Motion Control for Robot Manipulators with Flexible Joints

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Contents

Chapter 1

INTRODUCTION

Robotics is a relatively new field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of electrical engineering, mechanical engineering, industrial engineering, computer science, information engineering and mathematics. New disciplines of engineerings, such as manufacturing engineering, applications engineering and knowledge engineering, are beginning to emerge to deal with the complexity of the field of robotics and the larger area of factory automation.

1.1 Definition of Robots

The term robot was first introduced into our vocabulary by the Czech play writer Karel Capek in his 1920 play Rossums Universal Robots. Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, underwater vehicles, etc. Actually, anything which operates with some degree of autonomy, usually under computer control, has at some point been called a robot. An official definition of a robot comes from the Robot Institute of America (RIA): A robot is a reprogrammable multifunctional manipulator designed to move material parts, tools, or specialized devices through variable programmed motions for the performance of variety of tasks. The key element in the above definition is the reprogrammability. The so-called robotics revolution is, in fact, part of the large computer revolution.

A robot has several features that make it attractive in an industrial environment. The advantages of the introduction of robots are decreasing labor costs, increasing precision and productivity, and flexibility compared with specialized machines. And they can work in the conditions which are dull, repetitive, or hazardous for humans. It should be pointed out that the important applications of robots are by no means limited to those industrial jobs where the robots are directly replacing a human worker. There are many other applications of robots in areas where the use of human is impractical or undesirable. Among these are undersea and planetary exploration, satellite retrieval and repair, the defusing of explosive devices, and works in radioactive environments. Finally, prosthesis, such as artificial limbs, is robot device requiring method of analysis and design similar to those of industrial manipulators.

Overall robot system consists of the manipulator, external power source, end-of-arm tooling, external and internal sensors, servo, computer interface, control computer and programmed software.

1.2 Components and Structure of Robot Manipulators

In general, robot manipulators are composed of links connected by joints into an open kinematic chain. Joints are typically rotary (revolute) or linear (prismatic). A revolute joint is like a hinge and allows relative rotation between two links. A prismatic joint allows a linear relative motion between two links. We use the convention R for representing revolute joints and P for prismatic joints. The joints of a manipulator may be electrically, hydraulically or pneumatically actuated. The number of joints determines the degrees-of-freedom (DOF) of the manipulator.

1.2.1 Kinematic Structure

The manipulators are usually classified kinematically on the basis of the arms or 1st three joints. The majority of the manipulators fall into one of five geometric types: articulated (RRR), spherical (RRP), SCARA (RRP), cylindrical (RPP), or cartesian (PPP)

Articulated Configuration (RRR): The articulated manipulator is also called a revolute manipulator. A common design is the elbow type manipulator, such as PUMA.

Spherical Configuration (RRP): By replacing the third or elbow joint in the revolute configuration by a prismatic joint one obtains the spherical configuration.

SCARA Configuration (RRP): The so-called SCARA (Selective Compliant Articulated Robot for Assembly) is a popular configuration which is designed for assembly operation.

Cylindrical Configuration (RPP): The first joint is revolute while the second and third joints are prismatic. As the name suggests, the joint variables are the cylindrical coordinates of the end-effector with respect to the base.

Cartesian Configuration (PPP) A manipulator whose first three joints are prismatic is known as a cartesian manipulator.

1.2.2 Control of Robot Manipulators

We now focus on the problem of control synthesis of robot manipulators. In order to simplify the synthesis of and implementation of the control system, control is usually realized hierarchically in several levels, so that each control level solves its specific task. A higher level communicates with a lower level giving it instructions and receiving from it relevant information required for decision-making. After obtaining information from a lower level, each level makes decisions taking into account general decision obtained from a higher level and forwards them to a lower level for execution.

A hierarchical control system may have different numbers of level depending on robot type and the complexity of tasks for which the robot is intended. Here is a hierarchical control structure (as shown in Fig. 1.1), which are encountered often with three control levels. (Vukobratovic, 1989):

1. Strategical control level is the highest level which has to recognize the obstacles in the operating space and the conditions, and to plan the trajectories.

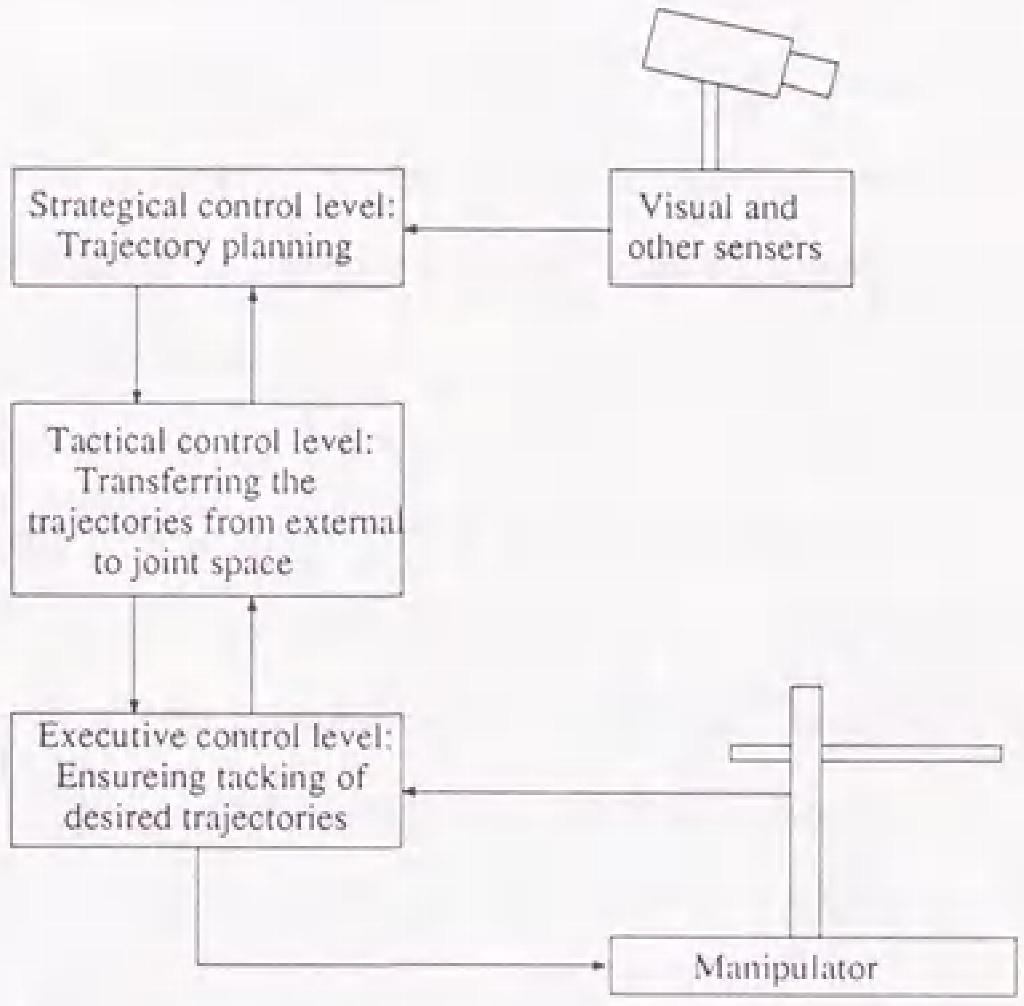


Fig. 1.1 Hierarchical structure of robot control

2. Tactical control level is the second level which has to map the trajectories from external into internal (joint) coordinates of the robot manipulator.

3. Executive control level is the lowest level which has to realize trajectories or (positions) of the robot manipulator joints.

This thesis is concerned with the problems of synthesizing the executive control levels for robot manipulators. The lowest control level should be synthesized (taking into account dynamic behavior) so as to provide the implementation of functional movement prescribed by higher levels. The executive control levels should guarantee the desired practical system stability, so as to provide the best possible system behavior through actuators.

To synthesize the simplest possible control, approximate system models are usually considered which allow the synthesis of simple and low-cost control. A linearized system model is frequently considered, on the basis of which control may easily be synthesized using the results of the extensively elaborated linear system theory. Since a chosen approximate model is assumed to be a sufficiently accurate approximation to the actual system model, it is usually considered in the literature that the control synthesized on the basis of an approximate model will also ensure the imposed requirement. However, this need not always be true since the validity of the approximate model is related to bounded regions in the system state space. It is therefore always necessary to test to what extent the control, synthesized using an approximate model, will prove satisfactory when applied to the actual system. Obviously, the closer the model to the actual system model, the higher the probability that synthesized control will satisfy the actual system as well. On the other hand, the closer the system model to the actual model, the more complex it becomes.

It is clear so far that the synthesis of the lowest control levels of robot manipulators requires careful consideration of the effect of system dynamics in order to synthesize a control law as simply and reliably as possible, which will satisfy the actual conditions of practical system stability.

The modern development of robot mechanisms is rapidly increasing. A linear composite speed of manipulator now achieves $5m/s$ and angular speed of their particular link surpasses $8rad/s$. This is not only the characteristics of robot manipulator with small reachability, but also that of robot manipulator possibility. Such an requirement leads to a significant dynamic effect in robot manipulator mechanisms, necessitating a careful study of their dynamics.

Most of the robot manipulators on the market today are not capable of ensuring precise tracking of fast trajectories, because they are just controlled by local servo-systems without introducing the compensation for dynamic forces. However, since the requirement imposed on robots concerning the speed and quality of operation (precision of tracking) is increasing, the control at the executive level must take into account the dynamics of the robot manipulator. The control which concerns all (or some) dynamic force in the robot manipulator system is called the *dynamic control* of robot manipulator. The basic problems when applying dynamic control lie in the fact that the dynamic forces acting within the robot manipulator are generally very complex function comprising coordinates, speed and acceleration of the joints. Thus, if we want the controller to compensate the effects of these forces, this usually leads to complex control law. Hereafter, the problem of the control of manipulator, in this thesis, is refer to that of dynamic control of robot manipulator in executive level.

1.3 Control of Robot Manipulator with Flexible Joints

Traditional, robot manipulators were modeled by rigid links with rigid transmission units, i.e. manipulators with rigid joints (MRJ), in order to simplify analysis and control. Many of today's robot manipulators typically with semi-rigid or flexible joints are driven by actuators. Also some robots are required to have light structures for high-speed performance and low energy consumption. In these cases, elasticity of the joints or links should be taken into consideration.

In fact, the joint flexibility can be observed, for example, in manipulator with harmonic drives; the elastic deformation for bearings and gear teeth introduces significant effect (Sweet and Good, 1984, Good *et al.* 1985). And manipulators with hydraulic actuators also exhibit joint flexibility because of the compliance in the hydraulic lines (Spong *et al.* 1987). In some applications, joint flexibility is introduced in order to comply with external objects and avoid damages due to accidental collision, as the flexible joints can absorb a certain amount of collision energy. In this thesis we explore the dynamic control of manipulators with flexible joints(MFJ).

From last decade, there have been many control schemes for MFJ proposed. They may roughly be divided into two categories. This categorization is based on the assumption related to the joint stiffness. The assumption determines the mathematic model of a MFJ.

If a MFJ has strong joint stiffness, then the order of its dynamic model may be reduced by applying the well-known singular perturbation techniques (SPT). In the singular perturbational formulation the control torques consist of a composite function which is the sum of the fast control and low control. The fast control torque is used to stabilize the fast subsystem, the motion of the actuators, while the slow control is used to stabilize the slow subsystem, the motion of the links. Several design strategies are proposed in term of representing the MFJs as the perturbation model of the MRJs. These include approximate linear feedback (Khorasani and Kokolovic, 1985, Nicosia *et al.* 1986), nonlinear dynamic feedback control (Spong 1987, Khorasani 1990) and integral manifold approach (Spong *et al.* 1987). Since by using SPT, the reduced model is rather similar to that of a MRJ, many control schemes developed for MRJs by Craig *et al.* (1986) and Slotine and Li (1987, 1989) can be modified for MFJs. For example, Campion and Bastin (1989), Spong (1989) had shown that adaptive control results for MRJs may be used to control MFJs provided some simple correction term is added to the control law to damp out the elastic oscillations at the joint. Khorasani (1992), Chang and Daniel (1992) have also shown the generalization of the adaptive inverse dynamics schemes and the generalization of dynamic control based on the method of Slotine and Li(1987). Ashoor *et al.*(1993) also gave a robust adaptive controller based on the combination of feedback linearization, singular perturbation, and recursive least-squares identification. A robust control with local stability is also proposed by Myszkorowski (1992).

However, SPT dose not necessarily apply to MFJs with arbitrary joint stiffness. A second category of controller is developed, with using a full-order dynamic model to design control schemes for general MFJs without any restriction on the joint stiffness. One example is the switching controller proposed by Mard and Ahmad (1991), which requires exact values of the inertia parameters to synthesize the control. Tomei (1991), Ailon and Ortega (1993) have respectively shown the PD controller and observer-based control are capable of stabilizing MFJs at set-point. Nicosia and Tomei (1993) also give a tracking controller for MFJs. Ramirez *et al.*(1992) derived a smooth dynamic linearization feedback controller with sliding model regulator. Adaptive or robust control schemes for MFJs with arbitrary joint stiffness are a more challenging problem. The high-order dynamics complicate the control structure and its stability analysis. Therefore, some researches have been made with the condition of known stiffness of the flexible joints. Mard and Ahmad (1992) developed a globally adaptive control law by an assumption of the bounded of the motor acceleration tracking error, and Yuan and Stepanenko (1993) derived a composite adaptive controller with asymptotic stability. An adaptive control law without *a priori* knowledge about the joint stiffness has been proposed by Lozano and Brogliato (1992), but in this method, possible singularities may be a problem in the parameter adaptation. In order to achieve the robustness to parameter uncertainty, some robust controllers have been proposed, for example, robust controllers with global feedback linearization and variable structure (Ramirez and Spong, 1988, Kwan and Yeung, 1993).

1.4 Outlines of the thesis

The aim in this thesis is to give some our researches in the advanced control methods for the MFJ. To increase the performance and accuracy of the manipulator, we have studied new adaptive controller, robust controller, non-adaptive and adaptive tracking controllers with sliding observer, and nonlinear H_∞ controller. All these methods are based on the full-order dynamic model for MFJ and have been proved to be stable by using Lyapunov stable theory. The effectiveness of these methods is checked by experimentation and simulation.

The thesis is organized as follows. After an introductory chapter, Chapter 2 gives a full-order dynamic model of manipulators with flexible joints under some reasonable assumptions. Some useful properties of the model are also given, which will be used in the design of the controllers later.

In Chapter 3, based on the model for MFJ and its linear parameterizability, a new adaptive control scheme is proposed. Under this controller law, the errors for the system are convergent to zero and all signals in the system are bounded. The robustness of the system to unmodeled dynamics has been analyzed. By some experiments in a one-link manipulator with a flexible joint, the effectiveness of the adaptive control will be shown.

In Chapter 4, a special linear parameterized representation for MFJ has been proposed. A new robust control scheme is proposed under an assumption in which the bounds of the parameter uncertainty are known *a priori*. The design is based on a lemma proposed by Qu *et al.*(1992). Under this control law, the errors for the system are also convergent to zero and all signals in the system are bounded. And the system is ultimately bounded if there exist unmodel dynamics. The control parameters are easier to be chosen than that in the adaptive control case. To show the effectiveness of the robust control, some simulations and expirements are made.

So far in many advanced control schemes for MFJ, including those proposed in Chapters 3 and 4, not only the information of motors but also those of links is used in the feedback. In Chapter 5, the non-adaptive and adaptive tracking controllers with sliding observer are given. In these controllers, only the information of the motors is used in the feedback, and the stability of the whole system, which include the manipulator, the controller and the observer, is guaranteed when the control parameters satisfy some sufficient conditions. In this chapter, the effects of noise to the controller, the observer, and the whole system has been analyzed. The effectiveness of the proposed tracking controllers has been shown by some simulations.

Based on quadratic optimal control, in Chapter 6, nonlinear H_∞ control for MFJ will be studied. A nonlinear H_∞ robust controller is proposed, in which only the state of the motor are used. The performance of the proposed controller is analyzed by some simulations. Chapter 7 will give a summary of the thesis.

Chapter 2

DYNAMIC MODEL FOR MANIPULATOR WITH FLEXIBLE JOINTS (MFJ)

2.1 Introduction

The proper choice of mathematical model for control system design is a crucial stage in the development of control strategies for any system. This is particularly true for robot manipulator due to their complicated dynamics. The problem of dynamic model for MFJ is the problem of establishing the relation between the input torques and joint/motor outputs(positions, velocities, and accelerations).

A robot manipulator can be thought of as a chain of bodies interconnected by revolute and/or prismatic joints. Each of the joint can be actuated by a motor. In most industrial applications, the links of manipulators are quite rigid and the manipulators employ DC or AC motors connected in series with high-ratio gear boxes used mainly for speed reduction and torque increase. In some applications, transmission belts or long shafts are also used in the driving system. When high performance at a high speed is required, the flexibility of joints (which is caused by the elasticity of gear teeth, shafts and so on) has to be taken into account in the modeling and the design of control laws(Sweet and Good 1984). For this reason

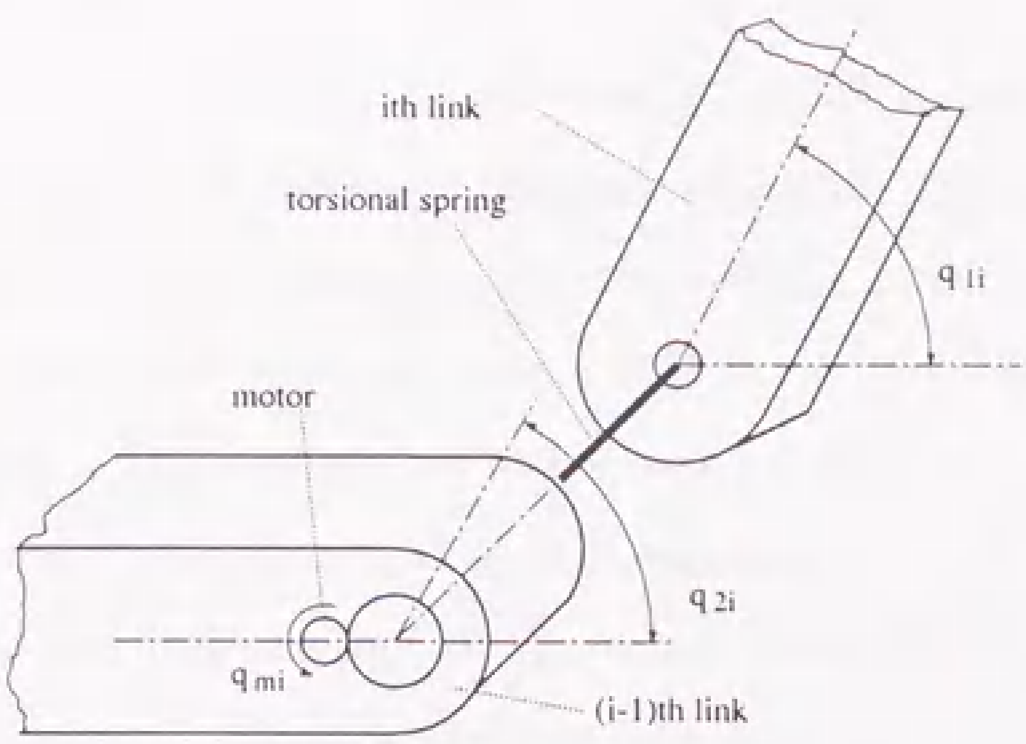


Fig. 2.1 the model of the *i*th flexible joint

we investigate the problem of modeling the dynamics of manipulators with flexible joints(MFJ). For notational simplicity we treat the case of revolute joints driven by DC-motors whose rotors are elastically coupled to the links. The objective of this chapter is to introduce Euler-Lagrange method, which can derive the dynamical equations describing the model of robot manipulators with flexible joints.

2.2 Dynamic Model for MFJ

In order to determine the dynamic model of the MFJ, we state following general assumptions which are made about the mechanical structure.

A2.1 The manipulator being studied has rigid links and interconnected by revolute flexible joints and the flexibility in each joint can be modeled as a linear torsional spring as shown in Figure 2.1

A2.2 The kinetic energy of rotor is due mainly to its own rotation. Equivalently, the motion of the motor's rotor is a pure rotation with respected to an principal inertia axis.

A2.3 The rotor/gear inertia is symmetric about the its axis of rotation.

Remark 2.2.1 By the assumption A2.1, we will obtain a dynamic model which has twice order as that of MRJ. The assumption A2.2 means that we can only consider the kinetic energy of the rotor rotating about its principal axis of rotation in modeling the energy of the motor, i.e., we neglect the kinetic energy of the rotor rotating about the other two principal axes and it is easy to justify for a large class of robot manipulators. The assumption A2.3 hardly needs any justification, which results in that the gravitational potential of the system and also the velocity of the rotor center of mass are independent of the rotor position. In fact, most existing models of MRJ are derived under the assumptions A2.2 and A2.3 (Paul 1981, Spong and Vidyasagar 1989) precisely. □

We now consider a manipulator with revolute joints actuated by DC-motors. Let $q_1 \in R^n$ represent the vector of the positions of the links, and let $q_2 \in R^n$ represent the vector of the *equivalent* motor positions (i.e. $q_{2i} = q_{mi}/G_{ri}, q_{mi}$ is the real position of the ith motor, G_{ri} is the gear ratio). The *equivalent* motor positions expresses the motor position in terms of the link position level. Then we define $q \stackrel{\text{def}}{=} [q_1^t, q_2^t]^t$. The external torques applied to the joints are represented by vector $u \in R^n$. For simplicity, we refer to the *equivalent* motor positions just as "motor position" hereafter.

Under the assumption A2.2, the total kinetic energy of the manipulator is

$$E_k = \frac{1}{2}\dot{q}_1^t M(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^t J_m \dot{q}_2, \tag{2.2.1}$$

where $M(q_1) \in R^n$ is positive symmetric inertia matrix of the "rigid" manipulator, i.e. $M(q_1) = (M_{ij}(q_1))$ which can be calculated by using standard techniques (Equation (6.66) in Paul, 1981). $J_m = diag[J_{mi}]$ is positive inertia matrix of the motor rotors and the diagonal elements are the motor inertias about their principal axes of rotation multiplied by the square of the gear ratios.

From the assumptions A2.1 and A2.3, the total potential energy of the manipulator is

$$E_p = E_{p1}(q_1) + \frac{1}{2}(q_1 - q_2)^t K_s(q_1 - q_2), \tag{2.2.2}$$

where, as in the case of the kinetic energy, the gravitational potential energy $E_{p1}(q_1)$ is found from standard formula for MRJ (e.g. Equation (6.54) in Paul, 1981). The term $\frac{1}{2}(q_1 - q_2)^t K_s(q_1 - q_2)$ is the potential energy caused by the flexibility in the joints(from A2.1). $K_s = diag[K_{si}]$ is positive stiffness matrix. The Lagrangian $L = E_k - E_p$ of the system is now given by

$$L = E_k - E_p = \frac{1}{2}\dot{q}_1^t M(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^t J_m \dot{q}_2 - E_{p1}(q_1) - \frac{1}{2}(q_1 - q_2)^t K_s(q_1 - q_2). \tag{2.2.3}$$

Following Lagrangian equation in which the dissipation is caused by the friction, we have the dynamic equations of manipulation motion as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} + \frac{\partial D_p}{\partial \dot{q}} = T, \tag{2.2.4}$$

where $T = [0, ..., 0, u_1, ..., u_n]$. $D_p = \frac{1}{2}(\dot{q}_1^t D\dot{q}_1 + \dot{q}_2^t D_m \dot{q}_2)$ is dissipation energy, $D = diag[D_i]$ and $D_m = diag[D_{mi}]$ are both positive semidefinite damping ratio matrices of the links and the motors, respectively. The diagonal elements of D_m are the motor damping ratios multiplied by the square of the gear ratios. Substituting (2.2.3) into (2.2.4) we get

$$M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + D\dot{q}_1 + K_s(q_1 - q_2) = 0 \tag{2.2.5}$$

and

$$J_m \ddot{q}_2 + D_m \dot{q}_2 + K_s(q_2 - q_1) = u, \tag{2.2.6}$$

in which

$$C(q_1, \dot{q}_1)\dot{q}_1 = \dot{M}(q_1)\dot{q}_1 - \frac{1}{2}\frac{\partial \dot{q}_1^t M(q_1)\dot{q}_1}{\partial q_1}$$

is the term containing coriolis, centripetal torques, and

$$G(q_1) = \frac{\partial E_{g1}(q_1)}{\partial q}$$

is the term containing the gravity torque acted on the manipulator.

Remark 2.2.2 (2.2.5) and (2.2.6) can also be considered as link subsystem and motor subsystem, respectively. We note that the gyroscopic forces (the inertial coupling) between each rotor and the other links are not included as a result of the assumption A2.2. In practice, the inertia coupling are much weaker than the spring torque coupling (Spong, 1987). Therefore, the dynamic model for link subsystem (2.2.5) and motor subsystem (2.2.6) are adoptable. □

Remark 2.2.3 The model given by (2.2.5) and (2.2.6) converges to the model of MRJ when the joint stiffness is infinite. In fact, when K_s tends to infinite (i.e. if the ith joint stiffness $k_{si} \to \infty$), the elastic displacement becomes to zero since $q_1 = q_2$, and the corresponding dynamics are given by

$$(M(q_1) + J_m)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + (D + D_m)\dot{q}_1 = u. \tag{2.2.7}$$

□

2.3 Properties of the Model

Although the (2.2.5) and (2.2.6) are complex, nonlinear equation for all but the simplest robot manipulators, they have several fundamental properties which can be exploited to facilitate control system design. We state these properties as follows.

P1. Null effect of fictitious forces(Spong 1989)

If the elements of $C(q_1, \dot{q}_1)$ are defined as

$$C_{kj} = \sum_{i=1}^{n} \frac{1}{2}\{\frac{\partial M_{kj}}{\partial q_{1i}} + \frac{\partial M_{ki}}{\partial q_{1j}} - \frac{\partial M_{ij}}{\partial q_{1k}}\}\dot{q}_{1i}, \tag{2.3.1}$$

then matrix $N = \dot{M} - 2C$ is skew-symmetric.

Proof: Given the inertia matrix $M(q_1)$, the kjth component of $\dot{M}(q_1)$ is given by

$$\dot{M}_{kj} = \sum_{i=1}^{n} \frac{\partial M_{kj}}{\partial q_i}\dot{q}_{1i}.$$

Therefore, the kjth component of N is given by

$$\begin{aligned} N_{kj} &= \dot{M}_{kj} - 2C_{kj} \\ &= \sum_{i=1}^{n}[\frac{\partial M_{kj}}{\partial q_i} - \{\frac{\partial M_{kj}}{\partial q_{1i}} + \frac{\partial M_{ki}}{\partial q_{1j}} - \frac{\partial M_{ij}}{\partial q_{1k}}\}]\dot{q}_{1i} \\ &= \sum_{i=1}^{n}(\frac{\partial M_{ij}}{\partial q_{1k}} - \frac{\partial M_{ki}}{\partial q_{1j}})\dot{q}_{1i}. \end{aligned}$$

The inertia matrix $M(q_1)$ is symmetric, i.e., $M_{ij} = M_{ji}$. By interchanging the indexes k and j, we can get

$$N_{kj} = -N_{jk},$$

which completes the proof. □

In the other words, for any vector $x \in R^n$, we have following relation

$$x^t(\dot{M} - 2C)x = 0. \tag{2.3.2}$$

This is an important property showing that $M(q_1)$ and $C(q_1, \dot{q}_1)$ are not independent.

P2. Linear exchangeble property of $C(q_1, \dot{q}_1)\dot{q}_1$ (Nicosia and Tomei, 1990)

If $C(q_1, \dot{q}_1)$ is defined as (2.3.1), we have

$$C(q_1, x)y = C(q_1, y)x, \tag{2.3.3}$$

where $x, y \in R^n$ are any vector.

Proof: Define $h(q_1, x, y) = C(q_1, x)y$. Since $M(q_1)$ is symmetric, we can write the ith component of $h(q_1, x, y)$ as

$$\begin{aligned} h_i(q_1, x, y) &= \sum_{j=1}^{n}[\frac{1}{2}\sum_{i=1}^{n}\{\frac{\partial M_{kj}}{\partial q_{1i}} + \frac{\partial M_{ki}}{\partial q_{1j}} - \frac{\partial M_{ij}}{\partial q_{1k}}\}x_i]y_j \\ &= \frac{1}{2}[\sum_{j=1}^{n}\sum_{i=1}^{n}\{\frac{\partial M_{kj}}{\partial q_{1i}} + \frac{\partial M_{ki}}{\partial q_{1j}} - \frac{\partial M_{ij}}{\partial q_{1k}}\}x_i y_j] \\ &= \sum_{i=1}^{n}[\frac{1}{2}\sum_{j=1}^{n}\{\frac{\partial M_{ki}}{\partial q_{1i}} + \frac{\partial M_{kj}}{\partial q_{1j}} - \frac{\partial M_{ji}}{\partial q_{1k}}\}y_j]x_i. \end{aligned}$$

By interchanging the indexes i and j, we have

$$h_i(q_1, x, y) = \sum_{j=1}^{n}[\frac{1}{2}\sum_{i=1}^{n}\{\frac{\partial M_{kj}}{\partial q_{1i}} + \frac{\partial M_{ki}}{\partial q_{1j}} - \frac{\partial M_{ij}}{\partial q_{1k}}\}y_i]x_j = h_i(q_1, y, x),$$

which completes the proof. □

This property is very useful in observation algorithms (Nicosia and Tomei, 1990). And it will be used in the design of the controllers with observer later.

P3. The passivity about the system (2.2.5) and (2.2.6) (Lozano and Brogliato, 1992)

In the case of the manipulator with rigid joints, the passivity property, i.e. passivity of the mapping: $u \rightarrow \dot{q}_1$, has been proved (Ortega and Spong, 1989). However, in the case of MFJ, the mapping: $u \rightarrow \dot{q}_1$ can not be proved to be passive but instead the mapping: $u \rightarrow \dot{q}_2$ is passive, i.e.,

$$\int_0^t u^t \dot{q}_2 dt \geq -\gamma^2 \quad T > 0, \gamma \in R. \tag{2.3.4}$$

Proof: From (2.2.4)(without considering the dissipation for simplification) we obtain

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = 0, \tag{2.3.5}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} = u. \tag{2.3.6}$$

Consider the adjoint Lagrangian function

$$L^* = p^t q - L, \tag{2.3.7}$$

where

$$p = \frac{\partial L}{\partial \dot{q}}$$

is the generalized momentum. It is well known that in the case of scleronomic system(i.e., holonomic system having a Lagrangian function independent of time) L^* is equal to the total energy in the system. From (2.3.7) we have

$$\frac{dL^*}{dt} = \frac{dp^t}{dt}\dot{q} + p^t\ddot{q} - \frac{\partial L^t}{\partial q}\dot{q} - \frac{\partial L}{\partial \dot{q}}\ddot{q}. \tag{2.3.8}$$

From (2.3.5) and (2.3.6) we obtain

$$\begin{aligned} \frac{dp^t}{dt}\dot{q} &= \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_1})^t\dot{q}_1 + \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_2})^t\dot{q}_2 \\ &= \frac{\partial L^t}{\partial q_1}\dot{q}_1 + [u + \frac{\partial L^t}{\partial q_2}]^t\dot{q}_2. \end{aligned} \tag{2.3.9}$$

Substitute it into (2.3.8), it follows that

$$\frac{dL}{dt} = \dot{q}_2^t u. \tag{2.3.10}$$

Integrating the above equation and noting that L^* is positive definite, the proof is completed. □

The passivity of the mapping: $u \rightarrow \dot{q}_1$ has been very useful in the design of controllers for MRJ (Ortega and Spong 1989, Slotine and Li 1987, 1989a, Sadagen and Horowitz 1990). Although the passivity of the mapping: $u \rightarrow \dot{q}_2$ is not directly used in the design of controllers for MFJ, it will be helpful in finding the Lyapunov function to be used in the control law synthesis.

P4. Boundedness property

For any physical parameters, there exist constants U_M, U_C and U_G, so that for any q_1 and $\dot{q}_1$ we have

$$\|M(q_1)\| \leq U_M,\ \|G(q_1)\| \leq U_G,\ \|C(q_1,\dot{q}_1)\| \leq U_C\|\dot{q}_1\|. \tag{2.3.11}$$

Matrix $M(q_1)$ and vector $G(q_1)$ are bounded by constants, since they contain trigonometric term in q_1. The centrifugal and Coriolis forces are quadratic in $\dot{q}_1$ and trigonometric in q. It should be noted that this property holds only on the case of the revolute joint manipulators.

P5. Regressor form of the link dynamics (Khosla and Kanade 1985, An *et al.* 1985, Sheu and Walker 1989)

The linear parametrizability of the dynamic model (2.2.5) enable us to write

$$\begin{aligned} M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + D\dot{q}_1 + K_s(q_1 - q_2) \\ = Y(q_1,\dot{q}_1,\ddot{q}_1)P_1 + K_s(q_1 - q_2) = 0, \end{aligned} \tag{2.3.12}$$

where $Y(q_1,\dot{q}_1,\ddot{q}_1) \in R^{n\times r}$ is regressor matrix of known function of $q_1,\dot{q}_1,\ddot{q}_1$ independent of the physical parameters of the manipulator. $P_1 \in R^r$ is a constant physical parameters vector of the link subsystem.

We define

$$T(q_1, \dot{q}_1, P_1) = \frac{1}{2}\dot{q}_1^t M(q_1)\dot{q}_1 + E_{p1}(q_1),$$

where P_1 is the inertial parameter vector, that contains all the inertial parameters of the link subsystem. So T is the sum of the kinetic and potential energy of the link subsystem. We can also get

$$T(q_1, \dot{q}_1, P_1) = E^t(q_1, \dot{q}_1)P_1,$$

$$E_{p1}(q_1, P_1) = T(q_1, 0, P_1) = E^t(q_1, 0)P_1,$$

where $E(q_1, \dot{q}_1)P$ is a nonlinear function of the q_1 and $\dot{q}_1$. The Lagrangian L_1 for the link subsystem can be defined as

$$\begin{aligned} L_1(q_1, \dot{q}_1, P_1) &= K(q_1, \dot{q}_1, P_1) - E_{p1}(q_1, P_1) \\ &= T(q_1, \dot{q}_1, P_1) - 2E_{p1}(q_1, P_1) \\ &= (E^t(q_1, \dot{q}_1) - 2E^t(q_1, 0))P_1 \\ &=: \Phi^t(q_1, \dot{q}_1)P_1. \end{aligned}$$

The Lagrangian form of the dynamic equation of motion is

$$\frac{d}{dt}\frac{\partial L_1}{\partial \dot{q}_1} - \frac{\partial L_1}{\partial q_1} + \frac{\partial D_p}{\partial \dot{q}_1} = Q,$$

where $Q = K_s(q_2 - q_1)$ is the torque acting on the link subsystem. We have

$$Q = \frac{d}{dt}(\frac{\partial \Phi}{\partial \dot{q}_1} - \frac{\partial \Phi}{\partial q_1})^t P_1 + \frac{\partial D_p}{\partial \dot{q}_1} = Y(q_1, \dot{q}_1, \ddot{q}_1)P.$$

Therefore, the Lagrangian dynamic equations are linear in the parameters. Because of this linearity, existing vector space theories are applicable to analyzing manipulator dynamics such that a more fundamental understanding of the manipulator dynamics is achieved.

2.4 Examples

Now two examples for dynamic model of MFJ, including a one-link and a two-link SCARA type manipulator with flexible joints, will be given. These two models will be used in the experimentation and simulation later.

