

PAPER

Diagram-Based Support for Collaborative Learning in Mathematical Exercise

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SUMMARY This paper focuses on the collaborative learning of mathematics in which learners effectively acquire knowledge of common exercises through discussion with other learners. During collaborative learning, learners sometimes cannot solve exercises successfully, because they cannot derive answers by themselves or they hesitate to propose answers through discussion. To cope with such situations, this paper proposes two support functions using diagrams to encourage active discussion, since diagrams are often used to graphically illustrate mathematical concepts. One function indicates the differences between learner diagrams and the group diagram in order to encourage participation in discussions. To compare the characteristics of diagrams drawn by different learners, *internal representation of the diagram*, which consists of types of figures and remarkable relations to other figures, is introduced. The other function provides hints in the group diagram so that all learners can consider their answers collaboratively through discussions. Since preparing hints for all exercises is difficult, *rules for drawing supplementary figures*, which are general methods for drawing supplementary figures that correspond to individual answering methods/formulas, are also developed. By applying available rules to current group diagram, appropriate supplementary figures that can solve current learning situations may be generated. The experimental results showed that the generated hints successfully increased the number of utterances in the groups. Moreover, learners were also able to derive answers by themselves and tended to propose more opinions in discussions when the uniqueness of their diagrams was indicated.

key words: collaborative learning, problem-based learning, knowledge acquisition, rule-based processing, diagram support, predicate-formed representation

1. Introduction

Computer-supported collaborative learning (CSCL) is one of progressive learning styles where plural learners located in physically distributed places communicate through networks and exist in a common learning space. In such learning environments, learners tackle the same exercises/problems collaboratively by exchanging opinions in a common learning space through networks. Based on the opinions derived in the common learning space, learners can acquire knowledge by extracting valuable information and deriving their own answers. Such private learning activities are not observed by other learners, so learning space for such activities is called *personal learning space*. Compared to personal learning space, common learning space is defined as a *public learning space*, in which all learning activities are grasped by all learners. As described in [1] and [2], much computer science researches focus on realizing dy-

namic interaction with other learners or with shared objects. However, learners' opinions that are posed in public learning spaces originated from their private learning activities in their personal learning spaces. On the other hand, their private learning activities may be invoked by other learners' ideas generated in public learning space. Therefore, to realize effective collaborative learning, not only to prepare public learning space, but also to coordinate between the public learning space and the personal learning spaces are important.

In collaborative learning in a public learning space, group discussion is the knowledge for individual learners. Based on the discussion, learners are able to sufficiently obtain the knowledge related to exercises/problems. At the same time, all learners are responsible for fruitful discussions. There are two possible situations that cause inactive discussion in public learning spaces: no learners can derive the answers or answering paths by themselves, and learners can derive the answers or answering paths but hesitate to express them. Therefore, the objective of this research is to develop a collaborative learning support mechanism to achieve successful discussions by encouraging learners individually to contribute to discussions and leading a group globally. In problem-based collaborative learning, not only answers but also various viewpoints for deriving answers should be discussed. So, the group is promoted to discuss effectively in which a variety of answering viewpoints is proposed and the number of utterances is large.

To promote knowledge derivation in public learning spaces, a mechanism that proposes hints to derive answers automatically is introduced. In order for learners to discuss knowledge of exercises/problems, opinions based on various viewpoints should be presented for individual situations. However, since discussion among learners is important in collaborative learning, such a support function should not provide the right answer. For the purpose of making learners derive various answering methods by themselves, the support function should behave more as "facilitator" than a "tutor" that performs a minimal but effective pedagogical intervention, e.g., to provide a hint, as described in [3]. Therefore, the mechanism holds the knowledge for all possible answering methods that can be applied to the situation and provides appropriate hints, but does not indicate the answer directly.

On the other hand, learners do not actively participate in discussions when they lack confidences in their opinions and often hesitate to express their answers. If learn-

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ers receive feedback that confirms the uniqueness of their answers, they might participate in group activities more actively to discuss their answers. To encourage learners, we developed a mechanism that points out the differences between learner and group answers. Of course, this mechanism does not estimate the correctness of learner answers generated in a personal learning space. However, it can indicate their uniqueness and differences.

Currently, we focus on the collaborative learning of mathematics, especially in subjects related to linear and quadratic functions. In mathematical exercises, since diagrams are often used to graphically illustrate the concepts, in our research the chat window and shared canvas for drawing diagrams are prepared in the public learning space to exchange opinions. Private canvases for drawing diagrams are also prepared individually in the personal learning spaces. Learners progress in their learning activities while drawing diagrams in their personal learning spaces. Opinions are provided to other learners by inputting utterances in the chat window or drawing diagrams on the shared canvas. In diagrams, equations that are derived by applying answering formulas are depicted as figures. On the contrary, by observing figures of underived equations, correct answering methods can be estimated.

This paper proposes two support functions using diagrams to encourage active discussion. One function indicates the uniqueness of learner answers by comparing learner diagrams with the group diagram. To compare the characteristics of diagrams drawn by different learners, *internal representation of the diagram*, which consists of types of figures and remarkable relations to other figures, is introduced. By handling diagrams with figures, differences of learning situations are recognized by comparing learner and group diagrams. The other function provides hints to the group diagram so that all learners can consider their answers collaboratively through discussions. Since preparing hints for all exercises is difficult, *rules for drawing supplementary figures*, which are general methods for drawing supplementary figures that correspond to individual answering methods/formulas, are also developed. By applying available rules to current group diagram based on forward reasoning, appropriate supplementary figures that indicate answering method to solve current learning situations may be generated.

Section 2 describes our approach that supports collaborative learning based on diagrams. In Sect. 3, the internal representation of a diagram is introduced. In Sect. 4, rules for drawing supplementary figures that are used to generate the hints of different viewpoints are proposed. The prototype system is shown in Sect. 5, and the experimental results are shown in Sect. 6. Finally, our paper is concluded in Sect. 7.

2. Approach

2.1 Learning Environment

Our research focuses on the collaborative learning of mathematical exercises. Such collaborative learning starts when learners tackling the same exercises/problems encounter impasses. Exercises/problems can be provided either by the Learning Management System (LMS) or by the teacher. First, learners attempt to solve exercises/problems by themselves. If they cannot, they organize a group to discuss them.

