固体電子工学

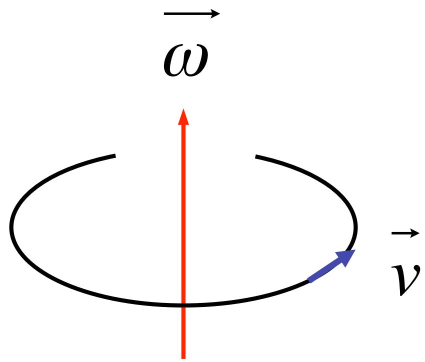
第１２回 磁場の中の電子

磁場の中の電子

自由な電子

運動方程式
$$m\frac{d\vec{v}}{dt} = -q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

電場が無いとき
$$\frac{d\vec{v}}{dt} = \vec{\omega_c}\times\vec{v} \qquad \vec{\omega_c} = \frac{q\vec{B}}{m}$$



角振動数 $\omega_c = qB/m$
の円運動を行う

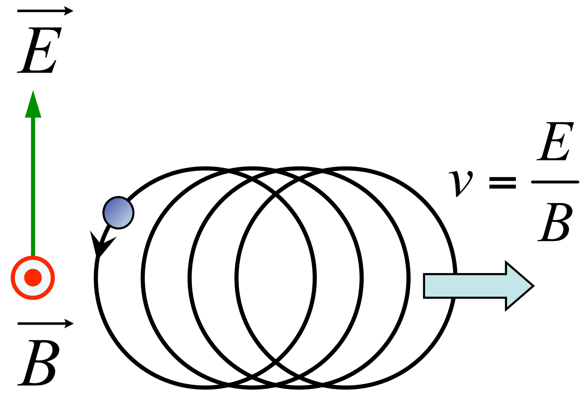
電場があるとき

磁場に垂直な電場

円運動の一周期で平均化

$$m\left\langle\frac{d\vec{v}}{dt}\right\rangle = -q\left(\vec{E} + \left\langle\vec{v}\right\rangle\times\vec{B}\right) = 0$$

$$\Longrightarrow \quad \left\langle\vec{v}\right\rangle = \frac{\vec{E}\times\vec{B}}{\left|\vec{B}\right|^2}$$



円運動しながらドリフト

散乱がある場合

$$m\frac{d\vec{v}}{dt}+\frac{m\vec{v}}{\tau}=-q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

定常状態、電流密度　$\vec{j}=-qn\vec{v}$

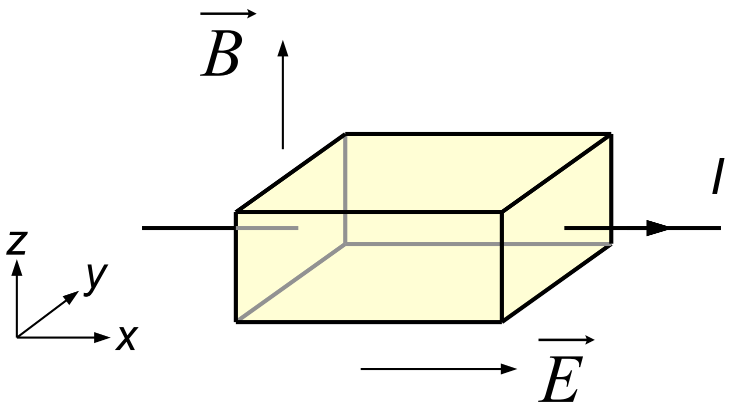
磁場の方向を z 軸にとると

$$\begin{pmatrix} 1, & \omega_c\tau, & 0 \\ -\omega_c\tau, & 1, & 0 \\ 0, & 0, & 1 \end{pmatrix}\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}=\sigma\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad \sigma=\frac{q^2\tau}{m}n$$

更に電場の方向を x 軸にとると

$$j_x=\frac{\sigma}{1+\omega_c^2\tau^2}E \qquad j_y=\sigma\frac{\omega_c\tau}{1+\omega_c^2\tau^2}E$$

