}{4T_k\rho} \cdot \frac{[\tilde{\Phi}_1(\tilde{q}^2)]^2}{1+\frac{n}{4T_k\rho}\tilde{\Phi}_2(\tilde{q}^2)} \tag{2.22}$$

$$Y(\tilde{q}_i) = \frac{\pi}{2}T\sum_{\omega}\frac{|\omega|\eta_3(\omega)+\frac{1}{8}D_2(\omega)[(\tilde{q}_1-\tilde{q}_3)^2+(\tilde{q}_2-\tilde{q}_4)^2]}{|\omega|^4[\prod_{j=1}^{4}\eta_2(\tilde{q}_j^2,\omega)]} . \tag{2.23}$$

3) Ginzburg-Landau current density

The second Ginzburg-Landau equation which relates the current to the vector potential is obtained by a similar procedure. Since the reduction involves methods already described, we will not take the space to give more than the final result,[6)]

$$j_s(r) = \frac{1}{4\pi} Z(\tilde{q}_i^2)(\tilde{q}_1-\tilde{q}_2)\Delta^+(r_1)\Delta(r_2)\Big|_{r_1=r_2=r} , \tag{2.24}$$

where

$$Z(\tilde{q}_i^2) = \frac{2\pi^2 Ne\tau_0}{m} T\sum_{\omega}\frac{\tau_1/(\tau_0+\tau_1)}{|\omega|^2[\prod_{j=1}^{2}\eta_2(\tilde{q}_j^2,\omega)]} . \tag{2.25}$$

Eqs. (2.21) and (2.24) are the basic equations for our discussions. As we will see later, the temperature dependence of the Ginzburg-Landau parameter $\kappa_2(T)$ is quite different from that of Maki's theory, mainly due to the behavior of $\chi(\omega)$ at low temperature and at low frequency. However, there remains some ambiguity in the form of $\alpha(\omega)$ and $\chi(\omega)$ in the region $\omega \lesssim T_k$. Fortunately no sharp transition between two limiting regions, and so we can believe such interpolation gives semi-quantitatively correct results.

§3. Magnetic properties

Using the basic equations in the preceding section, we can discuss quite generally the magnetic properties of superconductors near the upper critical field. In the following we describe several applications of the above equations.

1) the upper critical field $H_{c2}(T)$[1)]

If the transition from the superconducting state to the normal state in the presence of a magnetic field is of second order, the upper critical field $H_{c2}(T)$ is determined from the first term in Eq. (2.21) by

$$X(\tilde{q}^2)\Delta^+(r) = 0 \quad . \tag{3.1}$$

Taking for $\Delta^+(r)$ the Abrikosov solution in a magnetic field, the upper critical field is determined from

$$\tilde{\Phi}_0(\tilde{q}_0^2) - \frac{n}{4T_k\rho} \cdot \frac{[\tilde{\Phi}_1(\tilde{q}_0^2)]^2}{1+\frac{n}{4T_k\rho}\tilde{\Phi}_2(\tilde{q}_0^2)} = \frac{1}{|g|N\rho} \quad , \tag{3.2}$$

where q_0^2 is the lowest eigenvalue of the following equation:

$$D_2(\omega)\tilde{q}^2\Delta^+(r) = D_2(\omega)\; q_0^2\;\Delta^+(r)$$

$$q_0^2 = 2eH_{c2}(T) \quad . \tag{3.3}$$

If we confine ourselves to the case of alloys where the electronic mean free path is extremely short, then we can obtain the previous result[1)] using $D_0 = \frac{\tau_0 v_F^2}{3}$ instead of $D_2(\omega)$.