Since learners who are participating in the collaborative learning are tackling the same exercises/problems, they are expected to have the basic knowledge about the subjects in exercises/problems. Moreover, before joining into the group, they tried to solve the exercises/problems by themselves, so they may actively participating in the collaborative learning.

During the learning, learners discuss how to derive the common answers in the public learning space, while considering their own answers in their personal learning spaces. Answers in personal learning spaces are sometimes different from those discussed in the public learning space. However, unifying answers is not necessary. They may discuss different answers, but it is no need to pose specific answer to them.

2.2 Support for Collaborative Learning of Exercise/Problem-Based Learning

The goal in exercise/problem-based learning is not only to derive answers but also to acquire various knowledge related to exercises/problems. Such knowledge includes all applicable answering methods or the incorrect answering methods that tend to be applied to exercises/problems. Koschmann et al. modeled the process of problem-based learning in CSCL [4]. It consists of five components: problem formulating, self-directed learning, reflecting, abstracting, and applying knowledge. Of these five components, the group engages in abstracting and reflecting. In reflecting, learners critique the learning process and improve their answers by discussing and comparing their own approaches to the problem. During abstracting, learners articulate the knowledge they have acquired so as to explore similarities and differences or to discern generalizations. Reflecting occurs during knowledge transition from a public to a personal learning space, while abstracting corresponds to such transition from a personal to a public learning space. For learners to exchange various knowledge related to exercises/problems, many opinions should be derived during abstraction. In addition, since various ideas should be generated by each learner, supporting the reflecting may be effective.

There are plural contributions that support the reflecting of individual learners who participate in collaborative learning. Ayala proposed the formalization of agent's activity for managing the learning situation of all learners and

assigning appropriate tasks to them using Answer Sets Programming (ASP) [5]. Constantino-Gonzalez et al. developed a coaching agent in learning entity-relationship modeling, which points out the conflict between learner answers and the group answer [6]. These researches support individual learners based on their learning activities. However, they did not focus on the management issue of group activity but tried to promote the activities of individual learning, so learners may not discuss efficiently in the public learning space. Nakamura et al. also generated an agent that monitors the learning situation of all learners and makes utterances on their behalf [7]. This agent fuels the discussion from the viewpoint of each learner. Therefore, discussion topics sometimes become inconsistent. To enhance effective collaborative learning, such group activities as discussion should be effectively managed and organizationally controlled.

On the other hand, some contributions promote effective discussion by supporting the abstracting globally. Soh et al. extended I-MINDS and developed a multi-agent system for composing an effective collaborative learning group based on Jigsaw [8]. Also, Ikeda et al. proposed a mechanism to find appropriate co-learners based on the learner's understanding level [9]. These researches formed learning groups that may perform active discussion. Since a mechanism for monitoring or managing learning activities was not introduced in these researches, effective discussion was not ensured. Weinberger et al. analyzed the effects of scripts for collaborative learning that control arguments generated in group activities [10]. The scripts only manage the learning process, such as types of arguments, to generate each moment without observing the contents of the learning activities. Suebnukarn et al. constructed COMET, which grasps the learning situations of a group and learners using a Bayesian network and provides hints in medical problem-based collaborative learning [11]. This research focused on the performance of the group, but did not individually support learners who could not participate in the group activity. As Stahl et al. argued [12], analysis and management of both group and learner activities are necessary to promote knowledge transfer between personal and public learning spaces.

2.3 Mathematical Exercises

Mathematical exercises generally consist of one answer and several answering paths that are divided into several answering steps. Answering steps, which represent the answering scenes for deriving the answer, are characterized by derived equations. An answering step is changed to another answering step by applying answering methods or formulas.

Table 1 shows an example exercise and one of its answering paths related to a quadratic function. This answering path consists of five answering steps based on the applied answering methods. Each answering step is characterized by derived equations or values. In Table 1, the specific equations or values of the answering steps are underlined.

On the other hand, diagrams help learners visually un-

Table 1 Example of mathematical exercise.

<p>[Exercise] Derive an equation that is point symmetry to $y = x^2 - 4x + 7$ by the point $(1, 1)$.</p>
<p>[Answer] <i>Step 1:</i> $y = x^2 - 4x + 7 = (x - 2)^2 + 3$ The top of $y = x^2 - 4x + 7$ is $(2, 3)$. <i>Step 2:</i> Define a point that is point symmetry to $(2, 3)$ by point $(1, 1)$ as (a, b). <i>Step 3:</i> Since $(1, 1)$ is situated in the middle of $(2, 3)$ and (a, b), $(a + 2)/2 = 1, (b + 3)/2 = 1$ <i>Step 4:</i> The answers to these equations are $a = 0$ and $b = -1$. <i>Step 5:</i> Therefore, the target equation is $y = (x - 0)^2 - 1 = x^2 - 1$.</p>

derstand exercises that are described only by characters. Figures correspond to individual equations, and answering steps are identified by the shapes of all existing figures and their relations. Thus, the figures in the diagram represent the current progress of deriving answers. If one diagram holds figures that are not included in another diagram, more answering steps were derived by the learner who drew the latter diagram compared with those of the former learner. When two diagrams have different figures from each other, learners who drew these diagrams may have found answers based on different answering methods.

Moreover, visualization of current equations may help learners to understand the exercise content and their own learning situation. Since formulas or answering methods are applied based on specific shapes of figures or relations among figures, to illustrate existing figures and their relations promotes learners to find appropriate answering methods/formulas. So, visualization is effective for grasping current learning situation as well as solving the next answering step.

Lomas argued that a diagram was not only an intuitive tool, but was important means for some types of mathematical reasoning [13]. Ito et al. also analyzed the importance of diagrams in solving exercises of trigonometric functions [14]. They classified diagrams into eight types based on their roles in solving exercises. A diagram that includes supplementary figures of answering methods/formulas effectively promotes the derivation of new answering steps. Supplementary figures correspond to equations or values derived in the unsolved answering steps. The goals in unsolved answering steps and answering methods to be applied in such answering steps can be estimated based on the supplementary figures. Such supplementary figures are so effective that learners can understand what to derive and how to solve in the next answering steps.