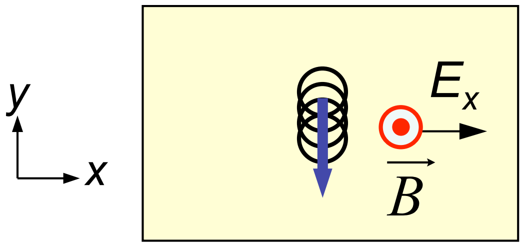
ホール効果



電流が x 方向にのみ流れるとき

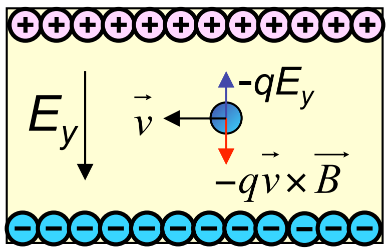
$$j_x = \sigma E_x$$

$$E_y = -\frac{j_x B}{qn}$$ ホール電場



ホール係数

$$R_H = \frac{E_y}{j_x B} = -\frac{1}{qn}$$



試料の境界に電荷ができ、ホール電場を形成

実験的に伝導電子密度、電子かホールか、を知る事が出来る

$$\mu = \sigma R_H$$

実験値から移動度も求まる

磁気抵抗

$$j_x = \sigma E_x$$

最もシンプルな近似　：　σ は磁場に依らない

実際には磁場に依存

（１）　電子が自由電子的でない
（２）　２つ以上のバンドに属する電子群、または１つのバンドでもフェルミ面の異なる部分に対応する電子群、またはホール群、が関与している
（３）　緩和時間が電子のエネルギー等に依存し、一定の仮定がなりたたない

一般的に弱磁場で次の磁場依存性をもつ

$$\sigma(B) = \sigma_0 + \sigma_2 B^2$$

磁場の中の電子（量子力学）

シュレディンガー方程式 $H\psi = E\psi$

磁場がある場合 $$H = \frac{\left(\vec{p} + q\vec{A}\right)^2}{2m}$$

$\vec{A}$ ：ベクトル・ポテンシャル

$\vec{B} = \nabla \times \overline{A}$

磁場の方向を z 軸にとると $\vec{A} = \left(0,\quad Bx,\quad 0\right)$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - i\frac{\hbar qB}{m}x\frac{\partial}{\partial y} + \frac{q^2B^2}{2m}x^2\right]\psi = E\psi$$

$$\psi = X\left(x\right)e^{ik_y y + ik_z z}$$

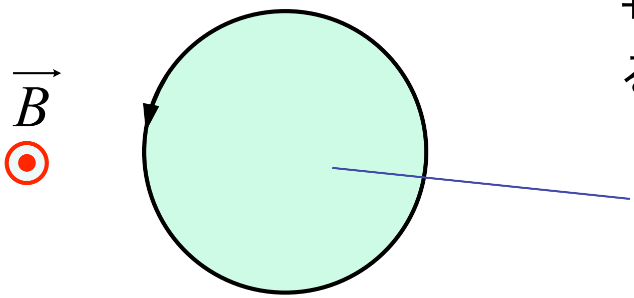
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega_c^2}{2}\left(x + \frac{\hbar k_y}{qB}\right)^2\right]X = \left(E - \frac{\hbar^2 k_z^2}{2m}\right)X$$

調和振動子の方程式 $$E = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m}$$ ランダウ準位

ランダウ準位

$$\left\langle \frac{m\omega_c^2}{2}\left(x+\frac{\hbar k_y}{qB}\right)^2 \right\rangle = \frac{1}{2}\hbar\omega_c\left(n+\frac{1}{2}\right) \quad \Rightarrow \quad \left\langle (x-x_0)^2 \right\rangle = \frac{\hbar}{qB}\left(n+\frac{1}{2}\right)$$

サイクロトロン運動の面積 $\pi r^2 = \pi\left(\left\langle (x-x_0)^2 \right\rangle + \left\langle (y-y_0)^2 \right\rangle\right) = \frac{2\pi\hbar}{qB}\left(n+\frac{1}{2}\right)$
=