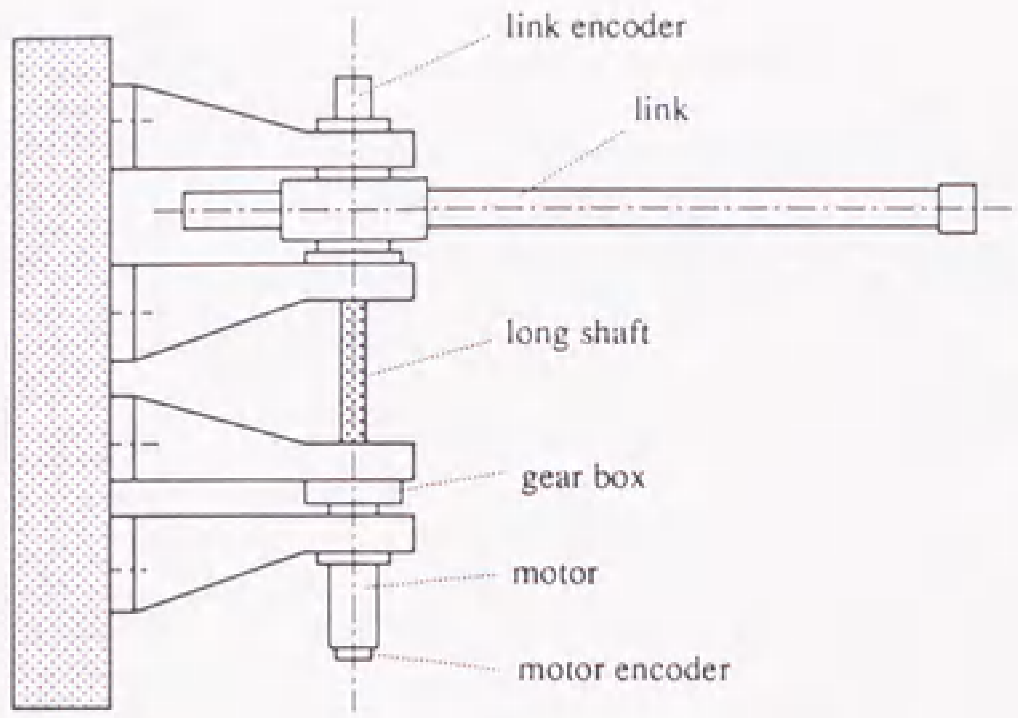


Fig. 2.2 The one-link manipulator with flexible joint

2.4.1 One-link Manipulator with Flexible Joint

Consider the one-link manipulator with flexible joint shown in Fig 2.2, consisting of a rigid link coupled through a gear box and a shaft to a DC-motor. The total kinetic energy of the manipulator is

$$E_k = \frac{1}{2}J_a\dot{q}_1^2 + \frac{1}{2}J_m\dot{q}_2^2, \tag{2.4.1}$$

where J_a and J_m are the rotational inertias of the link and motor, respectively. The potential energy is given as

$$E_p = \frac{1}{2}K_s(q_1 - q_2)^2, \tag{2.4.2}$$

where K_s is positive the constant stiffness. Therefore, the Lagrangian $L = E_k - E_p$ of the system is now given by

$$L = E_k - E_p = \frac{1}{2}(J_a\dot{q}_1^2 + J_m\dot{q}_2^2) - \frac{1}{2}K_s(q_1 - q_2)^2. \tag{2.4.3}$$

The dissipation energy D_p is given by

$$D_p = \frac{1}{2}(D_a\dot{q}_1^2 + D_m\dot{q}_2^2), \tag{2.4.4}$$

where D_a and D_m are the damping ratio of the links and the motors, respectively. Substituting (2.4.3) and (2.4.4) into the (2.2.4) yields the equation of motion

$$J_a\ddot{q}_1 + D_a\dot{q}_1 + K_s(q_1 - q_2) = 0 \tag{2.4.5}$$

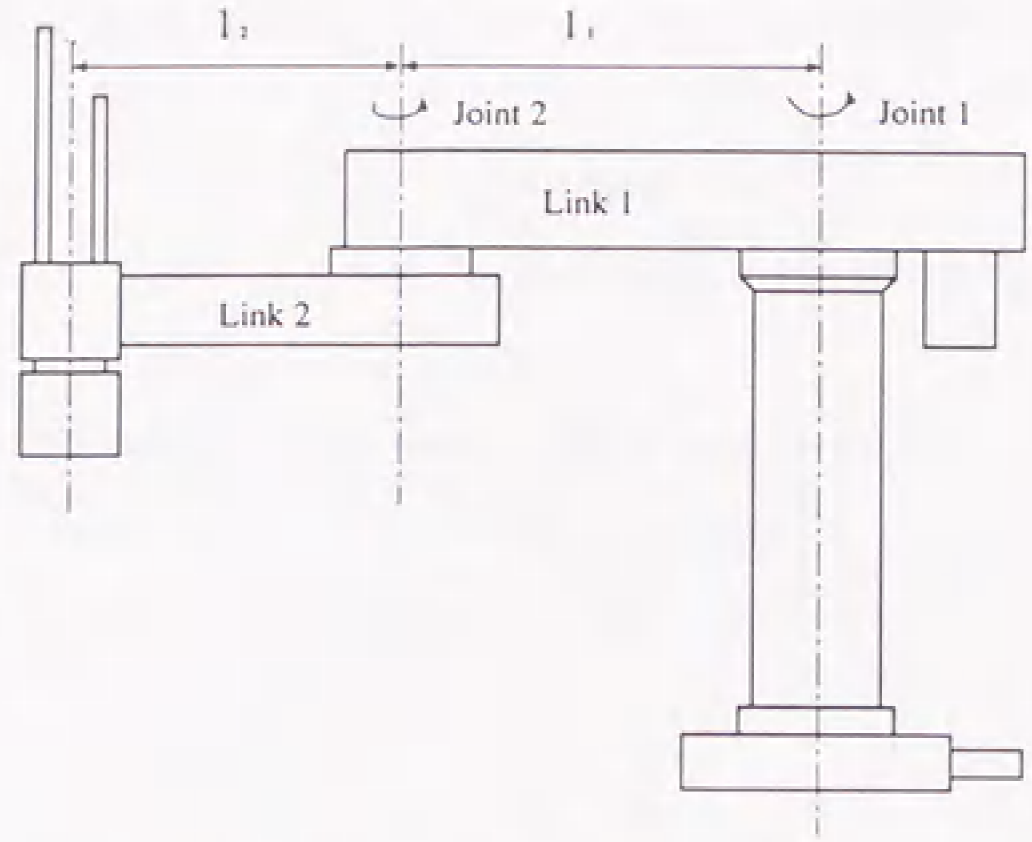


Fig. 2.3 The two-link SCARA type manipulator with flexible joints

and

$$J_m \ddot{q}_2 + D_m \dot{q}_2 + K_s(q_2 - q_1) = u. \tag{2.4.6}$$

Here, the mathematical model of the one-link manipulator with flexible joint is a couple of second order linear differential equations. The parameters for the one-link manipulator with flexible joint, which are actual parameters in our experiment device, are shown in Table 2.1.

2.4.2 Two-link SCARA type manipulator with flexible joints

Now consider the two-link SCARA type manipulator with flexible joints shown in Fig.2.3. Let us fix notation as follows: For $i = 1, 2$, m_i denotes the mass of the link i, l_i and l_{ci} denote the link length and the distance from the previous joint to the center of mass of the link i, respectively. The positions of the center of mass of the link 1 and 2 are

$$\left.\begin{array}{rcl} x_1 &=& l_{c1}cos(q_{11}) \\ y_1 &=& l_{c1}sin(q_{11}) \end{array}\right\} \tag{2.4.7}$$

and

$$\left.\begin{array}{rcl} x_2 &=& l_1 cos(q_{11}) + l_{c2}cos(q_{11} + q_{12}) \\ y_2 &=& l_1 sin(q_{11}) + l_{c2}sin(q_{11} + q_{12}) \end{array}\right\}. \tag{2.4.8}$$

Differentiating (2.4.7) and (2.4.8), the velocities of the center of mass of the link 1 and 2 are

$$\left.\begin{aligned} \dot{x}_1 &= -l_{c1}sin(q_{11})\dot{q}_{11} \\ \dot{y}_1 &= l_{c1}cos(q_{11})\dot{q}_{11} \end{aligned}\right\} \tag{2.4.9}$$

and

$$\left.\begin{aligned} \dot{x}_2 &= -l_1 sin(q_{11})\dot{q}_{11} - l_{c2}sin(q_{11}+q_{12})(\dot{q}_{11}+\dot{q}_{12}) \\ \dot{y}_2 &= l_1 cos(q_{11})\dot{q}_1 + l_{c2}cos(q_{11}+q_{12})(\dot{q}_{11}+\dot{q}_{12}) \end{aligned}\right\}. \tag{2.4.10}$$

Then the total kinetic energy of the manipulator is

$$\begin{aligned} E_k &= \frac{1}{2}\sum_{i=1}^{2}(m_i(x_i^2+y_i^2+z_i^2)+J_{mi}q_{2i}^2) \\ &= \frac{1}{2}(m_1 l_{c1}^2\dot{q}_{11}^2 + m_2(l_1^2\dot{q}_{11}^2 + l_{c2}^2(\dot{q}_{11}+\dot{q}_{12})^2 + 2l_1 l_{c2}cos(q_{12})(\dot{q}_{11}+\dot{q}_{12})) \\ &\quad + \sum_{i=1}^{2} J_{mi}q_{2i}^2). \end{aligned} \tag{2.4.11}$$

The potential energy is given as

$$E_p = \frac{1}{2}\sum_{i=1}^{2} K_{si}(q_{1i}-q_{2i})^2. \tag{2.4.12}$$

Thus, Lagrangian of the system is

$$L = E_k - E_p. \tag{2.4.13}$$

And the dissipation energy is

$$D_p = \frac{1}{2}\sum_{i=1}^{2}(D_{1i}\dot{q}_{1i} + D_{mi}\dot{q}_{2i}). \tag{2.4.14}$$

Substituting (2.4.11), (2.4.12), (2.4.13) and (2.4.14) into (2.2.4), we can obtain

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\ddot{q}_1 + \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}\dot{q}_1 + \begin{pmatrix} D_{11} & 0 \\ 0 & D_{22} \end{pmatrix}\dot{q}_1 + K_s(q_1-q_2) = 0 \tag{2.4.15}$$

and

$$\begin{pmatrix} J_{m1} & 0 \\ 0 & J_{m2} \end{pmatrix}\ddot{q}_2 + \begin{pmatrix} D_{m1} & 0 \\ 0 & D_{m2} \end{pmatrix}\dot{q}_2 + K_s(q_2-q_1) = u, \tag{2.4.16}$$

where

$$M_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c1}^2 + 2l_1 l_{c2}cos(q_{12})),$$

$$M_{12} = M_{21} = m_2(l_{c2}^2 + l_1 l_{c2} cos(q_{12})),$$

$$M_{22} = m_2 l_{c2}^2,\ C_{11} = -m_2 l_1 l_{c2} sin(q_{12})\dot{q}_{12},$$

$$C_{12} = -m_2 l_1 l_{c2} sin(q_{12})\dot{q}_{11} - m_2 l_1 l_{c2} sin(q_{12})\dot{q}_{12},$$

$$C_{21} = m_2 l_1 l_{c2} sin(q_{12})\dot{q}_{11},\ C_{22} = 0.$$

The gravity dose not affect the manipulator system since it only moves horizontally. A table of parameters for the two-link SCARA type manipulator with flexible joints, which are actual parameters in our simulation model, are shown in Figure 2.2.

2.5 Conclusion

In this chapter, we have given a simple model to represent the dynamics of the robot manipulator with flexible joints. The model gives the relation between the motions of the manipulator (including motors and links) and the applied torques. It can be discovered that the dynamic model is twice the order of the model of the same robot manipulator with rigid joints. We also have discussed some useful properties which will be used hereafter.

Parameter	*Value*	*unit*
K_s	8208.7	(Nm/rad)
J_a	0.618	(Kgm^2)
D_a	0.22	(Nms/rad)
J_m	0.45	(Kgm^2)
D_m	5.62	(Nms/rad)
G_r	53.5	

Table 2.1 The parameters of the one-link manipulator

Parameter	*Joint 1*	*Joint 2*	*unit*
m_i	14.9	12.04	kg
l_i	0.435	0.265	m
l_{ci}	0.0738	0.2115	m
D_i	3.154	3.154	Nms/rad
J_{mi}	1.8	0.45	kgm^2
D_{mi}	3.91	0.96	Nms/rad
G_{ri}	100.135	50.1775	
K_{si}	9.1×10^4	1.73×10^4	Nm/rad

Table 2.2 The parameters of the two-link SCARA type manipulator

Chapter 3

ADAPTIVE CONTROLLER FOR MFJ WITH POSITION AND VELOCITY FEEDBACK

3.1 Introduction

Advanced robotic application often requires effective controller in order to achieve accurate and fast tracking of fast desired motion. If the parameters of a manipulator are known *a priori*, many control approaches can be used for this purpose, and theoretically guarantee exact tracking. These approaches are based on nonlinear dynamic feedback linearization(Luca *et al.* 1985), pseudo-linearization technique (Nicosia *et al.* 1986), dynamic feedback with sliding mode regulators(Ramirez *et al.* 1992). Singular perturbation methods is used, and a velocity control law is designed to stabilize the fast modes of the system(Readman and Belanger 1992). A global nonlinear tracking control has been introduced by Nicosia and Tomei(1993).

If the physical parameters are not exactly known, however, we have to consider some advanced control schemes, e.g., adaptive controls. The problem of designing adaptive control law for manipulator, that ensures asymptotic trajectory tracking with boundedness of all internal signals, has been intensely studied. Several schemes are proposed by Slotine and Li(1987) and Ortega and Spong(1989). But they are suitable only to the robot manipulators with rigid joints. Recent years some researches about the control for robot manipulators with flexible joints have been done. Under the assumption that the joint stiffness is relatively large compared to the other parameters, some adaptive control laws have been studied by using singular perturbational analysis (Spong 1989, Ghorbel and Spong 1989, Chang and Daniel 1992). With arbitrary joint stiffness, Mard and Ahmad(1992), Yuan and Stepanenko (1993) also proposed some globally adaptive control law under the assumption that the joint stiffness is known. Adaptive indepentent joint controls has been also proposed by Seraji (1989) and Yang *et al.* (1992).

Here, an adaptive control scheme without the assumptions on the joint stiffness will be proposed. In the same way as Mard and Ahmad(1992), we also assume here that the link angles and velocities are measurable besides the motor angles and velocities. As the results, our control scheme guarantees that the tracking error converges to zero and that all the signals of the system remain bounded (Yang *et al.* 1995a).

3.2 Formulation of the Model, Some Assumptions and Notations

The model of the manipulators with flexible joints, which has been given in Chapter 2 (2.2.5) and (2.2.6), can be formulated as

$$M_1(q)\ddot{q} + C_1(q,\dot{q})\dot{q} + G_1(q) + D_1\dot{q} + Kq = T, \tag{3.2.1}$$

where $T = [0, u^t]^t$, $q = [q_1^t, q_2^t]^t$. $M_1(q)$, $C_1(q,\dot{q})$, $G_1(q)$, K and D_1 are structured as follows:

$$M_1(q) = \begin{pmatrix} M(q_1) & 0 \\ 0 & J_m \end{pmatrix}; C_1(q,\dot{q}) = \begin{pmatrix} C(q_1,\dot{q}_1) & 0 \\ 0 & 0 \end{pmatrix};$$

$$K = \begin{pmatrix} K_s & -K_s \\ -K_s & K_s \end{pmatrix}; G_1(q) = \begin{pmatrix} G(q_1) \\ 0 \end{pmatrix}; D_1 = \begin{pmatrix} D & 0 \\ 0 & D_m \end{pmatrix}.$$

In the design of the adaptive controller, this description of the model will be used. (3.2.1) is the mixed form of model for MFJ. It is easy to prove that the properties of null effect of fictitious force(P1), boundedness(P4) and parameter linearization(P5) hold. Before proceeding design of adaptive controller, we will have to make the following assumptions.

A3.1 The measurement of link position q_1 and velocity $\dot{q}_1$ and the motor position q_2 and velocity $\dot{q}_2$ are available.

A3.2 The desired link trajectory $q_{1d}(t) \in R^{n\times 1}$ is four times differentiable.

A3.3 The physical parameters of the manipulator are unknown, but they are constants.

We define the following notations:

$$\left.\begin{array}{rcl} e & \stackrel{\text{def}}{=} & q - q_d \\ \dot{q}_r & \stackrel{\text{def}}{=} & \dot{q}_d + \Lambda e \\ s & \stackrel{\text{def}}{=} & \dot{q}_r - \dot{q} \\ e_p & \stackrel{\text{def}}{=} & P - \hat{P} \end{array}\right\}, \tag{3.2.2}$$

where $q_d = [q_{d1}^t, q_{d2}^t]^t \in R^{2n}$, $e = [e_1^t, e_2^t]^t \in R^{2n}$, $q_r = [q_{r1}^t, q_{r2}^t]^t \in R^{2n}$, $s = [s_1^t, s_2^t]^t \in R^{2n}$. $e_1 \in R^n$ and $e_2 \in R^n$ are the errors for the link and motor, respectively. e_p is the parameters estimated error. By the linear parametrizability, we also define

$$Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)P \stackrel{\text{def}}{=} M_1(q)\ddot{q}_r + C_1(q, \dot{q})\dot{q}_r + G_1(q) + D_1\dot{q}_r + Kq_d, \tag{3.2.3}$$

where $Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) \in R^{2n\times r}$, and for simplification it can be rewritten as

$$Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) = \begin{bmatrix} Y_{d1}(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{d2}) \\ Y_{d2}(q_2, \dot{q}_2, q_{2d}, \dot{q}_{2d}, \ddot{q}_{2d}, q_{1d}) \end{bmatrix},$$

where Y_{d1} and $Y_{d2} \in R^{n\times r}$. Note that $Y_{d1}P$ is independent of $\dot{q}_{2d}, \ddot{q}_{2d}$ and $Y_{d2}P$ is also independent of $\dot{q}_{1d}, \ddot{q}_{1d}$.

3.3 Adaptive Control Law Design

The controller design problem is as follows: given any desired link trajectory $q_{1d}(t)$ as described in A3.2, derive a control law for the motor input torques and estimation law for the unknown parameters such that the manipulator output $q(t)$ tracks the desired trajectory.

Now we give the adaptive controller:

$$0 = Y_{d1}\hat{P} - K_{D1}s_1, \tag{3.3.1}$$

$$u = Y_{d2}\hat{P} - K_{D2}s_2 - \varepsilon(s_2,t)(s_1^t K_{D1}s_1), \tag{3.3.2}$$

where the superscript " ^ " expresses estimated value. The estimation law for the parameters is presented as follows

$$\dot{\hat{P}} = -M_p^{-1}Y_d^t s, \tag{3.3.3}$$

where

$$K_D = \begin{pmatrix} K_{D1} & 0 \\ 0 & K_{D2} \end{pmatrix}; \ \varepsilon(s_2,t) = \frac{s_2}{\|s_2\| + \lambda_1 e^{-\lambda_2 t}}.$$

K_{D1}, K_{D2} are $n \times n$ positive definite diagonal gain matrixes. λ_1 and λ_2 are positive real numbers. The motor's desired trajectory q_{2d} can be calculated by (3.3.1). M_p is a constant positive definite adaptation gain matrix.

The following theorem summarizes the stability property of the system under the adaptive control law (3.3.1),(3.3.2) and (3.3.3).

Theorem 3.3.1 *Consider the manipulator dynamics (3.2.1) under the adaptive control law (3.3.1),(3.3.2) and (3.3.3). Then the tracking error e and $\dot{e}$ converge to zero as $t \to \infty$ and all the signals in the system remain bounded.*

Proof: We substitute (3.3.1) and (3.3.2) into (3.2.1), we obtain the following error dynamics:

$$M_1\dot{s} + C_1 s = -(D_1 + K_D)s - Ke - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1} s_1) + Y_d e_p. \qquad (3.3.4)$$

Select Lyapunov function candidate as

$$V = \frac{1}{2}s^t M_1 s + \frac{1}{2}e^t K e + \frac{1}{2}e_p^t M e_p. \qquad (3.3.5)$$

Differentiating (3.3.5) along (3.2.1), noting $s^t(\dot{M}_1 - 2C_1)s = 0$ (the property P1 in Chapter 2), we can get

$$\dot{V} = s^t(M_1\dot{s} + C_1 s) + e^t K\dot{e} + e_p^t M_p \dot{e}_p. \qquad (3.3.6)$$

Substituting the error dynamics (3.3.4) into (3.3.6), and setting the parameter estimation law as (3.3.3) which result in

$$s^t Y_d^t e_p + e_p^t M_p \dot{e}_p = 0, \qquad (3.3.7)$$

we have

$$\begin{aligned} \dot{V} &= -s^t(K_D + D_1)s - e^t\Lambda K e - s^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1} s_1) \\ &\leq -s^t(K_D + D_1)s - e^t\Lambda K e. \end{aligned} \qquad (3.3.8)$$

Since $(K_D + D_1)$ and K are positive definite, we have $\dot{V} \leq 0$. This indicate that s and e converge to zero as time goes to infinity and the parameters estimated error e_p is bounded. In the calculation of control input u (3.3.2), we need q_{2d}, $\dot{q}_{2d}$ and $\ddot{q}_{2d}$ which are bounded (shown in Remark 3.3.2). We conclude that the tracking errors(e and $\dot{e}$) converge to zero as time goes to infinity and all the signals in the system remain bounded. □

Remark 3.3.1 In practice, we may simplify the algorithm by not estimating *a priori* known parameters (for example, the inertia and damp ratio of the motors can be known by their specifications). We can decompose the parameters P into

$[P_u^t, P_k^t]^t$: $P_u \in R^a(\ a \le r)$ are the unknown parameters; $P_k \in R^{(r-a)}$ are known. Correspondingly, $Y_d = [Y_{du}^t, Y_{dk}^t]^t$. The control law (3.3.1), (3.3.2) and (3.3.3) can be written as

$$
\begin{aligned}
0 &= Y_{du1}\hat{P}_u + Y_{dk1}P_k - K_{D1}s_1, \\
u &= Y_{du2}\hat{P}_u + Y_{du1}P_k - K_{D2}s_2 - \varepsilon(s_2,t)(s_1^t K_{D1}s_1), \\
\dot{\hat{P}}_u &= M_{pu}^{-1}Y_{du}^t s.
\end{aligned}
$$

With e_{p_u} and M_{pu} in the place of e_p and M_p in the Lyapunov function (3.3.5), we obtain

$$\dot{V} \le -s^t(K_D + D_1)s - e^t \Lambda K e.$$

We can conclude that Theorem 3.3.1 also holds in this case. □

Remark 3.3.2 Now we describe the property of the boundedness of q_{2d},$\dot{q}_{2d}$ and $\ddot{q}_{2d}$. The control input (3.3.2) involves q_{2d},$\dot{q}_{2d}$ and $\ddot{q}_{2d}$ which can be calculated by (3.3.1). From (3.3.5) and (3.3.8),we know that e, $\dot{e}$ and e_p is bounded. (3.3.1) can be written as

$$q_{2d} = q_{1d} + \hat{K}_1^{-1}(\tilde{Y}_{d1}\hat{\tilde{P}} + K_{D1}s_1), \tag{3.3.9}$$

where $\hat{\tilde{P}}$ is defined as one part of $\hat{P}$ except for the parameter of the stiffness. As one part of Y_{d1}, $\tilde{Y}_{d1}$ is corresponding to $\hat{\tilde{P}}$. From the estimation law (3.3.3) and (3.2.3), $\tilde{Y}_{d1}\hat{\tilde{P}}$ is the bounded function of q_{1d},$\dot{q}_{1d}$,$\ddot{q}_{1d}$,q_1 and $\dot{q}_1$. Then q_{2d} and q_2 are bounded. By using (2.2.5) and the property P4 in Chapter 2, we know that $\ddot{q}_1$ exists and is bounded. Differentiating (3.3.9), we can get

$$\dot{q}_{2d} = \dot{q}_{1d} + \hat{K}_1^{-1}(\dot{\tilde{Y}}_{d1}\hat{\tilde{P}} + \tilde{Y}_{d1}\dot{\hat{\tilde{P}}} + K_{D1}\dot{s}_1 + \dot{\hat{K}}_1(q_{1d} - q_{2d})).$$

For $\dot{\tilde{Y}}_{d1}\hat{\tilde{P}}$, $\tilde{Y}_{d1}\dot{\hat{\tilde{P}}}$ are the bounded functions of $q_1, \dot{q}_1, \ddot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}$ and $q_{1d}^{(3)}$, and $\dot{\hat{K}}_1$ is bounded. Therefore $\dot{q}_{2d}$ and $\dot{q}_2$ are also bounded. The same procedure can be repeated for $q_1^{(3)}$and$\ddot{q}_{2d}$ by differentiating (2.2.5) and (3.3.9). Therefore the control law (3.3.1), (3.3.2) and (3.3.3) involve the measurement of q_1, $\dot{q}_1, q_2$ and $\dot{q}_2$ and calculated $q_{2d}, \dot{q}_{2d}$, $\ddot{q}_{2d}$ which are all bounded. We will show later the calculation of $\dot{q}_{2d}, \ddot{q}_{2d}$ to an one-link manipulator with a flexible joint. □

Remark 3.3.3 Without the term $\varepsilon(s_2, t)(s_1^t K_{D1} s_1)$ in (3.3.2), we can also prove the stability of the system as done in the proof of Theorem 3.3.1. To increase the effect of the link signals feedback, adding a term $\varepsilon(s_2, t)(s_1^t K_{D1} s_1)$, We can get a better performance by a suitable choice of λ_1 and λ_2. □

3.4 Robustness Respect to the Unmodeled Dynamics

In practice, there are some unmodeled effects in the manipulator system, for example, the nonlinear effects of flexible joints and friction, etc. In the flexible joints, the stiffnesses of joints are variable rather than constant even though they can be approximated to a linear spring with small errors. Friction has a significant effect on the performance of the motion of the manipulator. The exact description of the nonlinear friction is very difficult. Considering the nonlinear effects of the flexibility, friction, and other unmodeled dynamics, we assume that the dynamic model (3.2.1) can be rewritten as

$$M_1(q)\ddot{q} + C_1(q, \dot{q})\dot{q} + G_1(q) + Kq + D_1\dot{q} = T + T_d, \tag{3.4.1}$$

where $T_d \in R^{2n}$ is the vector of unmodeled effects presenting friction, torque disturbance, etc. In variable structure system, the unmodeled dynamics is usually assumed to be bounded. For example, the colomb friction may be modeled as $F_c sgn(\dot{q})$. Therefore, the unmodeled effect T_d is bounded as (Rohrs *et al.*, 1985)

$$||T_d|| \leq d_0 + d_1||\dot{e}|| + d_2||e||, \tag{3.4.2}$$

with some positive constants d_0, d_1 and d_2.

The following theorem gives the stability of the system controlled by the adaptive control law (3.3.1),(3.3.2) and (3.3.3) under the unmodeled effect T_d.

Theorem 3.4.1 *Consider the manipulator dynamics (3.4.1) under the adaptive control law (3.3.1),(3.3.2) and (3.3.3). Then the tracking error e and $\dot{e}$ are ultimately bounded and all the signals in the system also remain bounded.*

proof: Substituting (3.3.1) and (3.3.2) into (3.4.1), we have the following error dynamics:

$$M_1\dot{s} + C_1 s = -(K_D + D_1)s - Ke - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2,t)(s_1^t K_{D1} s_1) + Y_d e_p - \tau_d. \quad (3.4.3)$$

For simplification, we choose Λ as diagonal matrix. We select Lyapunov function candidate as

$$V_2 = \frac{1}{2}s^t M_1 s + \frac{1}{2}e^t K e + \frac{1}{2}e_p^t M e_p + e^t\Lambda(K_D + D_1)e. \quad (3.4.4)$$

Differentiating (3.4.4) we get

$$\dot{V}_2 = s^t M_1 \dot{s} + \frac{1}{2}s^t \dot{B} s + e^t K \dot{e} + e_p^t M \dot{e}_p + 2e^t\Lambda(K_D + D_1)\dot{e}. \quad (3.4.5)$$

As the same way, simplifying $\dot{V}_2$ along (3.4.3) gives

$$\begin{aligned}\dot{V}_2 &= -\dot{e}^t(K_D + D_1)\dot{e} - e^t(\Lambda K + \Lambda^2(K_D + D_1))e - s^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2,t)(s_1^t K_{D1} s_1) \\ &\quad + s^t Y_d e_p + e_p^t M \dot{e}_p - s^t \tau_d. \quad (3.4.6)\end{aligned}$$

From (3.4.2) and (3.2.2), we have

$$\begin{aligned}\|s^t\tau_d\| &\le (\|\dot{e}\| + \|\Lambda\|\|e\|)(d_0 + d_1\|\dot{e}\| + d_2\|e\|) \\ &= d_0\|\dot{e}\| + d_1\|\dot{e}\|^2 + c_0\|e\| + c_1\|e\|^2 + c_2\|e\|\|\dot{e}\|, \quad (3.4.7)\end{aligned}$$

where $c_0 = d_0\|\Lambda\|, c_1 = d_2\|\Lambda\|$ and $c_2 = d_2 + d_1\|\Lambda\|$. Using the same estimation law for parameters (3.3.3), we obtain

$$\begin{aligned}\dot{V}_2 &\leq -\dot{e}^t K_D \dot{e} - e^t \Lambda^2 K_D e + \|s^t \tau_d\| \\ &\leq -a_1 \|\dot{e}\|^2 - a_2 \|e\|^2 + c_2 \|e\| \|\dot{e}\| + d_0 \|\dot{e}\| + c_0 \|e\|, \end{aligned} \tag{3.4.8}$$

where $a_1 = \lambda_k - d_1, a_2 = \lambda_k \lambda_\Lambda^2 - c_1$ and $\lambda_k, \lambda_\Lambda$ are the smallest eigenvalue of matrices of K_D and Λ. A further manipulation of (3.4.8) results

$$\begin{aligned}\dot{V}_2 &\leq -\frac{c_2}{2}(\|\dot{e}\| - \|e\|)^2 - p_1(\|\dot{e}\| - d_0/(2p_1))^2 - p_2(\|e\| - c_0/(2p_2))^2 + \lambda \\ &\leq -p_1(\|\dot{e}\| - d_0/(2p_1))^2 - p_2(\|e\| - c_0/(2p_2))^2 + \lambda, \end{aligned} \tag{3.4.9}$$

where

$$\begin{aligned} p_1 &= \lambda_k - d_1 - (d_2 + d_1\|\Lambda\|)/2, \\ p_2 &= \lambda_k \lambda_\Lambda^2 - d_1\|\Lambda\| - (d_2 + d_1\|\Lambda\|)/2, \\ \lambda &= (d_0^2/p_1 + c_0^2\|\Lambda\|^2/p_2)/4. \end{aligned}$$

If the smallest eigenvalue of the control gain matrix K_D is sufficiently large, we have $p_1 \geq 0$ and $p_2 \geq 0$. From (3.4.9), we can define an ellipse in a two-dimensional Euclidian space $\|\dot{e}\| - \|e\|$, i.e.,

$$p_1(\|\dot{e}\| - d_0/(2p_1))^2 + p_2(\|e\| - c_0/(2p_2))^2 - \lambda = 0, \tag{3.4.10}$$

within which the set $\dot{V}_2 \geq 0$. In other words, $\dot{V}_2 < 0$ whenever the norms of the tracking errors $(\|e\|, \|\dot{e}\|)$ is outside this ellipse. From the above discussion, the tracking errors of the system, under the unmodeled effect τ_d, are determined by the control gain matrix K_D and the τ_d's bounds, d_0, d_1 and d_2. □

Remark 3.4.1 The size of the ellipse (3.4.10) is determined by λ which can be made arbitrarily small by increasing the control gain λ_k. In fact, since sufficiently large control gain may not be available, only a minimum size of ellipse can be limited. □

3.5 Experimental results for the proposed adaptive controller

We will analyze the performance of the adaptive control scheme by the experiments using an one-link manipulator with a flexible joint(as shown in Fig.2.2). The dynamic model of the manipulator is given by (2.2.5) and (2.2.6). The description of the form (3.2.1) is as follows

$$\begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix}\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix}+\begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}+\begin{pmatrix} K_s & -K_s \\ -K_s & K_s \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}=\begin{pmatrix} 0 \\ u \end{pmatrix},$$

where J_1, J_2 are the inertias, and D_1, D_2 are the damping ratio of the link and motor, K_s is the stiffness of the joint, $q_2 = \theta_m/G_r(\theta_m$ is the real position of the motor) and G_r is the gear ratio of the gear box. The linear parametrized expression in terms of a suitably selected set of parameters is obtained as

$$Y(q,\dot{q},\ddot{q})P = \begin{pmatrix} -v & \ddot{q}_1 & \dot{q}_1 & 0 & 0 \\ v & 0 & 0 & \ddot{q}_2 & \dot{q}_2 \end{pmatrix}\begin{pmatrix} K_s \\ J_1 \\ D_1 \\ J_2 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix},$$

where $v = (q_2 - q_1)$. By the definition Eq.(3.2.3), we have

$$Y_d(q,\dot{q},q_d,\dot{q}_r,\ddot{q}_r)P = \begin{pmatrix} Y_{d1} \\ Y_{d2} \end{pmatrix} P = \begin{pmatrix} -v_d & \ddot{q}_{r1} & \dot{q}_{r1} & 0 & 0 \\ v_d & 0 & 0 & \ddot{q}_{r2} & \dot{q}_{r2} \end{pmatrix}\begin{pmatrix} K_s \\ J_1 \\ D_1 \\ J_2 \\ D_2 \end{pmatrix},$$

where $v_d = (q_{2d} - q_{1d})$. As described in Section 3.3, q_{2d} can be calculated by (3.3.9), i.e.,

$$q_{d2} = q_{d1} + \hat{K}_s^{-1}(\ddot{q}_{r1}\hat{J}_1 + \dot{q}_{r1}\hat{D}_1 + K_{D1}s_1).$$

Differentiating above equation, we can get

$$
\begin{aligned}
\dot{q}_{2d} &= \dot{q}_{d1} + \hat{K}_s^{-1}(q_{r1}^{(3)}\hat{J}_1 + \ddot{q}_{r1}\hat{D}_1 + \ddot{q}_{r1}\dot{\hat{J}}_1 + \dot{q}_{r1}\dot{\hat{D}}_1 + \dot{\hat{K}}_s(q_{1d} - q_{2d}) + K_{D1}\dot{s}_1), \\
\ddot{q}_{2d} &= \ddot{q}_{d1} + \hat{K}_s^{-1}(q_{r1}^{(4)}\hat{J}_1 + q_{r1}^{(3)}\hat{D}_1 + 2q_{r1}^{(3)}\dot{\hat{J}}_1 + 2\ddot{q}_{r1}\dot{\hat{D}}_1 + \ddot{q}_{r1}\ddot{\hat{J}}_1 + \dot{q}_{r1}\ddot{\hat{D}}_1 \\
&\quad + 2\dot{\hat{K}}_s(\dot{q}_{1d} - \dot{q}_{2d}) + \ddot{\hat{K}}_s(q_{1d} - q_{2d}) + K_{D1}\ddot{s}_1).
\end{aligned}
$$

In the calculation of $\dot{q}_{2d}$ and $\ddot{q}_{2d}$, we have to use $\ddot{q}_1$ and $q_1^{(3)}$ which are bounded as mentioned in Remark 3.3.2. To avoid the complication for calculating $\dot{q}_{2d}$ and $\ddot{q}_{2d}$, we use a digital differentiation in the experiments.

For simplification in adaptive control as mentioned in Remark 3.3.1, we assume that $P_k = [J_2, D_2]^t$ is the known parameter group which can be obtained from the motor specification, and that $P_u = [K_s, J_1, D_1]^t$ is the unknown parameter group. The actual parameters of the manipulator are shown in Table 2.1, which are from the motor specification and identified off-line.

In order to analyze the performance of our control scheme, the speed pattern of the desired trajectory is chosen as:

$$\ddot{q}_{1d} = -140t^3(t - T)^3/T_0^3, \quad 0 \le t \le T_0.$$

The accelerating time T_0 is 0.08 sec (the desired trajectory is shown in Fig.3.1). The purpose of the experiments is to demonstrate the stability and the performance characteristics inferred from the theoretical development.

Experiment 3.1. We use the control law with the adaptation mechanism switched off (i.e., the adaptation gain matrix $M_u^{-1} = 0$). The estimated parameters are set on their actual values (i.e., $\hat{P}(0) = P$, the non-adaptive controller-1). From the control result (shown in Fig.3.2), we can see that the angle of the link follows the desired trajectory quite well with a small vibration.

Experiment 3.2. We also use the control law with the adaptation mechanism switched off. But the estimated parameters are not set on their actual values (the non-adaptive controller-2), i.e., the estimated parameters are set as

$$\hat{P}_u(0) = [25000.0, 0.0, 0.0]^t.$$

It is reasonable that $\hat{J}_1(0)$ and $\hat{D}_1(0)$ are set to zeros and stiffness of joint $\hat{K}_s(0)$ is set to large value (almost same with the rigid joint), for assuming that the parameters are unknown. In the non-adaptive controller-2 (i.e., $\hat{P} \neq P$) the tracking error and vibration become large considerably (shown in Fig.3.3).

Experiment 3.3. We use the control law with the adaptation mechanism switch on (i.e., the adaptive controller). The initial estimated parameters $\hat{P}(0)$ are set to the same values as that in the experiment 3.2. For the adaptive controller, the adaptation gain matrix M_u, the constant gain matrixes Λ and K_D are chosen as diagonal (i.e., $M_{pu}^{-1} = diag(0.5, 0.3, 0.3)$, $\Lambda = 10I$,$K_D = 40I$).

Comparing with the non-adaptive controller-1 and 2 , from the result of the adaptive controller (shown in Fig.3.4), we know that:

1. At the initial, the tracking error is almost similar to that of the non-adaptive controller-2. However, it becomes smaller with the parameter adaptation driven by the tracking error, which are almost similar to that of the non-adaptive controller-1 finally.

2. Because of the adaptive control, the vibration of the link is quite small even though parameter uncertainty exists at the initial.

3. As expected, the estimated parameters in the adaptive controller are driven by tracking error and the parameter estimated errors are bounded. The changes of estimated parameters (shown in Fig.3.4) are seen to be very smooth, as expected from the integrator structure of the adaptive law. This smoothness is desirable for avoiding that adaptation excites the vibration modes of the manipulator with flexible joint.

4. The adaptive control law derived in Section 3.3 predicts that the tracking errors of the system converge to zero, while actually the tracking errors are not exactly converge to zero as seen in the experimental results. This different phenomenon is due to the unmodeled disturbance which is inherent in the experiments but ignored in the theoretical analysis. As analyzed in Section 3.4, the tracking

errors are determined by the control gain and the bounds of the unmodeled disturbance.

3.6 Conclusion

We proposed here an adaptive control scheme applicable to the robot manipulators with flexible joints. The asymptotic stability is ensured when all the physical parameters of the system are unknown. The link position and velocity tracking errors have been shown to converge to zero theoretically and all the signals in the system remain bounded. The robustness of the control system was analyzed under the presence of the bounded unmodeled disturbance. The experimental results have shown the effectiveness of the proposed adaptive control scheme.

The proposed controller used not only the information of the motors but also that of the links. However, there are some practical cases where the link information is not available. Therefore, it is necessary to study new control schemes which use only the information on the motors.