Figure 1 shows an example of supplementary figures corresponds to the exercise shown in Table 1. Figures in Fig. 1 (a) correspond to the quadratic equation and the point given in the exercise. Since the goal in *Step 1* is to derive the top of the given quadratic equation, the point is drawn to the top as the supplementary figure. Supplementary figure in *Step 2* is a point which is point symmetry to $(2, 3)$ by $(1, 1)$. This supplementary figure does not indicate that its

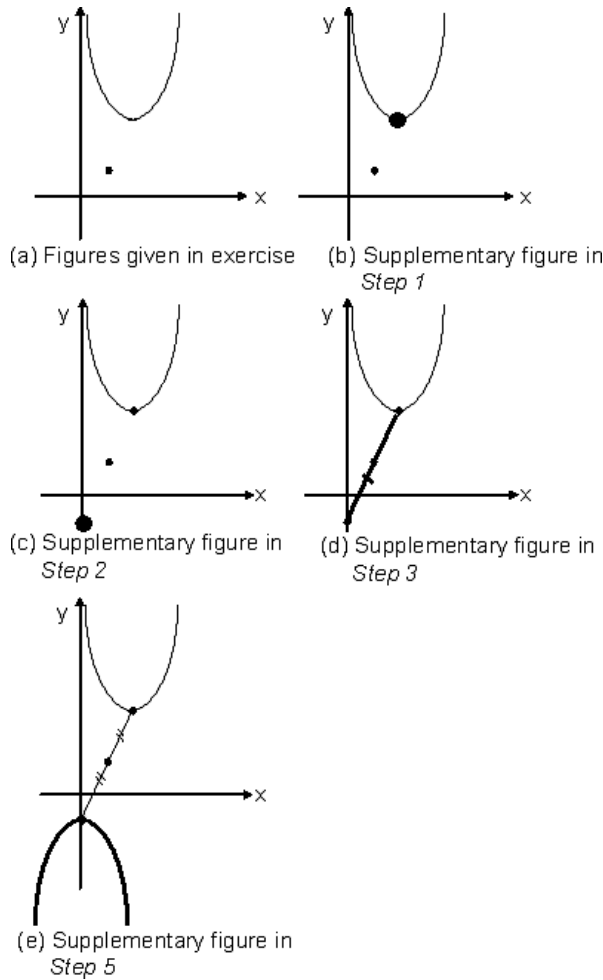


Fig. 1 Example of supplementary figures.

coordinate should be set by using variables such as a and b . However, it insists that the point should be derived. Supplementary figure in Step 3 indicates that the distances from $(1, 1)$ to $(2, 3)$ and $(1, 1)$ to the point illustrated in the former step are the same. Based on this supplementary figure, equations to derive the coordinate of the point are able to be generated. Since Step 4 is only to solve the equations, there are no appropriate supplementary figures. The supplementary figure in Step 5 is the quadratic equation which is the goal of the exercise. These supplementary figures do not directly indicate hints in the next answering steps, but imply goals and answering methods in the next answering steps.

2.4 Collaborative Learning Support Using Diagrams

Since learners acquire knowledge through discussion, they cannot acquire sufficient knowledge of exercises if the group failed to derive the answer. To encourage discussion based on various answering paths, the roles of monitoring the learning activities of groups and assisting a group are needed to accomplish effective learning. As a method for assisting a group, directly revealing the correct answers would stifle group discussion. The assistant function should give hints

to encourage group activity, if necessary. In mathematics, learners attain answers by applying formulas or answering methods in each step. These formulas or answering methods are indirectly indicated by the corresponding supplementary figures. Therefore, in our research, we developed a mechanism that generates supplementary figures for automatically deriving the next answering step.

Since it is difficult to prepare supplementary figures for all answering steps, the general method for drawing supplementary figures that correspond to individual answering methods/formulas, which are called the *rules for drawing supplementary figures*, is defined. These rules are selectively controlled based on forward reasoning. Although all answering methods whose conditions satisfy the current diagram can be selected even if they are not appropriate for the target exercise, learners can consider not only accuracy of the indicated formulas/answering methods but also the effectiveness of the proposed supplementary figures.

On the other hand, since learners acquire knowledge from other learners' utterances in discussions, they need to participate in group activities to stimulate discussion. If learners get new ideas and gain confidence, they may express their own opinions more easily. So, highlighting the differences between their answers and the group answer helps learners offer opinions to the group. Differences among answers can be grasped by comparing the diagrams. Meaningful differences between learner and group diagrams are detected and can be pointed out to learners. Nabil et al. proposed a method for specifying the differences between pictures based on the topological relations of objects [15]. In mathematical exercises, equations that correspond to figures and answering methods can be identified by the derived equations. However, the scales of the figures drawn in diagrams sometimes vary from learner to learner, so it is inappropriate to manage coordinates of individual figures. Our approach introduces the *internal representation of the diagram*, which consists of types of figures and remarkable relations to other figures. Since the internal representation of a diagram is used to identify the answering methods, learners do not handle figures by scales and coordinates, but only by types. Both learner and group diagrams are transformed into the corresponding internal representations and the differences between the corresponding internal representations are detected by comparing their components in the internal representations.

Figure 2 illustrates the outline processing of our collaborative learning support method using diagrams. A group diagram is manipulated to promote group discussion by adding supplementary figures that indicate an answering method. On the other hand, the uniqueness of the learner diagram is pointed out by detecting the differences between the learner and group diagrams to urge learners to propose their opinions.

2.5 System Overview

Currently, our research focuses on collaborative learning en-

vironments in which learners discuss their opinions through the chat window while drawing diagrams on the shared canvas. In addition, private canvases are provided for learners to draw their own diagrams. The server embeds the mechanism to encourage effective discussion. That is, to promote discussion of various answering paths, the server monitors the progress of learning based on a group diagram and generates hints, if necessary. The client assumes the role of urging individual learners to present their ideas to the group

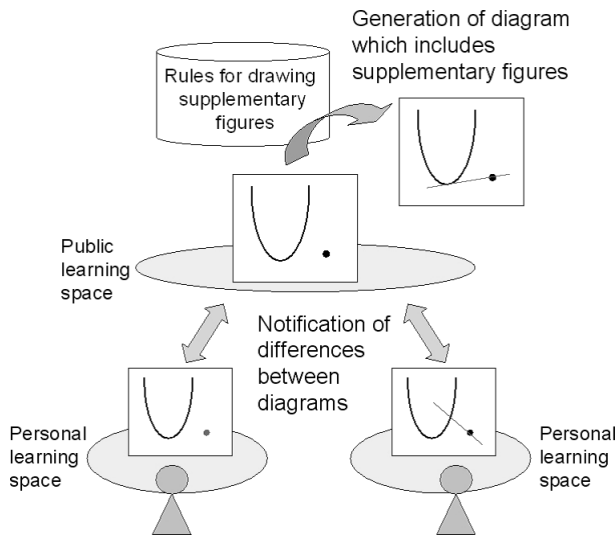


Fig. 2 Collaborative learning support based on diagrams.