サイクロトロン運動の軌道で囲まれる面積が量子化される

$$面積 = \frac{2\pi\hbar}{qB}\left(n+\frac{1}{2}\right)$$

k 空間でもサイクロトロン運動の軌道で囲まれる面積が量子化される

$$A = \frac{2\pi qB}{\hbar}\left(n+\frac{1}{2}\right)$$

状態密度

電子のエネルギーを磁場に垂直な運動による成分と磁場に平行な成分に別けて考える

$$E = E_{\perp} + E_{//} \qquad E_{\perp} = \left(n + \frac{1}{2}\right)\hbar\omega_c \qquad E_{//} = \frac{\hbar^2 k_z^2}{2m}$$

電子の状態密度

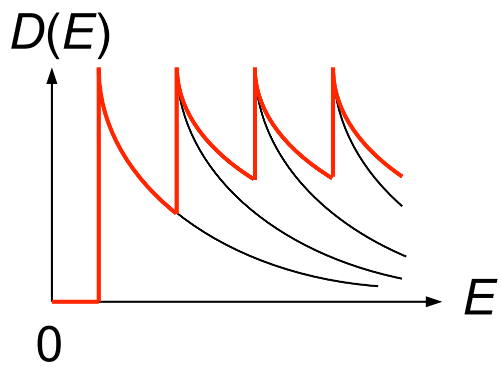
磁場が無いとき $\quad D(E)dE = \dfrac{2}{(2\pi)^3} dk_x\, dk_y\, dk_z$

$$E_{\perp} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right) \Longrightarrow dk_x dk_y = \frac{2\pi m}{\hbar^2} dE_{\perp}$$

磁場があるとき、１つのランダウ準位あたり

$$D(E)dE = \frac{2}{(2\pi)^3}\frac{2\pi m}{\hbar^2}\hbar\omega_c dk_z$$

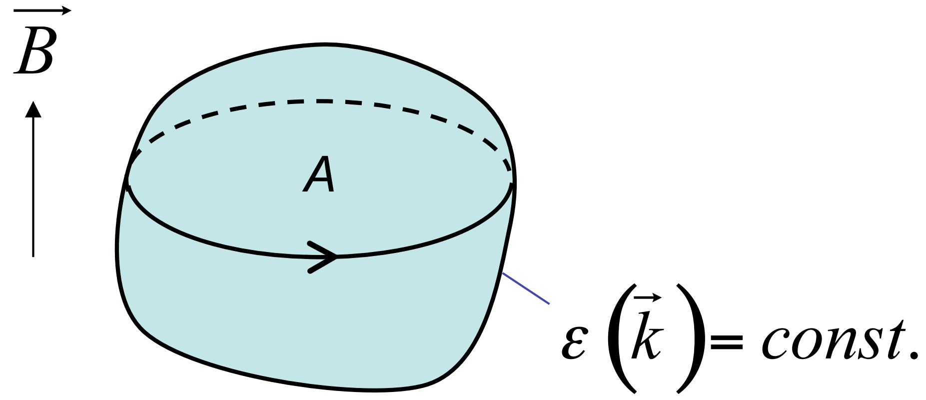
$$\Longrightarrow D(E) = \frac{\sqrt{2m}qB}{2\pi^2\hbar^2}\sum_{n=0}^{\infty}\frac{1}{\sqrt{E - \hbar\omega_c\left(n + \frac{1}{2}\right)}}$$



結晶の中の電子による
サイクロトロン運動

$$m\frac{d\vec{v}}{dt} = -q\vec{v}\times\vec{B}$$

$$\hbar\frac{d\vec{k}}{dt} = -q\frac{1}{\hbar}\frac{\partial\varepsilon}{\partial\vec{k}}\times\vec{B}$$