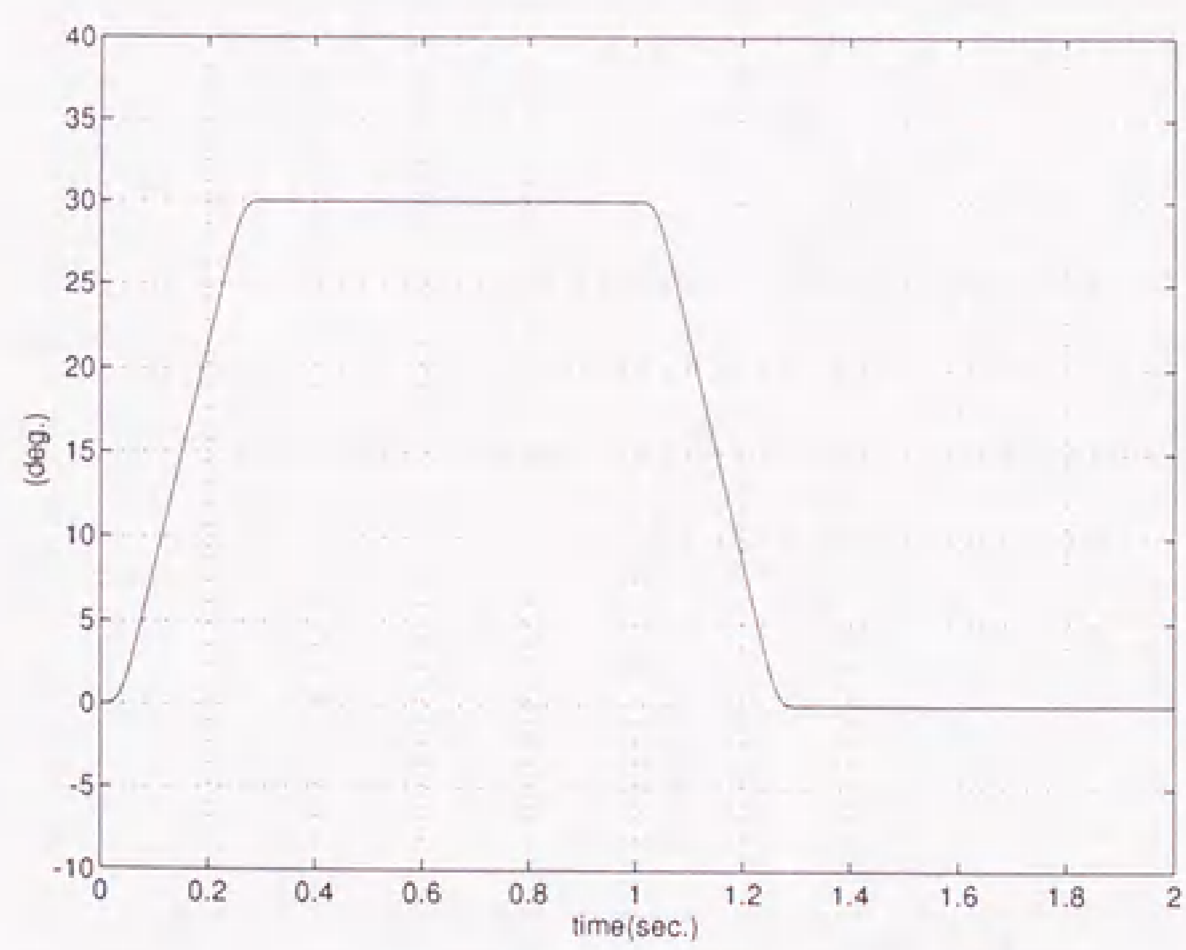


Fig. 3.1 The desired joint trajectories

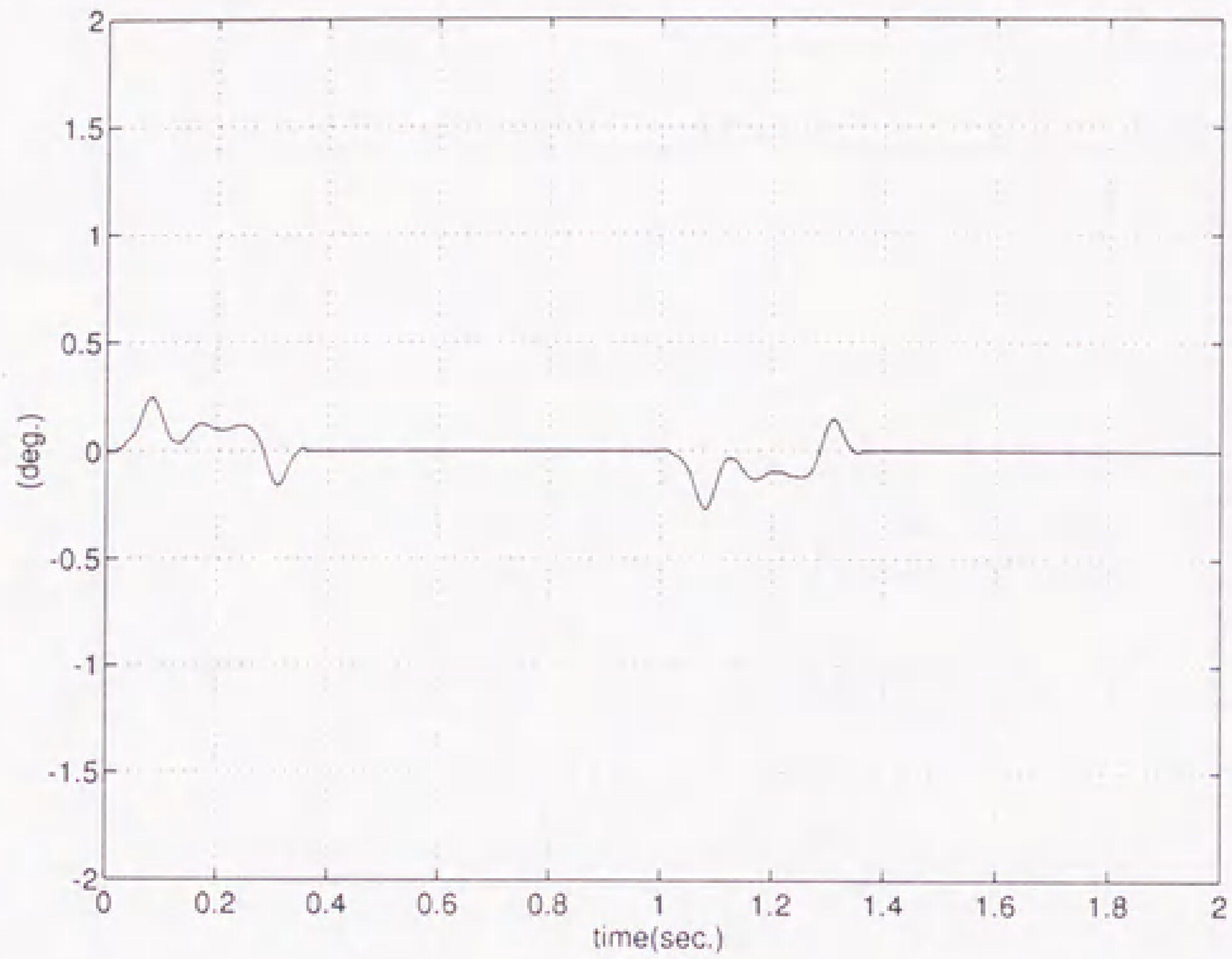


Fig. 3.2 The experimental result of the nonadaptive controller-1

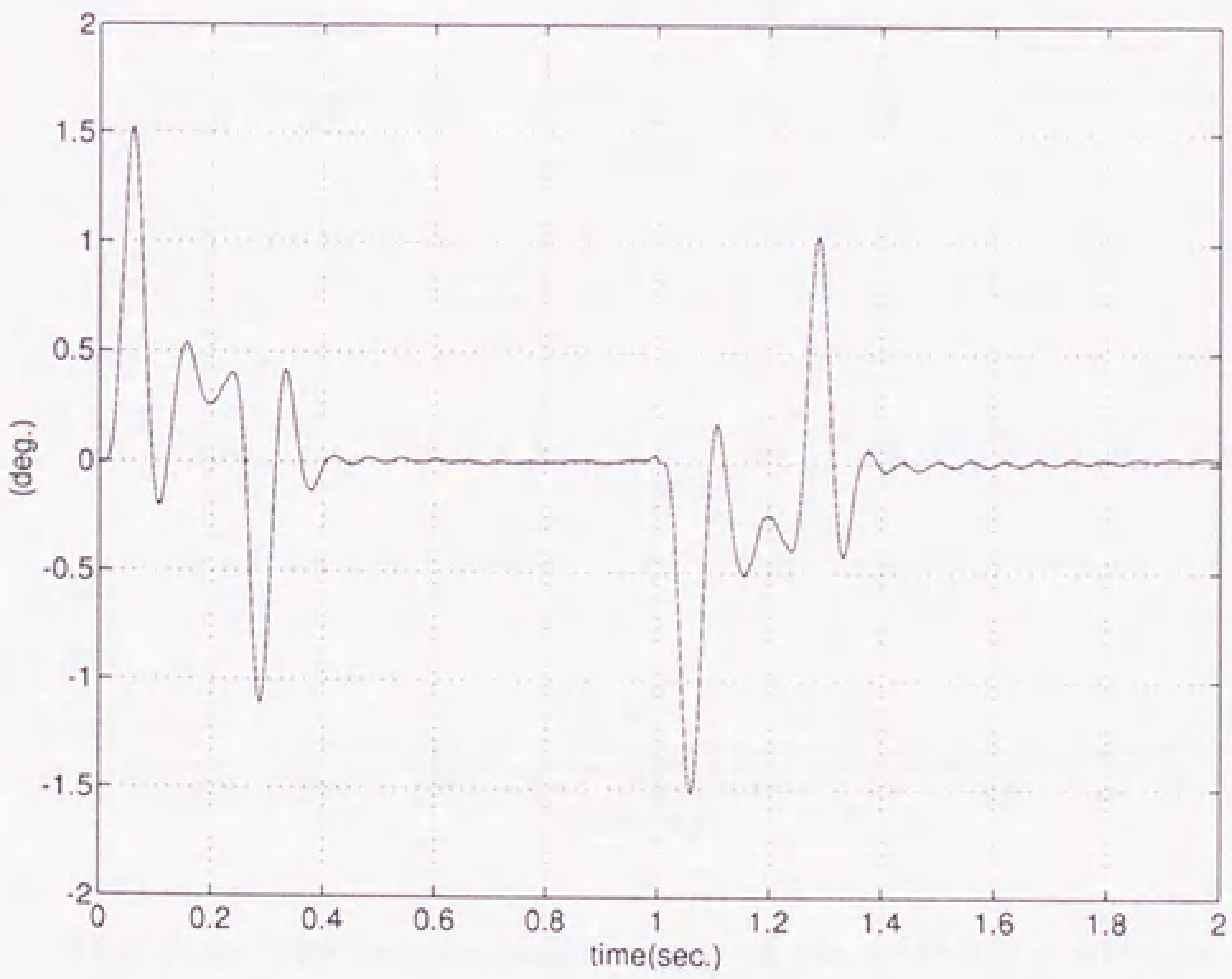


Fig. 3.3 The experimental result of the nonadaptive controller-2

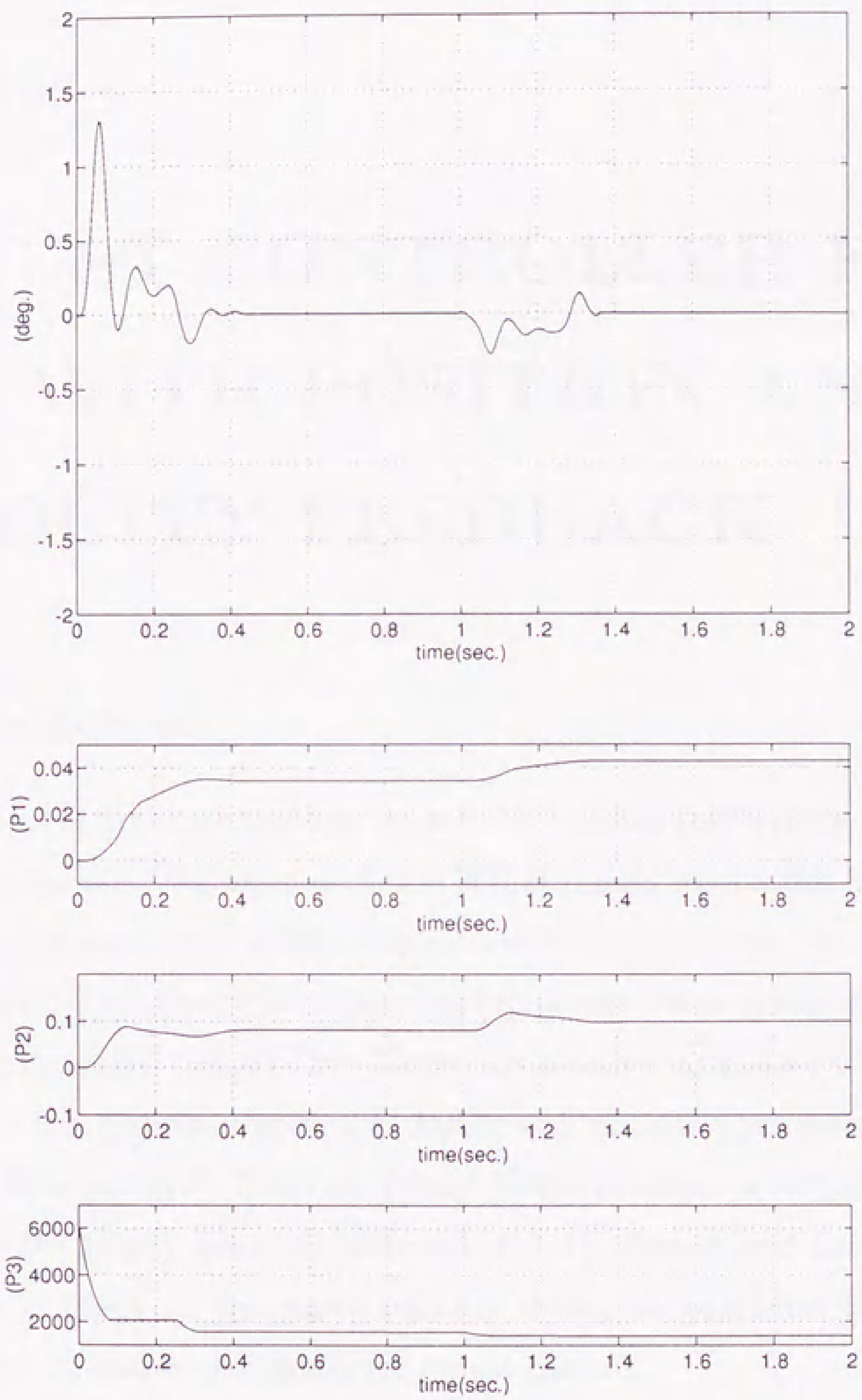


Fig. 3.4 The experimental results of the adaptive controller

Chapter 4

ROBUST CONTROLLER FOR MFJ WITH POSITION AND VELOCITY FEEDBACK

4.1 Introduction

Robust control of manipulators has attracted a considerable number of researchers over the past decade. The primary reason is that robust control can be used to handle uncertain parameters and time-varying load in manipulators. In other words, in the robust control, the controller has a fixed structure which yields an "acceptable" performance for a given uncertainty. Numerous robust control algorithms have been developed for the trajectory control of MRJs with uncertain parameters (Abdallah *et al.* 1990 for a surveys). Recently, Spong (1992) proposed a robust controller for MRJs using the theory given by Leitmann (1981), Corless and Leitmann (1981). This theory is based on Lyapunov stability theory to guarantee the stability of uncertain system and is very useful for robust control.

The introduction of joint flexibility in the manipulator modeling complicates considerably the equation of motion. In particular, the order of the related dynamic model becomes twice as much as that of MRJs. Consequently, the control law proposed for MFJs are more complex than those for MRJs. In order to achieve the robustness to parameter uncertainty, Ramires and Spong (1988) developed a robust scheme based on feedback linearization and variable structure method. Tomei (1991) also analyzed the robustness of a simple PD controller with a nonlinear feedback. With SPT, a robust tracking controller for MFJs have been given by Dawson *et al.* (1991).

Some robust adaptive control schemes for robot manipulators with flexible joints have also been proposed. Alshoor *et al.*(1993) gave robust adaptive control scheme by using singular perturbation analysis for the reduced order MFJ's model and recursive least-squares identification. Kwan and Yeung (1993) have used a controller with two control loops, in which one is adaptive control loop similar to that proposed by spong (1989) for rigid subsystem and another is sliding control loop for motor subsystem which resists against uncertainty of the system. But in this method, it is needed to assume that the link acceleration is measurable besides the link angle and velocity, and also the obtained controller makes discontinuous control inputs. In robust adaptive approaches described above, the adaptive parts are based on Spong(1992) (indirect adaptive controller) or least-squares dentification. These methods give no direct concern for the convergence of parameter etimation errors. Even thought they can guarantee the stability of MFJ system, they are difficult to get good tracking if the parameter estimation errors are not convergent to zero.

Here, we will present a new robust controller for MFJ with *a priori* known bound of uncertain parameters. A linear parameterized dynamic model is established which can describe the coupling between link and motor subsystems. We use the approach similar to that proposed by Spong(1992). But we derive continuous nonlinear control law which is suitable to robot manipulator with flexible joints. The same technique with Qu *et al.*(1992) is used to guarantee the exponential stability of the whole system (Yang *et al.* 1994a, 1995b).

4.2 Formulation of the Model, Some Notations and Assumption

The model of manipulator with flexible joints, which has been given in Chapter 2 (2.2.5) and (2.2.6), by the property P5 (i.e., the property of linear parametrizability), can be formulated as

$$\begin{pmatrix} Y_1(q_1, \dot{q}_1, \ddot{q}_1)P_1 + Y_{k1}P_k \\ Y_2(q_2, \dot{q}_2, \ddot{q}_2)P_2 + Y_{k2}P_k \end{pmatrix} = T, \tag{4.2.1}$$

where $Y_1(q_1, \dot{q}_1, \ddot{q}_1) \in R^{2nxr_1}$ and $Y_2(q_2, \dot{q}_2, \ddot{q}_2) \in R^{2nxr_2}$ are regressor matrix of known function of $q_1, \dot{q}_1, \ddot{q}_1, q_2, \dot{q}_2, \ddot{q}_2$ independent of the physical parameters of the manipulator. $T = [0, u^t]^t$,$Y_{k1} = diag[q_{1i} - q_{2i}]$, $Y_{k2} = diag[q_{2i} - q1i]$. $P_k = [k_1, k_2, ...k_n]^t$. $P_1 \in R^{r_1}, P_2 \in R^{r_2}$ are vectors of constant unknown physical parameters of link and motor respectively.

In the controller design, the assumptions, given in Chapter 3, about the measurement of the signals (the assumption A3.1) and the desired link trajectory (the assumption A3.2) are also used. Instead of the assumption (A3.3) about the unknown parameter, we will assume the following assumption.

A4.1 The parameter vectors P_1, P_2, and P_k may be uncertain and time-varying. But their nominal *constant* values $\bar{P}_1$, $\bar{P}_1$ and $\bar{P}_k$ are known *a priori*, and also with respect to the parameter error, $\tilde{P}_1 = P_1 - \bar{P}_1$, $\tilde{P}_2 = P_1 - \bar{P}_2$, and $\tilde{P}_k = P_k - \bar{P}_k$, their error bounds (positive constants) ρ_{p1},ρ_{p2} and ρ_k are known *a priori*, i.e.,

$$\left.\begin{aligned} \|\tilde{P}_1\| &\le \rho_1 \\ \|\tilde{P}_2\| &\le \rho_2 \\ \|\tilde{P}_k\| &\le \rho_k \end{aligned}\right\}. \tag{4.2.2}$$

We define the following notations (note that they are the same in Chapter 3):

$$\left.\begin{aligned} e &\stackrel{\text{def}}{=} q - q_d \\ \dot{q}_r &\stackrel{\text{def}}{=} \dot{q}_d + \Lambda e \\ s &\stackrel{\text{def}}{=} \dot{q}_r - \dot{q} \end{aligned}\right\}. \tag{4.2.3}$$

By the linear parametrizability, we also define

$$\left.\begin{aligned} Y_{d1}P_1 &\stackrel{\text{def}}{=} M(q)\ddot{q}_{r1} + C(q_1, \dot{q}_1)\dot{q}_{r1} \\ &\quad + G(q_1) + D\dot{q}_{r1} \\ Y_{d1}P_2 &\stackrel{\text{def}}{=} J_m\ddot{q}_{r2} + D_2\dot{q}_{r2} \\ Y_{kd1} &\stackrel{\text{def}}{=} diag[q_{1d} - q_{2d}] \\ Y_{kd2} &\stackrel{\text{def}}{=} diag[q_{2d} - q_{1d}] \end{aligned}\right\}. \tag{4.2.4}$$

Note that $Y_{d1}P_1$ is independent of $\ddot{q}_1$,q_{2d}, $\dot{q}_{2d}$ and $\ddot{q}_{2d}$ and $Y_{d2}P_2$ is independent of $\ddot{q}_2$, q_{1d}, $\dot{q}_{1d}$ and $\ddot{q}_{1d}$.

4.3 Robust Control Law Design

Before describing the robust controller, we modify a Lemma in Qu *et al.* (1992) into a suitable form for our purpose.

Lemma 4.3.1 *Let V be a Lyapunov function candidate for any given continuous time system satisfied the following conditions:*

$$(1)\lambda_1\|y\|^{m_1} \le V(y,t) \le \lambda_2\|y\|^{m_2}, \qquad \forall(y,t) \in R^n \times R \tag{4.3.1}$$

$$(2)\ \dot{V}(y,t) \le -\lambda_3\|y\|^{m_2} + \epsilon e^{-\beta t}, \qquad \forall(y,t) \in R^n \times R \tag{4.3.2}$$

where $m_2 \ge m_1 > 0$, $\lambda_2 \ge \lambda_1 > 0$, and λ_3, β, $\epsilon > 0$ are constants. Then, the exponential convergence of $V(y,t)$ satisfies the following inequality:

$$V(y,t) \le \begin{cases} V(y(t_0),t_0)e^{-\lambda(t-t_0)} + \epsilon(t-t_0)e^{-\lambda t}, & \lambda = \beta \\ V(y(t_0),t_0)e^{-\lambda(t-t_0)} + \epsilon_1 e^{-\beta t_0}/(\lambda-\beta), & \lambda \ne \beta \end{cases}$$

where $\lambda = \lambda_3/\lambda_2$, and $\epsilon_1 = \epsilon(e^{-\beta(t-t_0)} - e^{-\lambda(t-t_0)})$. Moreover, if $t \to \infty$, the system error $\|y\|$ is exponentially convergent to the origin in the large.

Proof: From (4.3.1) and (4.3.2), we have

$$\dot{V}(y,t) \le -\lambda V(y,t) + \epsilon e^{-\beta t}.$$

Then we get

$$V(y,t) \le V(y(t_0),t_0)e^{-\lambda(t-t_0)} + \epsilon e^{-\lambda t}\int_{t_0}^{t} e^{(\lambda-\beta)\tau} d\tau,$$

which gives the convergence of $V(y,t)$. Therefore, $\|y\|$ for any initial condition $y(t_0) = y_0$, is convergent to zero. □

The robust control design problem is as follows: given any desired link trajectory $q_{1d}(t)$ as described in A3.2, derive a control law for the motor input torques under the assumption A4.1 such that the manipulator output $q(t)$ tracks the desired trajectory.

Now we give the robust controller

$$\begin{aligned} T &= \begin{pmatrix} Y_{d1}P_{1o} \\ Y_{d2}P_{2o} \end{pmatrix} + \begin{pmatrix} Y_{kd1}P_{ko} \\ Y_{kd2}P_{ko} \end{pmatrix} + \begin{pmatrix} Y_{d1}U_{p1} \\ Y_{d2}U_{p2} \end{pmatrix} + \begin{pmatrix} Y_{kd1}U_{k1} \\ Y_{kd2}U_{k2} \end{pmatrix} + K_D s \\ &= \begin{pmatrix} Y_{d1}(P_{1o}+U_{p1}) \\ Y_{d2}(P_{2o}+U_{p2}) \end{pmatrix} + \begin{pmatrix} Y_{kd1}(P_{ko}+U_{k1}) \\ Y_{kd2}(P_{ko}+U_{k2}) \end{pmatrix} + K_D s, \end{aligned} \tag{4.3.3}$$

where

$$K_D = \begin{pmatrix} K_{D1} & 0 \\ 0 & K_{D2} \end{pmatrix},$$

$$\left.\begin{aligned} U_{p1} &= Y_{d1}^t s_1 \rho_1 / (\|Y_{d1}^t s_1\| + \gamma_1 e^{-\gamma_2 t}) \\ U_{p2} &= Y_{d2}^t s_2 \rho_2 / (\|Y_{d2}^t s_2\| + \gamma_1 e^{-\gamma_2 t}) \\ U_{k1} &= Y_{kd1}^t s_1 \rho_k / (\|Y_{kd1}^t s_1\| + \gamma_1 e^{-\gamma_2 t}) \\ U_{k2} &= Y_{kd2}^t s_2 \rho_k / (\|Y_{kd2}^t s_2\| + \gamma_1 e^{-\gamma_2 t}) \end{aligned}\right\}, \tag{4.3.4}$$

K_{D1}, K_{D2} and M are $n \times n$ positive definite diagonal gain matrixes. γ_1 and γ_2 are positive real numbers. The motor's input u and its desired trajectory q_{2d} can be calculated by (4.3.3).

The following theorem summarizes the exponential stability of the system under the robust controller (4.3.3) and (4.3.4).

Theorem 4.3.2 *Consider the manipulator dynamics (4.2.1) under the robust control law (4.3.3) and (4.3.4). The tracking error e and $\dot{e}$ converge to zero as $t \to \infty$, and all the signals in the system remain bounded.*

Proof: Selecting Lyapunov function candidate as

$$\begin{aligned} V &= \frac{1}{2}s_1^t M(q_1)s_1 + \frac{1}{2}s_2^t J_m s_2 \\ &+\frac{1}{2}(e_1 - e_2)^t K_s (e_1 - e_2), \end{aligned} \tag{4.3.5}$$

which can be rewritten as

$$V = \frac{1}{2}x^t \begin{pmatrix} M(q_1) & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & K_s \end{pmatrix} x, \tag{4.3.6}$$

where $x^t = [s_1^t, s_2^t, (e_1 - e_2)^t]$. From the property P4 in Chapter 2, we have

$$b_1\|x\|^2 \leq V \leq b_2\|x\|^2, \tag{4.3.7}$$

where $b_2 \geq b_1 > 0$. This means that the condition (1) in Lemma 4.3.1 holds. Differentiating (4.3.5) we get

$$\begin{aligned} \dot{V} &= s_1^t M(q_1)\dot{s}_1 + \frac{1}{2}s_1^t \dot{M}(q_1)s_1 + s_2^t J_m \dot{s}_2 \\ &+(\dot{e}_1 - \dot{e}_2)^t K_s (e_1 - e2). \end{aligned} \tag{4.3.8}$$

Using $s_1^t(\dot{M} - 2C)s_1 = 0$ (the property P1 in Chapter 2), (4.3.8) can be simplified along (4.2.1) as

$$\begin{aligned} \dot{V} &\leq s^t \begin{pmatrix} Y_{d1}P_1 + Y_{k1}P_k \\ Y_{d2}P_2 + Y_{k2}P_k \end{pmatrix} - s^t \begin{pmatrix} D & 0 \\ 0 & D_m \end{pmatrix} s - s^t T \\ &+(\dot{e}_1 - \dot{e}_2)^t K_s (e_1 - e_2). \end{aligned} \tag{4.3.9}$$

Substituting the control law (4.3.3)(4.3.4), we have

$$\begin{aligned} \dot{V} &\leq -s_1^t(K_{D1} + D)s_1 - s_2^t(K_{D2} + D_m)s_2 \\ &-(e_1 - e_2)^t \Lambda K_s (e_1 - e_2) \\ &+\|Y_{d1}^t s_1\|\rho_1 - \frac{\|Y_{d1}^t s_1\|^2 \rho_1}{\|Y_{d1}^t s_1\| + \gamma_1 e^{-\gamma_2 t}} \\ &+\|Y_{d2}^t s_2\|\rho_2 - \frac{\|Y_{d2}^t s_2\|^2 \rho_2}{\|Y_{d2}^t s_2\| + \gamma_1 e^{-\gamma_2 t}} \end{aligned}$$

$$+\|Y_{kd1}^t s_1\|\rho_k - \frac{\|Y_{kd1}^t s_1\|^2 \rho_k}{\|Y_{kd1}^t s_1\| + \gamma_1 e^{-\gamma_2 t}}$$
$$+\|Y_{kd2}^t s_2\|\rho_k - \frac{\|Y_{kd2}^t s_2\|^2 \rho_k}{\|Y_{kd2}^t s_2\| + \gamma_1 e^{-\gamma_2 t}}. \tag{4.3.10}$$

We can rewrite (4.3.10) as

$$\dot{V} \leq -x^t \bar{K} x + \rho\gamma_1 e^{-\gamma_2 t}, \tag{4.3.11}$$

where

$$\bar{K} = \begin{pmatrix} K_{D1} + D_1 & 0 & \\ 0 & K_{D2} + D_m & 0 \\ 0 & 0 & \Lambda_1 K_s \end{pmatrix},$$
$$\rho = \rho_1 + \rho_2 + \rho_k,$$

so that

$$\dot{V} \leq -\gamma_3 \|x\|^2 + \epsilon e^{-\gamma_2 t}, \tag{4.3.12}$$

where $\gamma_3 > 0$ which is the minimum of the eigenvalues of $\bar{K}$, and $\epsilon = \rho\gamma_1$. The condition (2) in Lemma 4.3.1 is also met. So, $\|x\|$ is exponentially convergent to the zero. In the calculation of control input u (4.3.3), we need q_{2d},$\dot{q}_{2d}$ and $\ddot{q}_{2d}$ which are bounded (shown in Remark 4.3.3). Therefore, we conclude that the tracking error e and $\dot{e}$ converge to zero as $t \to \infty$, and all the signals in the system remain bounded. □

Remark 4.3.1 Using ρ_1, ρ_2 and ρ_k to measure the parameter uncertainty of the motor and link subsystem may lead to overly conservative design. For this reason, we may use more precise bounds ρ_{1i}, ρ_{2i} and ρ_{ki} to measure the uncertainty of the ith parameter, e.g.,

$$|P_{1i} - \bar{P}_{1i}| = |\tilde{P}_{1i}| \leq \rho_{1i}, \quad i = 1, ..., r_1.$$

Let ξ_{1i} denote the ith component of the vector $Y_{d1}^t s_1$ and define the ith component of the U_{p1} as

$$U_{p1i} = \frac{\xi_{1i}\rho_{1i}}{|\xi_{1i}| + \gamma_1 e^{-\gamma_2 t}}.$$

Then using the same Lyapunov function candidate (4.3.5), we have

$$\dot{V} \leq -x^t \bar{K} x + \gamma_1 e^{-\gamma_2 t}(\sum_{i=1}^{r_1} \rho_{1i} + \sum_{j=1}^{r_2} \rho_{2i} + \sum_{l=1}^{n} \rho_{kl}).$$

In practice, we may simplify the algorithm by using *a priori* parameters exactly known (for example, the inertia and damping ratio of motor can be known from its catalogue data with reasonable precision) and by setting the their bounds ρ_{1i} or ρ_{2j} or ρ_{kl} to zero. □

Remark 4.3.2 In (4.3.4), we added the term $\gamma_1 e^{-\gamma_2 t}$ in U_{p1}, U_{p1}, U_{k1} and U_{k2} because the possible discontinuity of inputs can be avoided. □

Remark 4.3.3 Now we describe the property of the boundness of u, q_{2d}, $\dot{q}_{2d}$ and $\ddot{q}_{2d}$. From Theorem 4.3.2, we know that e and $\dot{e}$ are bounded and convergent to zero. From (4.3.7), we have

$$q_{2d} = q_{1d} + \hat{K}_s^{-1}(Y_{d1}(\bar{P}_1 + U_{p1}) + K_{D1} s_1). \tag{4.3.13}$$

From (4.2.4), $Y_{d1}(\bar{P}_1 + U_{p1})$ and $\hat{K}_s = \bar{K}_s + diag[U_{k1i}]$ are the bounded function of q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$, q_1 and $\dot{q}_1$. So, q_{2d} and q_2 are bounded. By using (2.2.5) we know that $\ddot{q}_1$ exists and is bounded. Differentiating (4.3.13), we can get

$$\dot{q}_{2d} = \dot{q}_{1d} + \hat{K}_s^{-1}(\dot{Y}_{d1}\hat{P}_u + Y_{d1}\dot{\hat{P}}_u + K_{D1}\dot{s}_1 + \dot{\hat{K}}_s(q_{1d} - q_{2d})), \tag{4.3.14}$$

where $\hat{P}_u = \bar{P}_1 + U_{p1}$. For $\dot{Y}_{d1}\hat{P}_u$, $Y_{d1}\dot{\hat{P}}_u$ and $\dot{\hat{K}}_s$ are bounded functions of q_1, $\dot{q}_1$, $\ddot{q}_1$, q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$ and $q_{1d}^{(3)}$. Therefore, $\dot{q}_{2d}$ and $\dot{q}_2$ are also bounded. The same procedure can be repeated for $q_1^{(3)}$ and $\ddot{q}_{2d}$ by differentiating (2.2.5) and (4.3.14). Therefore, the control law (4.3.3) and (4.3.4) involve the measurement of q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$ and calculated q_{2d}, $\dot{q}_{2d}$, $\ddot{q}_{2d}$ which are all bounded. We will show later the calculation of q_{2d}, $\dot{q}_{2d}$, $\ddot{q}_{2d}$ to a one-link manipulator with a flexible joint. □

4.4 Robustness Respect to Unmodeled Dynamics

In practice, there are some unmodeled effects in the manipulator system. Therefore, the robustness to unmodeled dynamics has to be analyized. As in Chapter 3, the dynamic model (4.2.1) can be rewritten as

$$\begin{pmatrix} Y_1(q_1, \dot{q}_1, \ddot{q}_1)P_1 + Y_{k1}P_k \\ Y_2(q_2, \dot{q}_2, \ddot{q}_2)P_2 + Y_{k2}P_k \end{pmatrix} = T + T_d, \qquad (4.4.1)$$

where $T_d \in R^{2n}$ is the vector of the unmodeled dynamics effect which is bounded as

$$\|T_d\| \leq d_0 + d_3\|s\|, \qquad (4.4.2)$$

with some positive constants d_0 and d_3. (4.4.2) is another representation of (3.4.2) proposed by Su and Stepanenko (1994).

The following theorem gives the stability of the system (4.4.1) with the robust control law (4.3.3) and (4.3.4) under the unmodeled effect T_d.

Theorem 4.4.1 *Consider the manipulator dynamics (4.4.1) under the robust control law (4.3.3) and (4.3.4). Then the tracking error e and $\dot{e}$ are ultimately bounded and all the signals in the system also remain bounded.*

proof: Using the same Lyapunov function candidate as in Theorem 4.3.2, its time derivative is shown as follows.

$$\begin{aligned} \dot{V} &\leq -x^t\bar{K}x + \rho\gamma_1 e^{-\gamma_2 t} + s^t T_d \\ &\leq -\gamma_3\|x\|^2 + \epsilon e^{-\gamma_2 t} + d_0\|x\| + d_3\|x\|^2 \\ &= -(\gamma_3 - d_3)\|x\|^2 + \epsilon e^{-\gamma_2 t} + d_0\|x\|. \end{aligned} \qquad (4.4.3)$$

If γ_3 is large enough, there exists $\gamma_3' = (\gamma_3 - d_3 - d_0) > 0$ such that

$$\begin{aligned} \dot{V} &\leq -\gamma_3'\|x\|^2 - d_0(\|x\| - \frac{1}{2})^2 + \frac{d_0}{4} + \epsilon e^{-\gamma_2 t} \\ &\leq -\gamma_3'\|x\|^2 + \frac{d_0}{4} + \epsilon e^{-\gamma_2 t}. \end{aligned}$$

(4.4.4)

According to (4.3.7), we obtain

$$\dot{V}(x,t) \leq -\lambda' V(x,t) + \frac{d_0}{4} + \epsilon e^{-\gamma_2 t}, \tag{4.4.5}$$

where $\lambda' = \gamma_3'/b_2$. Thus

$$V(x,t) \leq e^{-\lambda'(t-t_0)}(V(x(t_0),t_0) - \frac{d_0}{4\lambda'}) + \frac{d_0}{4\lambda'} + \epsilon e^{-\lambda_{\prime} t}\int_{t_0}^{t} e^{(\lambda_{\prime}-\gamma_2)} d\tau. \tag{4.4.6}$$

Therefore, from (4.3.7), therefore, we have

$$\|x\| \leq \frac{1}{\sqrt{b_1}}(e^{-\lambda'(t-t_0)}(V(x(t_0),t_0) - \frac{d_0}{4\lambda'}) + \frac{d_0}{4\lambda'} + \epsilon e^{-\lambda_{\prime} t}\int_{t_0}^{t} e^{(\lambda_{\prime}-\gamma_2)} d\tau)^{1/2} \tag{4.4.7}$$

and

$$\lim_{t\to\infty} \|x\| \leq \sqrt{\frac{d_0}{4\lambda' b_1}}.$$

From the above discussion, the tracking errors of the system, under the unmodeled effect T_d, are determined by the control gain matrix K_D and the T_d's bounds, d_0 and d_3. □

Remark 4.4.1 The bound error can be made arbitraily small by increasing the control gain λ'. In fact, since sufficiently large control gain may not be available, we are limited in the minimum size of the error bound. □

4.5 Simulation and Experiment

We will analyze the performance of the proposed control scheme by some simulations and experiments when the controller is used for the one link manipulator with flexible joint, which was shown in Chapter 3. By using the linear parametrized expression in terms of a suitable selected set of parameters, as mentioned in (4.2.1), we can obtain

$$\begin{pmatrix} Y_1 P_1 \\ Y_2 P_2 \end{pmatrix} + \begin{pmatrix} Y_{k1} P_k \\ Y_{k2} P_k \end{pmatrix} = \begin{pmatrix} \ddot{q}_1 & \dot{q}_1 & 0 & 0 \\ 0 & 0 & \ddot{q}_2 & \dot{q}_2 \end{pmatrix} \begin{pmatrix} J_1 \\ D_1 \\ J_2 \\ D_2 \end{pmatrix} + \begin{pmatrix} q_1 - q_2 \\ q_2 - q_1 \end{pmatrix} K_s = T.$$

By the definition (4.2.4), we have

$$
\begin{aligned}
Y_{d1}P_1 &= \ddot{q}_{r1}J_1 + \dot{q}_{r1}D_1, \\
Y_{d2}P_2 &= \ddot{q}_{r2}J_2 + \dot{q}_{r2}D_2, \\
Y_{kd1}P_k &= (q_{d1} - q_{d2})K_s, \\
Y_{kd2}P_k &= (q_{d2} - q_{d1})K_s.
\end{aligned}
$$

As described in Section 4.3, q_{2d} can be calculated by (4.3.13), i.e.,

$$
q_{d2} = q_{d1} + \hat{K}_s^{-1}(\ddot{q}_{r1}\hat{J}_1 + \dot{q}_{r1}\hat{D}_1 + K_{D1}s_1),
$$

where

$$
\begin{aligned}
\hat{J}_1 &= \bar{J}_1 + U_{J1}, \\
\hat{D}_1 &= \bar{D}_1 + U_{D1}.
\end{aligned}
$$

Differentiating (4.3.13) and (4.3.14), we can get

$$
\begin{aligned}
\dot{q}_{2d} &= \dot{q}_{d1} + \hat{K}_s^{-1}(q_{r1}^{(3)}\hat{J}_1 + \ddot{q}_{r1}\hat{D}_1 + \ddot{q}_{r1}\dot{\hat{J}}_1 + \dot{q}_{r1}\dot{\hat{D}}_1 \\
&\quad + \dot{\hat{K}}_s(q_{1d} - q_{2d}) + K_{D1}\dot{s}_1), \\
\ddot{q}_{2d} &= \ddot{q}_{d1} + \hat{K}_s^{-1}(q_{r1}^{(4)}\hat{J}_1 + q_{r1}^{(3)}\hat{D}_1 + 2q_{r1}^{(3)}\dot{\hat{J}}_1 \\
&\quad + 2\ddot{q}_{r1}\dot{\hat{D}}_1 + \ddot{q}_{r1}\ddot{\hat{J}}_1 \\
&\quad + \dot{q}_{r1}\ddot{\hat{D}}_1 + 2\dot{\hat{K}}_s(\dot{q}_{1d} - \dot{q}_{2d}) \\
&\quad + \ddot{\hat{K}}_s(q_{1d} - q_{2d}) + K_{D1}\ddot{s}_1).
\end{aligned}
$$

In the calculation of $\dot{q}_{2d}$ and $\ddot{q}_{2d}$, we have to use $\ddot{q}_1$ and $q_1^{(3)}$ which are bounded as mentioned in Remark 4.3.3. To avoid the complication for calculating $\dot{q}_{d2}$ and $\ddot{q}_{2d}$, we use a digital differentiation in the experiments. The purpose of simulations and experiments is to demonstrate the stability and performance properties inferred from the theoretical development. The parameters of the manipulator are the same in Chapter 3. The desired trajectory is also the same.