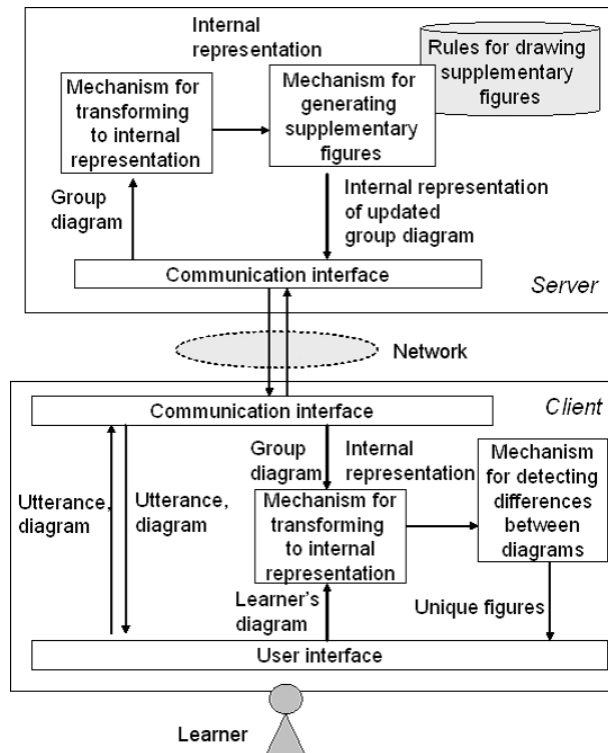


Fig. 3 System overview.

to activate group discussion and indicates the originality of learner answers by pointing out the differences between the group and learner diagrams.

Figure 3 shows the system overview. Utterances in the chat window and the diagrams on the shared canvas are sent to all clients through the server. When the server acquires a group diagram drawn on the shared canvas, it is transformed to an internal representation. When discussion encounters an impasse, the internal representation is updated by adding the supplementary figures based on the rules for drawing supplementary figures. On the other hand, in the client, the group and learner diagrams are transformed into internal representations. To encourage learners to join into the discussion, the differences between diagrams are extracted by comparing all figures and relations that consist of both diagrams. Then, diagrams are shown to the learners through a user interface that visually emphasizes unique figures.

3. Internal Representation

Our internal representation indicates the meaningful configuration of diagrams. Since the answering steps in mathematics are specified by derived equations that can be visualized by figures, diagrams are identified by drawn figures that correspond to the equations. Figures are characterized by their types and their locations. Therefore, internal representation consists of types of figures and their relations, which are represented in the predicate form. Predicates for types of figures are prepared heuristically by analyzing all possible figures that satisfy the answering methods/formulas from Japanese high school mathematics textbooks. The predicates for relations between figures are prepared for relations that are meaningful to determine answering methods. The types of figures and the relations of figures are represented as the following form. x and y denote each figure: *Predicate_of_Type(x)*, *Predicate_of_Relation(x,y)*.

Table 2 shows an example of predicates for the sub-

Table 2 Example of internal representation.

	Predicate	Meaning
Types of figures	$point(x)$	x is a point.
	$line(x)$	x is a line.
	$parabola(x)$	x is a parabola.
	$x-axis(x)$	x is an x-axis.
	$y-axis(x)$	x is a y-axis.
Relations between figures	$cross(x,y)$	x crosses y .
	$on(x,y)$	x is situated on y .
	$contact(x,y)$	x contacts with y .
	$apart(x,y)$	x is apart from y .
	$parallel(x,y)$	x is parallel to y .
	$vertical(x,y)$	x is vertical to y .
	$top(x,y)$	x is a top of y .
	$pivot(x,y)$	x is a pivot of y .
	$edge(x,y)$	x is an edge of y .
	$samelength(x,y)$	x has the same length as y .
	$sameshape(x,y)$	x has the same shape as y .
	$oppositeshape(x,y)$	x is upside-down to y .

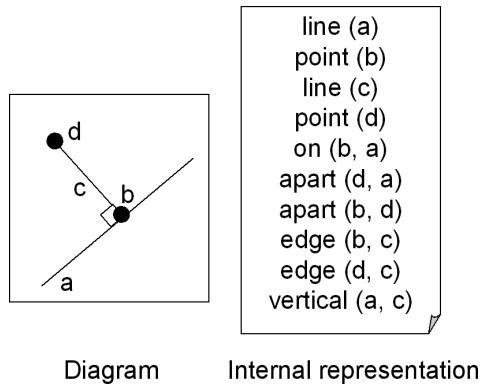


Fig. 4 Example of internal representation.

Table 3 Algorithm of comparing figures in different diagrams.

Step 1:	One figure on a private canvas is extracted as a target figure.
Step 2:	Types of target figure and figures in shared canvas are compared. Figures on shared canvas whose types are identical as the target figure are candidates of the same figure. If there is one candidate, it is regarded to represent the same equation as the target figure. If there is more than one candidate, go to Step 3.
Step 3:	Relations between other figures of the target figure and candidates are compared. Of all candidates, the one whose number of identical relations with the target figure is the largest is regarded as the same equation. Then go to Step 4.
Step 4:	If more figures exist on a private canvas, select one and go to Step 2. Otherwise, figures that do not match any figures in the other diagrams are detected as their differences.

ject of quadratic functions. Five types of figures and twelve relations between figures are arranged in Table 2.

Figure 4 is an example that represents internal representation, using the predicates listed in Table 2. In Fig. 4, the diagram that consists of two points and two lines is displayed. The two lines are orthogonal, one point is on the intersection of lines, and another point exists on one line. Therefore, its internal representation consists of ten predicates as shown in Fig. 4.