電子は *k* 空間で、磁場に垂直で
等エネルギー面に接するように運
動

サイクロトロン運動の周期

$$T = \frac{\hbar^2}{qB}\frac{dA}{d\varepsilon}$$

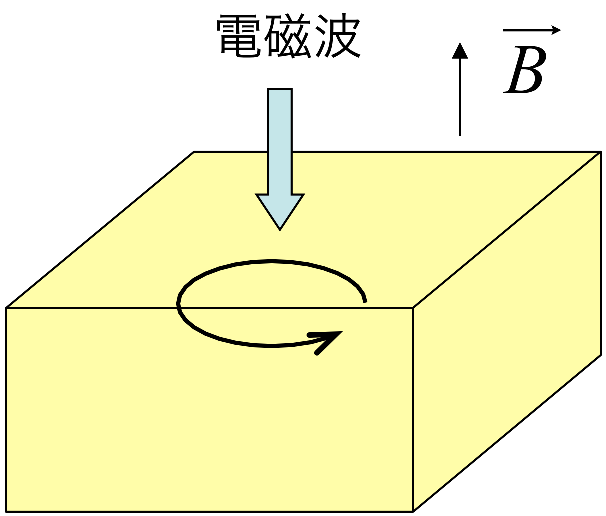
A : *k* 空間で軌道が囲む領域の面積

サイクロトロン共鳴

サイクロトロン角振動数 $\omega_c = \dfrac{2\pi}{T} = \dfrac{qB}{\hbar^2}\dfrac{2\pi}{dA/d\varepsilon}$

サイクロトロン質量 $m_c = \dfrac{qB}{\omega_c} = \dfrac{\hbar^2}{2\pi}\dfrac{dA}{d\varepsilon}$

電磁波の吸収



$\omega = \omega_c$ で電磁波の吸収が大きくなる

⇨ サイクロトロン質量が求まる

⇨ フェルミ面に関する情報が得られる

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Cyclotron Resonance of Electrons and Holes in Silicon and Germanium Crystals

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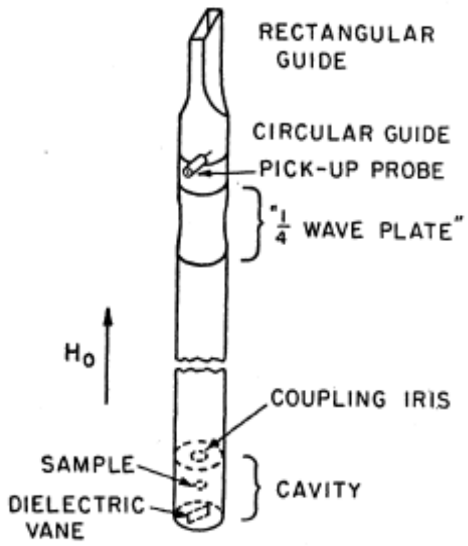


Fig. 4. Experimental arrangement for circular polarization studies of cyclotron resonance.

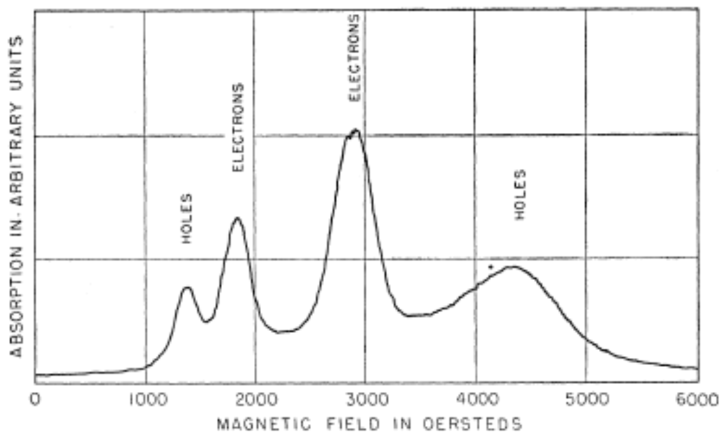


Fig. 3. Typical cyclotron resonance results in silicon near 24 000 Mc/sec and 4°K: static magnetic field orientation in a (110) plane at 30° from a [100] axis.