2) the Ginzburg-Landau parameter $\kappa_2(T)$

The general Genzburg-Landau parameter $\kappa_2(T)$ introduced by Maki, which may be determined from the slope of the magnetization curve near H_{c2}, is defined as follows:[6)]

$$\left(\frac{\partial M}{\partial H}\right)_{H_{c2}} = \frac{1}{4\pi\beta(2\kappa_2^{\,2}-1)} \tag{3.4}$$

where M is the magnetization and the constant β depends on the symmetry of the vortex line lattice. For the triangular lattice β=1.16. Using two fundamental equations (2.21) and (2.24), we have for $\kappa_2(T)$,[6]

$$\kappa_2^{\,2}(T) = \frac{1}{4e}\cdot\frac{Y(q_0^2)}{Z(q_0^2)X'(q_0^2)}\ , \tag{3.5}$$

where

$$X'(q_0^2) \equiv \left.\frac{\partial X(\tilde{q}^2)}{\partial \tilde{q}^2}\right|_{\tilde{q}^2=q_0^2} \tag{3.6}$$

$$Y(q_0^2) = -\left.\frac{T\sum_\omega\int d^3r\tilde{B}_\omega(\tilde{q}_i)\Delta^+(r_1)\Delta(r_2)\Delta^+(r_3)\Delta(r_4)}{N\rho\int d^3r|\Delta(r)|^4}\right|_{\tilde{q}^2=q_0^2}$$

$$= \frac{\pi}{2}T\sum_\omega\frac{|\omega|+\frac{n}{2\pi\rho}\chi(\omega)+\frac{1}{2}D_2(\omega)q_0^2}{[|\omega|\eta_2(q_0^2,\omega)]^4} \tag{3.7}$$

$$Z(q_0^2) = \frac{2\pi^2Ne\tau_0}{m}T\sum_\omega\frac{\tau_1/(\tau_0+\tau_1)}{[|\omega|\eta_2(q_0^2,\omega)]^2} \tag{3.8}$$

Deviation of $\kappa_2(T_c)/\kappa$ from the AG-or BCS-like behavior is most clearly observed by evaluating the quantity κ^* which is introduced as the initial slope of $\kappa_2(T)$ at the transition temperature:

$$\kappa^* = -\frac{1}{\kappa}\left(\frac{d\kappa_2(t_c)}{dt_c}\right)_{t_c=1} = -\frac{1}{2}\left[\frac{d}{dt_c}\ln\left(\frac{Y(0)}{X'(0)Z(0)}\right)\right]_{t_c=1}\ , \tag{3.9}$$

where κ is the Ginzburg-Landau parameter as defined by Gor'kov. Notice that κ^* depends exclusively on the single-impurity parameter and is one of the quanties sensitive to the magnetic nature of the single impurity imbedded in a superconductor.

Next we consider the behaviors of $\kappa_2(t)$, $\kappa_2(t_c)$ and κ^* for two limiting cases.

a) When $T \gg T_k$, $\tilde{\Phi}_1(q_0^2)$ and $\tilde{\Phi}_2(q_0^2)$ can be neglected. If we replace $\alpha(\omega)$ by $\alpha(T)$ and $\chi(\omega)$ by $\chi(T)$, then we find for $\kappa_2(t)$ near $H_{c2}(T)$

$$\left(\frac{\kappa_2(t)}{\kappa}\right)^2 \simeq \frac{\pi^4}{56\zeta(3)} \cdot \frac{-\psi^{(2)}(\frac{1}{2}+\lambda) - \frac{1}{3}\alpha(1-\delta)\psi^{(3)}(\frac{1}{2}+\lambda)}{[\psi^{(1)}(\frac{1}{2}+\lambda)]^2}, \tag{3.10}$$

where $\psi^{(n)}(z)$ is the poly-gamma function. Here α is defined by $\alpha \equiv \frac{n\alpha(T)}{2\pi T}$ with

$$\alpha(T) \simeq \frac{1}{2\pi\rho} \cdot \frac{\frac{3}{4}\pi^2}{(\ln\frac{T}{T_k})^2 + \frac{3}{4}\pi^2} \quad . \tag{3.11}$$