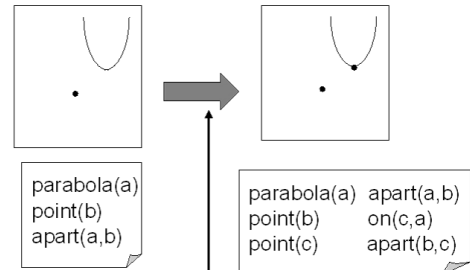
When diagrams are drawn on private and shared canvases, they are transformed into internal representations to detect their differences. Predicates that represent types of figures and their relations are compared individually by the algorithm shown in Table 3.

The internal representation of shared canvas is also analyzed in the server. Based on its internal representation, supplementary figures are generated. The details of the method are explained in Sect. 4.

4. Rules for Drawing Supplementary Figures

When learners cannot derive the answer, supplementary figures that fit to the learning situation are generated and at-

[Exercise]:
Derive an equation that is point symmetry to $y=x^2-4x+7$ by the point (1, 1).



[Rules for drawing supplementary figure for deriving top of parabola]
 $\exists x \exists y$ (IF parabola(x), THEN (point(y), top(x, y)))

Fig. 5 Example of applying rules to draw supplementary figures.

tached to the diagram. Supplementary figures should not reveal the answer, but provide hints to derive the next answering step.

Applicable answering methods are identified by the existing equations. In the diagram, equations are represented by figures, and answering methods are characterized by the existing figures. Then applicable answering methods are selected based on the existing figures on the shared canvas.

The following is the form to describe the rules for drawing supplementary figures. These rules are prepared for each answering method. In the IF part, figures needed to apply the answering method are described. In the THEN part, figures that indicate hints for the answering method are attached.

IF < existing_figures >
THEN < figures_to_be_added >

When a rule is selected, the figures in the THEN part are added to the group diagram as supplementary figures. Since their roles are to show the applicable answering methods and encourage the discussion in the next answering step, selecting correct rules at all the times is inappropriate. To generate supplementary figures in various answering methods, rules are chosen based on forward reasoning in which rules that match the IF part are applied. Sometimes plural available rules satisfy the diagram. The figures in the IF part represent a condition under which the corresponding answering method can be applied. As the number of figures in the IF part gets larger, the target situation of the answering method becomes more specific. Exercises may be prepared for particular answering methods. Thus, if plural rules are matched, the rule whose number of figures in the IF part is the largest is applied. When there are plural applicable rules whose IF parts have the same number of predicates, the rule whose ID is the smallest is selected. The IDs of rules represents their generalities, which is defined heuristically beforehand. Then figures described in the THEN part are visualized on the shared canvas and may become the next discussion topic among learners.

Figure 5 is an example of applying the rules to draw supplementary figures. It is assumed that the point and the parabola will be successfully drawn when the exercise is given. In this case, rules that correspond to the answering method for deriving the top of the parabola can be applied and a new figure that satisfies the added internal representation emerges.

5. Prototype System

A prototype system has been implemented in distributed collaborative environments for learning mathematics. A mechanism for generating supplementary figures is embedded in the server, and a mechanism for notifying learners the differences between diagrams is installed in the client. The prototype system is mainly coded using Java. Selecting and choosing rules for generating supplementary figures are realized using Prolog.

Figure 6 shows the interface in our prototype system. Learners exchange opinions through shared canvas and chat window where they can freely input utterances. On the other hand, only one learner can draw on a shared canvas at one time, because learners have to request access to draw a diagram. In addition to the shared canvas, learners also have their own private canvases for drawing diagrams. In drawing diagrams on both private and the shared canvases, learners indicate the types of figures and relations between adding and existing figures by selecting the corresponding buttons. Then they push the appropriate coordinates on the canvases to specify figures. Currently, buttons for requesting advices are provided to grasp the learning group’s impasse.

Once the button is pushed, an appropriate rule for drawing diagram is selected and supplementary figures are added to the diagram on the shared canvas. Figure 7 is an example of adding supplementary figures to the shared canvas. In this example, the IF part in the rule for deriving the distance between a point and a line is matched, and the figures in its THEN part are added to the diagram on the shared canvas. One of the figures that satisfy the updated internal representation is drawn in the shared canvas.

Differences between private and shared canvas are continuously emphasized to each learner. When the diagram on private or shared canvas is updated, the differences between these diagrams are examined. Then, the original figures equal to neither diagram are highlighted by changing their colors. Figure 8 is an example that indicates the differences between diagrams. Two learners whose diagrams on the shared canvases are identical but which are different on the private canvases are assumed. For learner A, two points and the line on the shared canvas are emphasized as original figures since they do not appear on the private canvas. In addition, parabolas on both canvases are also detected as different figures because they do not have the same relation with other figures. On the other hand, for learner B, none of the figures on the shared canvas are drawn on the private canvas, so they all are determined to be unique.

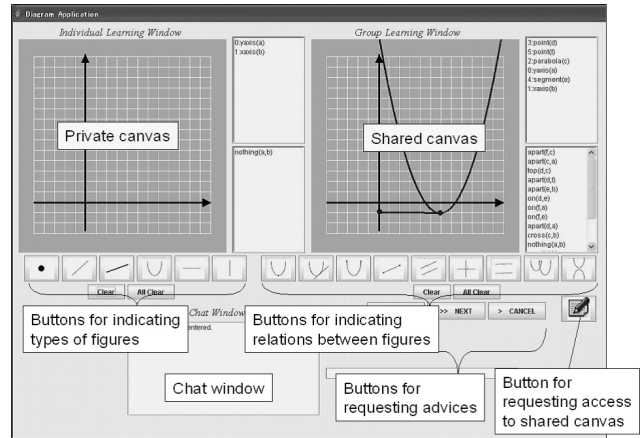


Fig. 6 Interface in prototype system.