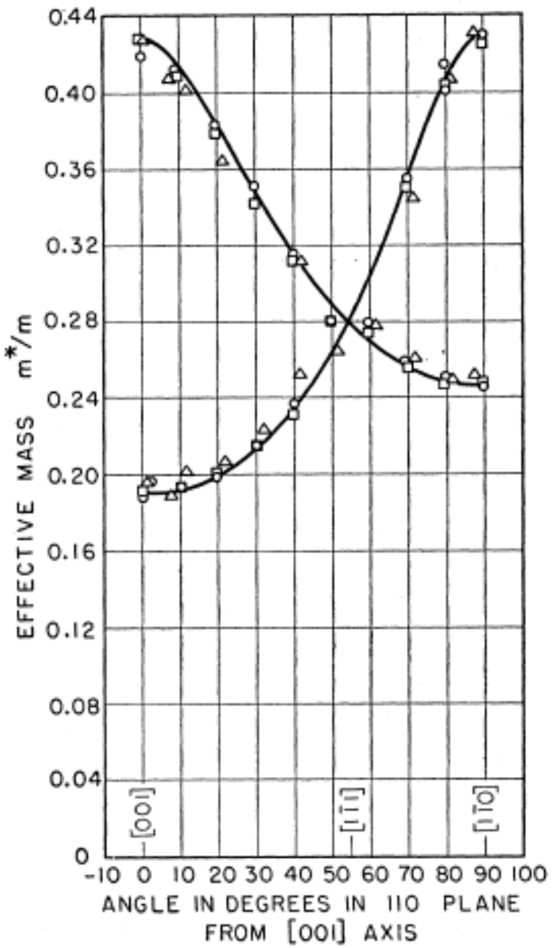
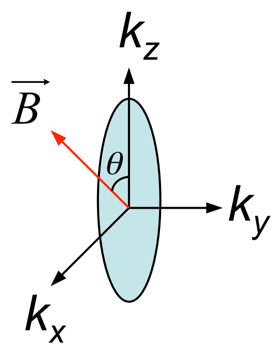


Fig. 6. Effective mass of electrons in silicon at 4°K for magnetic field directions in a (110) plane; the theoretical curves are calculated from Eq. (38), with $m_l = 0.98m$; $m_t = 0.19m$.



Eq.(38)

$$\left(\frac{1}{m^*}\right)^2 = \frac{\cos^2\theta}{m_t^2} + \frac{\sin^2\theta}{m_t m_l}$$