λ and δ are defined as follows:

$$\lambda = \alpha + \frac{eD_0 H_{c2}(T)}{2\pi T} \tag{3.12}$$

$$\delta = \frac{20}{9}\pi\rho\alpha(T) \quad . \tag{3.13}$$

Here use has been made of the approximation,

$$D_1(\omega) \simeq D_2(\omega) \simeq D_0 \quad . \tag{3.14}$$

When $T=T_c$, $\kappa_2(t_c)$ is given by

$$\left(\frac{\kappa_2(t_c)}{\kappa}\right)^2 \simeq \frac{\pi^4}{56\zeta(3)} \cdot \frac{-\psi^{(2)}(\frac{1}{2}+\alpha_c) - \frac{1}{3}\alpha_c(1-\delta)\psi^{(3)}(\frac{1}{2}+\alpha_c)}{[\psi^{(1)}(\frac{1}{2}+\alpha_c)]^2}, \tag{3.15}$$

where $\alpha_c = \frac{n\alpha(T_c)}{2\pi T_c}$. The calculation of κ^* can be simplified using the approximations

$$\alpha(T_{c0}) \simeq \frac{3\pi}{8\rho} \left(\ln\frac{T_k}{T_{c0}}\right)^{-2} \tag{3.16}$$

$$-\frac{dn}{dt_c}\Big|_{t_c=1} \simeq \frac{32\rho T_{c0}}{3\pi^2} \left(\ln\frac{T_K}{T_{c0}}\right)^2 \tag{3.17}$$

$$\delta \simeq \frac{\pi^2}{6} \left(\ln\frac{T_k}{T_{c0}}\right)^{-2} . \tag{3.18}$$

The result is given by

$$\kappa^* = \kappa^*_{AG} + \frac{\pi^4}{252\zeta(3)}\left(\ln\frac{T_k}{T_{c0}}\right)^{-2} , \tag{3.19}$$

where $\kappa^*_{AG}=-0.091$. One easily finds that κ^* approaches the AG value κ^*_{AG} in the limit $T_{c0}/T_k \to \infty$.

b) On the other hand, if $T \ll T_k$, we can put

$$\tilde{\Phi}_0(q_0^2) \simeq \tilde{\Phi}_1(q_0^2) \simeq \tilde{\Phi}_2(q_0^2) \simeq \frac{1}{|\tilde{g}|N\rho} \tag{3.20}$$

where $|\tilde{g}| = |g| - \dfrac{n}{4T_k N\rho^2}$.

Using the approximations

$$\begin{aligned} |\omega|\eta_3(\omega) &\simeq \left(1+\frac{n}{4T_k\rho}\right)|\omega| \\ |\omega| + n\alpha(\omega) &\simeq \left(1+\frac{n}{4T_K\rho}\right)|\omega| , \end{aligned} \tag{3.21}$$

we can simplify the calculations of $\kappa_2(T)$ as

$$\left(\frac{\kappa_2(t)}{\kappa}\right)^2 \simeq \frac{\pi^4}{56\,\zeta(3)} \cdot \left(1+\frac{n}{4T_k\rho}\right) \cdot \frac{-\psi^{(2)}(\frac{1}{2}+\tilde{\lambda})}{[\psi^{(1)}(\frac{1}{2}+\tilde{\lambda})]^2} , \tag{3.22}$$

where

$$\tilde{\lambda} = \frac{eD_0 H_{c2}(T)}{2\pi T} \cdot \frac{1}{1+\frac{n}{4T_k\rho}} .$$

When $T=T_c$, this result reduces to the simple formula,

$$\left(\frac{\kappa_2(t_c)}{\kappa}\right)^2 \simeq 1+\frac{n}{4T_k\rho} \qquad (3.23)$$

Then the T_k/T_{c0} - dependence of κ^* is given by

$$\kappa^* \simeq \frac{1}{2}\left(\ln\frac{T_k}{T_{c0}}\right)^{-2} . \qquad (3.24)$$