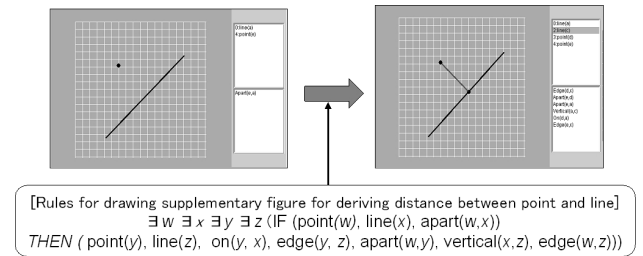


Fig. 7 Adding supplementary figures on shared canvas.

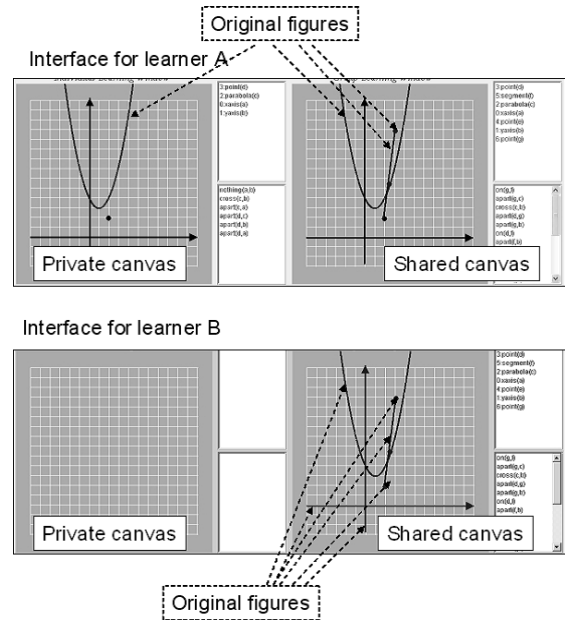


Fig. 8 Indicating differences between canvases.

6. Experiment and Consideration

Experiments were conducted separately for evaluating the effectiveness of the generated supplementary figures and the indicated differences between diagrams.

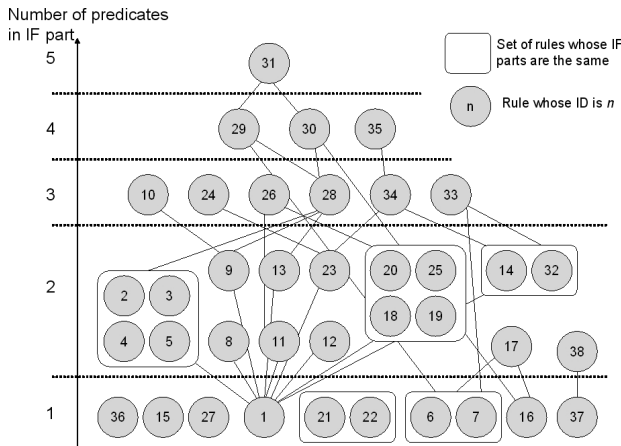


Fig. 9 Relations among rules defined from figures in IF part.

Table 4 Examples of rules for drawing supplementary figures.

ID	IF part	THEN part	Answering method
1	point(x)	parabola(y), top(x, y)	Draw parabola whose top is x
6	line(x)	line(y), parallel(y, x)	Draw line parallel to x
23	point(x) line(y)	point(v), line(w), edge(x, w), edge(v, w), vertical(y, w) on(v, y),	Draw line vertical to y from x
24	line(x) point(y) apart(y, x)	point(w), line(z), on(w, x), edge(w, z), apart(y, w), vertical(x, z), edge(y, z)	Derive distance between x and y

6.1 Experiment for Generating Supplementary Figures

6.1.1 Validity of Generated Supplementary Figures

The objective of the experiment was to evaluate the selection method of rules for generating supplementary figures. This experiment focused on quadratic functions in high school mathematics. 38 rules for drawing supplementary figures were prepared. The relations between the figures in the IF part of all the prepared rules were analyzed as shown in Fig. 9. Here, circles correspond to rules, and links between circles indicate the figures in the upper rules contain those in the lower rules. The numbers attached to the circles in Fig. 9 are IDs of rules. Table 4 shows some of the rules. The IF parts of supplementary rules 1 and 6 consist of only one predicate. Since the IF part of supplementary rule 23 includes predicates of point and line, links are attached between rules 23 and 1, and rules 23 and 6. Also, link is generated between rules 23 and 24. Based on Fig. 9, the IF parts in many rules are related to each other. Currently, if the IF parts of plural rules are matched with the current diagram,

Table 5 Analysis of applied rules.

Selected rules	Number
Appropriate rules	34
Inappropriate rules	5

Table 6 Analysis of inappropriately selected rules.

Characteristics of selected rules	Number
Figures in the IF part are identical as those in correct rules.	4
Figures in the IF part include those in correct rules.	1

then the rule with the largest number of figures is selected. If the numbers of predicates in the IF parts are the same, the rule with smaller ID is applied.

Using these rules, supplementary figures are generated by investigating the 39 answering steps in 25 exercises. Table 5 shows the analysis of the selected rules. Rules whose answering methods are appropriate for solving the exercise were applied to 87% of the cases. Inappropriate rules were derived for only 13% of the cases. This result shows that the selection of rules for drawing supplementary figures is effective for the most cases. Table 6, which is the detailed analysis of inappropriately selected rules, indicates the relations of the figures in the IF part of the inappropriately selected rules with those of the correct rule. The figures in the IF part of all inappropriate rules contain those of the correct rules. Since our objective is not to select appropriate rules but rather to indicate the answering methods applicable to the situation, inappropriate rules are also important to enhance fruitful discussion. Therefore, if learners cannot improve their learning based on the generated supplementary figures, selecting another rule may be effective whose figures in the IF part are the subset of those in the applied rules. Based on the provided diagram, discussion for different answering methods may be encouraged.

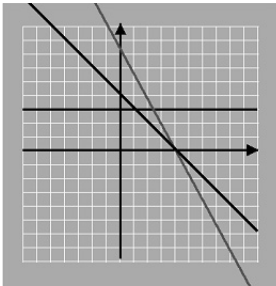
6.1.2 Effectiveness of Supplementary Figures

Two groups of undergraduate and graduate students in our laboratory were asked to study collaboratively with our prototype system in order to evaluate its effectiveness for generating supplementary figures. They all studied the target exercise when they were high school students, but they found themselves difficult in deriving the answer. Since they all have similar levels of understanding, the groups were formed randomly. This experiment investigated if the generated supplementary figures can help members derive a group answer. In this experiment, differences between private and shared canvases were not indicated. Group 1 consisted of four members (denoted as A to D), and group 2 consisted of three members (denoted as E to G). Both groups tackled the same exercise that consisted of four answering steps, which is shown in Table 7.