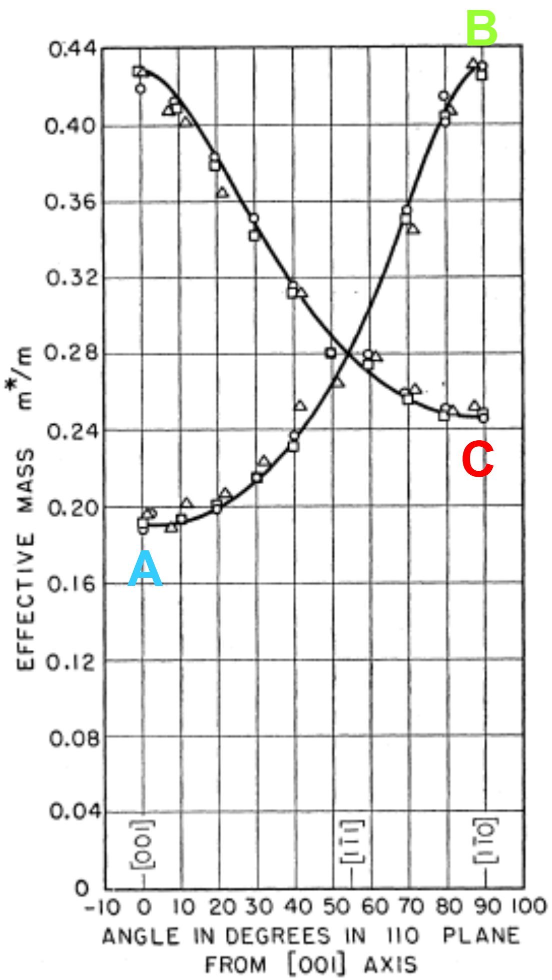


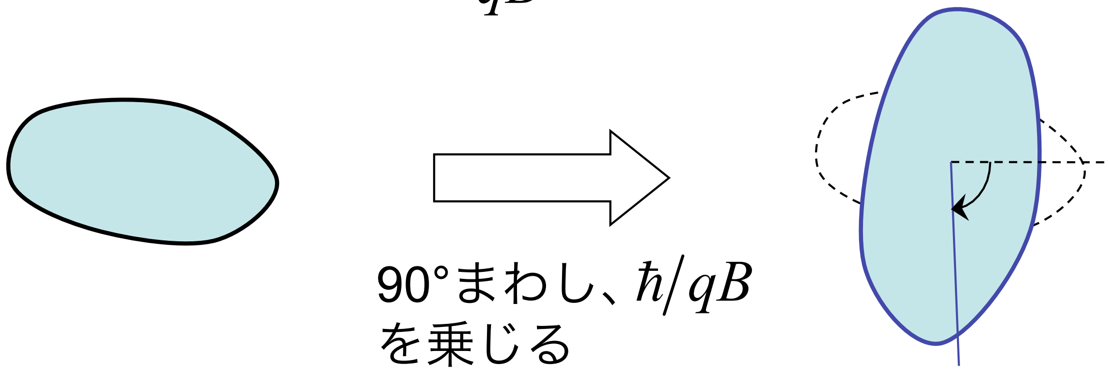
FIG. 6. Effective mass of electrons in silicon at 4°K for magnetic field directions in a (110) plane; the theoretical curves are calculated from Eq. (38), with $m_l = 0.98m$; $m_t = 0.19m$.