In the simulations, we give some results under three cases.

Case 1 All the parameters P_1, P_2 and K_s in the controller are with 30% of uncertainty, e.g., $|P_{\alpha i} - \bar{P}_{\alpha i}|/\bar{P}_{\alpha i} < 0.3, \alpha = 1, 2, k$. The control results without and with the robust terms U_{p1},U_{p2} and U_k are shown in Fig.4.1 and Fig.4.2, respectively.

Case 2 All the parameters P_1, P_2 and K_s in the controller are with 50% of uncertainty. The control results without and with the robust terms U_{p1},U_{p2} and U_k are shown in Fig.4.3 and Fig.4.4, respectively.

Case 3 To test the robustness of the controller respect to time-varying parameters further, a load with $2Kg$ weight was added to the end-effector of the manipulator (it is similar to the condition in which the manipulator moves to place without a object, picks up the object and returns). The control results without and with the robust terms are shown in Fig.4.5 and Fig.4.6, respectively.

In the experiments, as stated in Remark 4.3.1, we use some different bounds ρ_{1i} ($i = 1, 2, 3$) and ρ_k from ρ_{2i} ($i = 1, 2$). ρ_{1i} and ρ_k (for the link parameter) are set to 50%, i.e., $|P_{1i} - \bar{P}_{1i}|/\bar{P}_{1i} < 0.5$ and $|P_k - \bar{P}_k|/\bar{P}_k < 0.5$. ρ_{21} and ρ_{22} (for motor parameters) are set to zero, because the parameters of motor are known with reasonable accuracy from its catalogue data.

First, we use the control law without the robust term U_{p1},U_{p2} and U_k and with $\hat{K}_s = \bar{K}_s$. From the control result (shown in Fig.4.7), it can be seen that the tracking error and vibration are very large (sometimes, the system became unstable). Thus, it is desirable to use the robust control scheme. The experimental result is shown in Fig.4.8. For the robust controller, we set the γ_1 and γ_2 as 0.01 and 0.1, respectively.

Comparing with the results of the non-robust controller ($U_p = 0$), the results of the proposed robust controllers show that:

1. Thanks to the robust control, the tracking error and vibration of the link are rather small even though there exists parameter uncertainty.

2. From Fig.4.5 and Fig.4.6, the robust controller are with a good robustness to quick changing load, besides that to the time-invariable uncertainty.

3. The robust control law derived in Section 4.3 predicts that the tracking errors of the system converge to zero, while actually the tracking errors are not ex-

actly converge to zero as seen in the simulation and experimentation. This different phenomenon may be due to the digital differentiation and the unmodeled dynamics which is inherent in the simulation and experiment but ignored in the theoretical analysis. As analyzed in Section 4.3, the tracking errors are determined by the control gain and the bounds of the unmodeled dynamics.

4. The control scheme is quite simple, for the control parameter are easy to be determined.

Therefore, this scheme can accurately control the motion of the one flexible-joint manipulator along the desired trajectory with good robustness.

4.6 Conclusion

In this chapter, we present a robust control scheme applicable to manipulators with flexible joints. Exponential stability is ensured theoretically even when the parameters of the system is not exactly known. As an alternative to adaptive control, this scheme may be used in certain case. In particular, the scheme is very attractive in an environment where the uncertainty is not too large. Moreover, the present scheme is in very easy choice for control parameters. However, the proposed controller used not only the information of the motors but also that of the links. Therefore, it is necessary to study new robust controls which use only the information on the motors.

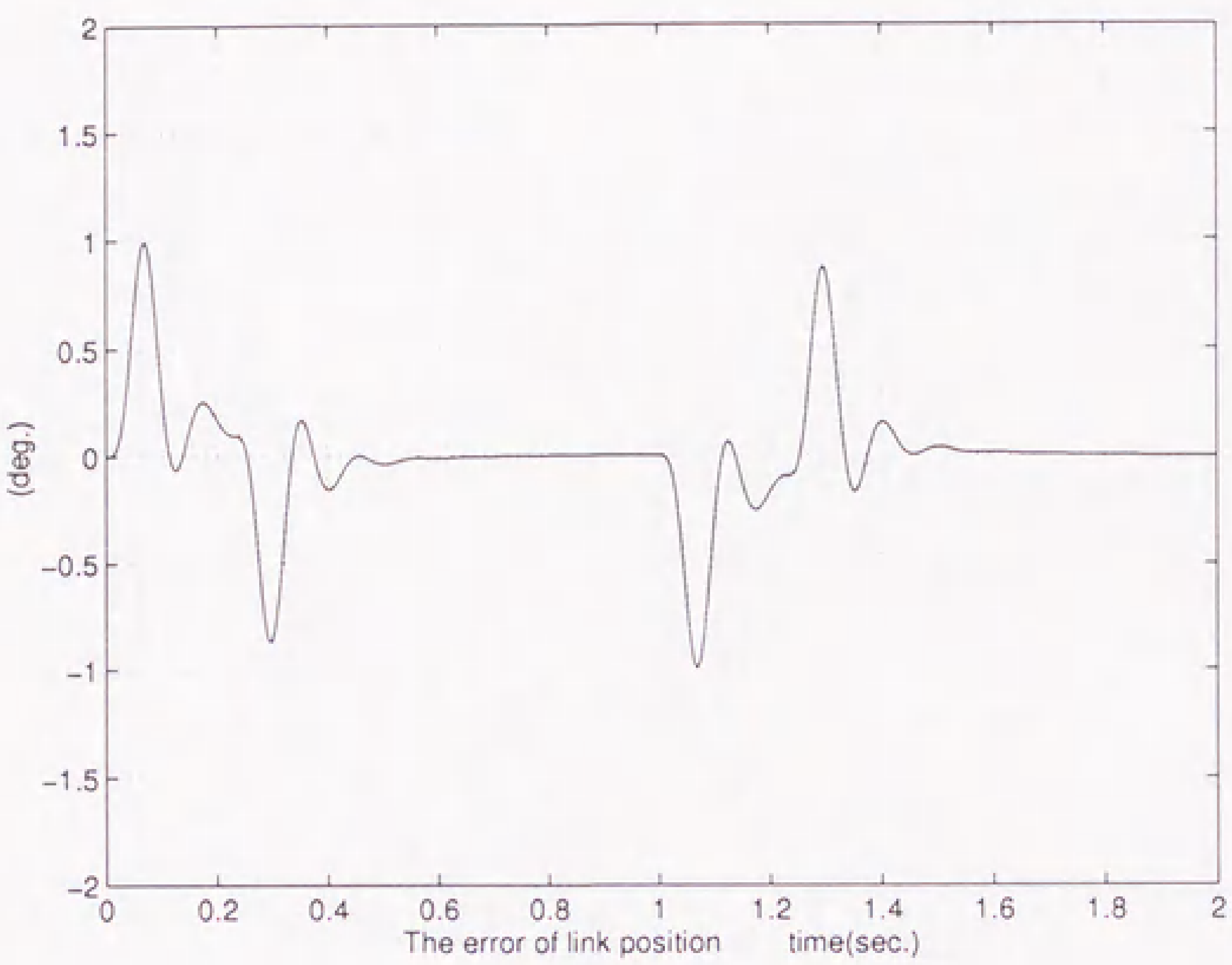


Fig. 4.1 The simulation result of the non-robust controller (30% uncertainty)

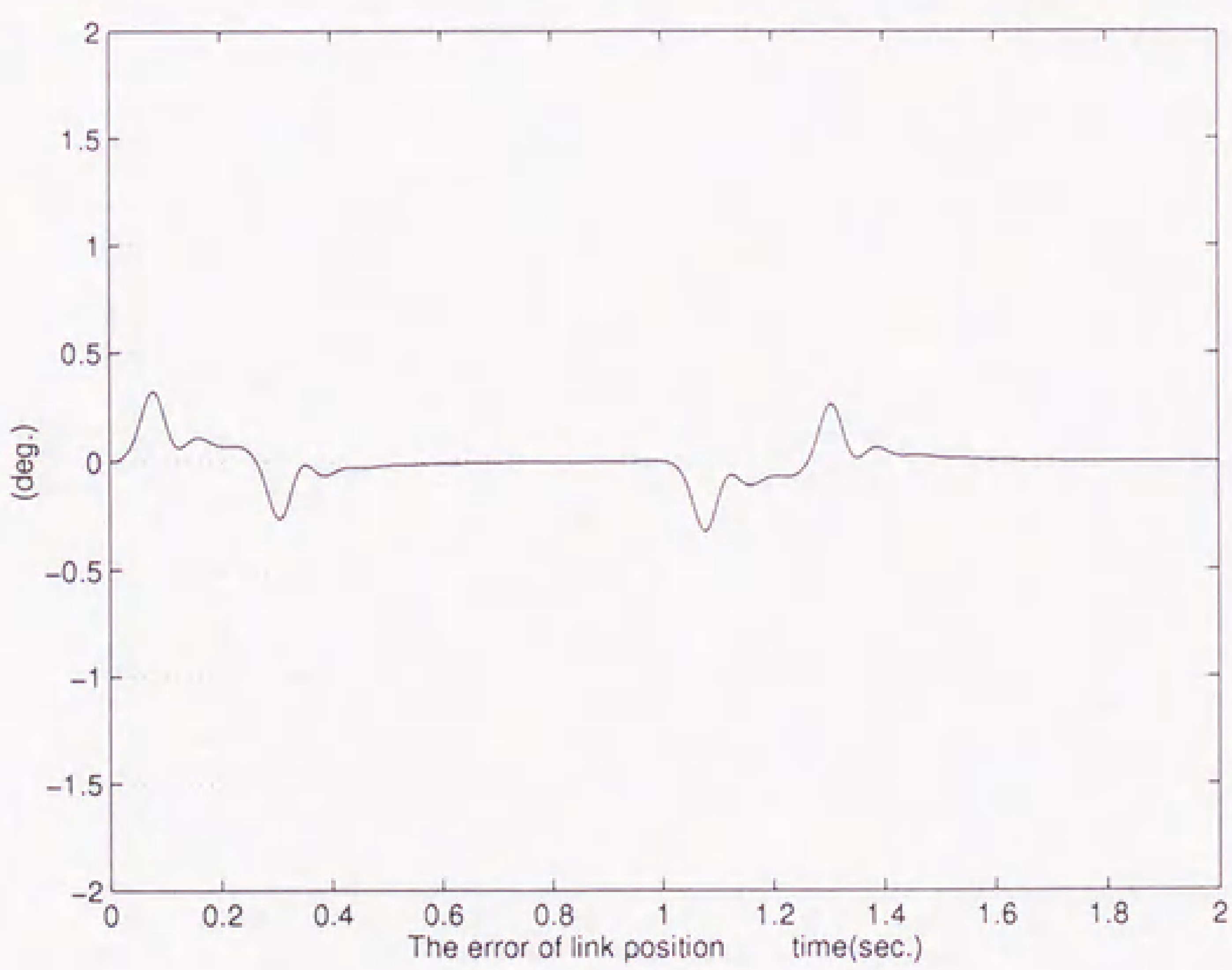


Fig. 4.2 The simulation result of the robust controller (30% uncertainty)

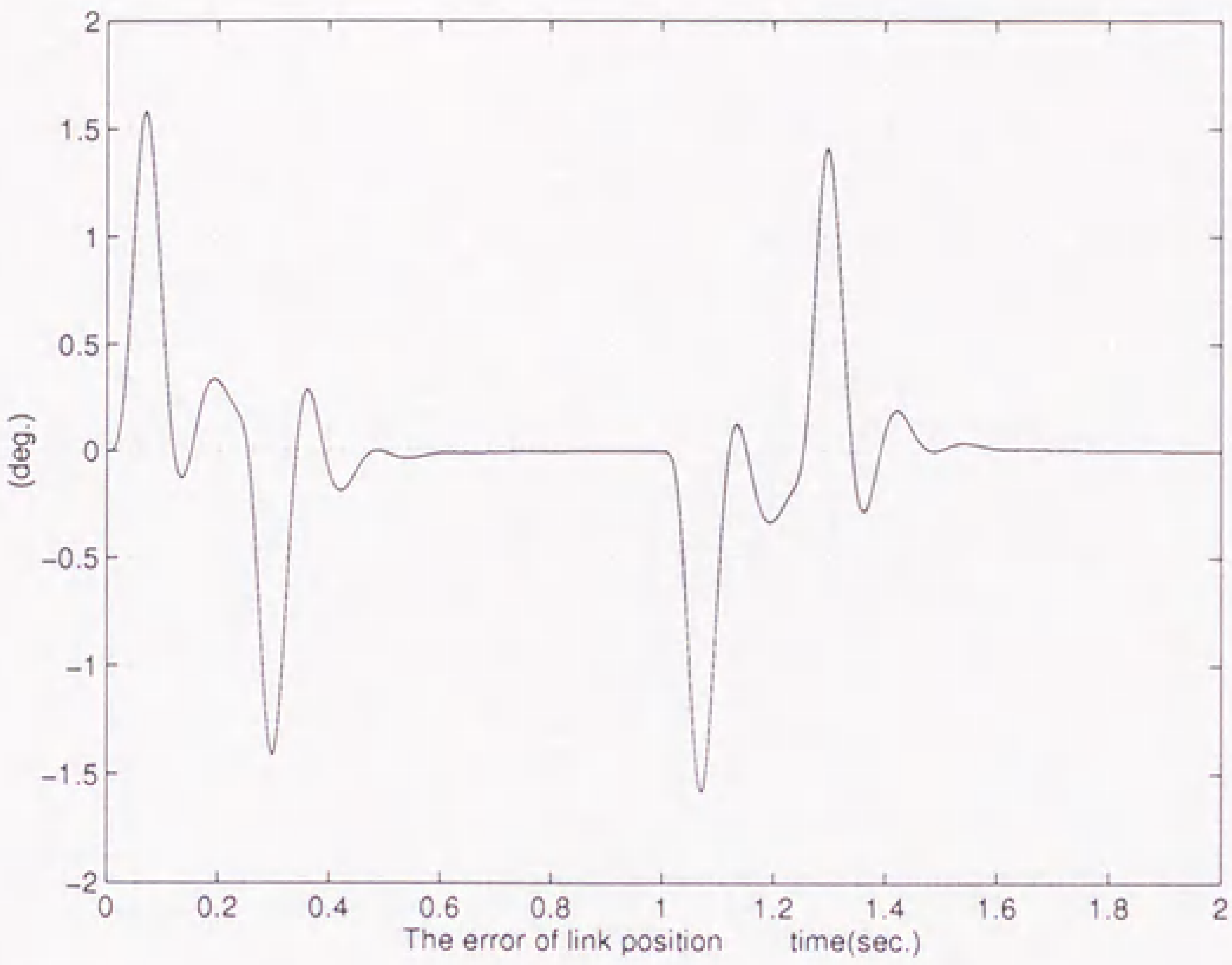


Fig. 4.3 The simulation result of the non-robust controller (50% uncertainty)

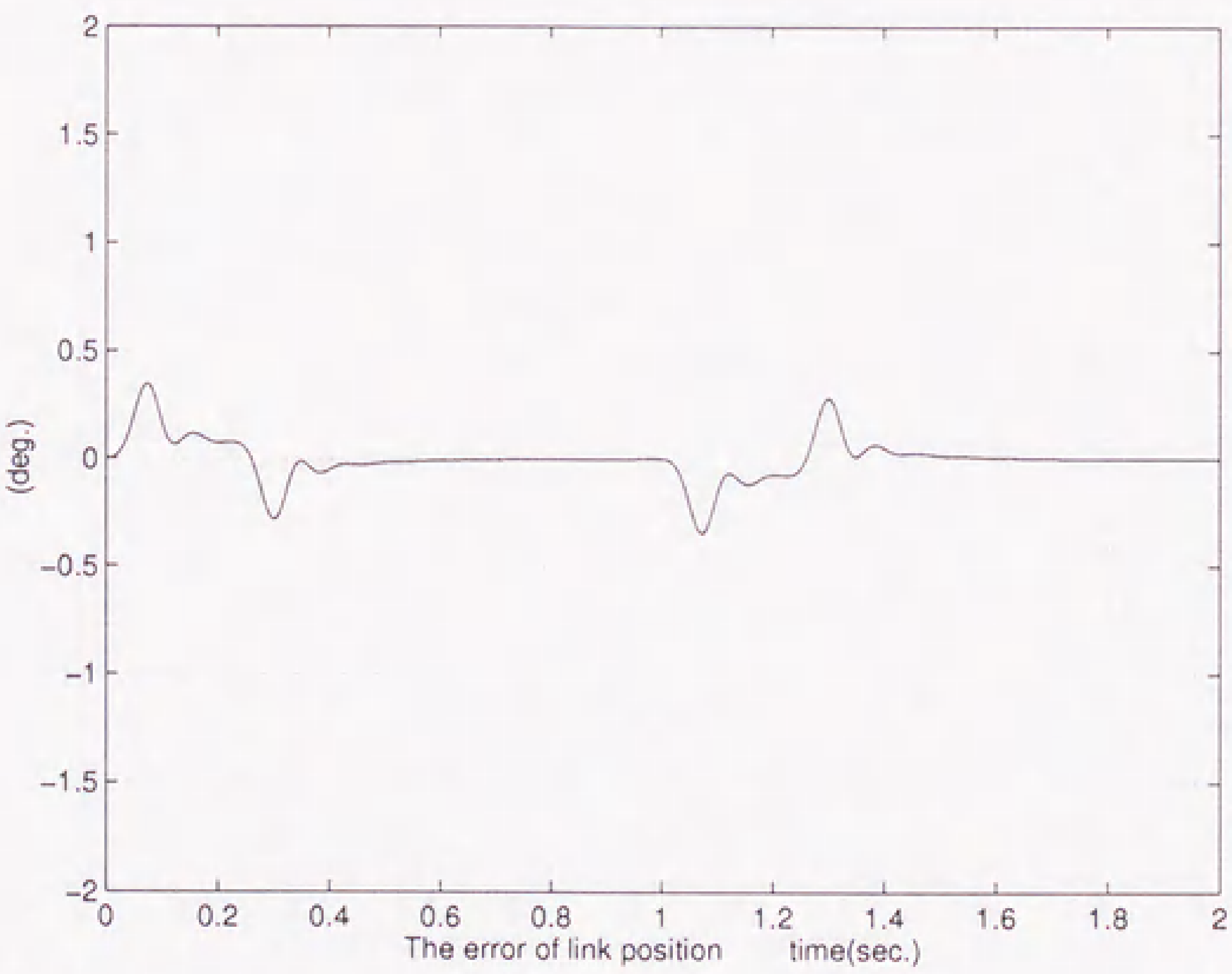


Fig. 4.4 The simulation result of the robust controller (50% uncertainty)

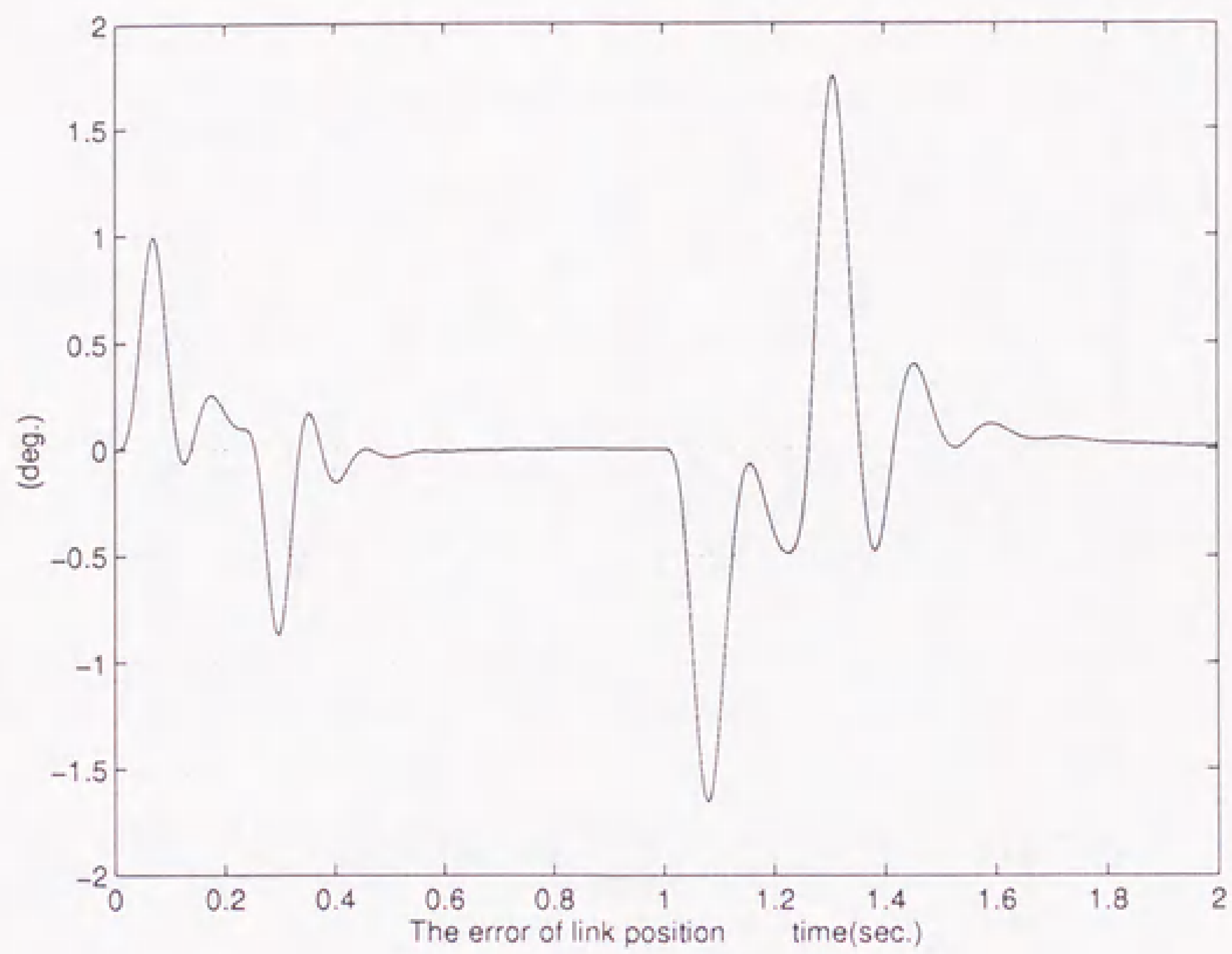


Fig. 4.5 The simulation result of the non-robust controller (with a sudden loaded)

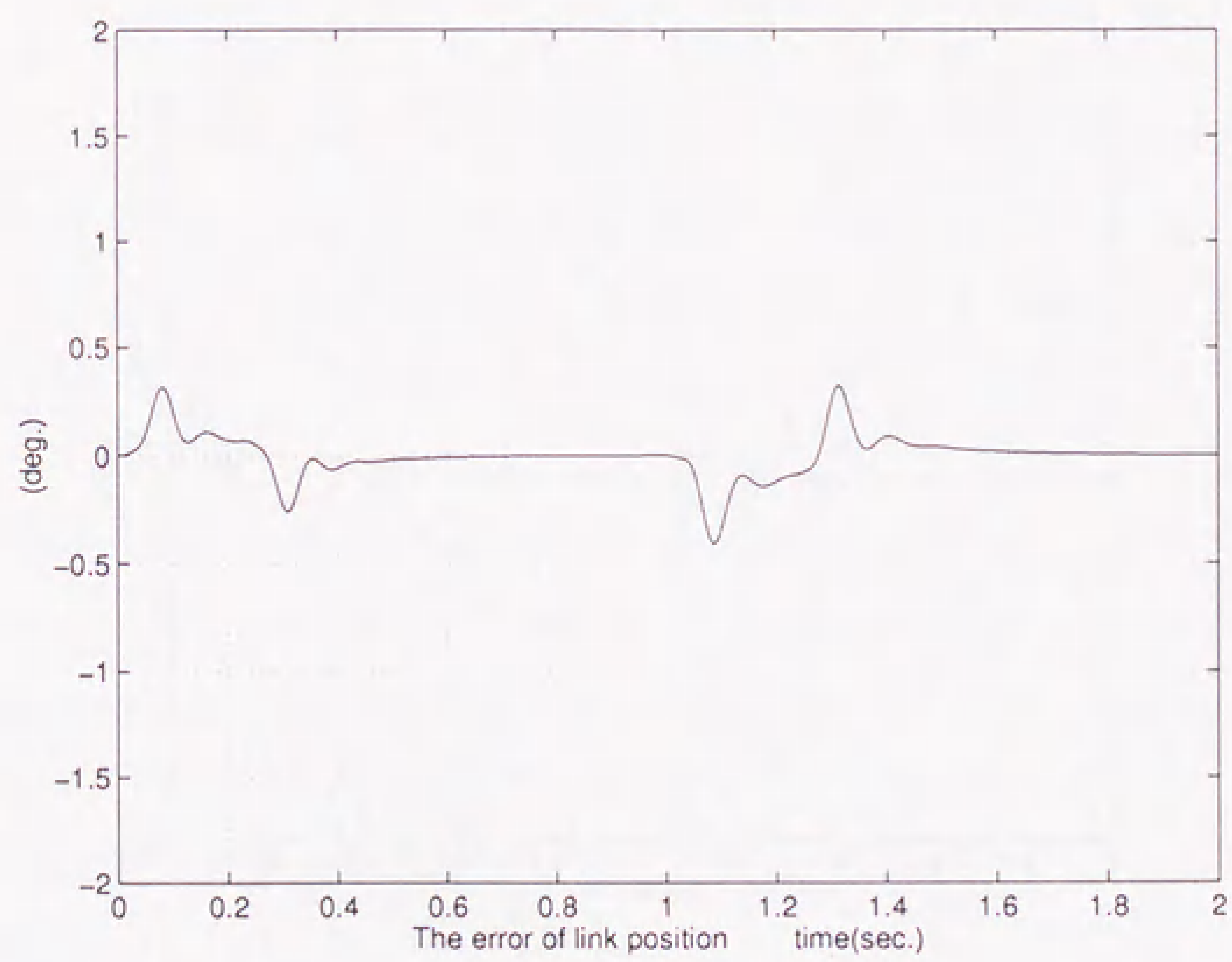


Fig. 4.6 The simulation result of the robust controller (with a sudden loaded)

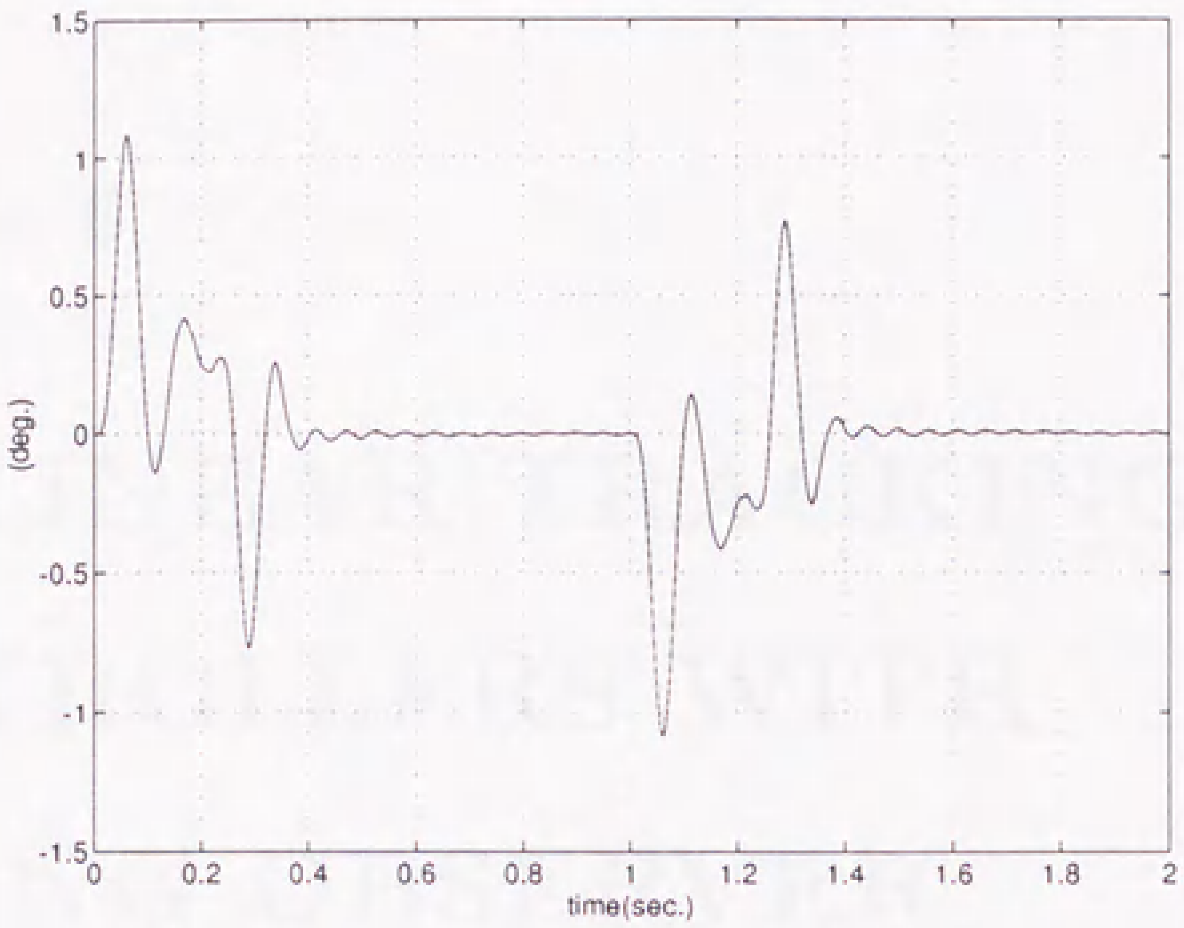


Fig. 4.7 The experimental result of the non-robust controller

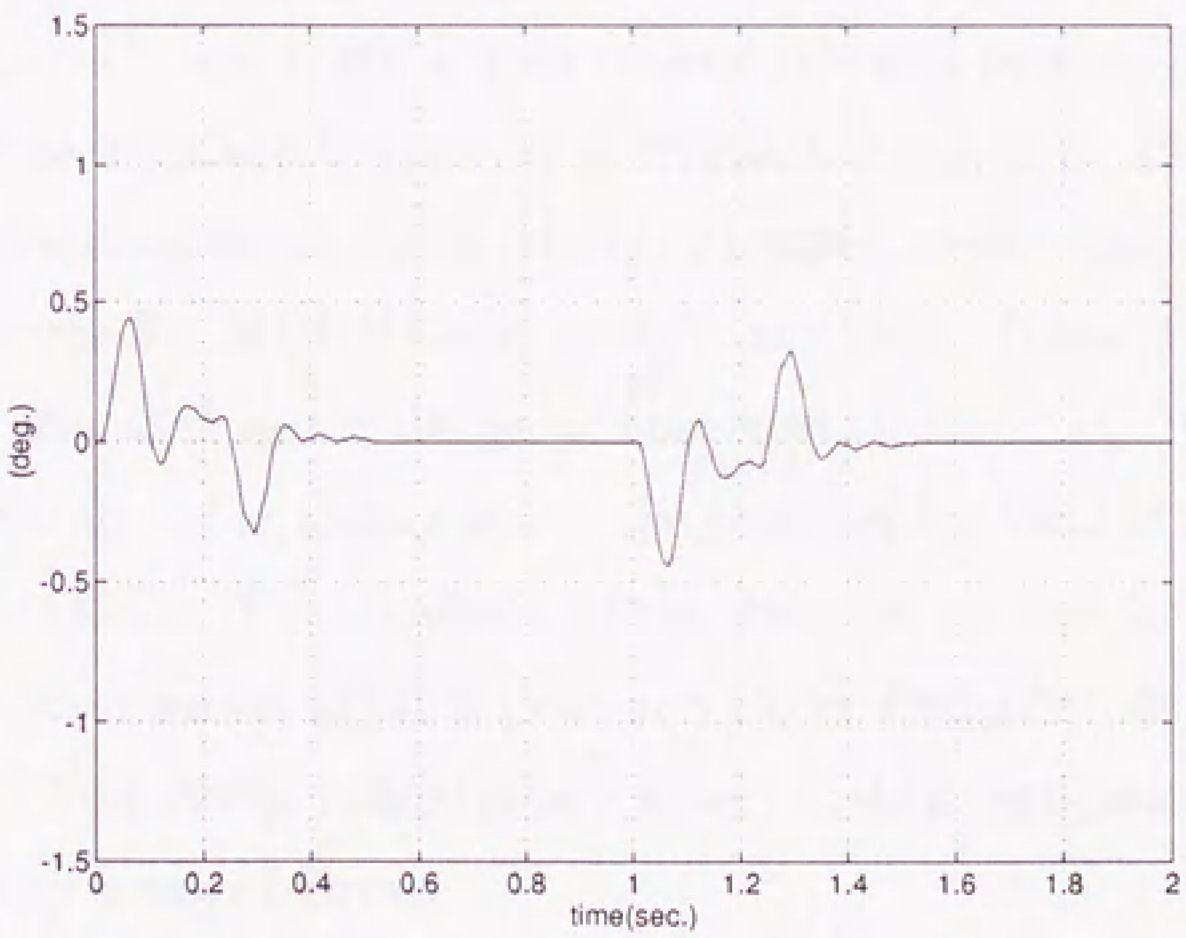


Fig. 4.8 The experimental result of the robust controller

Chapter 5

NONLINEAR TRACKING CONTROLLERS WITH SLIDING OBSERVER

5.1 Introduction

Although many adaptive and robust controllers for MFJ have been studied, as described in Chapters 3 and 4, the almost control schemes there require the position and velocity of the links and the motors in feedback (Yang *et al.* 1995a, 1995b). For decreasing the requirement of measurements of states, some researchers established nonlinear observers for MFJ (Nicosia and Tomei 1988, Tomei 1990). High-gain techniques are also used in the design of observers (Deza *et al.* 1992, Nicosia and Tornambe 1989). By using sliding mode, the observer for MRJ is also proposed by Wit and Slotine (1991). The stabilities of the dynamic control for MRJ combining with nonlinear observer are studied (Wit and Fixot 1991,1992, Nicosia and Tomei 1990, Singh and Yim 1992). Indeed, there is very little investigation concerning the control of MFJ by using observer.

Here, we will present nonlinear tracking controllers for MFJ by using sliding observers. Only the information of the motors is used for feedback. The stability is guaranteed by using Lyapunov stability theory. The design of controller has two steps: the first, by using the observed states, is to design a "desired motor position" which can make the link subsystem asymptotically stable. The second is to design an input of the motor subsystem which can realize the "desired motor position" and guarantee the stability of the whole system (Yang *et al.* 1994b, 1995d). The control design is modified to get an adaptive scheme in order to deal with the uncertainty of the parameters (Yang *et al.* 1994c, 1995c).

5.2 Some Notations and Assumptions

In the design of the tracking controllers, we define a new variable θ (fictional motor position)

$$\theta \stackrel{\text{def}}{=} K_s q_2. \tag{5.2.1}$$

Then we define the following notations.

$$\left.\begin{array}{rcc} e_q & \stackrel{\text{def}}{=} & q_1 - q_{1d} \\ \dot{q}_r & \stackrel{\text{def}}{=} & \dot{q}_{1d} - K_1 e_q \\ s_q & \stackrel{\text{def}}{=} & \dot{q}_1 - \dot{q}_r \\ e_\theta & \stackrel{\text{def}}{=} & \theta - \theta_d \\ \dot{q}_{r\theta} & \stackrel{\text{def}}{=} & \dot{\theta}_d - K_1 e_\theta \\ s_\theta & \stackrel{\text{def}}{=} & \dot{\theta} - \dot{q}_{r\theta} \end{array}\right\}, \tag{5.2.2}$$

where $K_1 = diag[K_{1i}]$ which is positive definite matrix. θ_d is the desired θ which will be defined later. By the linear parametrizability, we define

$$Y_d(q_1, \dot{q}_1, \dot{q}_r, \ddot{q}_r)P \stackrel{\text{def}}{=} M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + D\dot{q}_r. \tag{5.2.3}$$

By definition (5.2.1) we can rewrite (2.2.5) and (2.2.6) as

$$M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + D\dot{q} + K_s q_1 = \theta \tag{5.2.4}$$

and

$$J_m K_s^{-1}\ddot{\theta} + D_m K_s^{-1}\dot{\theta} + \theta = u + K_s q_1. \tag{5.2.5}$$

Therefore, the model of MFJ can be regarded as a combination of two subsystems, i.e., the link subsystem (5.2.3) and the motor subsystem (5.2.4). In the link subsystem, the input is fictional motor position θ and the output is q_1. In the motor subsystem, with a disturbance "$K_s q_1$" from the link subsystem, the input is u and the output is θ. The tracking controllers are designed based on the model described by (5.2.4) and (5.2.5). We make the following assumptions:

A5.1 The equivalent motor position q_2 ,velocity $\dot{q}_2$ and acceleration $\ddot{q}_2$ are measurable.

A5.2 The desired trajectory $q_{1d}(t) \in R^n$ is given and it is three times differentiable.

A5.3 All the physical parameters of the motor subsystem (5.2.5) are known *a priori*.