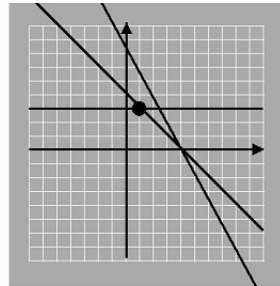
During the learning, supplementary figures were generated once for group 2 and twice for group 1. For group 1,

Table 7 Exercise used in experiment.

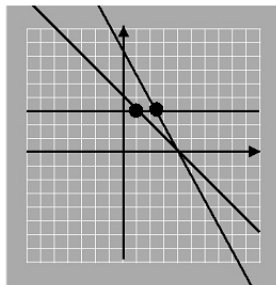
If point (x, y) moves the area which satisfies $y \leq 3 \cdots (1)$, $x + y - 4 \geq 0 \cdots (2)$, and $2x + y - 8 \leq 0 \cdots (3)$, derive the maximum and minimum values of $k = y + x^2 - 4x$.



(a) Before supplementary figure is generated.

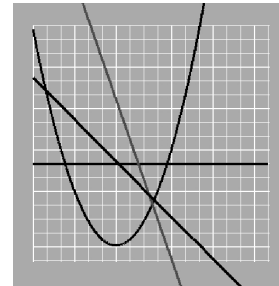


(b) After first supplementary figure is generated.

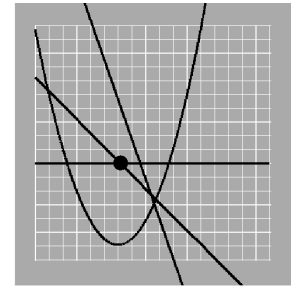


(b) After second supplementary figure is generated.

Fig. 10 Group diagrams of group 1.



(a) Before supplementary figure is generated.



(b) After supplementary figure is generated.

Fig. 11 Group diagrams of group 2.

Table 8 Applied rule.

IF part	THEN part	Answering method
line(x), line(y)	point(z), on(z, x), on(z, y)	Derive intersection of two lines.

Table 9 The numbers of appropriate/inappropriate rules for diagrams.

	Appropriate rule	Inappropriate rule
group 1(1)	1	1
group 1(2)	1	3
group 2	2	3

one member accidentally pushed the button consecutively. After the button was pushed twice, the members could see the supplementary figures generated by both rules. Since the button was pushed in such a short interval, they discussed both supplementary figures together as if they were derived by one rule. The answers were successfully derived after the supplementary figures had been derived for both groups.

All the selected rules were appropriate for the learning situation. Figures 10 and 11 are group diagrams of group 1 and group 2 before and after the supplementary figures were generated. Figures 10(a) and 11(a) were correct but insufficient to derive the answer of the exercise. For both diagrams, rule for deriving the intersections of two lines was applied, which is shown in Table 8. One of the solutions to the exercise was deriving the intersections of every three lines and substituting the results to the quadratic equation for the parabola. Therefore, the selected rule was appropriate for the solution. In both diagrams, other rules were able to be applied. The numbers of appropriate and inappropriate rules that can be applied for diagrams and have the same number of predicates in the IF part are shown in Table 9. The group 1 (1) refers to the first supplementary figure for group 1. Based on the result, our system could find appropriate rule even if the number of applicable rules was more than one.

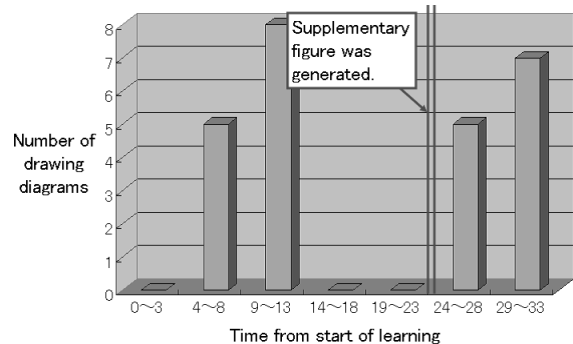


Fig. 12 Number of drawing diagrams for group 1.

Figures 12 and 13 indicate the numbers of members' drawing diagrams. Members began to draw diagrams after the supplementary figures had been generated. The drawn diagrams were able to provide effective hints for deriving the answer. Therefore, generating hints using supplementary figures may be helpful to lead members to the right answers.

On the other hand, the numbers of utterances for each interval time are shown in Figures 14 and 15. The numbers of utterances for individual members are identified by colors. In both groups, the numbers of utterances dramatically increased after the supplementary figures were generated. Moreover, the numbers of utterances for each member also increased. For groups 1 and 2, after the supplementary figure was visualized, discussion started related to the coordinates of the drawn point. This result confirms that generated

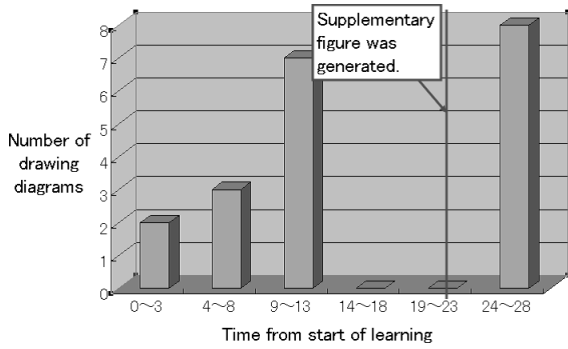


Fig. 13 Number of drawing diagrams for group 2.

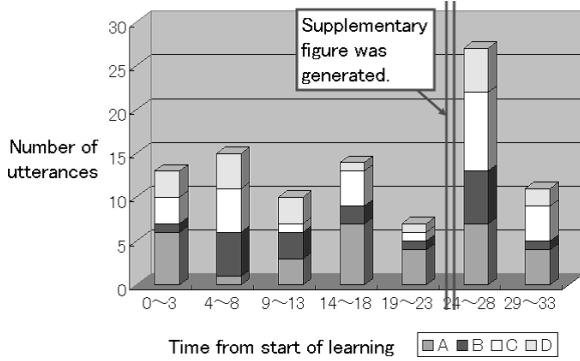


Fig. 14 Number of utterances for group 1.