Here use has been made of the approximation

$$-\left.\frac{dn}{dt_c}\right|_{t_c=1} \simeq 4T_k\rho\left(\ln\frac{T_k}{T_{c0}}\right)^{-2} . \qquad (3.25)$$

One easily finds that κ^* approaches the BCS-value $\kappa^*_{BCS}=0$ in the limit of high Kondo temperature, $T_k/T_{c0}\to\infty$, in contrast to Maki's theory. The behavior of $\kappa_2(t_c)/\kappa$ as a function of T_c/T_{c0} is given by the numerical calculation for the wide region of T_k/T_{c0}. Results are given in Fig. 1. Here the parameter used for the

Fig. 1

numerical calculation is $\frac{\omega_D}{\pi T_{c0}} = 1169$. Results of numerical calculations for κ^* are given in Fig. 2. The initial slope of the

Fig. 2

Ginzburg-Landau parameter at T_c becomes maximum where $T_k/T_{c0} \simeq 1$. The numerical value of T_k/T_{c0} at the maximum is not so much meaningful, since it depends on the detail of our interpolation.*) Throughout this section we have considered only the equilibrium properties of type-II superconductors with Kondo effect near the transition point. Finally we comment that using the Ginzburg -Landau parameter $\kappa_2(t)$, we can also discuss the dynamical properties of the vortex state near the transition point (e.g. the flux-flow resistivity arising from the motion of the order parameter).

§4. Discussion

In this paper we have discussed how the Kondo effect affects the magnetic properties of type-II superconducting alloys based on the interpolation approximation. Though our results contain the already known limiting cases: the AG- or BCS- like behavior, they are different from those of Maki's theory. In particular, the initial slope κ^* of the Ginzburg-Landau parameter tends to the BCS -value at the end of an infinite Kondo temperature. Therefore we can say that the study of the magnetic properties near H_{c2} in type-II superconductors with Kondo effect will also provide important information on the Kondo effect in superconductors. Throughout this paper we have neglected completely the effect of the magnetic field on magnetic impurities. Since the magnetic field characteristic of the Kondo effect is given by $H_k = k_B T_k / \mu_B$, this approximation is allowed only when

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*) The definition of the Kondo temperature in the Yamada-Yosida theory is different from that of the most divergent approximation. Since we made an interpolation between them, our calculation contains some ambiguity about T_k. Therefore, quantitative comparison of our results with experiments does not seem so much meaningful.

$H_{c2} \ll H_k$. In general, the magnetic-field effect on magnetic impurities is expected to appear as $(H/H_k)^2$ in the lowest order. Therefore, the Ginzburg-Landau parameter $\kappa_2(t_c)$ at T_c and κ^* are not affected by this effect, and our approximation of neglecting it is justified.

Experimentally, the Kondo effect in type II superconductors has not been studied extensively. Most of the previous work[7] was carried out in alloy systems with $T_k/T_{co} < 1$. For the most interesting case of $T_k/T_{co} \gtrsim 1$ there exist no reliable experimental results. However, if alloy systems with $T_k/T_{co} \gtrsim 1$ are found in the future, we believe that a continuous change in the quantity κ^* from the negative value to the positive ones can be observed experimentally.

Acknowledgements

Thanks are due to Professor Y. Nagaoka for critical reading of the manuscript.