5.3 Design of the Nonadaptive Controller without Observer

For better illustrating the method which we propose later, at first, we give a nonlinear nonadaptive tracking controller without observer. In other words, besides the assumptions A5.1, A5.2 and A5.3, we assume that the measurements of the link position q_1 and velocity $\dot{q}_1$ are available and the physical parameters of the link subsystem (5.3.5) are known. In this study, the concept of "desired fictional motor position" θ_d is introduced. θ_d is a synthesized vector intended to be applied to the right side of (5.2.4) which is the desired output of (5.2.5). θ_d is defined as

$$\theta_d \stackrel{\text{def}}{=} Y_d(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d})P + K_s q_{1d} - K_2 s_q, \tag{5.3.1}$$

where $K_2 = diag[K_{2i}]$ is positive definite matrix. θ_d can be realized by the desired reference trajectories and the measurable information. Now we give a lemma to show that if θ_d as defined in (5.3.1) is the output of the motor subsystem (5.2.5) exactly, i.e., $\theta = \theta_d$, the link subsystem (5.2.4) is globally exponentially stable.

lemma 5.3.1 *Consider the link subsystem (5.2.4) and give the input θ as θ_d in Eq.(5.3.1). Then, the equilibrium point $(e_q, \dot{e}_q) = 0$ is globally exponentially stable.*

Proof: Substituting (5.3.1) into (5.2.4), we obtain the following error dynamics:

$$M\dot{s}_q + Cs_q = -(K_2 + D)s_q - K_s e_q. \tag{5.3.2}$$

We choose a Lyapunov function candidate as

$$V_1 = \frac{1}{2}(s_q^t M s_q + e_q^t K_s e_q)$$

and its time derivative leads to (by $s_q^t(\dot{M} - 2C)s_q = 0$)

$$\dot{V}_1 = s_q^t(M\dot{s}_q + Cs_q) + \dot{e}_q K_s e_q. \tag{5.3.3}$$

Substituting (5.3.2) into (5.3.3), we have

$$\dot{V}_1 = -s_q^t(K_2 + D)s_q - e_q^t K_1 K_s e_q.$$

Because K_1, K_s are positive definite, $\dot{V}_1$ is negative. Therefore e_q and $\dot{e}_q$ are exponentially convergent to zero. □

Remark 5.3.1 For link subsystem, the "desired fictional motor position" θ_d defined in (5.3.1) is similar to the computed torque control for MRJ. From the definition of θ_d, $\dot{\theta}_d$ and $\ddot{\theta}_d$ can be calculated as

$$\begin{aligned} \dot{\theta}_d &= \dot{Y}_d P + K_s \dot{q}_{1d} - K_2 \dot{s}_q, \\ \ddot{\theta}_d &= \ddot{Y}_d P + K_s \ddot{q}_{1d} - K_2 \ddot{s}_q. \end{aligned}$$

By (5.2.4) we obtain

$$\ddot{q}_1 = M^{-1}(q_1)(-C(q_1, \dot{q}_1)\dot{q}_1 - K_s(q_1 - q_2) - G(q_1) - D\dot{q}_1).$$

$\dot{\theta}_d$ is the function of $q_{1d}^{(3)}$, $\ddot{q}_{1d}$, $\dot{q}_{1d}$, q_1, $\dot{q}_1$ and q_2. $\ddot{\theta}_d$ is the function of $q_{1d}^{(4)}$, $q_{1d}^{(3)}$, $\ddot{q}_{1d}$, $\dot{q}_{1d}$, q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$. □

Due to the dynamics of (5.2.5), the output of the motor subsystem θ is not exactly equal to θ_d. A substitution of $\theta = \theta_d - e_\theta$ into (5.2.4) leads to the following error dynamics

$$M\dot{s}_q + Cs_q = -(K_2 + D)s_q + K_s(e_{k\theta} - e_q), \tag{5.3.4}$$

where $e_{k\theta} = K_s^{-1} e_\theta$. If we choose the control input u in (5.2.5) as

$$u = J_m K_s^{-1} \ddot{q}_{r\theta} + D_m K_s^{-1} \dot{q}_{r\theta} - K_4 K_s^{-1} s_\theta + K_s (K_s^{-1} \theta_d - q_{1d}), \tag{5.3.5}$$

where $K_4 = diag[K_{4i}]$ is a diagonal positive definite matrix, substitution of (5.3.5) into (5.2.5) leads to the error dynamics of the motor subsystem:

$$J_m K_s^{-1} \dot{s}_\theta = -(D_m + K_4) K_s^{-1} s_\theta - K_s (e_{k\theta} - e_q). \tag{5.3.6}$$

The following theorem summarizes the stability property of the system described by (5.2.4) and (5.2.5), under the control law (5.3.1) and (5.3.5).

Theorem 5.3.2 *Consider the closed-loop system described by (5.3.4) and (5.3.6). The equilibrium point* $(e_q, \dot{e}_q, e_\theta, \dot{e}_\theta) = 0$ *is globally exponentially stable.*

Proof: Selecting a Lyapunov function candidate as

$$V = \frac{1}{2} \{ s_q^t M s_q + s_\theta^t K_s^{-1} J_m K_s^{-1} s_\theta + (e_q - e_{k\theta})^t K_s (e_q - e_{k\theta}) \}. \tag{5.3.7}$$

Its time derivative leads to (by $s_q^t(\dot{M} - 2C) s_q = 0$)

$$\dot{V} = s_q^t (M \dot{s}_q + C s_q) + \dot{e}_q K_s e_{q\cdot} + s_\theta^t K_s^{-1} J_m K_s^{-1} \dot{s}_\theta + (\dot{e}_q - \dot{e}_{k\theta})^t K_s (e_q - e_{k\theta}) \}. \tag{5.3.8}$$

Substituting (5.3.4) and (5.3.6) into (5.3.8) results

$$\dot{V} = -s_q^t (K_2 + D) s_q - s_\theta^t K_s^{-1} (D_m + K_4) K_s^{-1} s_\theta - (e_q - e_{k\theta})^t K_1 K_s (e_q - e_{k\theta}) \}.$$

It is easy to see that some suitable values of K_1, K_2, K_3 and K_4 make $\dot{V}$ negative definite. Hence the equilibrium point $(e_q, \dot{e}_q, \dot{e}_\theta, e_\theta) = 0$ is globally exponentially stable. □

Remark 5.3.2 The control law described by (5.3.1) and (5.3.5) is a nonadaptive tracking controller in which all the parameters of the system have to be known and the measurements of position and velocity of the links and motors are required. This result is almost the same as that of Nicosia and Tomei (1993). □

5.4 Design of the Nonadaptive Controller with Sliding Observer

In the present industrial robot manipulators with flexible joins, the measurement devices are mounted in the driving motor only, such that the measurement of the link information are not available. Here we will give a nonadaptive controller with observer, under the assumptions A5.1-A5.3 and

A5.4 All the physical parameters of the link subsystem (5.2.5) are known *a priori.*

We can use (5.2.5) to calculate the link position q_1 by using $q_2, \dot{q}_2$ and $\ddot{q}_2$, if the parameters of motor subsystem are known. A sliding observer is used to estimate both q_1 and $\dot{q}_1$.

In nonlinear systems, the controller and observer cannot, in general, be independently designed (as shown Fig.5.1). When an observer is inserted in the feedback control loop, the controller will use the estimates of link positions and velocities. In this way, the estimation errors will affect the control torques. The control torques are input vectors to the state estimation error equation of the observer and the manipulator. The interactions between the observer and controller give rise to the problem of stability, thus the controller and observer should be regarded as a whole control system in order to ensure asymptotic tracking of the desired reference.

5.4.1 The Sliding Observer

Now, we use a nonlinear observer to estimate state $x_1 = q_1$ and $x_2 = \dot{q}_1$. Define $\beta(x_1, x_2)$ as

$$\beta(x_1, x_2) = -M(x_1)^{-1}[C(x_1, x_2) + G(x_1) + Dx_2 + K_s x_1]. \qquad (5.4.1)$$

The sliding observer is given by the following structure proposed by Wit and Slotine (1991).

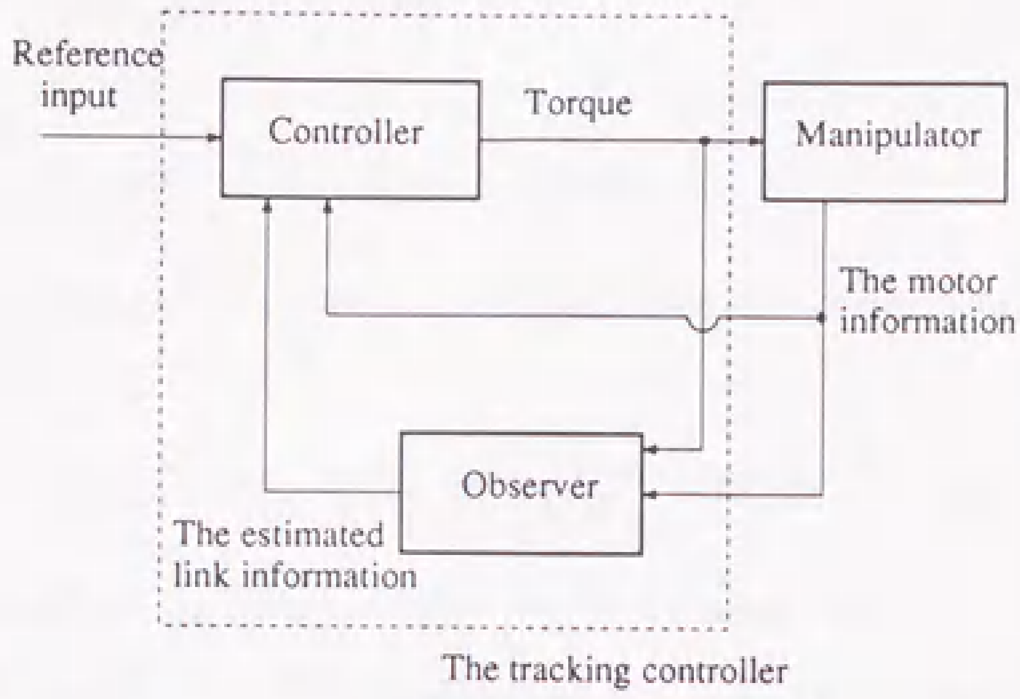


Fig. 5.1 The block diagram of the controller with observer

$$\left.\begin{array}{rcl}\dot{\hat{x}}_1 &=& \Gamma_1\tilde{x}_1 + \Lambda_1 sgn(\tilde{x}_1) \;+\; \hat{x}_2 \\ \dot{\hat{x}}_2 &=& \Gamma_2\tilde{x}_1 + \Lambda_2 sgn(\tilde{x}_1) \;+\; \beta(x_1, \hat{x}_2) + M^{-1}\theta\end{array}\right\}, \tag{5.4.2}$$

where $\hat{x}_i$ is the estimate of x_i, $\tilde{x}_i = x_i - \hat{x}_i$ is the estimated error of x_i, Γ_1, Γ_2 are positive definite constant diagonal matrices, i.e., $\Gamma_1 = diag[\gamma_{1i}], \Gamma_2 = diag[\gamma_{2i}]$. Λ_1, Λ_2 are state-dependent matrices to be defined later. Subtracting (5.4.2) from (5.2.4) and using the property P2 in Chapter 2, we obtain

$$\left.\begin{array}{rcl}\dot{\tilde{x}}_1 &=& -\Gamma_1\tilde{x}_1 - \Lambda_1 sgn(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &=& -\Gamma_2\tilde{x}_1 - \Lambda_2 sgn(\tilde{x}_1) \\ && -M^{-1}(C(x_1, x_2) + C(x_1, \hat{x}_2) + D)\tilde{x}_2\end{array}\right\}. \tag{5.4.3}$$

Wit and Slotine (1991) showed that under the mild suitable initial conditions, all the trajectories $[\tilde{x}_1, \tilde{x}_2]$ enter into the hyperplane defined by $\tilde{x}_1 = 0$, and the behavior of the resulting reduced order manifold is given by

$$\begin{array}{rcl}\dot{\tilde{x}}_2 &=& -M^{-1}(C(x_1, x_2) + D + C(x_1, \hat{x}_2) + M\Lambda_2\Lambda_1^{-1})\tilde{x}_2 \\ &=& -M^{-1}(C(x_1, x_2) + Q)\tilde{x}_2,\end{array} \tag{5.4.4}$$

where

$$Q = C(x_1, \hat{x}_2) + M\Lambda_2\Lambda_1^{-1} + D.$$

Defining a Lyapunov function candidate as

$$V_0 = \frac{1}{2}\tilde{x}_2^t M \tilde{x}_2, \tag{5.4.5}$$

then, by using the property P2, we can obtain the time derivative of V_0 as

$$\dot{V}_0 = -\tilde{x}_2^t(C(x_1, \hat{x}_2) + M\Lambda_2\Lambda_1^{-1} + D)\tilde{x}_2 = -\tilde{x}_2^t Q \tilde{x}_2. \tag{5.4.6}$$

Therefore, the estimated velocity error $\tilde{x}_2$ converges to zero exponentially if the matrices Q is positive definite. It is suggested by Wit and Slotine (1991) that Q and Λ_1 can be chosen to be diagonal positive definite(e.i. $Q = diag[Q_i], \Lambda_1 = diag[\lambda_{1i}]$). Then, we have

$$\Lambda_2 = M^{-1}(Q - C(x_1, \hat{x}_2) - D)\Lambda_1. \tag{5.4.7}$$

Remark 5.4.1 In the sliding hypersurface, the observer given by (5.4.2) is exponentially stable and the estimated error $\tilde{x}_2$ is

$$\tilde{x}_2 = \Lambda_1 sgn(\tilde{x}_1)$$

so that it is bounded by

$$|\tilde{x}_{2i}| < \lambda_{1i}; \quad i = 1, ..., n.$$

□

5.4.2 Nonadaptive Tracking Control

The control problem is given as follows: given any desired link trajectory q_{1d} under in the assumptions A5.1∼A5.4, derive a nonlinear tracking control law, in which the observer is used to estimate the information of the link, such that the manipulator output $q_1(t)$ tracks the desired trajectory stably. Here, the "desired fictional motor position" θ_d is defined as

$$\theta_d \stackrel{\text{def}}{=} Y_d(\hat{q}_1, \dot{\hat{q}}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d})P + K_s q_{1d} - K_2 s_q', \tag{5.4.8}$$

where $K_2 = diag[K_{2i}]$ is positive definite matrix. $s'_q \stackrel{\text{def}}{=} (\dot{q}_{1d} - \hat{x}_2) + K_1(q_{1d} - \hat{q}_1) = s_q + \tilde{x}_2 + K_1\tilde{x}_1$. θ_d can be realized by the desired reference trajectories and the outputs of the sliding observer. (5.4.8) can be rewritten as

$$\theta_d = Y_d(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d})P + K_s q_{1d} + W_1\tilde{x}_1 - K_2 s_q + W_2\tilde{x}_2, \tag{5.4.9}$$

where $W_2 = (B(\hat{q}_1)K_1 + C(\hat{q}_1, \dot{q}_r) - K_2)$, W_1 (the explicit expression is omitted) is bounded. Now we give a lemma to show that if θ_d defined in (5.4.8) is the output of the motor subsystem (5.2.5) exactly, i.e, $\theta = \theta_d$, the link subsystem (5.2.4) is globally exponentially stable.

lemma 5.4.1 *Consider the link subsystem (5.2.4) and give the input θ as θ_d in (5.4.8) where the sliding observer (5.4.2) is used. Then, the equilibrium point $(e_q, \dot{e}_q, \tilde{x}_1, \tilde{x}_2) = 0$ is globally exponentially stable.*

Proof: Substituting (5.4.9) into Eq.(5.2.4), we obtain the following error dynamics

$$M\dot{s}_q + Cs_q = -(D_1 + K_2)s_q - K_s e_q - W_1\tilde{x}_1 - W_2\tilde{x}_2. \tag{5.4.10}$$

Notice that all the trajectories of the sliding observer enter into the hyperplane defined by $\tilde{x}_1 = 0$ if the initial condition (Wit and Slotine 1991) is met. Hence, we have

$$M\dot{s}_q + Cs_q = -(D_1 + K_2)s_q - K_s e_q - W_2\tilde{x}_2. \tag{5.4.11}$$

We choose a Lyapunov function candidate as

$$V_1 = \frac{1}{2}(s_q^t M s_q + \tilde{x}_2^t M \tilde{x}_2 + e_q^t K_s e_q).$$

Its time derivative leads to

$$\dot{V}_1 = s_q^t(\dot{M}\dot{s}_q^t + Cs_q) + \tilde{x}_2^t(\dot{M}\dot{\tilde{x}}_2 + C\tilde{x}_2) + \dot{e}_q^t K_s e_q.$$

Substituting (5.4.4) and (5.4.11) into $\dot{V}_1$ results

$$\dot{V}_1 = -s_q^t(K_2 + D)s_q - \tilde{x}_2^t Q\tilde{x}_2 - s_q^t W_2\tilde{x}_2 - e_q^t K_1 K_s e_q.$$

By suitable choice of K_1, K_2 and Q, $\dot{V}_1$ can be made negative. Therefore e_q, $\dot{e}_q$ and $\tilde{x}_2$ are exponentially convergent to zero. □

Remark 5.4.2 From the definition of θ_d, $\dot{\theta}_d$ can be calculated as

$$\dot{\theta}_d = \dot{Y}_d P + K_s \dot{q}_{1d} - K_2 \dot{s}'_q, \tag{5.4.12}$$

where $\dot{Y}_d$ and $\dot{s}'_q$ are functions of $q_{1d}^{(3)}$, $\ddot{q}_{1d}$, $\dot{q}_{1d}$, $\ddot{\hat{q}}_1$, $\dot{\hat{q}}_1$, $\hat{q}_1$. $q_{1d}^{(3)}$, $\ddot{q}_{1d}$ and $\dot{q}_{1d}$ are known from the desired reference trajectories. $\ddot{\hat{q}}_1$, $\dot{\hat{q}}_1$ and $\hat{q}_1$ can be evaluated by the observer (5.4.2). □

Due to the dynamics of (5.2.5), the output of the motor subsystem θ is not exactly equal to θ_d, so that a substitution of $\theta = \theta_d - e_\theta$ into (5.2.4) leads to the following error dynamics

$$M\dot{s}_q + Cs_q = -(D_1 + K_2)s_q + K_s(e_{k\theta} - e_q) - W\tilde{x}_2. \tag{5.4.13}$$

If we choose the control input u in (5.4.5) as

$$\begin{aligned} u &= K_s(K_s^{-1}\theta_d - q_{1d}) + D_m K_s^{-1}\dot{\theta}_d \\ &\quad -K_3 e_{k\theta} - K_4 \dot{e}_{k\theta} + J_m K_s^{-1}\ddot{\theta}, \end{aligned} \tag{5.4.14}$$

where $K_3 = diag[K_{3i}]$ and $K_4 = diag[K_{4i}]$ are positive definite diagonal matrices, substitution of (5.4.14) into (5.2.5) leads to the error dynamics for the motor subsystem:

$$(D_m + K_4)\dot{e}_{k\theta} = -K_s(e_{k\theta} - e_q) - K_3 e_{k\theta}. \tag{5.4.15}$$

The following theorem summarizes the stability property of the system described by (5.2.4) and (5.2.5), under the control law (5.4.8) and (5.4.14) and the observer (5.4.2).

Theorem 5.4.2 *Consider the closed-loop system described by (5.4.4), (5.4.13) and (5.4.15). Suppose that the following conditions are satisfied*

$$\Delta_{1i}\Delta_{2i} > \Delta_{3i}^2/4 \quad i = 1, 2, ...n; \tag{5.4.16}$$

$$(K_2 + D)_e(Q)_e > (W_2)_E^2/4, \tag{5.4.17}$$

where

$$\begin{aligned}
\Delta_{1i} &= K_{1i} + (D_{mi} + K_{4i})^{-1}K_{si}, \\
\Delta_{2i} &= K_{3i}/K_{si} + 1 + (D_{mi} + K_{4i})^{-1}K_{si} \\
&\quad +(D_{mi} + K_{4i})^{-1}K_{3i}, \\
\Delta_{3i} &= K_{1i} + 1 + 2(D_{mi} + K_{4i})^{-1}K_{si} \\
&\quad +(D_{mi} + K_{4i})^{-1}K_{3i},
\end{aligned}$$

$(\cdot)_e$ and $(\cdot)_E$ mean the minimum and maximum eigenvalues, respectively. Then the equilibrium point $(e_q, \dot{e}_q, \tilde{x}_2, e_\theta) = 0$ is globally exponentially stable.

Proof: Selecting a Lyapunov function candidate as

$$\begin{aligned}
V &= \frac{1}{2}\{s_q^t M s_q + \tilde{x}_2^t M \tilde{x}_2 + e_{k\theta}^t (D_m + K_4) e_{k\theta} \\
&\quad +(e_q - e_{k\theta})^t K_s (e_q - e_{k\theta})\}.
\end{aligned} \tag{5.4.18}$$

Differentiating (5.4.18) along (5.4.4), (5.4.13) and (5.4.15) results

$$\begin{aligned}
\dot{V} &= -s_q^t(K_2 + D)s_q - \tilde{x}_2^t Q \tilde{x}_2 - s_q^t W_2 \tilde{x}_2 \\
&\quad -e_q^t K_s \Delta_1 e_q - e_{k\theta}^t K_s \Delta_2 e_{k\theta} + e_{k\theta}^t K_s \Delta_3 e_q,
\end{aligned} \tag{5.4.19}$$

where $\Delta_1 = K_1 + (D_m + K_4)^{-1}K_s$, $\Delta_2 = K_3/K_s + 1 + (D_m + K_4)^{-1}K_s + (D_m + K_4)^{-1}K_3$, $\Delta_3 = K_1 + 1 + 2(D_m + K_4)^{-1}K_s + (D_m + K_4)^{-1}K_3$. Considering that K_s, D_m, K_1, K_3 and K_4 are all diagonal positive definite matrices, (5.4.19) can be simplified as

$$\dot{V} = -v^t A v - \sum_{i=1}^{n} K_{si}\{e_{qi}^2 \Delta_{1i} + e_{k\theta i}^2 \Delta_{2i} - e_{qi} e_{k\theta i} \Delta_{3i}\}, \tag{5.4.20}$$

where $v = [\|s_q\|, \|\tilde{x}_2\|]^t$ and

$$A = \begin{pmatrix} (K_2 + D)_e & -(W_2)_E/2 \\ -(W_2)_E/2 & (Q)_e \end{pmatrix}.$$

It is easy to see that suitable values of K_1, K_2K_3, K_4 and Q make $\dot{V}$ negative definite. In other words, if the choices of K_1, K_3, K_4 and Q satisfy the conditions (5.4.16) and (5.4.17), then $\dot{V}$ is negative definite, hence the equilibrium point $(e_q, \dot{e}_q, \tilde{x}_2, e_\theta) = 0$ is globally exponentially stable. □

Remark 5.4.3 Theroem 5.4.2 only gives a sufficient condition of stability of the whole system including the manipulator with flexible joints, the sliding observer and the controller under the assumptions A5.1-A5.3 and A5.4. Fortunately, this sufficient condition can be satisfied easily in most applications. □

5.5 Design of the Adaptive Controller with Robust Sliding Observer

The controller (5.4.8) and (5.4.14) can be viewed as a feedforward computed torque control with respect to the M, C, D and G. Therefore, the effectiveness of this controller is dependent on the adequate knowledge of the physical parameters of the link subsystem. In practice, these parameters may be time variable and uncertain. Then, it is needed to design adaptive controller. Instead of A5.4, we make another assumption:

A5.5. The physical parameters of the link subsystem are not exactly known. But the nominal values and the bounds of the uncertainty for the parameters of the link subsystem are known *a priori*.

Hereafter, an adaptive tracking control scheme will be presented under the assumptions A5.1 ~ A5.3 and A5.5.

5.5.1 The Robust Sliding Observer

Now, we use an robust nonlinear observer to estimate state $x_1 = q_1$ and $x_2 = \dot{q}_1$ which is given as

$$\left.\begin{aligned} \dot{\hat{x}}_1 &= \Gamma_1 \tilde{x}_1 + \Lambda_1 sgn(\tilde{x}_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= \Gamma_2 \tilde{x}_1 + \Lambda_2 sgn(\tilde{x}_1) + \hat{\beta}(x_1, \hat{x}_2) + \hat{M}^{-1}\theta + t \end{aligned}\right\}, \tag{5.5.1}$$

where "ˆ" represents the estimate value. t is a robust compensation term against the uncertainty of link parameters. Subtracting (5.5.1) from (5.2.4), we obtain

$$\left.\begin{aligned} \dot{\tilde{x}}_1 &= -\Gamma_1 \tilde{x}_1 - \Lambda_1 sgn(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\Gamma_2 \tilde{x}_1 - \Lambda_2 sgn(\tilde{x}_1) \\ &\quad -\hat{M}^{-1}(\hat{C}(x_1, x_2) + \hat{C}(x_1, \hat{x}_2)\tilde{x}_2 + D\tilde{x}_2 + \Delta\beta - t \end{aligned}\right\}, \tag{5.5.2}$$

where $\Delta\beta = \beta(x_1, x_2) + \hat{M}^{-1}(\hat{C}(x_1, \hat{x}_2)x_2 + \hat{G} + \hat{D}\hat{x}_2 + K_s x_1)$. We laso have

$$\left.\begin{aligned} \dot{\tilde{x}}_2 &= -(\hat{M}^{-1}(\hat{C}(x_1, \hat{x}_2) + \hat{D}) \\ &\quad +\Lambda_1\Lambda_2^{-1})\tilde{x}_2 + \Delta\beta - t \\ &= -Q_1\tilde{x}_2 + \Delta\beta - t \end{aligned}\right\}, \tag{5.5.3}$$

where

$$Q_1 = \hat{M}^{-1}(\hat{C}(x_1, \hat{x}_2) + \hat{D} + \hat{M}\Lambda_2\Lambda_1^{-1}).$$

Defining a Lyapunov function candidate as

$$V_0 = \frac{1}{2}\tilde{x}_2^t \tilde{x}_2, \tag{5.5.4}$$

the time derivative of V_0 is given as

$$\dot{V}_0 = -\tilde{x}_2^t \hat{M}^{-1}(\hat{C}(x_1, \hat{x}_2) + \hat{M}\Lambda_2\Lambda_1^{-1} + D)\tilde{x}_2 + \tilde{x}_2^t \hat{M}(\Delta\beta - t). \tag{5.5.5}$$

By A5.5 and the property P4 in Chapter 2, we can bound $\Delta\beta$ as

$$\begin{aligned} \Delta\beta &= (\hat{M}^{-1}\hat{C} - M^{-1}C)x_2 + (\hat{M}^{-1}\hat{D} - M^{-1}D)x_2 \\ &\quad +(\hat{M}^{-1}\hat{G} - M^{-1}G) + (\hat{M}^{-1} - M^{-1})(K_s x_1 - \theta) \\ &\le \rho_3\|x_2\|^2 + \rho_2\|x_2\| + \rho_1\|K_s x_1 - \theta\| + \rho_0, \end{aligned}$$

where ρ_0, ρ_1, ρ_2 and ρ_3 are known constants. In the sliding path, we have $x_2 = \hat{x}_2 - \tilde{x}_2 = \hat{x}_2 - \Lambda_1 sgn(\tilde{x}_1)$ (see Remark 5.4.1). Hence, we obtain

$$\begin{aligned}\Delta\beta &\leq \rho_3\|\hat{x}_2\|^2 + (\rho_2 + 2\rho_3\lambda_1)\|\hat{x}_2\| + \rho_1\|K_s x_1 - \theta\| \\ &\quad +(\rho_0 + \rho_2\lambda_1 + \lambda_1^2) \\ &= \zeta(\hat{x}_2, x_1, \theta). \end{aligned} \tag{5.5.6}$$

The robust compensation term is defined as:

$$t = \begin{cases} \zeta(\hat{x}_2, x_1, \theta)\tilde{x}_2/\|\tilde{x}_2\|, & \|\tilde{x}_2\| \geq 0 \\ 0, & \|\tilde{x}_2\| = 0 \end{cases}. \tag{5.5.7}$$

Then

$$\dot{V}_0 \leq -\tilde{x}_2^t Q_1 \tilde{x}_2. \tag{5.5.8}$$

As shown in Section 5.4, the estimated velocity error $\tilde{x}_2$ converges to zero exponentially with suitable choice of Q_1, Λ_1 and Λ_2.

Remark 5.5.1 As shown by Wit and Slotine (1991), if the uncretainty of the parameters is larger than a certain range, the trajectories of the observer may run out of the sliding path. Therefore, we add the robust compensation term t in the observer for increasing the robustness of the sliding observer. □

5.5.2 Adaptive Tracking Control

The control problem can be described as follows: given any desired link trajectory q_{1d} under the assumptions A5.1∼A5.2 and A5.5, derive a adaptive tracking control law, in which the robust observer is used to observe the information of the link, such that the manipulator output $q(t)$ tracks the desired trajectory stably.

Using the same procedure, we define θ_d as

$$\theta_d \stackrel{\text{def}}{=} Y_d(\hat{q}_1, \dot{\hat{q}}_1, q_d, \dot{q}_d, \ddot{q}_d)\hat{P} + K_s q_d - K_2 s_q' \tag{5.5.9}$$

and

$$\dot{\hat{P}} = -M_p^{-1} Y_d^t s_q, \tag{5.5.10}$$

where M_p is a constant positive definite adaptation gain matrix. In (5.5.9), θ_d is also a synthesized vector which can be realized by the desired reference and the outputs of the robust observer and estimated parameter. (5.5.9) can be rewritten as:

$$\theta_d = Y_d P + K_s q_d + W_1 \tilde{x}_1 - K_2 s_q + W_2 \tilde{x}_2 + Y_d e_p, \tag{5.5.11}$$

where $e_p = P - \hat{P}$. By substituting (4.5.11) into (5.2.5) and considering $\theta = \theta_d - e_\theta$, we also can obtain

$$M\dot{s}_q + Cs_q = -(D_1 + K_2)s_q + K_s(e_{k\theta} - e_q) - W_2\tilde{x}_2 + Y_d e_p. \tag{5.5.12}$$

If we choose the control input u as in Section 5.4, we can also get the following theorem, which summarizes the stability property of the system described by (5.2.4) and (5.2.5), under the control law (5.5.9), (5.5.10), (5.4.14) and the observer (5.5.1).

Theorem 5.5.1 *For the closed-loop system described by (5.4.15), (5.5.12) and (5.5.3), if the following conditions are satisfied*

$$\Delta_{1i}\Delta_{2i} > \Delta_{3i}^2/4 \quad i = 1, 2, ...n; \tag{5.5.13}$$

$$(K_2 + D)_e(Q_1)_e > (W_2)_E^2/4, \tag{5.5.14}$$

then the equilibrium point $(e_q, \dot{e}_q, \tilde{x}_2, e_\theta) = 0$ *is globally exponentially stable. Morever, the errors of the estimated parameters* e_p *are bounded.*

Proof: Selecting a Lyapunov function candidate as

$$\begin{aligned} V &= \frac{1}{2}\{s_q^t M s_q + \tilde{x}_2^t \tilde{x}_2 + e_{k\theta}^t (D_m + K_4) e_{k\theta} \\ &\quad + (e_q - e_{k\theta})^t K_s (e_q - e_{k\theta}) + e_p^t M_{p1} e_p\}. \end{aligned} \tag{5.5.15}$$

Differentiating (5.5.15) along (5.5.3), (5.5.12) and (5.4.15) results

$$\begin{aligned} \dot{V} &= -s_q^t(K_2 + D)s_q - \tilde{x}_2^t Q_1 \tilde{x}_2 - s_q^t W_2 \tilde{x}_2 + e_p^t(Me_p - s_q^t Y_d) \\ &\quad - e_q^t K_s \Delta_1 e_q - e_{k\theta}^t K_s \Delta_2 e_{k\theta} + e_{k\theta}^t K_s \Delta_3 e_q + \tilde{x}_2^t \hat{M}(\Delta\beta - t). \end{aligned} \tag{5.5.16}$$

By the (5.5.7) and (5.5.10), (5.5.16) can be simplified as

$$\dot{V} \leq -v^t A v - \sum_{i=1}^{n} K_{si}\{e_{qi}^2 \Delta_{1i} + e_{k\theta i}^2 \Delta_{2i} - e_{qi} e_{k\theta i} \Delta_{3i}\}, \qquad (5.5.17)$$

where $v = [\|s_q\|, \|\tilde{x}_2\|]^t$ and

$$A = \begin{pmatrix} (K_2 + D)_e & -(W_2)_E/2 \\ -(W_2)_E/2 & (Q_1)_e \end{pmatrix}.$$

It is easy to see that some suitable values of $K_1, K_2 K_3, K_4$ and Q_1 make $\dot{V}$ negative definite. In other words, if the choices of K_1, K_3, K_4 and Q_1 satisfy the conditions (5.5.13) and (5.5.14), then $\dot{V}$ is negative definite, hence the equilibrium point $(e_q, \dot{e}_q, \tilde{x}_2, e_\theta) = 0$ is globally exponentially stable. The errors of the estimated parameters e_p are bounded. □

5.6 The Effects of Noise

In the proposed tracking controllers using sliding observer, the measurement of motor acceleration is required. However, in practice, this request is difficult to be met. The acceleration can only be obtained by numerical differentiation from the motor velocity. To decrease high frequency disturbance, some low-pass filter can also be used actually (this procedure is illustrated in Section 5.7). Because of the numerical differentiation and filter, some noise or error will be introduced. We should analyze the stability of the system under some measurement noise or error. Taking the non-adaptive tracking controller as an example, we analyze the robustness of the system with respect to noise. We assume that the motor acceleration, which is obtained by certain numerical methods, is described by the following form

$$\ddot{q}_{2c} = \ddot{q}_2 + v(t),$$

where $v(t) \in R^n$ is noise or error on the $[-v_0/2, v_0/2]$, $v_0 > 0$ is a bounded constant. It is also assumed that $\dot{v}(t)$ are also bounded by a constant ρ_v. In calculation of the link position q_1, $u, q_2, \dot{q}_2$ and $\ddot{q}_{2c}$ are used. The calculated link position is given as

$$
\begin{aligned}
q_{1c} &= q_2 + K_s^{-1}(J_m \ddot{q}_{2c} + D_m \dot{q}_2 - u) \\
&= q_2 + K_s^{-1}(J_m \ddot{q}_2 + D_m \dot{q}_2 - u) + K_s^{-1} J_m v(t) \\
&= q_1 + v_1,
\end{aligned}
$$

where $v_1 = K_s^{-1} J_m v(t)$. Considering the noise of the calculated link position, the error dynamics for sliding observer can be written as

$$
\left.
\begin{aligned}
\dot{\tilde{x}}_1 &= -\Gamma_1 \tilde{x}_{1c} - \Lambda_1 sgn(\tilde{x}_{1c}) + \tilde{x}_2 \\
\dot{\tilde{x}}_2 &= -\Gamma_2 \tilde{x}_{1c} - \Lambda_2 sgn(\tilde{x}_{1c}) \\
& \quad -M^{-1}(C(x_1, x_2) + C(x_1, \hat{x}_2) + D)\tilde{x}_2
\end{aligned}
\right\},
$$

where $\tilde{x}_{1c} = x_{1c} - \hat{x}_1 = \tilde{x}_1 + v_1$. With the similar conditions proposed by Wit and Slotine (1991), the sliding behavior can then only occur on the surface (Slotine *et al.* 1987)

$$
\tilde{x}_{1c} = \tilde{x}_1 + v_1 = 0.
$$

Therefore, the behavior of the resulting reduced order manifold is given by

$$
\dot{\tilde{x}}_2 = -M^{-1}(C(x_1, x_2) + Q)\tilde{x}_2 - \Lambda_2 \Lambda_1^{-1} \dot{v}_1.
$$

Because the "desired fictional motor position" (5.4.9) has used the estimated link position and velocity, the error dynamics for the link subsystem can be written as

$$
M\dot{s}_q + Cs_q = -(D_1 + K_2)s_q + K_s(e_{k\theta} - e_q) - W_1 v_1 - W_2 \tilde{x}_2.
$$

The design of the control law u is similar to (5.4.13), except $\ddot{q}_{2c}$ is used in feedback directly instead of $\ddot{q}_2$. So that the error dynamics for motor subsystem becomes

$$
(D_m + K_4)\dot{e}_{k\theta} = -K_s(e_{k\theta} - e_q) - K_3 e_{k\theta} - v_2,
$$

where vector $v_2 = J_m v(t)$. If the same Lyapunov function as (5.4.18) is selected, we can get its time derivation as

$$
\begin{aligned}
\dot{V} &= -s_q^t(K_2 + D)s_q - \tilde{x}_2^t Q \tilde{x}_2 - s_q^t W_2 \tilde{x}_2 - s_q^t \omega_1 - \tilde{x}_2^t \omega_2 \\
& \quad -\sum_{i=1}^{n} K_{si}\{e_{qi}^2 \Delta_{1i} + e_{k\theta i}^2 \Delta_{2i} - e_{qi} e_{k\theta i} \Delta_{3i}\} - e_{k\theta}^t v_2,
\end{aligned}
$$

where vector $\omega_1 = W_1 v_1$ and vector $\omega_2 = B\Lambda_2\Lambda_1^{-1}\dot{v}_1$ which are all bounded. $\dot{V}$ can be rewritten as

$$\begin{aligned}\dot{V} &= -y^t A' y - \rho_1(\|s_q\| - \beta_1)^2 + \varphi_1 - \rho_2(\|\tilde{x}_2\| - \beta_2)^2 + \varphi_2 \\ &\quad -\sum_{i=1}^{n} K_{si}\{e_{qi}^2\Delta_{1i} + e_{k\theta i}^2\Delta_{2i}(1-\gamma_3) - e_{qi}e_{k\theta i}\Delta_{3i}\} - \rho_3(\|e_{k\theta}\| - \beta_3)^2 + \varphi_3,\end{aligned}$$

where $y = [s_q^t, \tilde{x}_2^t]$,

$$A' = \begin{pmatrix} (1-\gamma_1)(K_2+D)_e & -(W_2)_E/2 \\ -(W_2)_e/2 & (1-\gamma_2)Q_e \end{pmatrix};$$

with some suitable constants $0 < \gamma_1, \gamma_2, \gamma_3 < 1$.

$$\begin{aligned}\rho_1 = \gamma_1(K_2+D)_e;\quad \beta_1 = \|\omega_1\|/(2\rho_1);\quad \varphi_1 = \|\omega_1\|^2/(4\rho_1);\\ \rho_2 = \gamma_2(Q)_e;\quad \beta_2 = \|\omega_2\|/(2\rho_2);\quad \varphi_2 = \|\omega_2\|^2/(4\rho_2);\\ \rho_3 = \gamma_3(\Delta_2 K_s)_e;\quad \beta_3 = \|v_2\|/(2\rho_3);\quad \varphi_{3i} = \|v_2\|^2/(4\rho_3).\end{aligned}$$

If the following conditions are satisfied

$$\Delta_{1i}\Delta_{2i}(1-\gamma_3) \geq \Delta_{3i}^2/4 \quad i = 1, 2, ...n;$$

$$(K_2+D)_e(Q)_e(1-\gamma_1)(1-\gamma_2) \geq (W_2)_E^2/4,$$

$\dot{V}$ can be simplified as

$$\dot{V} \leq -\rho_1(\|s_q\| - \beta_1)^2 + \varphi_1 - \rho_2(\|\tilde{x}_2\| - \beta_2)^2 + \varphi_2 - \rho_3(\|e_{k\theta i}\| - \beta_3)^2 + \varphi_3.$$

Supposing the control gains are suitably chosen, we have $\rho_1 \geq 0$, $\rho_2 \geq 0$ and $\rho_3 \geq 0$. From the above inequality, we can define an ellipse in a three-dimensional Euclidian space $(\|s_q\|, \|\tilde{x}_2\|, \|e_{k\theta}\|)$, i.e.,

$$\rho_1(\|s_q\| - \beta_1)^2 + \rho_2(\|\tilde{x}_2\| - \beta_2)^2 + \rho_3(\|e_{k\theta i}\| - \beta_3)^2 - (\varphi_1 + \varphi_2 + \varphi_3) = 0,$$

within which the set $\dot{V} \geq 0$. In other words, $\dot{V} < 0$ whenever the norms of the tracking errors $(\|s_q\|, \|\tilde{x}_2\|, \|e_{k\theta i}\|)$ is outside this ellipse. From the above discussion, the tracking errors of the system, under the measurement error $v(t)$, are determined by the control gains and the bounds of $v(t)$ and $\dot{v}(t)$ (or v_0 and ρ_v).