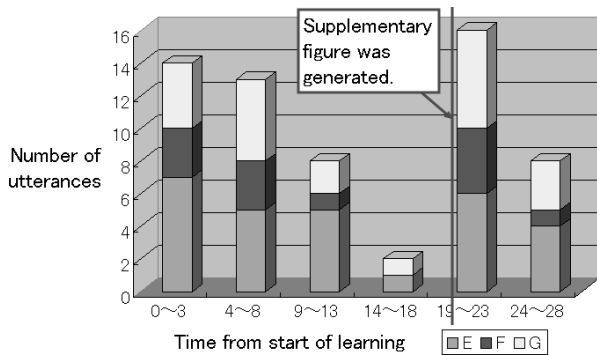


Fig. 15 Number of utterances for group 2.

supplementary figures can encourage active discussion for specific answering methods indicated by supplementary figures.

6.2 Experiment for Indicating Differences between Diagrams

Two additional groups (group 3 and group 4) of three members (denoted as *H* to *M*) in our laboratory were formed to study two mathematical exercises collaboratively. As the members in the former experiment, they all had the basic knowledge required for the exercises. For one exercise, members used a system in which the mechanism that indicated the differences between diagrams was embedded

Table 10 Exercises used in experiment.

<i>Learning 1</i> Derive the maximum and minimum values of $x - 4y$, when x and y are real numbers and satisfy $x \geq 0$, $0 \leq y \leq 6$, and $(x - 2y + 8)(3x + y - 18) \leq 0$.
<i>Learning 2</i> Derive the maximum and minimum values of $x + y$, when x and y satisfy $x \geq -1$, $y \geq 0$, $x + 2y \leq 3$, and $2x + y \leq 4$.

Table 11 The number of manipulating each canvas in *learning 1*.

Target canvas	Former canvas	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
private	private	12 57%	6 43%	1 17%	6 40%	3 43%	3 50%
private	shared	3 14%	5 36%	2 33%	3 20%	2 29%	2 33%
shared	shared	3 14%	2 14%	2 33%	3 20%	1 14%	1 17%
shared	private	3 14%	1 7%	1 17%	3 20%	1 14%	0 0%

Table 12 The number of manipulating each canvas in *learning 2*.

Target canvas	Former canvas	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
private	private	10 50%	0 0%	2 40%	2 33%	7 39%	2 33%
private	shared	2 10%	0 0%	1 20%	1 17%	5 28%	1 17%
shared	shared	6 30%	1 50%	2 40%	2 33%	4 22%	2 33%
shared	private	2 10%	1 50%	0 0%	1 17%	2 11%	1 17%

(*learning 1*), and for another exercise they only studied with a chat system (*learning 2*). The effect of indicating differences between diagrams was evaluated by investigating member activities in both learnings situations. Exercises for each learning are shown in Table 10.

The numbers of drawn figures on shared and private canvases were counted for individual members in both learnings. Tables 11 and 12 show the numbers of drawing diagrams for each canvas with target canvases of the former manipulations. *H* to *M* correspond to individual members. *Target canvas* indicates the canvas that each member manipulated, while *former canvas* refers to the canvas which was updated just before the target canvas. Upper number corresponds to the number of drawing each canvas, and lower number indicates its ratio in all manipulations of the member. If members draw diagrams on their private canvases after the shared canvas was manipulated, they might successfully acquire a new idea from the group to solve answering steps. On the other hand, when they manipulated the shared canvas after they updated their own private canvas, they might propose their own answers to the group.

The ratios of manipulating the shared canvas after drawing private canvas did not show significant differences between *learning 1* and *learning 2*. However, although the target exercises were different, the ratios of manipulating the

private canvas after the shared canvas were increased for all members in *learning 1* compared to *learning 2*. It indicates that members were able to proceed their answering activities if they can notice of the new idea from the group diagram. This shows the same result as the report of Error-Based Simulation of Horiguchi et al. [16]. They argued that visualizing learners' incorrect answers by Error-Based Simulation was effective for promoting the reflection. In our research, members' diagrams in their private canvases correspond to their insufficient answers, while the group diagram is regarded as the correct answer. By indicating the differences between group and learner diagrams, members were able to find new solutions or ideas and reorganize their answers. Thus, our system is likely to promote learners' reflections on their answers and support them of deriving answers by themselves.

7. Conclusion

This paper proposed two support functions for enhancing the successful discussion of collaborative learning. Concrete mechanisms for realizing such functions were developed for mathematical exercises. To increase learner awareness of the uniqueness of their answers and encourage them to propose opinions, differences between the diagram drawn in the learner's personal learning space and group diagram were specified. Moreover, supplementary figures that indicate answering methods were generated automatically by selecting appropriate rules for drawing supplementary figures to solve the situations when learners could not derive answers by themselves. Based on the experimental results, our proposed mechanisms contributed to successful discussions: the private learning activities were encouraged, and the amount of participation for group activities was increased by indicating the differences between group and learner diagrams. The number of utterances also increased after supplementary figures were generated. However, since we had only small number of experiments, further experiments is needed to evaluate the effectiveness of our system.

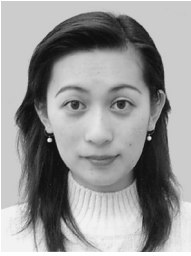
Currently, supplementary figures are generated every time learners push a button for requiring advice. The system does not evaluate whether the learning situation was actually need advices. Therefore, supplementary figures were sometimes generated inappropriately, when some learners could still contribute to derive the group answer. We previously proposed a system that grasps the learning situation and detects an impasse based on chat window discussions [17], [18]. Future work will address the integration of these mechanisms, so that supplementary figures are generated unconsciously and at appropriate times.

The internal model is represented by figures drawn intentionally by learners. The number of figures drawn in the diagram, especially the points that correspond to edges of the line, vary by learners. This difference sometimes causes a different internal representation. If the internal representation is different, the figures between the group and learner diagrams are not matched correctly. To cope with the personalization of drawing diagrams, a method needs to be de-

veloped to regulate the internal representation.

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