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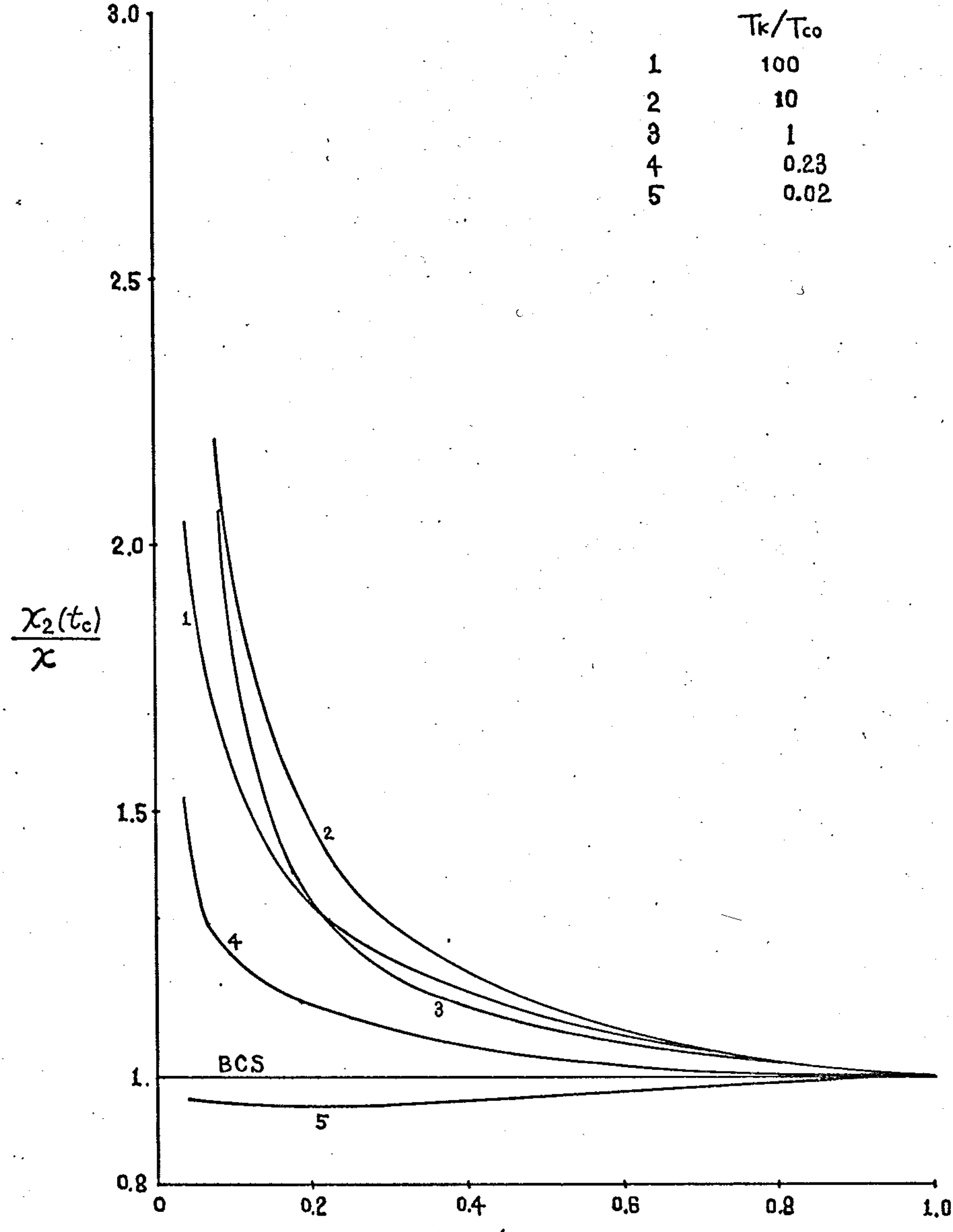
Figure Captions

Fig. 1 The temperature dependence of the $\kappa_2(T_c)$ parameter is shown in the absence of magnetic field for the various values of T_k/T_{c0}.