5.7 Simulation Results

By some simulations, we will analyze the performance of the proposed control scheme when it is used to control a two-link SCARA type manipulator with flexible joints. Each link of the manipulator is driven by a DC motor through a gear box. The dynamic model of the two-link manipulator with flexible joints is given by (2.4.15) and (2.4.16).The coefficient matrics of the link subsystem are obtained as

$$M(q_1) = \begin{pmatrix} 2.43 + 2.17cos(q_{12}) & 0.538 + 1.108cos(q_{12}) \\ 0.538 + 1.108cos(q_{12}) & 0.538 \end{pmatrix},$$

$$C(q_1, \dot{q}_1) = \begin{pmatrix} -1.108sin(q_{12})\dot{q}_{12} & -1.108sin(q_{12})\dot{q}_{11} - 1.108sin(q_{12})\dot{q}_{12} \\ 1.108sin(q_{12})\dot{q}_{11} & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} 3.154 & 0 \\ 0 & 3.154 \end{pmatrix}.$$

Therefore, we have $0.4I < M < 4.24I$ and $C \leq 1.64\|\dot{q}_1\|$. In terms of a suitably selected set of parameters, as mentioned in Chapter 2, we can parameterize the link subsystem dynamics linearly as

$$YP + K_s(q_1 - q_2) = 0,$$

where

$$Y(q_1, \dot{q}_1, \ddot{q}_1) = \begin{pmatrix} \ddot{q}_{11} & \ddot{q}_{11} + \ddot{q}_{12} & (v_1 cos(q_{12}) - v_2 sin(q_{12})) & \dot{q}_{11} & 0 \\ 0 & \ddot{q}_{11} + \ddot{q}_{12} & (\ddot{q}_{11} cos(q_{12}) - \dot{q}_1^2 sin(q_{12})) & 0 & \dot{q}_{12} \end{pmatrix},$$

$$P = \begin{pmatrix} m_1 l_{c1}^2 + m_2 l_1^2 \\ m_2 l_{c2}^2 \\ m_2 l_1 l_{c2} \\ D_1 \\ D_2 \end{pmatrix},$$

$$v_1 = 2\ddot{q}_{11} + \ddot{q}_{12}; \quad v_2 = 2\dot{q}_{11}\dot{q}_{12} + \dot{q}_{12}^2.$$

The physical parameters of the manipulator are shown in Table 2.2. The actual parameter vector $\bar{P}$ can be got as $\bar{P} = [2.307, 0.539, 1.108, 3.154, 3.154]^t$.

In order to analyze the performance of the control scheme, the desired reference trajectory is composed of four straight lines which forms a square represented by $p(0)[0.2m, 0.2m]$, $p(1)[0.2m, 0.4m]$, $p(2)[0.4m, 0.4m]$ and $p(3)[0.4m, 0.2m]$ in Cartesian space (shown in Fig.5.2a). For every straight line, the acceleration segment and deceleration segment are fifth-order polynomials constructed with zero velocity and zero acceleration at the start and the end of the path. The maximum velocity of motion is $1.0m/s$, and the time of the acceleration segment and deceleration segment are all $0.2sec.$. The joint trajectories for the straight line can be obtained by using the inverse kinematics(shown in Fig.5.2b,c). The maximum desired joint velocities are $q_{1d_{max}} = 2.44rad/s$ and $q_{2d_{max}} = 4.71rad/s$, respectively.

The purpose of the simulations is to demonstrate the stability and the performance of the controller proposed in this chapter, and furthermore to compare the performance of the tracking controller with those of PD controller. We have three controllers.

1. A conventional PD controller is given by

$$u_i = K_{pi}(q_{1di}G_i - q_{mi}) + K_{vi}(\dot{q}_{1di}G_i - \dot{q}_{mi}),$$

which is a local proportional-derivative controller. The parameters for PD control are chosen as $K_d = diag(100, 100)$ and $K_v = diag(2, 2)$.

2. In the nonadaptive tracking controller, the parameters for the observer are chosen as

$$Q = diag(30, 30); \quad \Lambda_1 = diag(0.01, 0.1);$$
$$\Gamma_1 = diag(200, 200); \quad \Gamma_2 = diag(100, 100).$$

The parameters for the controller are chosen as

$$K_1 = diag(1, 1); \quad K_2 = diag(50, 50);$$
$$K_3 = diag(5K_{s1}, 5K_{s2}); K_4 = diag(1.0K_{s1}, 1.0K_{s2}).$$

Thus $\Delta_{11} = \Delta_{12} = 2.0$, $\Delta_{21} = \Delta_{22} = 12.0$, $\Delta_{31} = \Delta_{32} = 9.0$, $(K_2+D)_e(Q)_e = 1528.8$ and $(W_2)_E = 62.93$. The sufficient conditions (5.4.16) and (5.4.17) are satisfied.

At first, the parameters P in the controller are set to the real manipulator's parameters $\bar{P}$. The simulation results for the nonadaptive tracking controller are shown in Fig.5.5. Next, we set the parameters P in the controller with only about 10% parameter errors (e.g. $P = [2.12, 0.49, 0.99, 2.48, 2.48]$). The simulation results are shown in Fig.5.6

When parameter uncertainty becomes larger (e.g. $P = (1.18, 0.27, 0.56, 1.58, 1.58)^t \neq \bar{P}$), as known in section 5.5, the system will be unstable because the observer run out the sliding path. For this reason, it is desirable to use the adaptive tracking controller.

3. In the adaptive tracking controller, the adaptation gain matrix is selected as $M^{-1} = 0.4I_{5\times5}$ and the initial values of the parameters P are set to

$$P = (1.18, 0.27, 0.56, 1.58, 1.58)^t,$$

which are about 50% parameter errors (i.e.$|P_i - \bar{P}_i| \leq 0.5\bar{P}_i$).

In simulation, the sampling time is set as $T = 0.002sec$. The motor accelerations are obtained by digital differentiation of the motor velocities

$$\ddot{q}_{mci} = \frac{\dot{q}_{mi}(k) - \dot{q}_{mi}(k-1)}{T},$$

where $\ddot{q}_{mc}$ is the calculated acceleration. In practice, we have to face to the noise as analyzed in Section 5.6. We assume that the motor velocities are measured by the sensors with white noises in standard deviation 0.1. Considering the mechanical motor subsystem, we design a low-pass Butterworth digital filter with a third order. The cut-frequency of the filter is 0.3 Nyquist frequency. The frequency response of the filter is shown in Fig.5.3. In absence of the measurement noise, the simulation results are shown in Fig.5.4– Fig.5.7. In presence of the measurement noise(non-adaptive case), we get the simulation results in the cases of no using and using the digital filter as shown in Figs.5.8 and 5.9. From the simulation results we can see that:

R1. In the case of PD control (Fig.5.4), the system is stable. But the maximum link position tracking errors are quite large(almost 2.0°), and the maximum tracking errors in Cartesian space are $1.0 \times 10^{-3}m$.

R2. In the case of the nonadaptive controller, if the parameters are exactly known ($P = \bar{P}$), we can get very good performances as shown in Fig.5.5. The maximum link position tracking errors (in Joint space) are smaller than $0.05°$ (Fig.5.5a), and the maximum Cartesian path errors of the end point are about $2.0 \times 10^{-4}m$(Fig.5.5b). The performances are much better than that of PD controller. But if there exists some parameter uncertainty, the performances of the nonadaptive controller become worse (as shown in fig.5.6). The system will be unstable under larger parameter uncertainty.

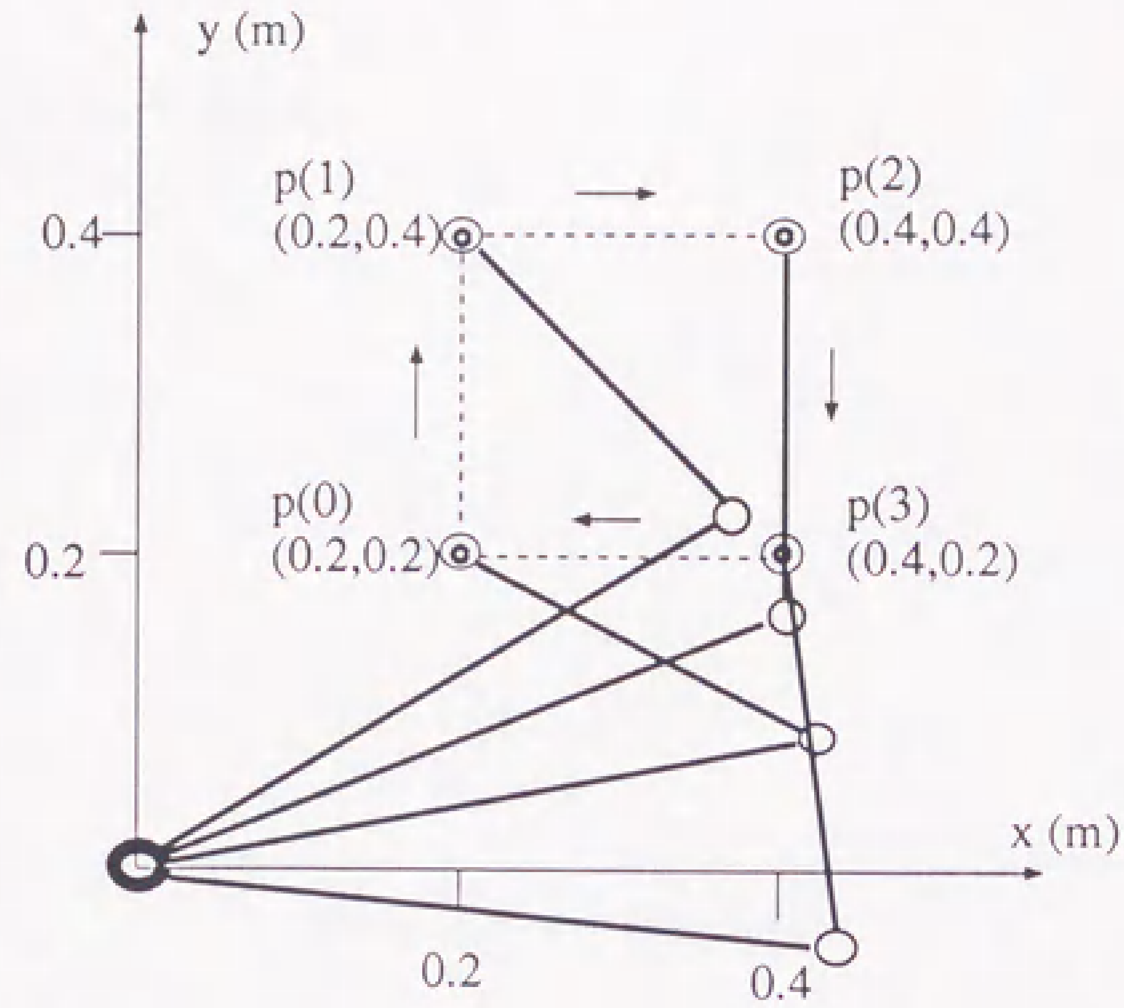
R3. In the case of the adaptive controller(Fig.5.7), even though there exist the parameter uncertainties, the system is stable. The tracking errors become small with the parameter adaptation, which are almost the same with that shown in Fig.5.5 finally. The boundedness of the estimation of the parameters are shown in Fig.5.7d.

R4. Fig.5.5c and Fig.5.7c shows the errors of estimated velocities by the sliding observer. The sliding observer is working within the hypersurface (the estimated link position errors of the observer are equal to zero), and the estimated link velocity errors of observer are bounded.

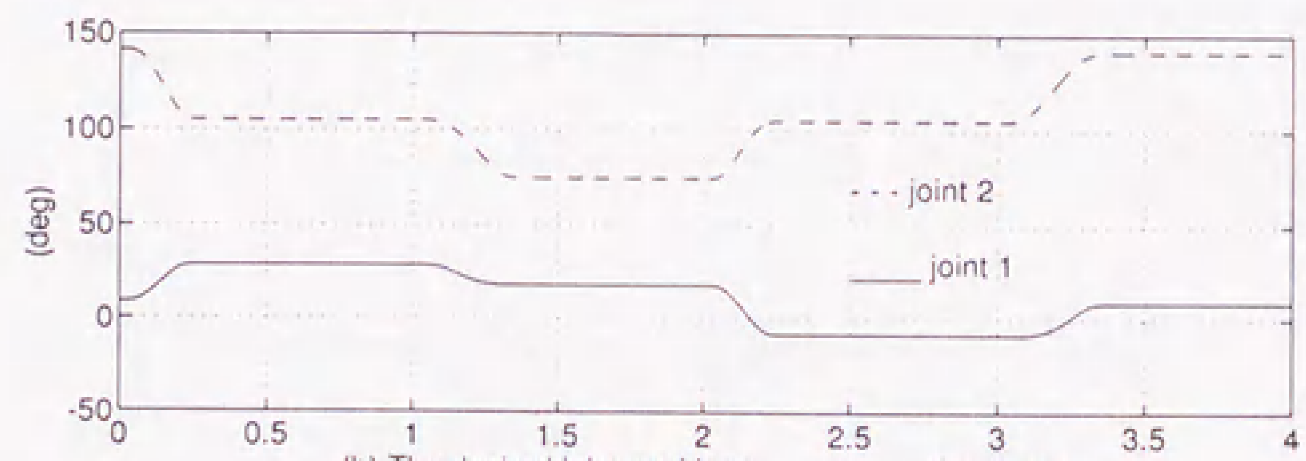
R5. In presence of the measurement of noise, without the digital filter, the tracking errors become larger obviously. The whole system is shaking about its equilibrium point. But in the case of using the filter(Fig.5.8), in spite of the noise, the system is stable, and the tracking errors are also rather small. In the cases of no using and using the filter, the real acceleration $\ddot{q}_{mr}$ and the errors of the calculated acceleration $\ddot{q}_{mc}$ (or $\ddot{q}_{mr} - \ddot{q}_{mc}$) are shown in Fig.5.10 and 5.11 respectively.

5.8 Conclusion

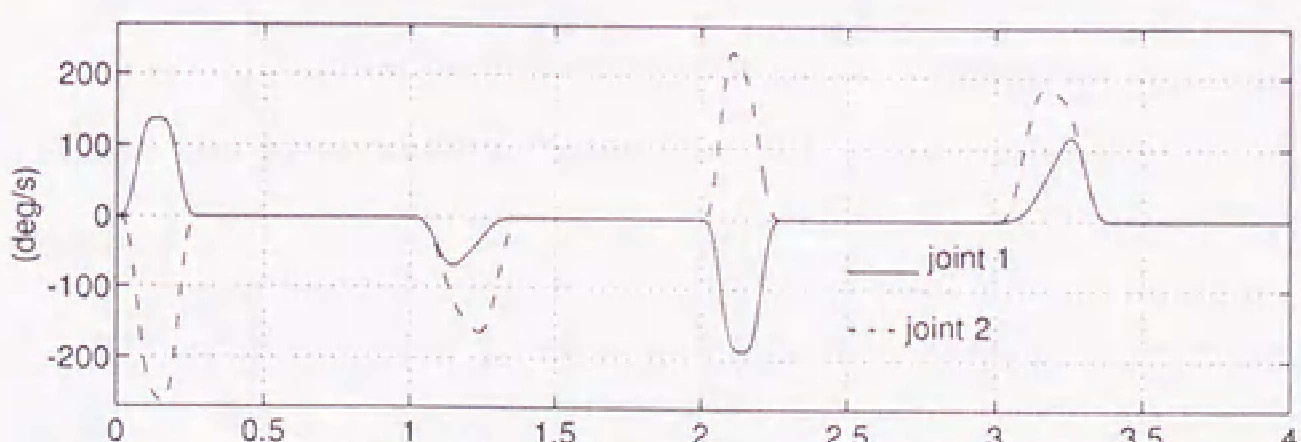
In this chapter, the tracking control schemes combine sliding observers with controllers, which are proposed for controlling manipulator with flexible joints. The design of the controllers consists of two steps. The first, motivated by the studies of control for MRJ, is to design a "desired motor position" which can be calculated from the desired trajectories and observed states. The second, to guarantee the stability of the whole system, is to design the input of the motor subsystem. We gave not only a nonadaptive tracking controller under the condition of the exactly known parameters, but also an adaptive tracking controller when there exist the uncertainty of the parameters. From this research, we have conclusion that the proposed controller is stable and effective. By using this algorithms, we can control manipulators with flexible joints without using the link information. The theorems presented in this chapter state only sufficient conditions of stability of the whole system. Fortunately, these sufficient conditions can be satisfied easily in most applications. In actual control, the control laws are all realized by a digital computer. We also analyzed the stability theoretically by considered the noise caused by digital sampling, calculating and measuring. Finally, some simulations had been made to confirm the effectiveness of the proposed controller.



(a) The desired trajectories in Cartesian Space



(b) The desired joint positions



(c) The desired joint velocities

Fig. 5.2 The desired trajectories

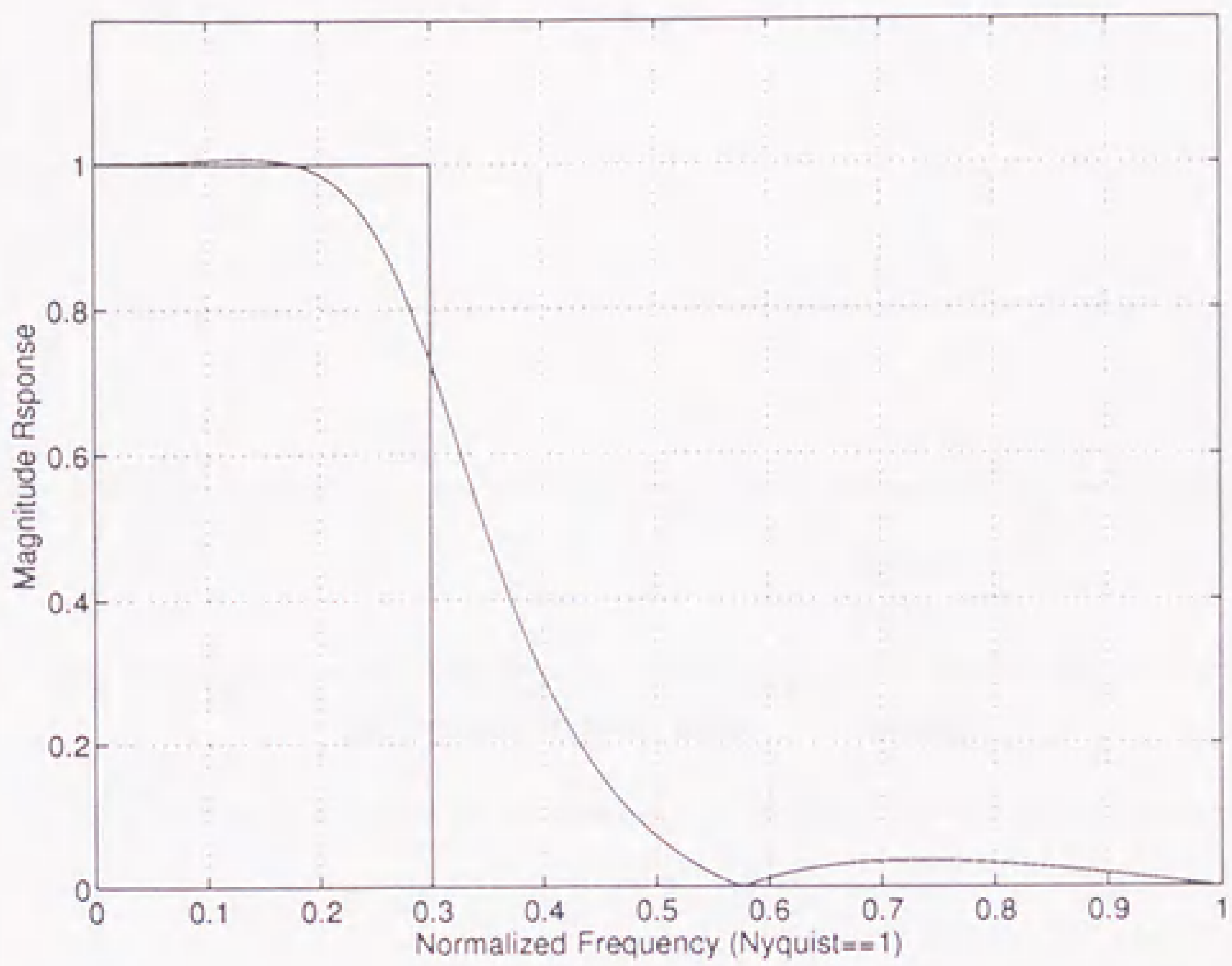


Fig. 5.3 The frequency response of the filter

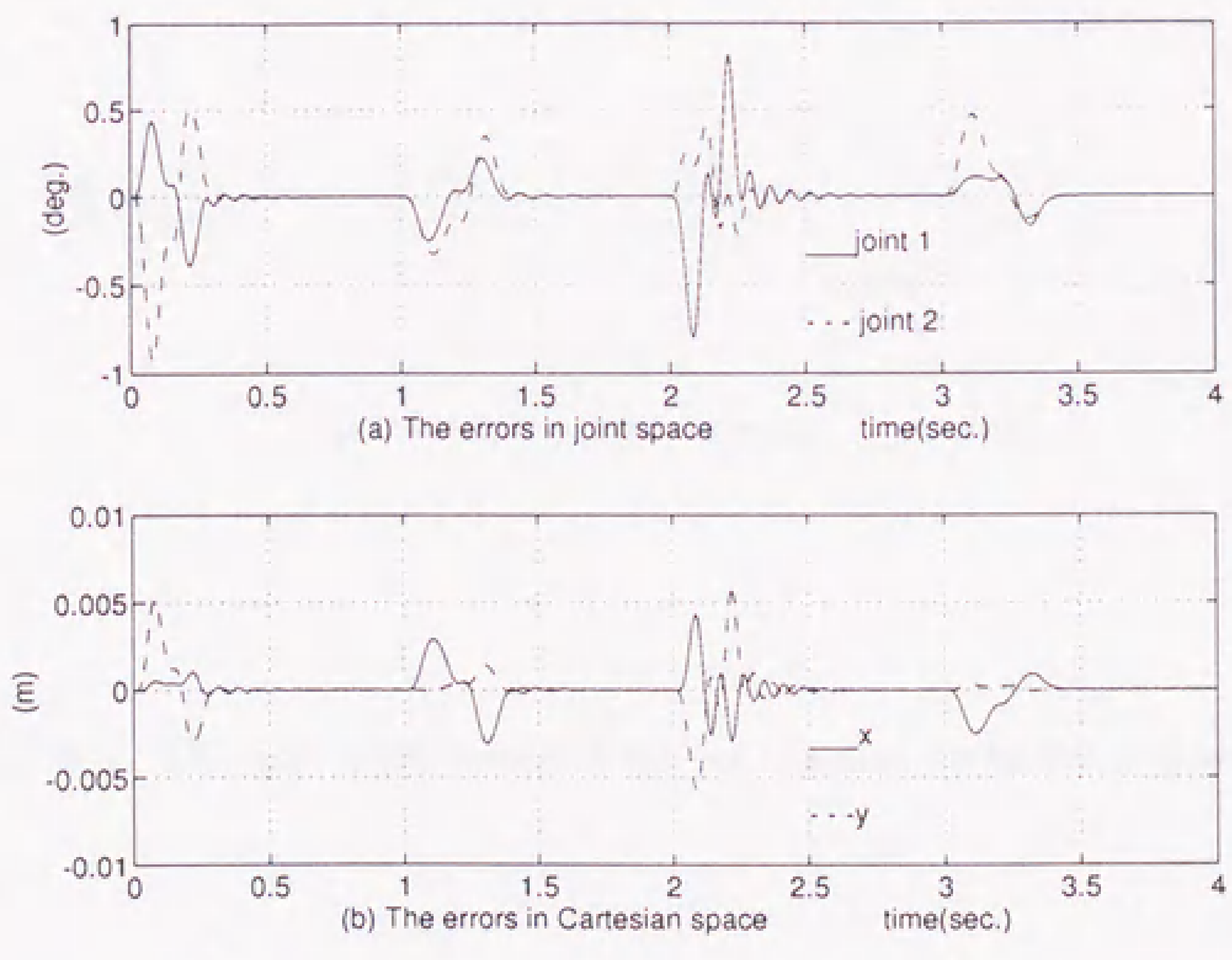


Fig. 5.4 The simulation results of PD controller

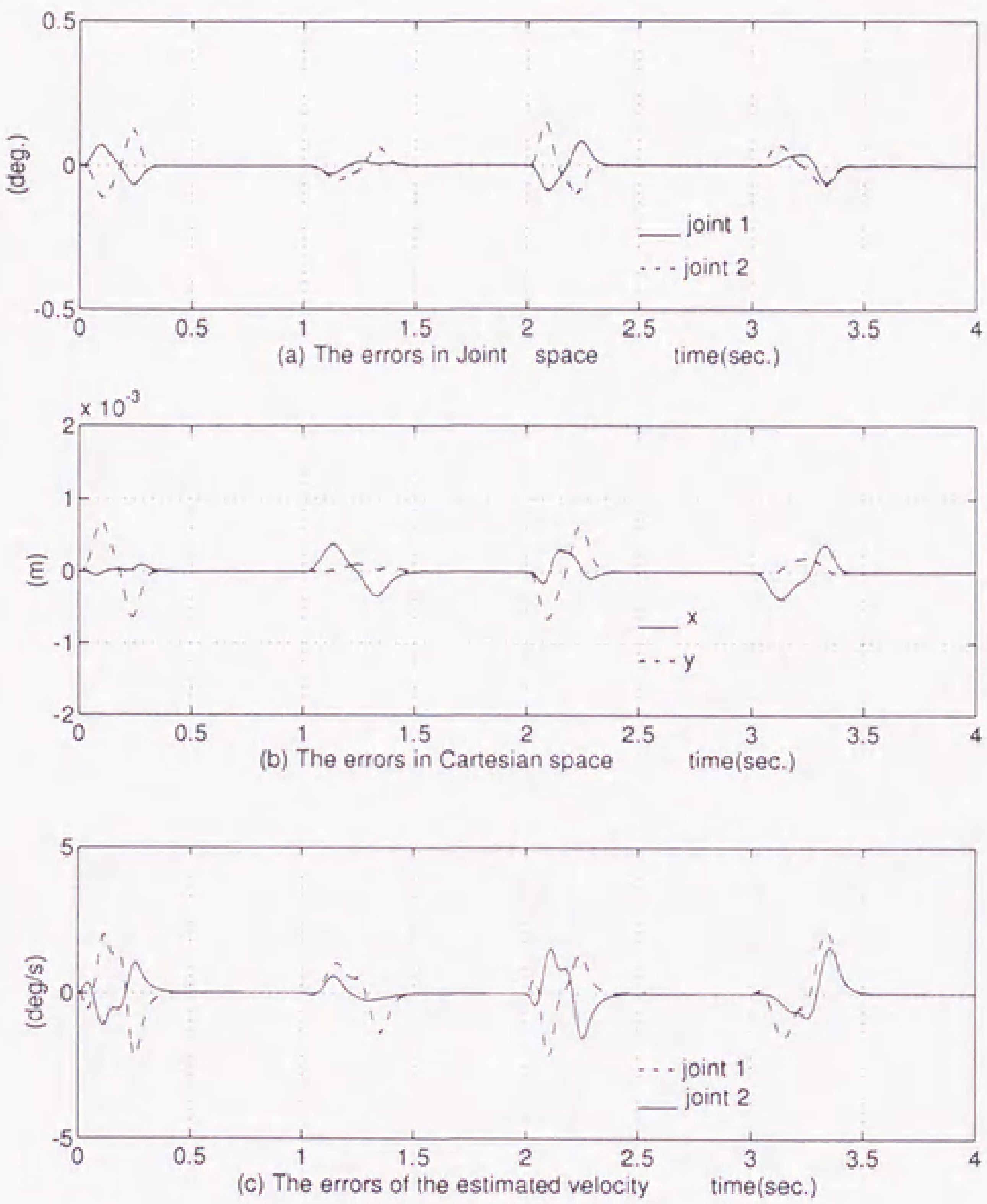


Fig. 5.5 The simulation results of the nonadaptive controller without noise

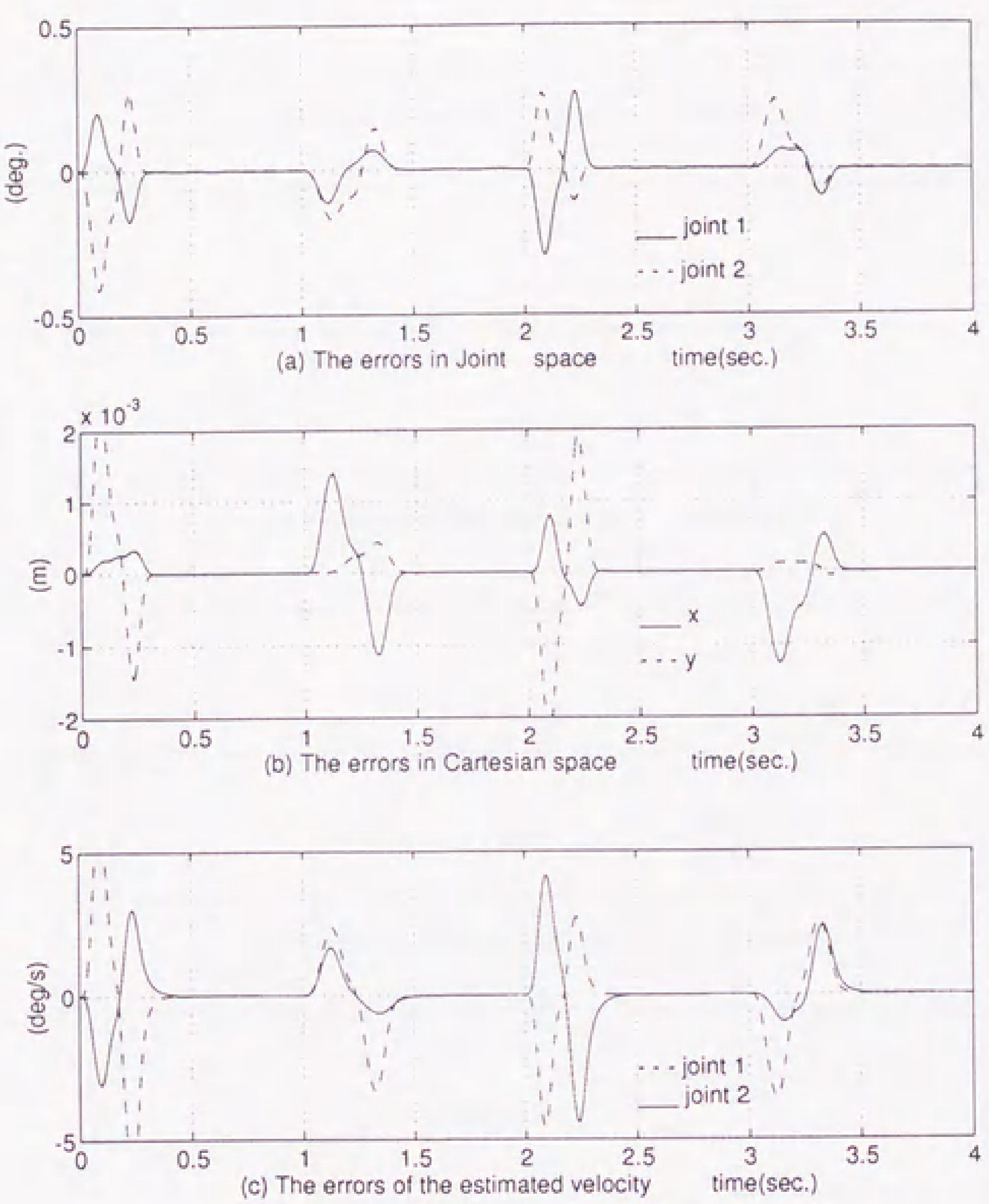


Fig. 5.6 The simulation results of the nonadaptive controller without noise

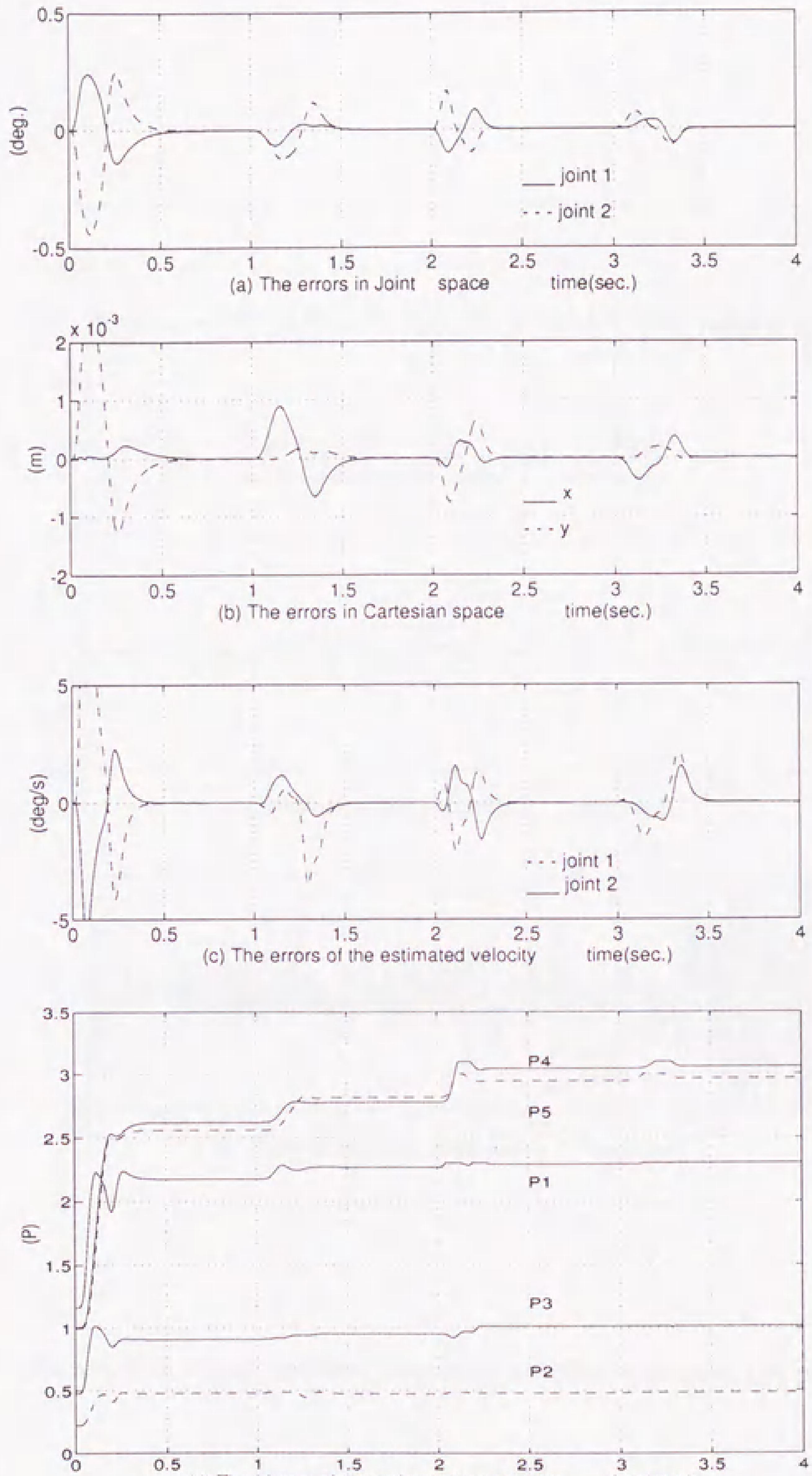


Fig. 5.7 The simulation results of the adaptive controller without noise

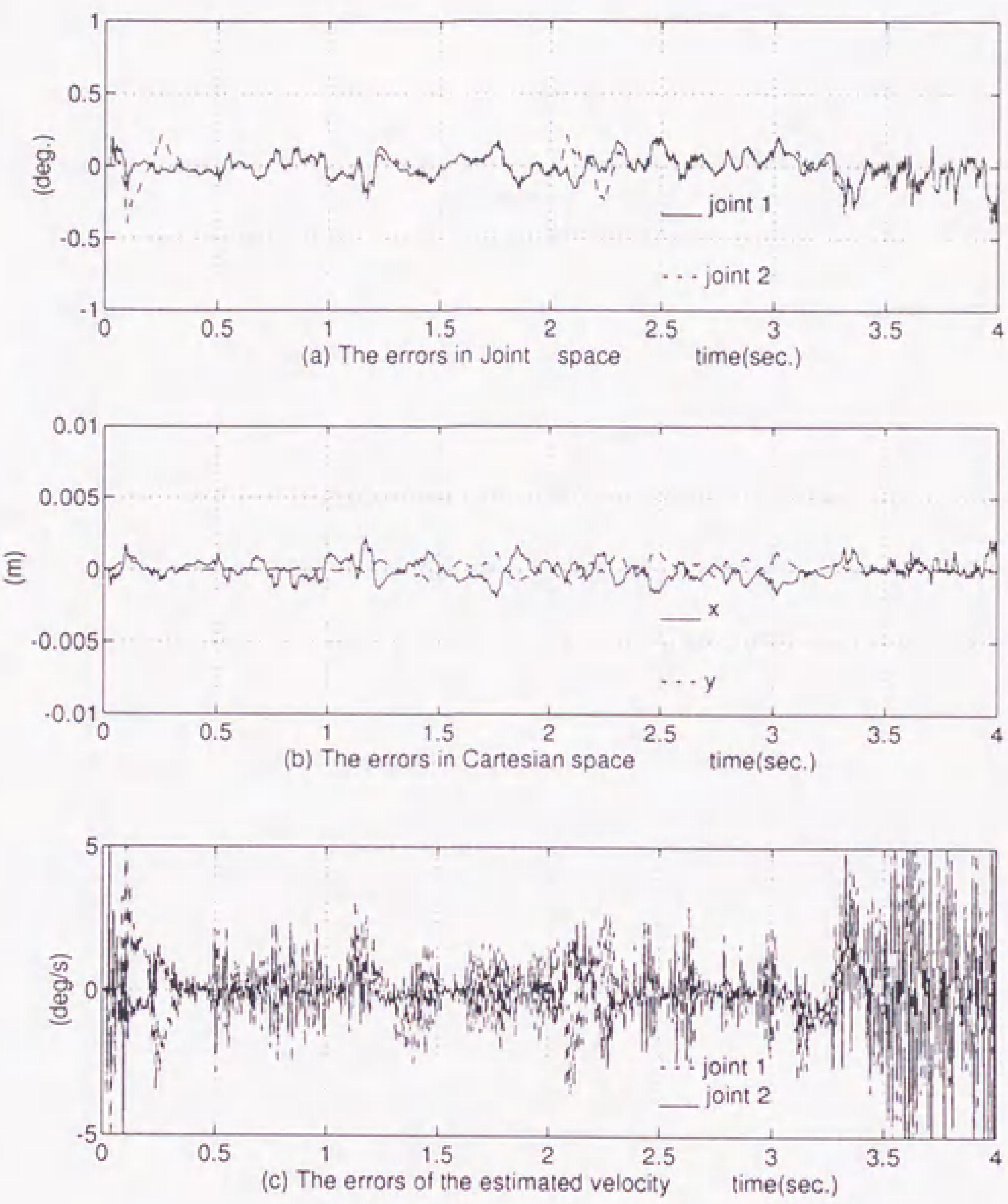


Fig. 5.8 The simulation results of the proposed controller with noise (no using the filter)

