LETTER

# Construction of Petri Nets from a Given Partial Language

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**SUMMARY** This paper deals with the synthesis of Petri nets. Partial languages adequately represent the concurrent behaviors of Petri nets. We first propose a construction problem for Petri nets, in which the objective is to synthesize a Petri net to exhibit the desired behavior specified as a partial language. We next discuss the solvability of this problem and last present the outline of a solution technique.
***key words:*** *Petri net, partial language, synthesis, concurrency, abstraction*

## 1. Introduction

To carry out the design of systems in terms of Petri nets, we must adequately specify the desired behaviors of the systems. Although Petri net languages have frequently been used to describe the behaviors of Petri nets, they cannot precisely describe the concurrent behaviors. For example, the transitions $a$ and $b$ in $N_1$ shown in Fig. 1 can fire concurrently, while the transitions $a$ and $b$ cannot in $N_2$. However, Petri net languages of these two nets are identical, namely, $\{\epsilon, a, b, ab, ba\}$. Petri net languages cannot draw an exact line between the two net behaviors, because they force all the transition firings into a linear ordering. To describe concurrency, Grabowski applied partial languages to description of Petri net behaviors [1].

If the desired behavior is denoted by a partial language, it may be possible to construct a Petri net which generates the given partial language. This paper aims at formulating a Petri net construction problem and discussing its solvability and solution technique.

## 2. Preliminaries [1]–[3]

Let $U$ be a set, $R$ be a partial ordering on $U$, and $\lambda : U \to \Sigma$, $\Sigma$ being an alphabet. Then $[U, R, \lambda]$, the isomorphism class of $(U, R, \lambda)$ is called a **partial word over** $\Sigma$ and a subset of the set of all partial words over $\Sigma$, $PW(\Sigma)$, is called a **partial language over** $\Sigma$. Graphically, a partial word is represented using the Hasse diagram for $(U, R)$ but replacing each node $k$ by its label $\lambda(k)$ as shown in Fig. 2.

Let $p = [U, R, \lambda]$ be a partial word over $\Sigma$. A subset $V$ of $U$ is said to be a **left-monotonic set of** $p$ if and only if $\forall u \in U : v \in V \wedge uRv \Rightarrow u \in V$. Moreover, a subset $Q$ of $U$ $(V \cap Q = \emptyset)$ is called the **adjacent set of** $V$ if and only if $V \cup \{q\}$ is a left-monotonic set of p for all $q \in Q$.

The **restriction of a partial language** $PL$ **to** $\Sigma'(\subseteqq \Sigma)$ is defined by

$$PL|\Sigma' := \bigcup_{p \in PL} \{ \ [U', R \cap U'^2, \lambda|U'] \mid U' = \{x \in U \mid \lambda(x) \in \Sigma'\} \ \}.$$

For a partial word $p$ and a left-monotonic set of $p$, $V$, $[V, R \cap V^2, \lambda|V]$ is said to be a **prefix of** $p$. The set of all prefixes of $p$ is denoted by $p^{\mathrm{PREF}}$, and that of $PL$ is defined by

$$PL^{\mathrm{PREF}} := \bigcup_{p \in PL} p^{\mathrm{PREF}}.$$

Let $p$ and $p'$ be partial words over $\Sigma$. $p'$ is said to be **smoother than** $p$, denoted by $p' \leqq p$, if and only if there exist representatives $(U, R, \lambda) \in p$ and $(U', R', \lambda') \in p'$ such that $U = U', \lambda = \lambda'$, and $R \subseteqq R'$. $p' < p$ implies that $p' \leqq p$ and $p' \neq p$. The set of all partial words smoother than $p$ is denoted by $We(p)$, and that smoother than $PL$ is defined by

$$We(PL) := \bigcup_{p \in PL} We(p).$$

A **Petri net** is a tuple $N = (S, T, F, K, W, M_0)$, where $S$ is the finite set of places, $T$ is the finite set of transitions, $F \subseteqq (S \times T) \cup (T \times S)$ is the incidence relation, $K : S \to \mathbb{N}$ (set of natural numbers) is the