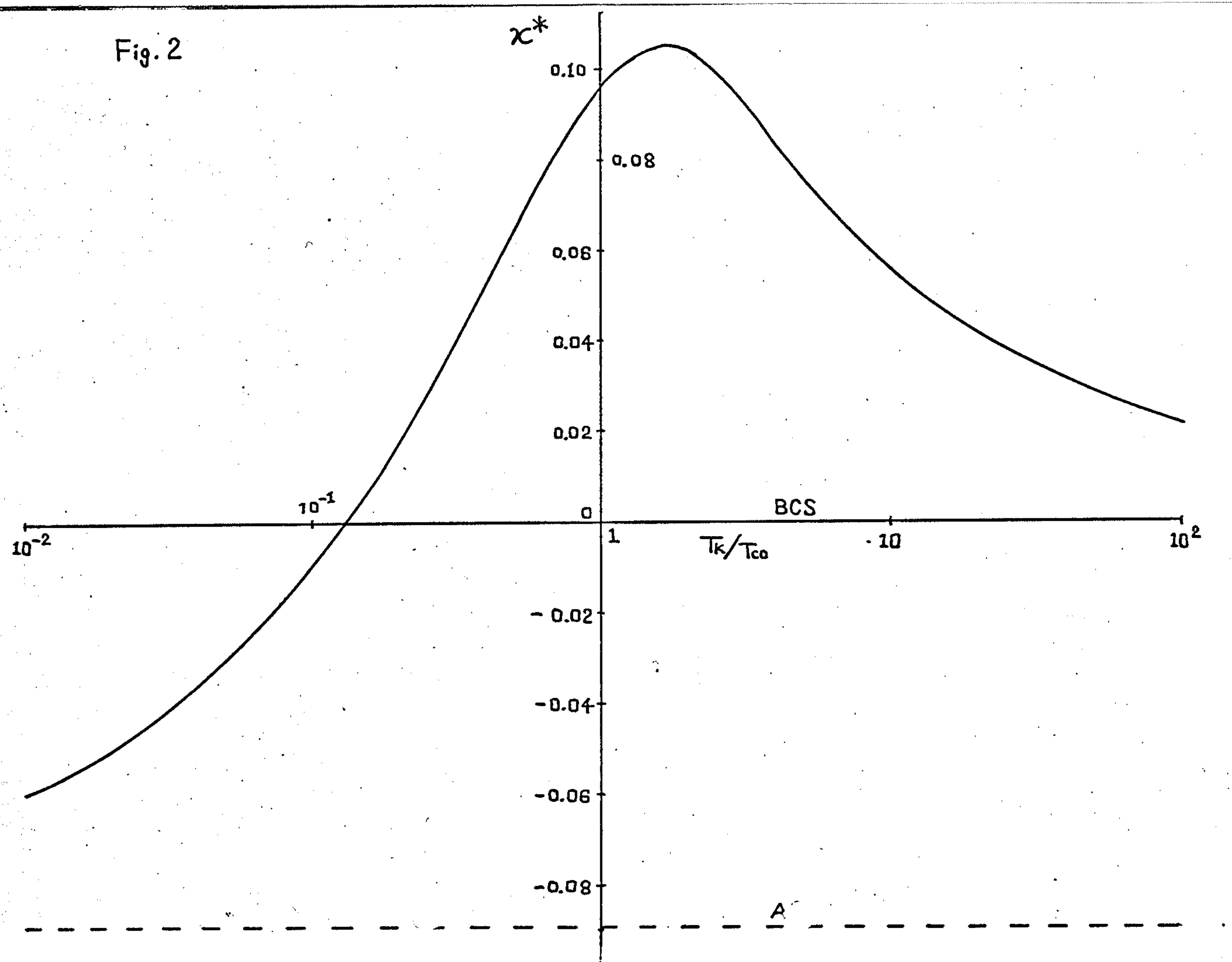
Fig. 2 The initial slope κ^* of the Ginzburg-Landau parameter as a function of T_k/T_{c0}.

Fig. 1



Tk/Tco
1 100
2 10
3 1
4 0.23
5 0.02
3.0
2.5
2.0
1.5
1.
0.8
$\frac{\chi_2(t_c)}{\chi}$
BCS
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
Tc/Tco

Fig. 2



x*
0.10
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
10^-2
10^-1
1
10
10^2
Tk/Tco
BCS
A