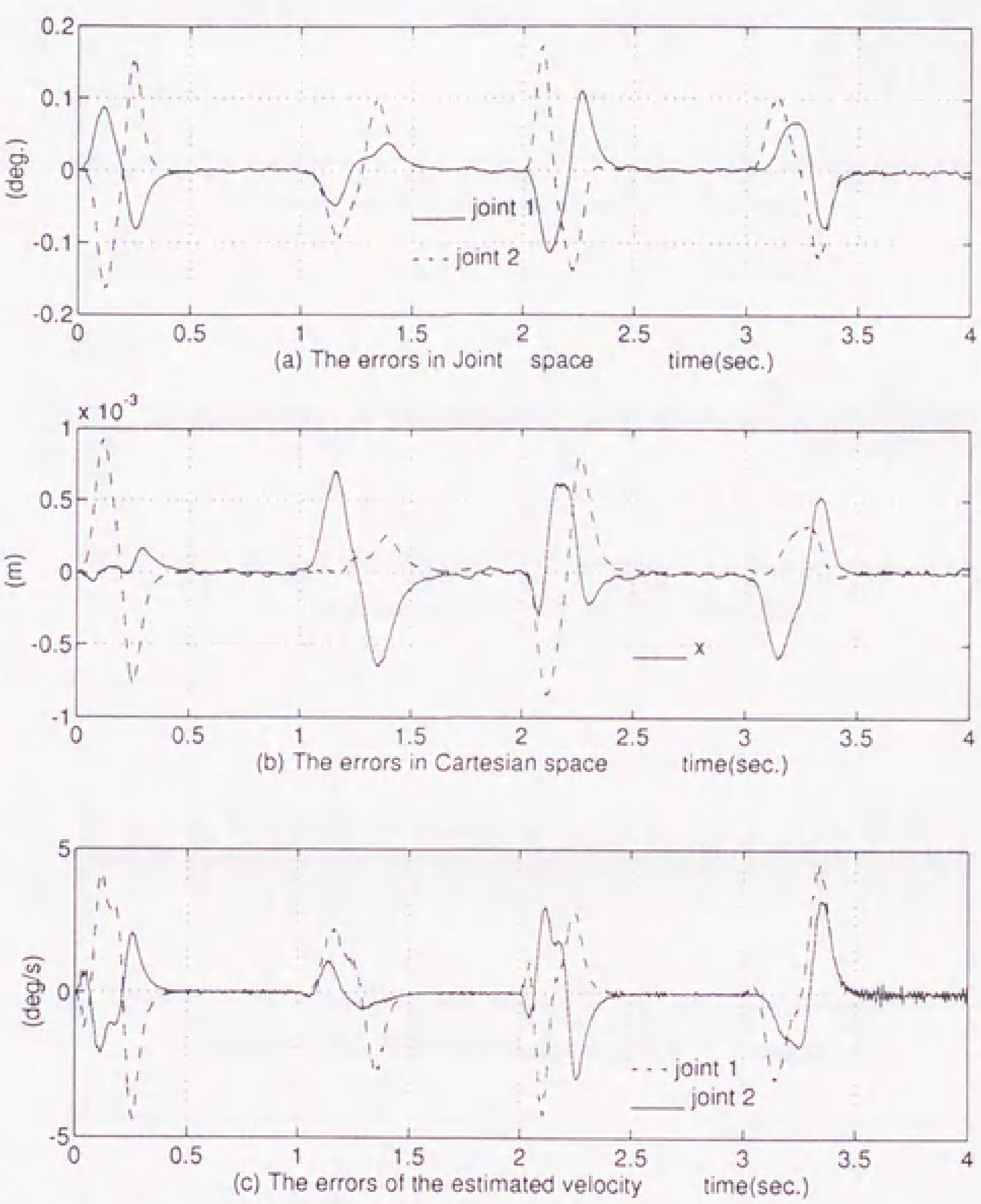


Fig. 5.9 The simulation results of the proposed controller with noise (using the filter)

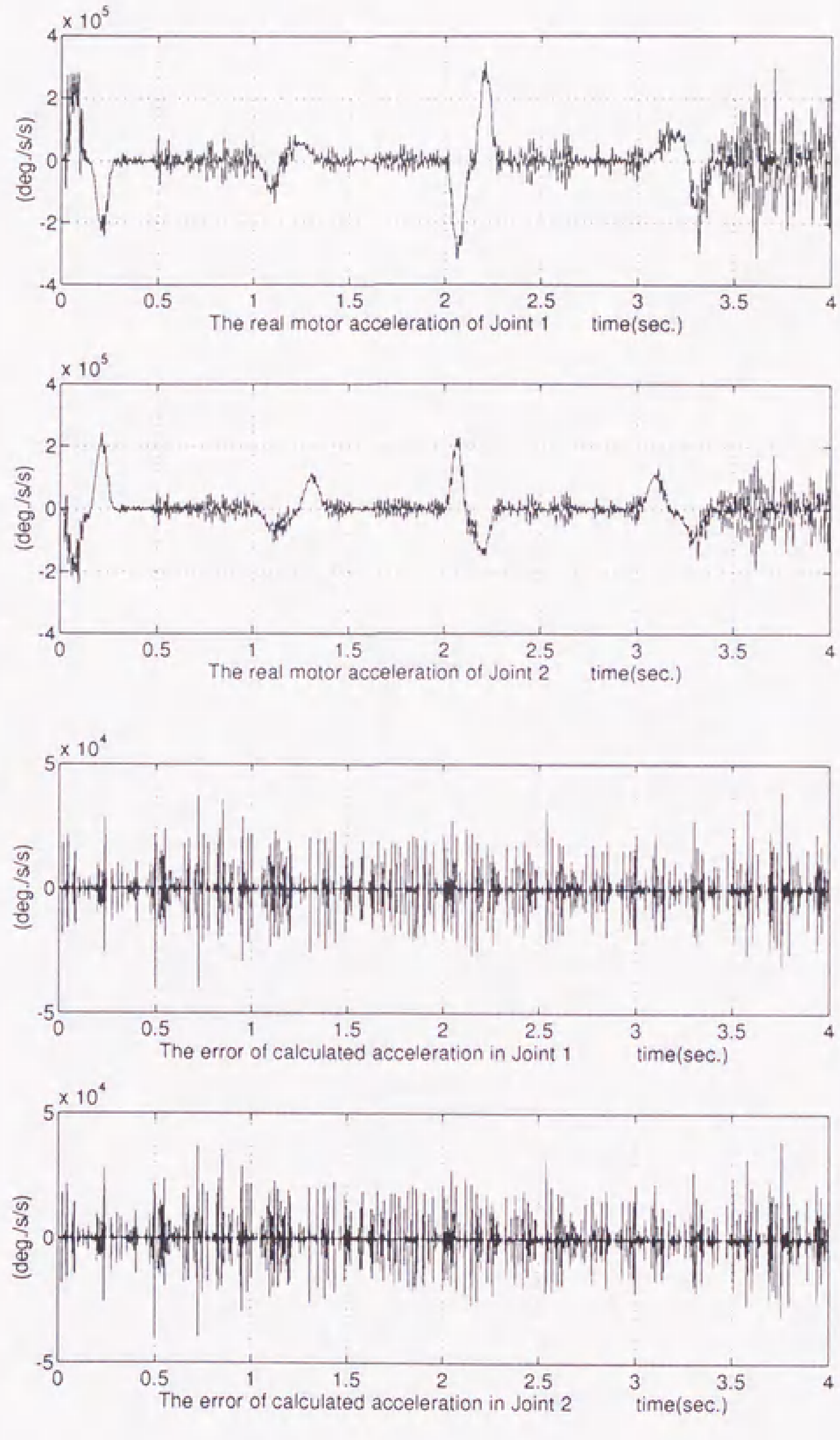


Fig. 5.10 The real motor acceleration and error of calculated acceleration with noise (no using the filter)

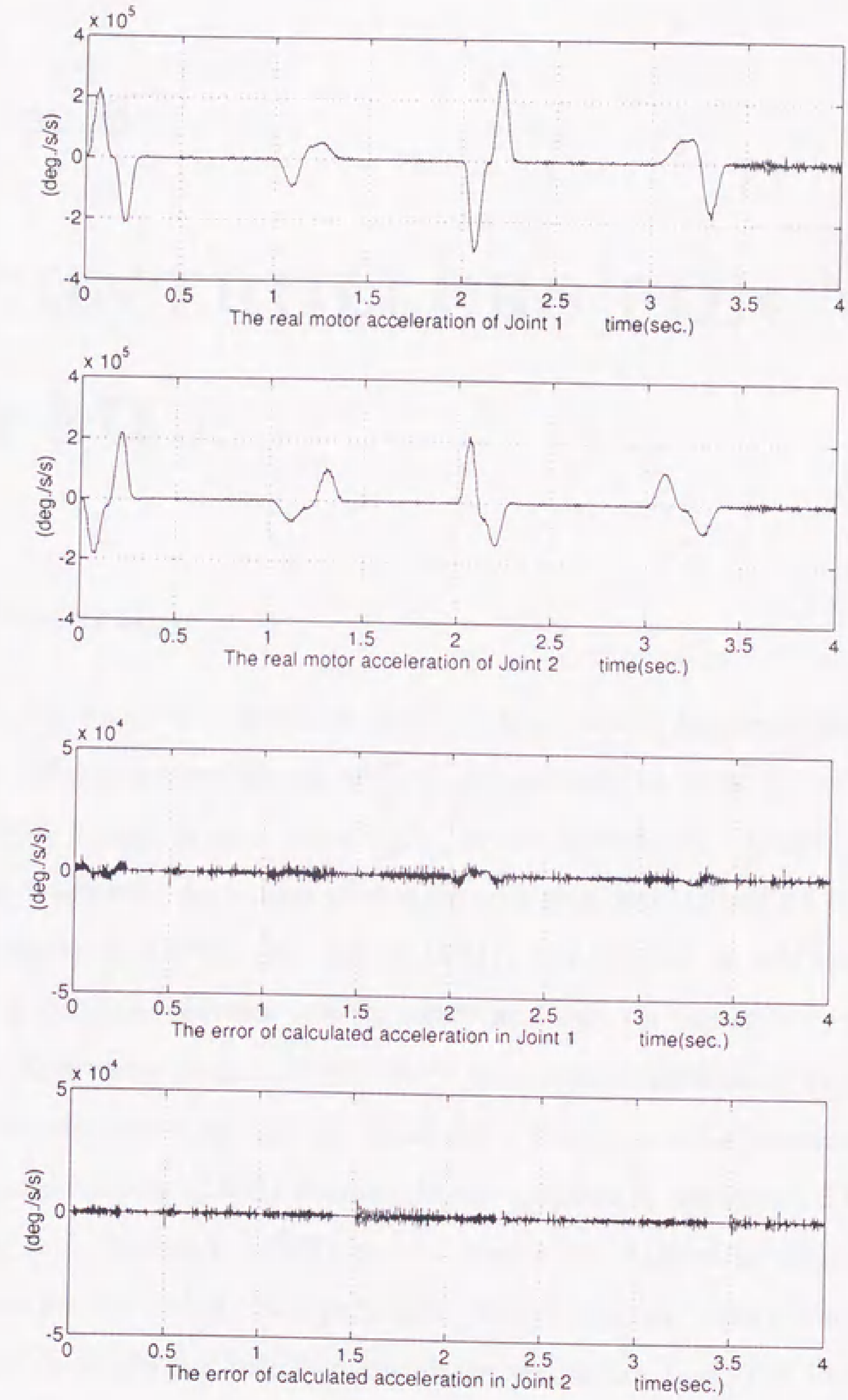


Fig. 5.11 The real motor acceleration and error of calculated acceleration with noise (using the filter)

Chapter 6

H_∞ CONTROLLERS FOR MRJ AND MFJ

6.1 Introduction

The control of a robot manipulator, as described before, has received a great deal of attention. Many approaches have been introduced to solve this problem. The optimal control design is also meaningful to be considered. Linear optimal control based on linearized equations of motion is a standard approach to the solution of such problems (Anderson and Moore 1971). For control of robot manipulators, linear optimal control solutions are standard and rely on linearized equations with regard to an operating point. There have been some researches which related to this topic. Saridis and Lee (1979) made early work on self-optimizing control in robot. Luo and Saridis (1985) studied linear quadratic design of PID controller. Vukobratovic and Kircanski (1982) gave a controller realized by designing the optimal trajectories for robot manipulators. Some studies, concerning suboptimal control based on nonlinear robot manipulator dynamics from the viewpoint of an approximate solution, have also been proposed, for example, Lee and Chen (1983) presented a suboptimal nonlinear control design based on quasi-linearization and linear optimization. Johansson (1990) has studied the optimal motion control with minimization of the applied torques (forces) by finding an explicit solution to the corresponding Hamilton-Jacobi equation. Chen and Feng (1994) has given a H_∞ control design for MRJ, But they did not describe the stability of the system.

In this chapter, we will discuss the H_2 controller for the MRJ first. Then we derived a H_∞ control law which will control the MRJ along the desired trajectory with minimizing index function stably. Finally, the H_∞ control law will be modified to control the MFJ.

6.2 H_2 Controller for MRJ

To better explain the H_∞ controller for MRJ, we begin with H_2 Controller based on the quadratic optimal control problem.

6.2.1 Quadratic Optimization Problem

To derive optimal feedback, the control problem is formulated as a quadratic optimization problem with a performance index $J(x, u)$. Consider a system

$$\dot{x} = f(x,t) + G(x)u \tag{6.2.1}$$

$$u \in R^m, x \in R^n, f(x_0) = 0.$$

Given the performance index

$$J(x,u) = \int_{t_0}^{\infty} L(x,u)dt, \tag{6.2.2}$$

where

$$L(x,u) = \frac{1}{2}(x^t Q x + u^t R u), \tag{6.2.3}$$

we find an optimal control $u = u^*$ that will transfer the system from an initial state $x(t_0)$ to the origin while minimizing $J(x, u)$. The control variable u is weighted with the matrix $R = R^t > 0$, and the state variable x is weighted with $Q = Q^t > 0$. Define the Hamiltonian of the optimization as

$$H(\frac{\partial V(x,t)}{\partial x}, x, u) = (\frac{\partial V(x,t)}{\partial x})^t \dot{x} + L(x,u), \tag{6.2.4}$$

where V satisfies the partial differential equation

$$-\frac{\partial V(x,t)}{\partial t} = (\frac{\partial V(x,t)}{\partial x})^t \dot{x} + L(x,u). \qquad (6.2.5)$$

A necessary and sufficient condition for optimality (Lee 1967, Sage 1968) is to choose a value function V that satisfies the Hamilton-Jacobi equation

$$\begin{aligned} -\frac{\partial V(x,t)}{\partial t} &= min_u H(\frac{\partial V(x,t)}{\partial x}, x, u) \\ &= H(\frac{\partial V(x,t)}{\partial x}, x, u^*). \end{aligned} \qquad (6.2.6)$$

The following lemma gives the solution of the problem stated above.

lemma 6.2.1 *Consider the system described in (6.2.1). Suppose there exists a smooth solution $V(x,t) \geq 0$ with $V(x_0) = 0$. The Hamilton-Jacobi equation (6.2.6) becomes:*

$$\frac{\partial V}{\partial t} + (\frac{\partial V}{\partial x})^t f(x,t) - \frac{1}{2}(\frac{\partial V}{\partial x})^t G R^{-1} G^t \frac{\partial V}{\partial x} + \frac{1}{2} x^t Q x = 0. \qquad (6.2.7)$$

The optimal feedback control law $u = u^*$ that minimizes $J(u)$ is

$$u = u^* = -R^{-1} G^t \frac{\partial V}{\partial x}. \qquad (6.2.8)$$

Proof: The proof of this lemma can be easily obtained by substituting (6.2.1), (6.2.3) and (6.2.8) into (6.2.6). □

The key step in applying the above lemma successfully to any control problem is to find the appropriate function $V(x,t) > 0$ which satisfies the Hamilton-Jacobi equation (6.2.7).

6.2.2 Formulation of the Model of MRJ

In the design of the H_2 controller for MRJ, we have to assume that the desired trajectory $q_{1d}(t) \in R^n$ is given and three times differentiable, the positions and velocities of the links are available for measurement, and the parameter of the system are known *a priori*. The model for the MRJ can be written as

$$M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + D\dot{q}_1 + G(q_1) = \tau, \qquad (6.2.9)$$

where τ is the input torque vector for MRJ. The state of tracking error is defined as

$$e_l \stackrel{\text{def}}{=} \begin{pmatrix} \dot{e}_l \\ e_l \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} \dot{q}_1 - \dot{q}_{1d} \\ q_1 - q_{1d} \end{pmatrix}. \tag{6.2.10}$$

The tracking problem of generalized position q_1 is reduced to the regulation problem of error state e_l. Now, we divide input torque τ into two parts, τ_1 and τ_2, i.e.,

$$\tau = \tau_1 + \tau_2. \tag{6.2.11}$$

τ_1 is aimed at compensating the gross nonlinear terms in manipulator system, and a special choice of τ_1 is

$$\begin{aligned} \tau_1 &= M\ddot{q}_{1d} + C\dot{q}_{1d} + D\dot{q}_1 + G \\ &\quad -T_{22}^{-1}MT_{21}e_{l2} - T_{22}^{-1}CT_{21}e_{l1}, \end{aligned} \tag{6.2.12}$$

where $T_2 = [T_{21}T_{22}]$ with $T_{21} \in R^{n\times n}$ and $T_{22} \in R^{n\times n}$ are diagonal positive matrices.

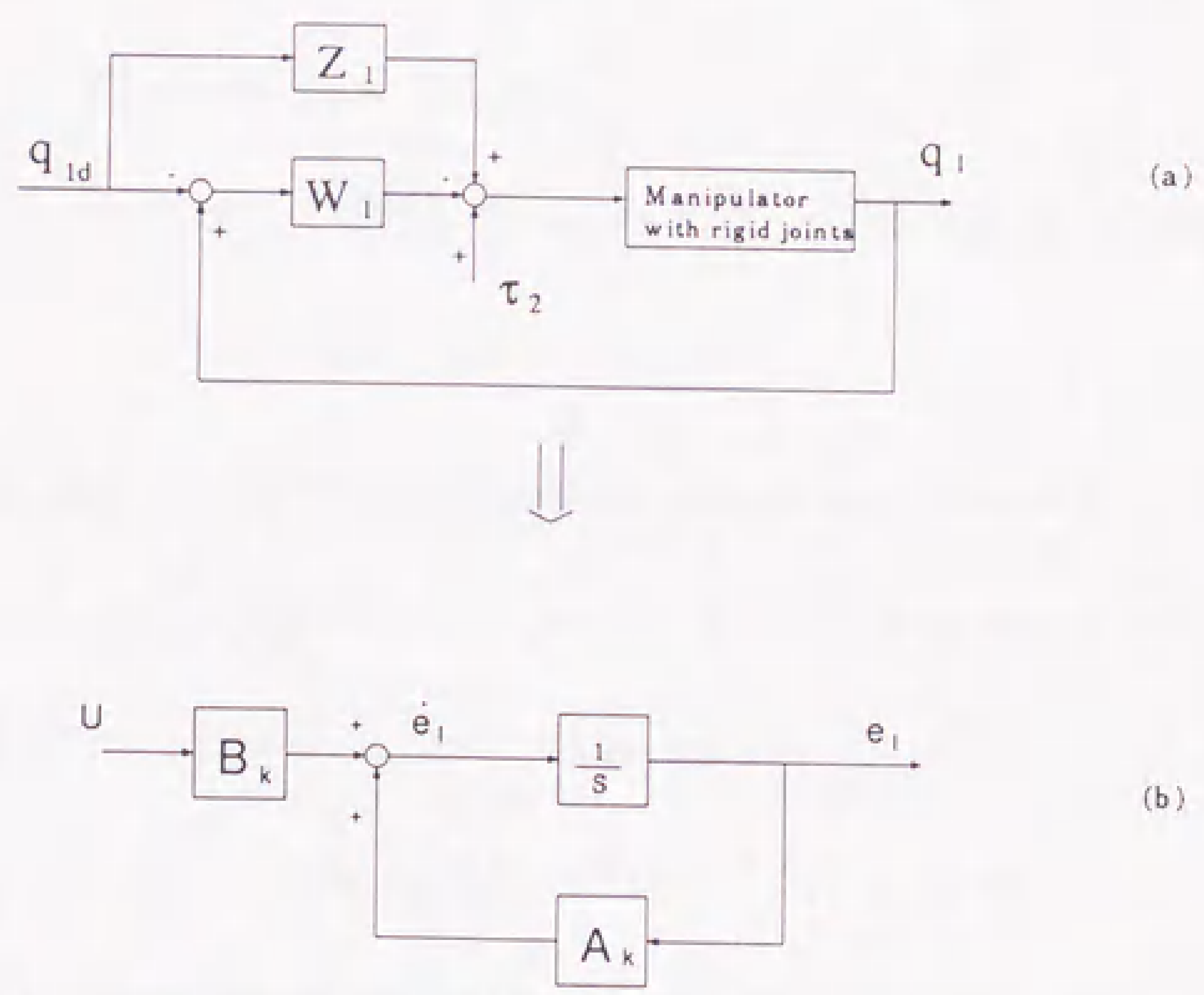


Fig. 6.1 The block diagram of the MRJ system
$Z_1 = M\ddot{q}_{1d} + C\dot{q}_{1d} + D\dot{q}_1 + G$;
$W_1 = T_{22}^{-1}(CT_{21}, MT_{21}e_{l2})e_l$.

Substituting (6.2.11) and (6.2.12) into (6.2.9) (as shown in Fig.6.1(a)), we can obtain

$$MT_2\dot{e}_l + CT_2e_l = U, \tag{6.2.13}$$

where $U = T_{22}\tau_2$. The equation of motion (6.2.13) from U to e_l (as shown in Fig.6.1(b)) are then

$$\dot{e}_l = A_k e_l + B_k U, \tag{6.2.14}$$

where $B_k = T_0^{-1} B M^{-1}$

$$A_k = T_0^{-1} \begin{pmatrix} -T_{22}^{-1} T_{21} & T_{22}^{-1} \\ 0_{n \times n} & -M^{-1} C \end{pmatrix} T_0,$$

$$B = \begin{pmatrix} 0_{n \times n} \\ I_{n \times n} \end{pmatrix}; T_0 = \begin{pmatrix} I_{n \times n} & 0_{n \times n} \\ T_{21} & T_{22} \end{pmatrix}.$$

Remark 6.2.1 In (6.2.12), the term $M\ddot{q}_{1d} + C\dot{q}_{1d} + D\dot{q}_1 + G$ is designed to compensate the nonlinearity in the MRJ system. This term also can be viewed as a feedforward in terms of the exactly known physical parameters.

6.2.3 The H_2 Control Law Design

As proposed by Johansson (1990), we chose a value function V in the Hamiltonian as

$$V = \frac{1}{2} e_l^t P(t) e_l. \tag{6.2.15}$$

Along (6.2.13), the corresponding Hamilton-Jacobi equation (6.2.7) becomes

$$e_l^t (\frac{\partial P}{\partial t} + P A_k + A_k^t P - P B_k R^{-1} B_k^t P + Q) e_l = 0. \tag{6.2.16}$$

Therefore, a Riccati equation is obtained as follows

$$\frac{\partial P}{\partial t} + P A_k + A_k^t P - P B_k R^{-1} B_k^t P + Q = 0. \tag{6.2.17}$$

In general, it is not easy to solve the Riccati equation. However, in the manipulator system, the Riccati equation can be further simplified to an algebraic matrix equation with an suitable choice of the matrix function $P(t)$

$$P(t) = T_0^t \begin{pmatrix} S & 0 \\ 0 & M(q_1) \end{pmatrix} T_0,$$

where S is positive definite symmetric constant matrix. From the property P1, the Riccati equation becomes to an algebraic matrix equation

$$\begin{pmatrix} 0 & S \\ S & 0 \end{pmatrix} - T_0^t B R^{-1} B^t T_0 + Q = 0. \tag{6.2.18}$$

Then, from (6.2.8) we have

$$U = U^* = -R^{-1} T_2 e_l, \tag{6.2.19}$$

where U^* is the optimal input for (6.2.13). From the above analysis, we know that if the matrices S and T_0 solve the algebraic matrix equation (6.2.18), the value function V of (6.2.15) is a solution to the Hamilton-Jacobi equation. The sufficient condition for stable optimal control has been solved by Johansson (1990). However the design dose not guarantee any robustness with respect to the uncertainty of the physical parameters. Hereafter, we try to design a H_∞ optimal controller.

6.3 H_∞ optimal controller for MRJ

For linear time-invariant finite dimensional systems, the H_∞ norm is well know and commonly described in the frequency domain (Francis 1987). A variety of practical problems can be formulated as the optimization of the H_∞ norm of closed-loop transfer function. In addition, the reliable state-space solution for H_∞ optimization is already available as in Doyle *et al.* (1989). It is therefore interesting to see if the H_∞ optimization can also be applied to nonlinear systems with the same benefits of robustness carried over from the linear case. Researchers are already moving in this direction, for example Van der Schaft (1991). The generalized notations of H_∞ norm and H_∞ optimization are given as follows.

6.3.1 H_∞ optimization Problem

Let us now consider a smooth system which is additionally affected by an unknown disturbance d in the following way

$$\dot{x} = f(x,t) + G(x,t)u + E(x,t)d, \quad y = x, \tag{6.3.1}$$

$$u \in R^m, x \in R^n, d \in R^q, f(x_0) = 0.$$

Then a H_∞ optimization problem can be described as follows.

Definition 6.1 Find the smallest value $\gamma^* \geq 0$ such that for any $\gamma \geq \gamma^*$ there exists a smooth state feedback $u = l(x)$, $l(x_0) = 0$ which results in that the L_2-gain from d to z ($z = ((Q^{1/2}x)^t, (R^{1/2}u)^t)^t$) is less than or equal to γ, i.e.,

$$\int_0^T \|z\|^2 dt \leq \gamma^2 \int_0^T \|d(t)\|^2 dt,$$

where $u \in L_2$ and $d \in L_2$. □

Rhee and Speyer (1991) and Limrbeer *et al.* (1992) proposed a performance criterion which includes a desired disturbance attenuation level γ for the perturbed tracking dynamics (6.3.1)

$$min_{u\in L_2} max_{0\neq d\in L_2} \frac{\int_{t_0}^{\infty} \frac{1}{2}(x^tQx + u^tRu)dt}{\int_{t_0}^{\infty} \frac{1}{2}(d^td)dt} \leq \gamma^2. \tag{6.3.2}$$

So the performance criterion is equivalent to the following minmax problem

$$min_{u\in L_2} max_{0\neq d\in L_2} \int_{t_0}^{\infty} \frac{1}{2}(x^tQx + u^tRu - \gamma^2 d^t d)dt \leq 0.$$

To derive optimal feedback, the control problem is formulated as a quadratic optimization problem by defining a performance index $J(x, u, d)$ as

$$J(x, u, d) = \int_{t_0}^{\infty} L(x, u, d)dt, \tag{6.3.3}$$

where

$$L(x, u, d) = \frac{1}{2}(x^tQx + u^tRu - \gamma^2 d^t d). \tag{6.3.4}$$

By introducing the value function $V(x, t) = min_u max_d J(x, u, d)$, the performance criterion (6.3.2) is equivalent to $V(x(0), 0) \leq 0$ (when $x(0) = 0$) and $\dot{V}(x(0), 0) \leq 0$. In other words, given the performance index $J(x, u, d)$, we should find an optimal control $u = u^*$ that moves the system from an arbitrary initial state $x(t_0)$ to the origin of the error space while minimizing $J(x, u, d)$ under the worst case disturbance $d = d^*$.

The control variable u is weighted with the matrix $R = R^t > 0$, and the vector of velocity and position errors x are weighted with $Q = Q^t > 0$. Define the Hamiltonian of the optimization as

$$H(\frac{\partial V(x,t)}{\partial x}, x, u, d) = (\frac{\partial V(x,t)}{\partial x})^t \dot{x} + L(x,u,d). \qquad (6.3.5)$$

A necessary and sufficient condition for optimality is to choose a value function V that satisfies the Hamilton-Jacobi equation

$$-\frac{\partial V(x,t)}{\partial t} = min_u max_d H(\frac{\partial V(x,t)}{\partial x}, x, u, d) = H(\frac{\partial V(x,t)}{\partial x}, x, u^*, d^*). \qquad (6.3.6)$$

The following lemma gives the solution of the problem stated above.

Lemma 6.3.1 *Consider the system described in (6.3.1). Let $\gamma \geq 0$. Suppose there exists a smooth solution $V(x,t) \geq 0$ with $V(x_0) = 0$ to the Hamilton-Jacobi equation:*

$$\frac{\partial V}{\partial t} + (\frac{\partial V}{\partial x})^t f(x,t) + \frac{1}{2}(\frac{\partial V}{\partial x})^t (\frac{1}{\gamma^2} EE^t - GR^{-1}G^t)\frac{\partial V}{\partial x} + \frac{1}{2}x^t Q x = 0. \qquad (6.3.7)$$

Then the closed-loop system with

$$\left.\begin{array}{rl} u = u^* = & -R^{-1}G^t \frac{\partial V}{\partial x} \\ d = d^* = & \frac{1}{\gamma^2} E^t \frac{\partial V}{\partial x} \end{array}\right\} \qquad (6.3.8)$$

has L_2-gain, from d to $z = ((Q^{1/2}x)^t, (R^{1/2}u)^t)^t$, and Q, R are symmetric positive definite.

Proof: The proof of this lemma can be easily obtained by substituting (6.3.1), (6.3.4) and (6.3.8) into (6.3.6). □

In the following sections, we will apply the above theorem to the control of MRJ, i.e., to solve the partial differential equation (6.3.7) for $V(x,t)$. Then the solution, substituted into (6.3.8), would yield the optimal control law under the worst case disturbance.

6.3.2 The Formulation of the Model (6.2.9) with Uncertainty

The controller corresponding to (6.2.11), (6.2.12) and (6.2.19) can be viewed as a feedforward computed torque control with respect to the M, C, D and G. The effectiveness of the controller is thus dependent on the adequate knowledge of the physical parameters of the system. In practice, this parameters are time variable and uncertain. Therefore, it is needed to design robust H_∞ controller. In the design of the H_∞ controller, we have to assume that the desired trajectory $q_{1d}(t) \in R^n$ is given and three times differentiable, the positions and velocities of the links are available for measurement. Instead of assuming that the parameter of the system are known *a priori* in Section 6.2, we further assume that the physical parameters of the MRJ are not exactly known, but the nominal values and the bounds of the uncertainty for the parameters are known *a priori*.

In other words, we can divide the each parameter matrix in the model (6.2.9) into a nominal part and uncertain part

$$\begin{aligned} M(q_1) &= M_0(q_{1d}) + \Delta M(q_1) + \delta_m(e_{l1}), \\ C(q_1, \dot{q}_1) &= C_0(q_{1d}, \dot{q}_{1d}) + \Delta C(q_1, \dot{q}_1) + \delta_c(e_{l1}, e_{l2}), \\ D &= D_0 + \Delta D, \\ G(q) &= G_0(q_{1d}) + \Delta G(q_1) + \delta_g(e_{l1}), \end{aligned}$$

where M_0, C_0, D_0 and G_0 are the functions of nominal parameter and the desired trajectory which are assumed to be known *a priori*, whereas ΔM, ΔC, ΔD and ΔG are determined by the bounds of the parameter uncertainty and δ_m, δ_c, and δ_g are determined by the system error. It should be noted that matrics of the nominal part are time variant along the desired trajectory. The dynamic model (6.2.9) can be taken as follows

$$M_0\ddot{q}_1 + C_0\dot{q}_1 + D_0\dot{q}_1 + G_0 = \tau + d_w, \tag{6.3.9}$$

where

$$\begin{aligned} d_w = & -(\Delta M + \delta_m)\ddot{q}_1 - (\Delta C + \delta_c)\dot{q}_1 - \Delta D\dot{q}_1 \\ & -(\Delta G + \delta_g). \end{aligned} \tag{6.3.10}$$

The input torque τ are also divided into two parts, τ_1 and τ_2, i.e.,

$$\tau = \tau_1 + \tau_2. \tag{6.3.11}$$

τ_1 is aimed at compensating the gross nonlinear terms in the system. A special choice of τ_1 is

$$\begin{aligned} \tau_1 &= M_0\ddot{q}_{1d} + C_0\dot{q}_{1d} + D_0\dot{q}_{1d} + G_0 \\ &\quad -T_{22}^{-1}M_0T_{21}e_{l2} - T_{22}^{-1}C_0T_{21}e_{l1}. \end{aligned} \tag{6.3.12}$$

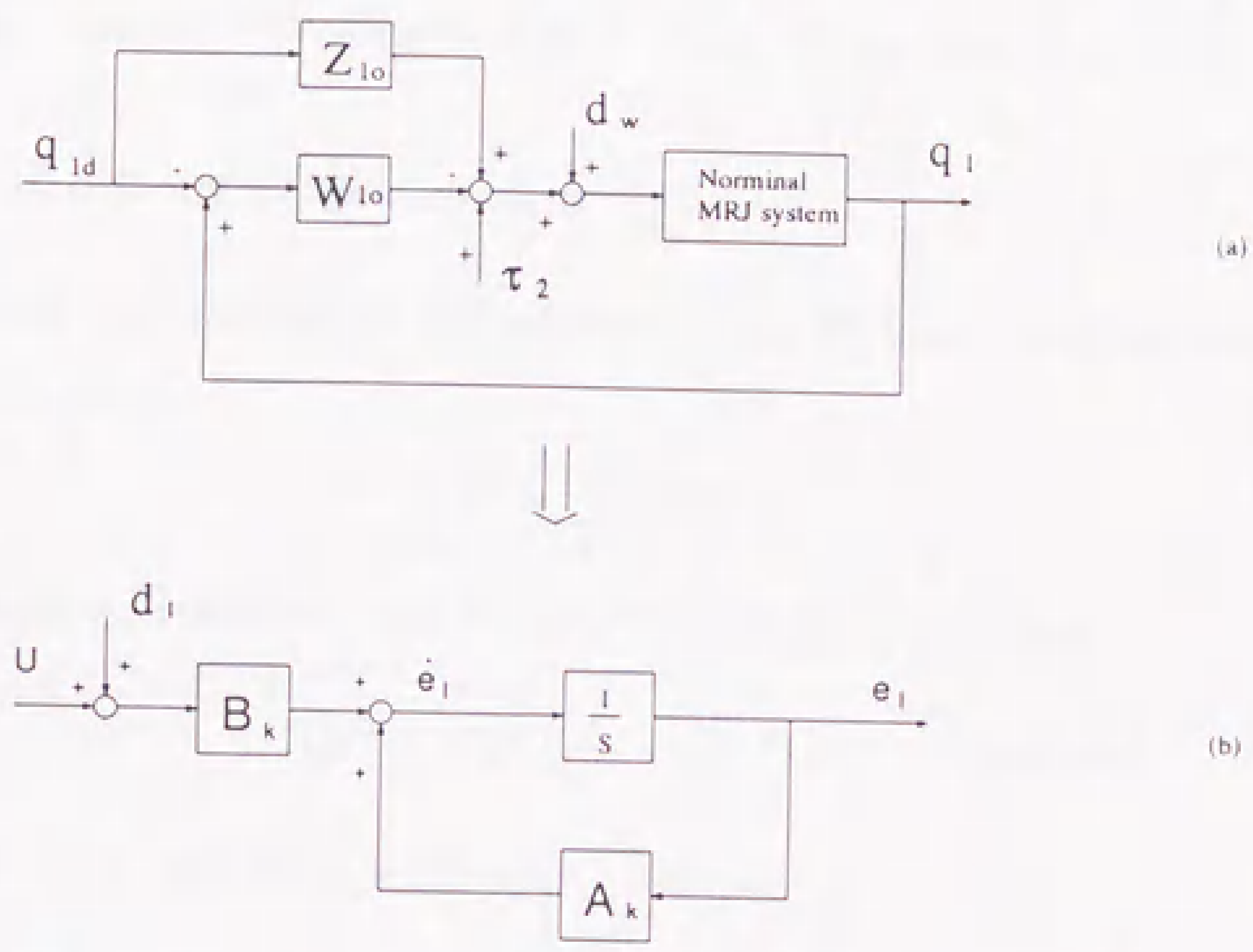


Fig. 6.2 The block diagram of the MRJ with uncertainty.
$Z_{1o} = M_0\ddot{q}_{1d} + C_0\dot{q}_{1d} + D_0\dot{q}_{1d} + G_0$;
$W_{1o} = T_{22}^{-1}(C_0T_{21}, M_0T_{21})e_l$.