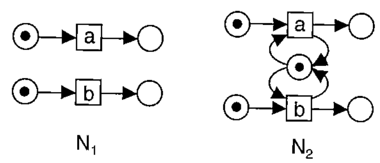


**Fig. 1** Petri nets with the same language.

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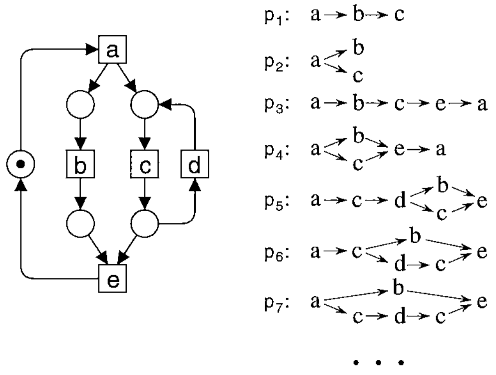



**Fig. 2** Enabled partial words.

place capacity function, $W : F \to \mathbf{N}$ is the arc weight function, and $M_0 : S \to \mathbf{N} \cup \{0\}$ is the initial marking. A special Petri net in which place capacities and arc weights are equal to one is called a **condition/event net** (C/E net). We consider only the places with finite capacities.

A partial word $p = [U, R, \lambda]$ over $T$ is said to be **enabled in** $N$ if and only if

$$\sum_{a \in Q} W(s, \lambda(a)) \leqq M_0(s) + \sum_{a \in V} \big(W(\lambda(a), s) - W(s, \lambda(a))\big) \leqq K(s) - \sum_{a \in Q} W(\lambda(a), s), \quad \text{for all } s \in S,$$

where $V$ is a left-monotonic set of $p$ and $Q$ is the adjacent set of $V$. The set of all partial words enabled in $N$ is called the **Petri net partial language of** $N$, denoted by $PL(N)$.

A subset $FP(N)$ of $PL(N)$ such that

$$FP(N) := \{p \in PL(N) | \not\exists q \in PL(N) : q > p\}$$

is called the **activity of** $N$. $FP(N)$ is the set of enabled partial words richest in concurrency. $PL(N)$ can be known from $FP(N)$, that is, for any Petri net $N$,

$$PL(N) = We(FP(N)).$$

Figure 2 shows a C/E net and some of its enabled partial words. The partial word $p_1$ implies that the transitions $a$, $b$, and $c$ fire sequentially. The partial word $p_2$ implies that $a$ first fires and then $b$ and $c$ fire concurrently. The choice as to which partial word is selected is made in a nondeterministic manner. The partial words $p_2$, $p_4$ and $p_7$ are elements of $FP(N)$.

## 3. Formulation of Petri Net Construction Problem

We formulate a construction problem for Petri nets in the following form:

**Construction Problem P(X):** Given a partial language $X$ over $T$ as a specification, construct a Petri net $N$ such that

$$X = FP(N)|T \qquad \square$$

If $p \in PL(N)$ then $p^{\mathrm{PREF}} \subseteqq PL(N)$. Therefore, the specification is given by a partial language such that $X = X^{\mathrm{PREF}}$. It should be noted that $X$ is not equal to $FP(N)$, but to $FP(N)|T$. An 'extra' transition which appears in $N$ but does not in $T$ is called an **auxiliary transition**. It may be possible to construct a Petri net such that $X = FP(N)|T$ by using auxiliary transitions even if we cannot construct a net such that $X = FP(N)$.

## 4. Solvability of Petri Net Construction Problem

We call a labeled antichain comprising multiple nodes with the same label a **multiantichain**. Unlike a C/E net, general Petri nets allow concurrent firings of the same transition. Such a behavior is represented by a multiantichain.

The set of specifications can be devided into the following two cases.

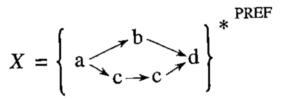
Case 1: If a specification does not contain any concurrent firings of the same transition, we can present the following theorem:

**Theorem 1:** Suppose that every partial word in $X$ over $T$ has no multiantichains. Then $P(X)$ has solutions if and only if there exists a C/E net $N_{\mathrm{CE}}$ such that $X = FP(N_{\mathrm{CE}})|T$.

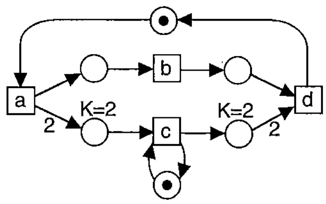
**Proof:**

Sufficiency : The 'if' part is trivial because C/E nets are a subclass of Petri nets.