軌道の量子化

量子化　　*k* 空間で軌道が囲む領域の面積 A が量子化される

$$A = \frac{2\pi qB}{\hbar}\left(n + \frac{1}{2}\right)$$

$$x - \frac{\hbar}{qB}k_y = const. \qquad y + \frac{\hbar}{qB}k_x = const.$$



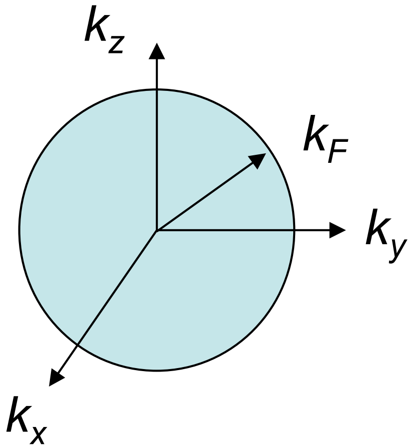
k 空間　　　　　　　　　　　　実空間

実空間でもサイクロトロン運動の軌道で囲まれる面積が量子化される

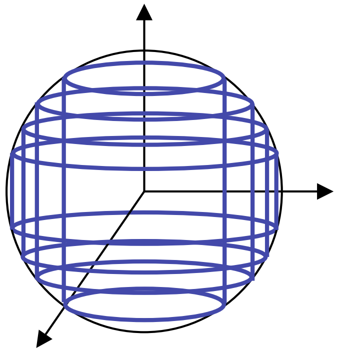
A : k 空間で軌道が囲む領域の面積

$$A = \frac{2\pi q B}{\hbar}\left(n + \frac{1}{2}\right)$$

k 空間のフェルミ面上の電子が多くの物性を決める

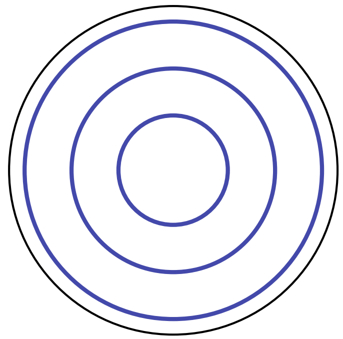


$B = 0$



$B \neq 0$

上から見た図



磁場を大きくしていくと

軌道面積が大きくなりフェルミ面からはずれる

なんらかの異常が観測される

実験によるフェルミ面の観測

$$\frac{\hbar A}{2\pi qB} = n + \frac{1}{2}$$

物理量が 1/*B* に対して振動

帯磁率　：　de Haas – van Alphen 効果

電気伝導　：　Schbnikov – de Haas 効果

振動の周期からフェルミ面の断面積 *A* が求まる

フェルミ面の形状を実験的に調べることができる

The De Haas–Van Alphen Effect in Copper

THE de Haas – van Alphen effect in copper has been observed in a whisker made by the Brenner method[1] and oriented with the [111] axis along the field (see Fig. 1); the period is about $1 \cdot 7 \times 10^{-9}$ gauss^{-1}, which is close to the value for a free electron sphere of one electron/atom and not inconsistent with the Fermi surface proposed by Pippard[2]; the effective electron mass as estimated from the temperature variation of the effect is about 1·3 times the free electron mass.

Previous negative results[3,4] may have been due to (*a*) crystal imperfections, (*b*) eddy currents associated with the impulsive field, (*c*) non-occurrence of the effect in certain crystal directions. Copper whiskers are usually very well oriented along one of the principal crystal directions and are of good quality as crystals; thus they are particularly favourable as regards (*a*), since the influence of a given degree of imperfection is least along a direction of symmetry. Moreover, a whisker provides in a natural way a very thin crystal wire and thus helps as regards (*b*); the diameter of the whisker used was about 0·2 mm. It is interesting to note that three other whiskers, two oriented along [100] and one along [110], showed no de Haas – van Alphen effect; this may well be merely because of inadequate sensitivity (in the successful observation the oscillations had only about ten times the noise amplitude), but it is possible that the magneto-resistance effect is particularly high for the [111] direction[5–7] and not for the other directions, thus making the [111] whisker particularly favourable as regards reducing eddy currents.

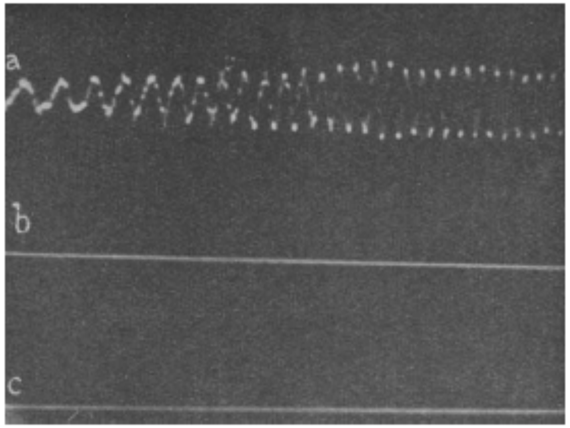


Fig. 1. Trace *a* shows the amplified output from a pick-up coil containing the [111] whisker at 1·1° K. in a magnetic field *H* of about 72,000 gauss, which drops by about 250 gauss across the picture during a time of sweep of about 0·6 msec. This drop is indicated by the slight decrease of separation between traces *b* and *c*, which is a measure of the variation of ($H - 67,000$) gauss. The shortening of the interval occupied by each successive period is due to the increasing rate of decrease of the magnetic field from left to right

The discovery of a positive effect in copper removes the suspicion that the earlier negative results in monovalent metals may have been due to fundamental rather than technical reasons, and thus opens the way to a detailed investigation of the electronic structure of these metals.

It is a pleasure to acknowledge the hospitality of the General Electric Research Laboratory, Schenectady, N.Y., during last September, which provided the opportunity of obtaining the whiskers, and I should like particularly to thank Mrs. Ethel Fontanella, of General Electric, for her assistance in preparing the whiskers for use, and Mr. Harry Davies for his assistance in the experiments.

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University of Cambridge.
Dec. 22.

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