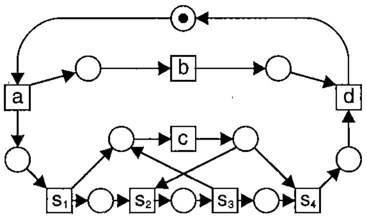
Necessity : Suppose that a Petri net $N$ is a solution of $P(X)$. Vogler proved that there exists a labeled C/E net $N_{l1}$ such that $PL(N) = PL(N_{l1})$ if $PL(N)$ does not have any partial words with multiantichains [4]. The transitions in $N_{l1}$ are labeled by the transitions in $N$. It is not required that all transitions of $N_{l1}$ are labeled distinctly. Since $PL(N) = PL(N_{l1})$, $FP(N) = FP(N_{l1})$. For each $t \in T$ in $N$, let $T_t$ be a set of transitions with label $t$ in $N_{l1}$. We first add two new places $s_i$ and $s_o$ to $N_{l1}$. For each $e \in T_t$ in $N_{l1}$, we also add two transitions $e_i$ and $e_o$ and one place $s$ to $N_{l1}$ so that ${}^\bullet e_i := {}^\bullet e$, $e_i^\bullet := \{s_i, s\}$, ${}^\bullet e_o := \{s_o, s\}$ and $e_o^\bullet := e^\bullet$, where ${}^\bullet x$ and $x^\bullet$ are the preset of $x$ and the postset of $x$, respectively. We next construct a new labeled C/E net $N_{l2}$, by labeling $e_i$ and $e_o$ by $\epsilon$, removing $e$ from $N_{l1}$, and adding a new transition with label $t$ so that ${}^\bullet t := \{s_i\}$ and $t^\bullet := \{s_o\}$. Then $FP(N_{l2})|T = FP(N_{l1})$. If we regard the transitions with label $\epsilon$ as auxiliary transitions, we finally get a C/E net $N_{\mathrm{CE}}$ such that $X = FP(N_{\mathrm{CE}})|T$, because $FP(N_{\mathrm{CE}})|T = (FP(N_{\mathrm{CE}})|T)|T = (FP(N_{l2})|T)|T = FP(N_{l1})|T = FP(N)|T = X$. $\square$




(a) Specification $X$




(b) Solution Petri net of $P(X)$




$s_1, s_2, s_3, s_4$ : auxiliary transitions

(c) Solution C/E net of $P(X)$

**Fig. 3** Example 1.

A C/E net $N_{\mathrm{CE}}$ such that $X = FP(N_{\mathrm{CE}})|T$ exists if and only if $X$ can be described by an **extended regular expression**. The detailed discussion of C/E net construction problem can be found in [3], [5].

**Example 1:** Consider the specification $X$ shown in Fig. 3 (a), where $PL^*$ represents the indefinite concatenation of $PL$. The Petri net $N$ in Fig. 3 (b) is one of the solutions of $P(X)$, and the C/E net in Fig. 3 (c) is obtained from $N$ by the procedure stated in the proof of Theorem 1. □

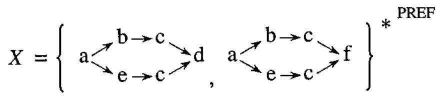
<u>**Case 2:**</u> If a specification contains concurrent firings of the same transition, we can verify the following two theorems :

**Theorem 2:** Suppose that at least one partial word in $X$ over $T$ has multiantichains. Let $X'$ be a partial language generated by forcing nodes in every multiantichain into a linear ordering. If $P(X)$ has solutions, then

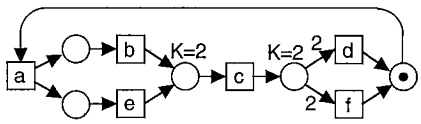
(A) there exists a C/E net $N_{\mathrm{CE}}$ such that $X' = FP(N_{\mathrm{CE}})|T$.

**Proof:** Suppose that a Petri net $N$ is a solution of $P(X)$. Let $N'$ be a Petri net constructed by adding a new place $s$ to $N$ so that ${}^\bullet s = s^\bullet := \{t\}$ and $M_0(s) = 1$ for each $t \in T$ in $N$. Then $N'$ satisfies $X' = FP(N')|T$. Therefore, by Theorem 1, there exists a C/E net $N_{\mathrm{CE}}$ such that $X' = FP(N_{\mathrm{CE}})|T$. □

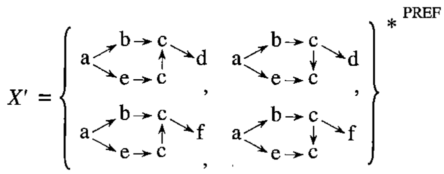
**Example 2:** For the specification $X$ shown in Fig. 4 (a), the Petri net in Fig. 4 (b) is a solution of $P(X)$. Figure 4 (c) shows the partial language $X'$ generated by forcing the nodes with label c in $X$ into a linear ordering. The activity over $T$ of the C/E net in Fig. 4 (d) coincides with $X'$, that is, Condition (A) holds. □



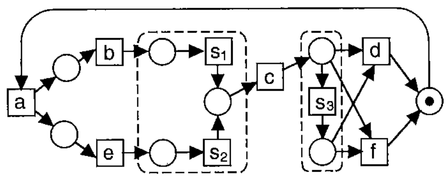

(a) Specification $X$




(b) Solution Petri net of $P(X)$




(c) Partial language $X'$




(d) C/E net such that $X' = FP(N_{\mathrm{CE}})|T$

**Fig. 4** Example 2.

**Theorem 3:** Suppose that partial words $p_i$s in $X$ have multiantichains. For every $i$, let $p_i'$ be a partial word generated by mutually interchanging the successor nodes of the multiantichain of $p_i$. If $P(X)$ has solutions, then

(B) $X$ contains $p_i'$, too, for every $i$.

**Proof:** Suppose that a Petri net $N$ is a solution of $P(X)$. Let $p_i \in X$ be a partial word with a multiantichain $\{\lambda(u_1), \cdots, \lambda(u_m)\}$, where $\lambda(u_j) = t$ for all $j$. Whichever transition $\lambda(u_j)$ fires, $N$ yields the same marking. This imples that the same partial word succeeds to every $\lambda(u_j)$. Therefore, $X$ contains $p_i'$. □

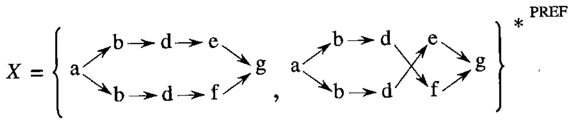
**Example 3:** Consider the specification $X$ shown in Fig. 5 (a). The Petri net shown in Fig. 5 (b) is one of the solutions of $P(X)$ and $X$ satisfies Condition (B). □

For Conditions (A) and (B), one is not contained in the other. The conjunction of these conditions is necessary for $P(X)$ to be solvable, too.

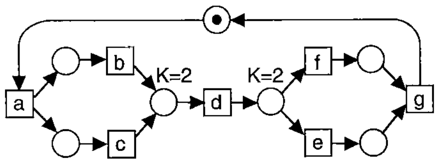
## 5. Discussion

The discussion in the previous section leads to the outline of a solution technique for $P(X)$.

In Case 1, a C/E net whose activity over $T$ coincides with the specification $X$ is one of the solutions




(a) Specification $X$




(b) Solution Petri net of $P(X)$

**Fig. 5** Example 3.

of $P(X)$. A solution techniqe for C/E net construction problem has been developed [3], [5].

In Case 2, Theorems 2 and 3 suggest a solution technique whose gross steps are as follows:

**Step1:** If $X$ satisfies Condition (B), proceed to Step 2, otherwise $P(X)$ has no solutions.

**Step2:** Construct a C/E net $N_{\mathrm{CE}}$ such that $X' = FP(N_{\mathrm{CE}})|T$, where $X'$ is a partial language generated by forcing every multiantichain in $X$ into a linear ordering, and proceed to Step 3.

**Step3:** Construct a Petri net $N$ such that $X = FP(N)|T$ from $N_{\mathrm{CE}}$ obtained in Step 2.

A lot of difficulty in this technique is found in Step 3. Desel et al. formalized the relation called **abstraction** between C/E nets and Petri nets , and presented the association of a Petri net abstraction to a C/E net [6]. Since the behavior of a C/E net is always included in the behavior of its Petri net abstraction, their abstraction can be used in Step 3. The dotted lines shown in Fig. 4 (d) represent the subnets abstracted to the places in the Petri net shown in Fig. 4 (b). There, however, exist many Petri net abstractions to a C/E net. How to choose an appropriate abstraction whose activity over $T$ coincides with $X$, still remains for a future study.

## 6. Conclusion

In this paper, we investigated the synthesis of Petri nets. A general solution technique for the formulated problem has not been developed yet. This paper, however, indicates the possibility of applications of Petri nets to the design of systems.

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