Substituting (6.3.11) and (6.3.12) into (6.3.9) (as shown in Fig.6.2(a)), we can obtain

$$M_0T_2\dot{e}_l + C_0T_2e_l = U + d_1, \tag{6.3.13}$$

where $U = T_{22}\tau_{fd2}$, $d_1 = T_{22}(d_w + D_0e_{l2})$. The equation of motion (6.3.13) from U to e_l (as shown in Fig.6.2(b)) is then

$$\dot{e}_l = A_ke_l + B_k(U + d_1), \tag{6.3.14}$$

where $B_k = T_0^{-1} B M_0^{-1}$,

$$A_k = T_0^{-1} \begin{pmatrix} -T_{22}^{-1}T_{21} & T_{22}^{-1} \\ 0_{n\times n} & -M_0^{-1}C_0 \end{pmatrix} T_0,$$

$$B = \begin{pmatrix} 0_{n\times n} \\ I_{n\times n} \end{pmatrix}, T_0 = \begin{pmatrix} I_{n\times n} & 0_{n\times n} \\ T_{21} & T_{22} \end{pmatrix}.$$

Remark 6.3.1 In (6.3.12), the term $M_0\ddot{q}_{1d} + C_0\dot{q}_{1d} + D_0\dot{q}_{1d} + G_0$ is a feedforward by using the known nominal parameters and desired link trajectories, such that it is bounded and can be calculated off-line when the trajectories are given. □

6.3.3 The H_∞ Control law design

We will describe the design of the control input U based on Lemma 6.3.1. As in Section 6.2, we choose

$$V = \frac{1}{2} e_l^t P(t) e_l. \tag{6.3.15}$$

The corresponding Hamilton-Jacobi equation (6.3.7) becomes

$$e_l^t(\frac{\partial P}{\partial t} + PA_k + A_k^t P + PB_k(\frac{1}{\gamma^2} - R^{-1})B_k^t P + Q)e_l = 0. \tag{6.3.16}$$

Therefore, a Riccati equation is obtained follows

$$\frac{\partial P}{\partial t} + PA_k + A_k^t P + PB_k(\frac{1}{\gamma^2} - R^{-1})B_k^t P + Q = 0. \tag{6.3.17}$$

Similar to that in Section 6.2 the matrix function $P(t)$ is chosen as

$$P(t) = T_0^t \begin{pmatrix} S & 0 \\ 0 & M_0(q_{1d}) \end{pmatrix} T_0.$$

The Riccati equation becomes to an algebraic matrix equation

$$\begin{pmatrix} 0 & S \\ S & 0 \end{pmatrix} + T_0^t B(\frac{1}{\gamma^2} I - R^{-1}) B T_0 + Q = 0. \tag{6.3.18}$$

Then we have

$$U = U^* = -R^{-1} T_2 e_l, \quad d_1 = d_1^* = \frac{1}{\gamma^2} T_2 e_l, \tag{6.3.19}$$

where U^* ans d_1^* are optimal input and worst case disturbance, respectively. From the above analysis, we know that if the matrices S and T_0 solve the algebraic matrix equation (6.3.17), the value function V of (6.3.15) is a solution to the Hamilton-Jacobi equation. Let the weighting matrix Q with Cholesky factors Q_1 and Q_2 be chosen such that

$$Q = Q^t = \begin{pmatrix} Q_1^t Q_1 & Q_{12} \\ Q_{12}^t & Q_2^t Q_2 \end{pmatrix}.$$

Therefore, the matrix eqaution (6.3.17) can be split into the following four equations.

$$Q_1^t Q_1 + T_{21}^t(\frac{1}{\gamma^2}I - R^{-1})T_{21} = 0, \qquad (6.3.20)$$

$$S + Q_{12} + T_{21}^t(\frac{1}{\gamma^2}I - R^{-1})T_{22} = 0, \qquad (6.3.21)$$

$$S + Q_{12}^t + T_{22}^t(\frac{1}{\gamma^2}I - R^{-1})T_{21} = 0, \qquad (6.3.22)$$

$$Q_2^t Q_2 + T_{22}^t(\frac{1}{\gamma^2}I - R^{-1})T_{22} = 0. \qquad (6.3.23)$$

From (6.3.20) and (6.3.23), $(R^{-1} - 1/\gamma^2 I)$ must be positive since $Q_1^t Q_1$ and $Q_2^t Q_2$ are positive definite. Let

$$(R^{-1} - \frac{1}{\gamma^2}I) = R_1^t R_1 > 0.$$

Then $T_{21} = R_1^t Q_1$ and $T_{22} = R_1^t Q_2$, i.e.,

$$T_0 = \begin{pmatrix} I & 0 \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} I & 0 \\ R_1^t Q_1 & R_1^t Q_2 \end{pmatrix}.$$

So, from (6.2.21) and (6.2.22), we have

$$S = \frac{1}{2}[(Q_1^t Q_2 + Q_2^t Q_1) - (Q_{12}^t + Q_{12})].$$

The sufficient condition for stable optimal control requires that $S = S^t > 0$, which is almost the same with that proposed by Johasson (1990) in solving the H_2 control problem.

6.3.4 Stability of the System

In this subsection, we analyze the stability and robustness of the whole system. From (6.2.9), we have

$$\ddot{q}_1 = M^{-1}\tau - M^{-1}(C\dot{q}_1 + D\dot{q}_1 + G).$$

So

$$\begin{aligned} d_w &= -(\Delta M + \delta_m)M^{-1}\tau + (\Delta M + \delta_m)M^{-1}(C\dot{q}_1 + D\dot{q}_1 + G) \\ &\quad -(\Delta C + \delta_c)\dot{q}_1 - \Delta D\dot{q}_1 - (\Delta G + \delta_g). \end{aligned}$$

Because the inertia matrix is positive definite and bounded, we have

$$(\Delta M + \delta_m)M^{-1} = (I - M_0 M^{-1}) < I.$$

Therefore d_1 can be bounded as

$$\|d_1\| \leq \gamma_1\|e_l\| + \gamma_2\|e_l\|^2 + \gamma_3\|\tau\| + \gamma_0 = \gamma_1'\|e_l\| + \gamma_2'\|e_l\|^2 + \gamma_0'. \qquad (6.3.24)$$

By using the property P4, we know that γ_1', γ_2', and γ_0' are known positive numbers. The following theorem gives the stability of the nominal system (6.3.9) under the control law (6.3.12) and (6.3.19).

Theorem 6.3.2 *Consider the system (6.3.14) with disturbance d_1 which is bounded as (6.3.24). If the control law is chosen as (6.3.12) and (6.3.19), the equilibrium point $e_l = 0$ of the system is ultimately bounded stable.*

Proof: Select the value function V as (6.3.15). From (6.3.6), the time derivative of V is

$$\begin{aligned} \frac{\partial V}{\partial t} &= -L(e_l, U_f^*, d^*) - (\frac{\partial V(e_l,t)}{\partial e_l})^t \dot{e}_l|_{U^*,d^*} \\ &= -L(e_l, U^*, d^*) - e_l^t T_0^t B\frac{1}{\gamma^2}B^t T_0 e_l + e_l^t T_0^t B d_1 \\ &\leq -\frac{1}{2}e_l^t(Q + T_0^t B R^{-1} B^t T_0)e_l + \|e_l\|\|T_0^t B\|(\gamma_1'\|e_l\| + \gamma_2'\|e_l\|^2 + \gamma_0') \\ &\leq -\lambda_{QR}\|e_l\|^2 + \|e_l\|(\rho_1\|e_l\| + \rho_2\|e_l\|^2 + \rho_0) \\ &= -\lambda_1\|e_l\|^2 + \rho_0\|e_l\|, \qquad (6.3.25) \end{aligned}$$

where λ_{QR} is the minimum eigenvalue of $(Q + T_0^t B R^{-1} B^t T_0)$ and $\lambda_1 = \lambda_{QR} - \rho_1 - \rho_2\|e\|$. The first inequality is obtained by using (6.3.24). Since λ_1 should be larger than zero, we have the region of attraction as

$$\Omega = \{e \in R^{2n} : \|e_l\| < \frac{\lambda_{QR} - \rho_1}{\rho_2}\}.$$

Now, we can define an ellipse in Euclidian space, i.e.,

$$\lambda_1(\|v\| - \varepsilon_1)^2 - \varepsilon_0 = 0,$$

where $\varepsilon_1 = \rho_0/(2\lambda_1)$ and $\varepsilon_0 = \rho_0^2/(4\lambda_1)$. Within this ellipse the set $\dot{V} \geq 0$. In other words, $\dot{V}_1 < 0$ whenever the norm of the tracking errors $\|v\|$ is outside this ellipse. The size of the ellipse is determined by λ_1 (which is related to the control gain) and the bound of the uncertainty. □

Remark 6.3.2 The H_∞ controller has robustness respect to not only the parameter uncertainty but also the unmodeled dynamics. For example, if the unmodeled dynamics T_d has the form of (3.4.2), the error of the system is also ultimately bounded, but the size of the convergent ellipse will change. □

6.4 Robust H_∞ control for MFJ

In the design of the H_∞ controller for MFJ, we have to assume that the desired trajectory $q_{1d}(t) \in R^n$ is given and four times differentiable, the physical parameters of the MFJ are not exactly known, but the nominal values and the bounds of the uncertainty for the parameters are known *a priori*, the positions and velocities of the links are available for measurement. Similar to that in Chapter 5, we separate the model of MFJ into two subsystem, i.e., the link subsystem and the motor subsystem. The model (2.2.5) and (2.2.6) are rewritten as

$$M_0\ddot{q}_1 + C_0\dot{q}_1 + D_0\dot{q}_1 + G_0 = \tau_f + d_w, \tag{6.4.1}$$

$$Y_2(\ddot{q}_2, \dot{q}_2)P_2 + Y_{k2}P_k = u, \tag{6.4.2}$$

where $\tau_f = K_s(q_2 - q_1)$ is a fictional input torque vector. Y_2 is regressor matrix of known function of $\ddot{q}_2$, $\dot{q}_2$, and $Y_{k2} \stackrel{\text{def}}{=} diag[q_1 - q_2]$, P_2 and P_k are motor parameter and stiffness parameter vectors respectively (this definition is the same with that in Chapter 4). First, we assume that the link subsystem is controlled by a desired "fictional input torque" τ_{fd}, which is defined in the same way as that in (6.3.11). This desired "fictional input torque" τ_{fd} will make the link subsystem stable with robustness respect to the parameter uncertainty and unmodeled disturbance as described in section 6.3.

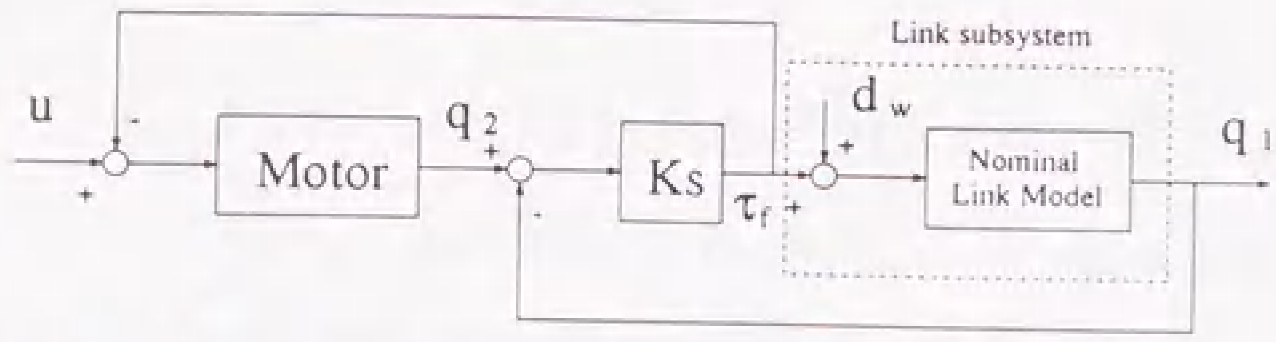


Fig. 6.3 The block diagram of the manipulator with flexible joints

Now let us consider that the input torque τ_f is applied through the motor subsystem. The manipulator with flexible joints can be shown in block diagram as Fig.6.3. The desired motor trajectory can be computed as follows.

$$q_{2d} = q_{1d} + K_{s0}^{-1}(M_0\ddot{q}_{1d} + C_0\dot{q}_{1d} + D_0\dot{q}_{1d} + G_0). \tag{6.4.3}$$

It is important to notice that the desired motor trajectories q_{2d}, $\dot{q}_{2d}$ and $\ddot{q}_{2d}$ are dependent on the link trajectories (q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$, $q_{1d}^{(3)}$ and $q_{1d}^{(4)}$) and on the nominal link physical parameters. The state tracking error for motor subsystem is defined as

$$e_m = \begin{pmatrix} e_{m1} \\ e_{m2} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} q_2 - q_{2d} \\ \dot{q}_2 - \dot{q}_{2d} \end{pmatrix}.$$

Then, the desired "fictional input torque" can be rewritten as

$$\begin{aligned} \tau_{fd} &= \tau_{fd1} + \tau_{fd2} \\ &= M_0\ddot{q}_d + C_0\dot{q}_d + D_0\dot{q}_d + G_0 \\ &\quad -T_{22}^{-1}(M_0T_{21}e_{l2} + C_0T_{21}e_{l1} + R^{-1}T_2e_l) \\ &= K_{s0}(q_{2d} - q_{1d}) - We_l, \end{aligned} \tag{6.4.4}$$

where $W = T_{22}^{-1}(M_0 T_{21} + R^{-1}T_{21}, C_0 T_{21} + R^{-1}T_{22})$. Because of the dynamics of the motor subsystem, $\tau_f = K_s(q_2 - q_1)$ is not equal to τ_{fd}. A substitution of

$$\tau_f = \tau_{fd} + \Delta\tau_f = \tau_{fd} + K_s(e_{m1} - e_{l1}) + We_l + (K_s - K_{s0})(q_{2d} - q_{1d})$$

into link subsystem (6.4.1) leads

$$M_0 T_2 \dot{e} + C_0 T_2 e = U_f + d_1 + T_{22}(K_s(e_{m1} - e_{l1}) + We_l + (K_s - K_{s0})(q_{2d} - q_{1d})). \quad (6.4.5)$$

The actual control input u in (6.4.2) is chosen as

$$u = Y_{d2}^t P_{2o} + Y_{kd2}^t P_{k0} - K_p s_m - We_l - t_m - t_k, \quad (6.4.6)$$

where $s_m \overset{\text{def}}{=} e_{m2} + T_{21}T_{22}^{-1}e_{m1}$, $\dot{q}_{rm} \overset{\text{def}}{=} q_{2d} - T_{21}T_{22}^{-1}e_{m1}$ and K_p is a positive definite diagonal matrix, $Y_{d2}^t P_{2o} \overset{\text{def}}{=} J_{m0}\ddot{q}_{rm} + D_{m0}\dot{q}_{rm}$, $Y_{kd2}^t P_{k0} \overset{\text{def}}{=} K_{s0}(q_{2d} - q_{1d})$. P_{2o} and P_{k0} are nominal vectors of the motor parameters and stiffness parameters, respectively. t_m and t_k are robust compensation terms for the uncertainty in the motor parameters and stiffness of the flexible joints, respectively, which are defined as

$$t_m \overset{\text{def}}{=} \begin{cases} \frac{Y_{d2}^t s_m \rho_2}{\|Y_{d2}^t s_m\|} & \|Y_{d2}^t s_m\| > 0 \\ 0 & \|Y_{d2}^t s_m\| = 0 \end{cases}; \quad t_k \overset{\text{def}}{=} \begin{cases} \frac{Y_{kd2}^t s_m \rho_k}{\|Y_{kd2}^t s_m\|} & \|Y_{kd2}^t s_m\| > 0 \\ 0 & \|Y_{kd2}^t s_m\| = 0 \end{cases}; \quad (6.4.7)$$

where ρ_2 and ρ_k are the upper bounds of the uncertainties of the motor parameters and stiffness parameters. The whole control system is shown in Fig.6.4.

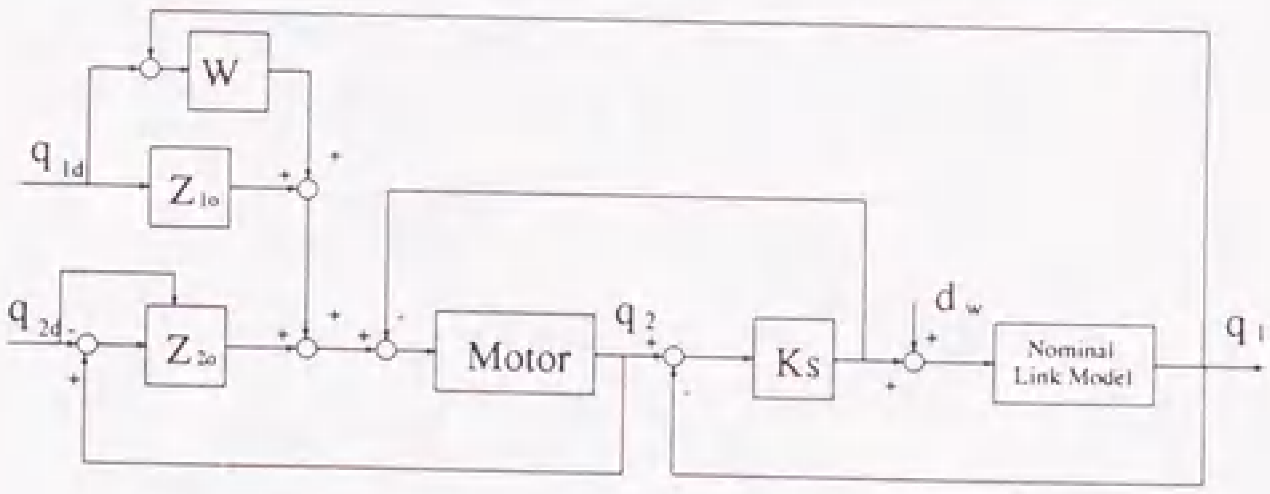


Fig. 6.4 The block diagram of the whole system
$Z_{1o} = M_0\ddot{q}_{1d} + C_0\dot{q}_{1d} + D_0\dot{q}_{1d} + G_0$;
$W_{1o} = T_{22}^{-1}(C_0T_{21}, M_0T_{21})e_l$;
$Z_{2o} = Y_{dm}^t P_{m0} - K_p s_m - We_l - t_m$.

Substitution of (6.4.6) into (6.4.2) leads to

$$J_m \dot{s}_m = -(D_m + K_p)s_m - K_s(e_{m1} - e_{l1}) - We_l + Y_{d2}(P_2 - P_{2o}) + Y_{kd2}(P_k - P_{k0}) - t_m - t_k. \quad (6.4.8)$$

The following theorem summarizes the stability of the whole system

Theorem 6.4.1 *Considering the closed-loop control system defined by (6.4.5) and (6.4.8), the equilibrium point* $(e_l, e_m) = 0$ *of the system is uniformly bounded stable.*

Proof: Select a Lyapunov function candidate as

$$V_1 = V + \frac{1}{2}(s_m^t T_{22}^2 J_m s_m + (e_{m1} - e_{l1})^t T_{22}^2 K_s (e_{m1} - e_{l1})), \tag{6.4.9}$$

where V is shown in (6.3.15). From (6.3.6), we have

$$\frac{\partial V}{\partial t} = -L(e_l, U_f^*, d^*) - (\frac{\partial V(e_l, t)}{\partial e_l})^t \dot{e}_l|_{U^*, d^*}.$$

Then the time derivative of (6.4.9) becames

$$\begin{aligned}
\frac{dV_1}{dt} &= -L(e_l, U^*, d^*) - e_l^t T_0^t B \frac{1}{\gamma^2} B^t T_0 e_l + e_l^t T_0^t B d_1 \\
&\quad + e_l^t T_0^t B T_{22} K_s (e_{m1} - e_{l1}) + e_l^t T_0^t B T_{22} W e_l \\
&\quad + e_l^t T_0^t B T_{22} (K_s - K_{s0})(q_{2d} - q_{1d}) \\
&\quad - s_m^t T_{22}^2 ((D_m + K_p) s_m + K_s (e_{m1} - e_{l1}) + W e_l) \\
&\quad + (e_{m2} - e_{l2})^t T_{22}^2 K_s (e_{m1} - e_{l1}) \\
&\quad + s_m^t T_{22}^2 (Y_{d2}^t (P_2 - P_{20}) - t_m) \\
&\quad + s_m^t T_{22}^2 (Y_{kd2}^t (P_k - P_{k0}) - t_k) \\
&\le -\frac{1}{2} e_l^t (Q + T_0^t B R^{-1} B^t T_0) e_l + e_l^t T_0^t B (d_1 + d_2) \\
&\quad - e_m^t T_0^t B (D_m + K_p) B^t T_0 e_m \\
&\quad - (e_{m1} - e_{l1})^t T_{12} T_{22} K_s (e_{m1} - e_{l1}) \\
&\quad + (e_l - e_m)^t T_0^t B T_{22} W e_l,
\end{aligned} \tag{6.4.10}$$

where $d_2 = T_{22}(K_s - K_{s0}) K_{s0} (M_0 \ddot{q}_d + C_0 \dot{q}_d + D_0 \dot{q}_d + G_0)$. From the property P4, we have

$$d_1 \le \gamma_1' \|e_l\| + \gamma_2' \|e_l\|^2 + \gamma_3' \|e_{m1} - e_{l1}\| + \gamma_0', \quad d_2 \le \gamma_k.$$

So

$$e_l^t T_0^t B (d_1 + d_2) \le \|e_l\| (\rho_1' \|e_l\| + \rho_2' \|e_l\|^2 + \rho_3' \|e_{m1} - e_{l1}\| + \rho_{k0})$$

$$(e_l - e_m)^t T_0^t B T_{22} W e_l \leq \beta_1 \|e_l\|^2 + \beta_2 \|e_l\| \|e_m\|.$$

Then

$$\begin{aligned}
\frac{dV_1}{dt} &\leq -\lambda_{QR}\|e_l\|^2 - \lambda_{DK}\|e_m\|^2 - \lambda_{TK}\|e_{m1} - e_{l1}\|^2 \\
&\quad + \|e_l\|((\rho_1' + \beta_1)\|e_l\| + \rho_2'\|e_l\|^2 + \rho_{k0} \\
&\quad + \rho_3'\|e_{m1} - e_{l1}\| + \beta_2\|e_m\|) \\
&= -\lambda_1\|e_l\|^2 + \rho_{k0}\|e_l\| \\
&\quad - \lambda_{DK}\|e_m\|^2 - \lambda_{TK}\|e_{m1} - e_{l1}\|^2 \\
&\quad + \beta_2\|e_l\|\|e_m\| + \rho_3'\|e_l\|\|e_{m1} - e_{l1}\|, \qquad (6.4.11)
\end{aligned}$$

where λ_{QR}, λ_{DK} and λ_{TK} are the minimum eigenvalues of $(Q + T_0^t B R^{-1} B^t T_0)$, $T_0^t B(D_m + K_p) B^t T_0$ and $T_{12} T_{22} K_s$ respectively, $\lambda_1 = \lambda_{QR} - \rho_1' - \beta_1 - \rho_2'\|e_l\|$. We have the region of attraction as

$$\Omega = \{e \in R^{2n} : \|e_l\| < \frac{\lambda_{QR} - \rho_1' - \beta_1}{\rho_2'}\}.$$

Further we can rewrite (6.4.11) as

$$\frac{dV_1}{dt} \leq -v^t \Phi v + \rho_{k0} v \qquad (6.4.12)$$

where $v = (\|e_l\|, \|e_m\|, \|e_{m1} - e_{l1}\|)$ and

$$\Phi = \begin{pmatrix} \lambda_1 & -\beta_2/2 & -\rho_3'/2 \\ -\beta_2/2 & \lambda_{DK} & 0 \\ -\rho_3'/2 & 0 & \lambda_{TK} \end{pmatrix}.$$

If λ_1, λ_2 and λ_{TK} are large enough, Φ can be made positive. Now, we can define an ellipse in Euclidian space, i.e.,

$$\lambda_\Phi(\|v\| - \varepsilon_1)^2 - \varepsilon_0 = 0,$$

where λ_Φ is the the minimum eigenvalue of Φ, $\varepsilon_1 = \rho_{k0}/(2\lambda_\Phi)$ and $\varepsilon_0 = \rho_{k0}^2/(4\lambda_\Phi)$. Within this ellipse the set $\dot{V}_1 \geq 0$. In other words,$\dot{V}_1 < 0$ whenever the norms of the tracking errors $\|v\|$ is outside this ellipse. □

In the input of the motor subsystem u (6.4.6), not only the motor information but the link information are required. If we use only the motor information in the feedback, i.e., instead of (6.4.6), we can use

$$u = Y_{d2}^t P_{2o} + Y_{kd2}^t P_{k0} - K_p s_m - W e_m - t_m - t_k. \tag{6.4.13}$$

The whole control system is shown in Fig.6.5. Substitution of (6.4.13) into (6.4.2) leads to

$$J_m \dot{s} = -(D_m + K_p)s_m - K_s(e_{m1} - e_{l1}) - W e_m + Y_{d2}(P_2 - P_{2o}) + Y_{kd2}(P_k - P_{k0}) - t_m - t_k. \tag{6.4.14}$$

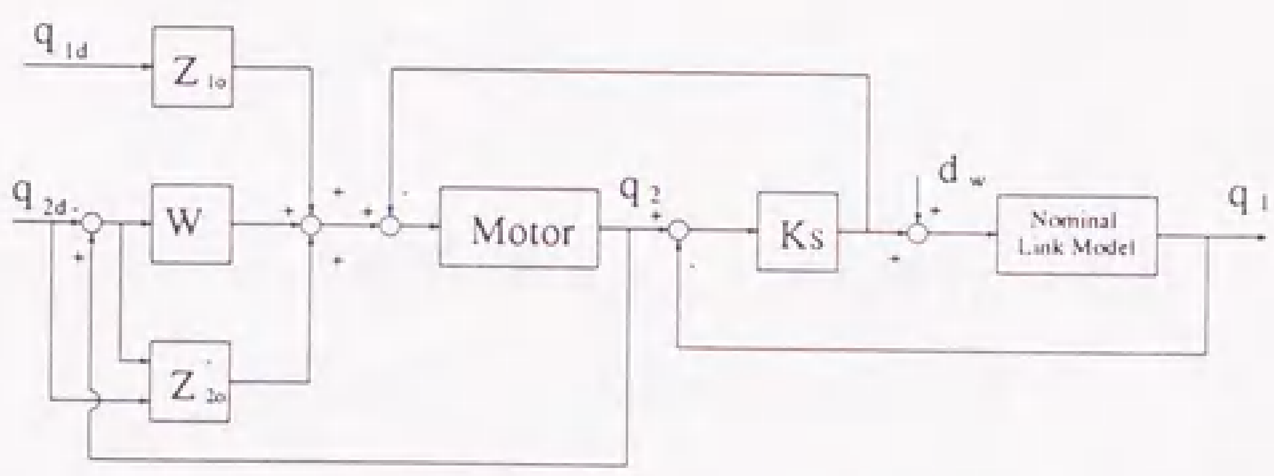


Fig. 6.5 The block diagram of the whole system
$Z_{1o} = M_0 \ddot{q}_{1d} + C_0 \dot{q}_{1d} + D_0 \dot{q}_{1d} + G_0;$
$W_{1o} = T_{22}^{-1}(C_0 T_{21}, M_0 T_{21})e_l;$
$Z_{2o}' = Y_{dm}^t P_{m0} - K_p s_m - W e_m - t_m.$

The following theorem summarizes the stability of the whole system by using only the position and velocity of the motors in feedback.

Theorem 6.4.2 *Considering the closed-loop control system defined by (6.4.5) and (6.4.14), the equilibrium point $(e_l, e_m) = 0$ of the system is uniformly bounded stable.*

Proof: Select a Lyapunov function candidate as

$$V_1 = V + \frac{1}{2}(s_m^t T_{22}^2 J_m s_m + (e_{m1} - e_{l1})^t T_{22}^2 K_s (e_{m1} - e_{l1})). \tag{6.4.15}$$

Then the time derivative of (6.4.15) becames

$$\begin{aligned} \frac{dV_1}{dt} &= -L(e_l, U^*, d^*) - e_l^t T_0^t B \frac{1}{\gamma^2} B^t T_0 e_l + e_l^t T_0^t B d_1 \\ &\quad + e_l^t T_0^t B T_{22} K_s (e_{m1} - e_{l1}) + e_l^t T_0^t B T_{22} W e_l \end{aligned}$$

$$
\begin{aligned}
&+e_l^t T_0^t B T_{22}(K_s - K_{s0})(q_{2d} - q_{1d}) \\
&-s_m^t T_{22}^2((D_m + K_p)s_m + K_s(e_{m1} - e_{l1}) + W e_m) \\
&+(e_{m2} - e_{l2})^t T_{22}^2 K_s(e_{m1} - e_{l1}) \\
&+s_m^t T_{22}^2(Y_{d2}^t(P_2 - P_{20}) - t_m) \\
&+s_m^t T_{22}^2(Y_{kd2}^t(P_k - P_{k0}) - t_k) \\
\leq\ & -\frac{1}{2}e_l^t(Q + T_0^t B R^{-1} B^t T_0)e_l + e_l^t T_0^t B(d_1 + d_2) \\
&-e_m^t T_0^t B(D_m + K_p)B^t T_0 e_m \\
&-(e_{m1} - e_{l1})^t T_{12} T_{22} K_s(e_{m1} - e_{l1}) \\
&+(e_l - e_m)^t T_0^t B T_{22} W e_l \\
&+e_m^t T_0^t B T_{22} W(e_l - e_m). \qquad (6.4.16)
\end{aligned}
$$

We also have

$$
\begin{aligned}
&(e_l - e_m)^t T_0^t B T_{22} W e_l + e_m^t T_0^t B T_{22} W(e_l - e_m) \\
&\leq \beta_1' \|e_l\|^2 + \beta_2' \|e_l\| \|e_m\| + \beta_3' \|e_m\|^2.
\end{aligned}
$$

Then

$$
\begin{aligned}
\frac{dV_1}{dt} \leq\ & -\lambda_{QR}\|e_l\|^2 - \lambda_{DK}\|e_m\|^2 - \lambda_{TK}\|e_{m1} - e_{l1}\|^2 \\
&+\|e_l\|((\rho_1' + \beta_1')\|e_l\| + \rho_2'\|e_l\|^2 + \rho_{k0} \\
&+\rho_3'\|e_{m1} - e_{l1}\| + \beta_2'\|e_m\|) + \beta_3'\|e_m\|^2 \\
=\ & -\lambda_1'\|e_l\|^2 + \rho_{k0}\|e_l\| \\
&-\lambda_2'\|e_m\|^2 - \lambda_{TK}\|e_{m1} - e_{l1}\|^2 \\
&+\beta_2'\|e_l\|\|e_m\| + \rho_3'\|e_l\|\|e_{m1} - e_{l1}\|, \qquad (6.4.17)
\end{aligned}
$$

where λ_{QR}, λ_{DK} and λ_{TK} are the minimum eigenvalues of $(Q + T_0^t B R^{-1} B^t T_0)$, $T_0^t B(D_m + K_p)B^t T_0$ and $T_{12}T_{22}K_s$ respectively, $\lambda_1' = \lambda_{QR} - \rho_1' - \beta_1' - \rho_2'\|e_l\|$ and $\lambda_2' = \lambda_{DK} - \beta_3'$. We have the regin of attraction as

$$
\Omega = \{e \in R^{2n} : \|e_l\| < \frac{\lambda_{QR} - \rho_1' - \beta_1'}{\rho_2}\}.
$$

Further we can rewrite Eq.(6.4.17) as

$$
\frac{dV_1}{dt} \leq -v^t \Phi' v + \rho_{k0} v, \qquad (6.4.18)
$$

where $v = (\|e_l\|, \|e_m\|, \|e_{m1} - e_{l1}\|)$ and

$$\Phi' = \begin{pmatrix} \lambda_1' & -\beta_2'/2 & -\rho_3'/2 \\ -\beta_2'/2 & \lambda_2' & 0 \\ -\rho_3'/2 & 0 & \lambda_{TK} \end{pmatrix}.$$

If λ_1', λ_2' and λ_{TK} are large enough, Φ' can be made positive. Now, we can define an ellipse in Euclidian space, i.e.

$$\lambda_{\Phi'}(\|v\| - \varepsilon_1')^2 - \varepsilon_0' = 0,$$

where $\lambda_{\Phi'}$ is the the minimum eigenvalue of Φ', $\varepsilon_1' = \rho_{k0}/(2\lambda_{\Phi'})$ and $\varepsilon_0' = \rho_{k0}^2/(4\lambda_{\Phi'})$. Within this ellipse the set $\dot{V}_1 \geq 0$. In other words,$\dot{V}_1 < 0$ whenever the norm of the tracking errors $\|v\|$ is outside this ellipse. □

6.5 Simulation Results

By some simulations, we will analyze the performance of the proposed control scheme when it is used to control a two-link SCARA type manipulator with flexible joints. Each link of the manipulator is driven by a DC motor through a gear box. The model of the two-link manipulator with flexible joints is the same with that in Chapter 5. The desired link trajectories are also the same with that in Chapter 5.

The purpose of the simulations is to demonstrate the stability and the performance of the controllers proposed in this chapter. The nominal parameters and the uncertainties are given as shown in Table 6.1

Parameter	*Joint 1*	*Joint 2*	*unit*
m_{i0}	10.0	8.0	kg
D_{i0}	2.0	2.0	Nms/rad
J_{mi0}	1.2	0.3	kgm^2
D_{mi0}	2.6	0.64	Nms/rad
K_{si0}	1.2×10^5	2.3×10^4	Nm/rad
Δm_{i0}	4.9	4.04	kg
ΔD_{i0}	1.154	1.154	Nms/rad
ΔJ_{mi0}	0.6	0.15	kgm^2
ΔD_{mi0}	1.31	0.32	Nms/rad
ΔK_{si0}	3.1×10^4	0.5×10^4	Nm/rad

Table 6.1 The nominal parameters and uncertainties

The coefficient matrices of the nonimal parameters and uncertainty in the link subsystem are obtained as

$$M_0(q_1) = \begin{pmatrix} 1.615 + 1.427cos(q_{12}) & 0.3576 + 0.736cos(q_{12}) \\ 0.376 + 0.736cos(q_{12}) & 0.357 \end{pmatrix},$$

$$\Delta M(q_1) = \begin{pmatrix} 0.815 + 0.743cos(q_{12}) & 0.181 + 0.372cos(q_{12}) \\ 0.181 + 0.372cos(q_{12}) & 0.181 \end{pmatrix},$$

$$C_0(q_1, \dot{q}_1) = \begin{pmatrix} -0.736sin(q_{12})\dot{q}_{12} & -0.736sin(q_{12})\dot{q}_{11} - 0.736sin(q_{12})\dot{q}_{12} \\ 0.736sin(q_{12})\dot{q}_{11} & 0 \end{pmatrix},$$

$$\Delta C(q_1, \dot{q}_1) = \begin{pmatrix} -0.372sin(q_{12})\dot{q}_{12} & -0.372sin(q_{12})\dot{q}_{11} - 0.372sin(q_{12})\dot{q}_{12} \\ 0.372sin(q_{12})\dot{q}_{11} & 0 \end{pmatrix},$$

$$D_0 = \begin{pmatrix} 2.0 & 0 \\ 0 & 2.0 \end{pmatrix}, \quad \Delta D = \begin{pmatrix} 1.154 & 0 \\ 0 & 1.154 \end{pmatrix}.$$

We also have $0.27I < M_0 < 3.5I$, $0.13I < \Delta M(q_1) < 1.74I$, $C_0 < 1.14\|q_1\|$ and

$\Delta C < 0.55\|q_1\|$. The errors of modeling along the trajectory are

$$\delta_m < \begin{pmatrix} 1.427 & 0.736 \\ 0.736 & 0 \end{pmatrix} < 1.03,$$

$$\delta_c < \begin{pmatrix} -0.736\dot{e}_{l2} & -0.736\dot{e}_{l1} - 0.736\dot{e}_{l2} \\ -0.736\dot{e}_{l1} & 0 \end{pmatrix} < 1.104\|\dot{e}_l\|.$$

The linear parameterized expression of the motor subsystem is

$$Y_2P_2 + Y_{k2}P_k = \begin{pmatrix} \ddot{q}_{21} & \dot{q}_{21} & 0 & 0 \\ 0 & 0 & \ddot{q}_{21} & \dot{q}_{21} \end{pmatrix} \begin{pmatrix} J_{m1} \\ D_{m1} \\ J_{m2} \\ D_{m2} \end{pmatrix} + \begin{pmatrix} q_{21} - q_{11} & 0 \\ 0 & q_{22} - q_{12} \end{pmatrix} \begin{pmatrix} K_{s1} \\ K_{s2} \end{pmatrix} = u.$$

Using the same technique which has been proposed in Chapter 4, i.e., by different uncertainty bounds for $P_{2i}(i = 1,2,3,4)$ and $P_{kj}(j = 1,2)$, we have

$$\bar{\rho}_2 = (0.6, 1.31, 0.15, 0.32)^t; \qquad \bar{\rho}_k = (3.0 \times 10^4, 0.5 \times 10^4)^t.$$

Therefore the robust compensation term (6.4.7) can be rewritten as

$$t_{mi} \stackrel{\text{def}}{=} \begin{cases} \frac{\eta_{2i}\bar{\rho}_{2i}}{|\eta_{2i}|} & |\eta_{2i}| > 0 \\ 0 & |\eta_{2i}| = 0 \end{cases}; \qquad t_k \stackrel{\text{def}}{=} \begin{cases} \frac{\eta_{kj}\bar{\rho}_{kj}}{|\eta_{kj}|} & |\eta_{kj}| > 0 \\ 0 & |\eta_{kj}| = 0 \end{cases};$$

where η_{2i} and η_{kj} denote the ith and jth component of the vector $Y_{d2}^t s_m$ and $Y_{kd2}^t s_m$, respectively.

In order to illustrate the capability of attenuation with different choices of γ, we deliberately design the control law to achieve the following three different levels of disturbance attenuation. To consider the feedback gains for the positions and velocities respectively, we set

$$Q = \begin{pmatrix} \beta_1 I_{2\times2} & 0 \\ 0 & \beta_2 I_{2\times2} \end{pmatrix},$$

where β_1 and β_2 are positive constants.

Case 1: $\gamma = 0.2$, $R = 0.01I_{2\times2}$, $\beta_1 = 500.0$ and $\beta_2 = 10.0$. So that $R_1 = 0.1155I_{2\times2}$.

$$T_0 = \begin{pmatrix} I_{2\times2} & 0 \\ 57.8I_{2\times2} & 1.155I_{2\times2} \end{pmatrix}.$$

Case 2: $\gamma = 0.05$, $R = 0.001 I_{2\times2}$, $\beta_1 = 500.0$ and $\beta_2 = 10.0$. So that $R_1 = 0.041 I_{2\times2}$

$$T_0 = \begin{pmatrix} I_{2\times2} & 0 \\ 25.5 I_{2\times2} & 0.41 I_{2\times2} \end{pmatrix}.$$

Case 3: $\gamma = 0.02$, $R = 0.0002 I_{2\times2}$, $\beta_1 = 500.0$ and $\beta_2 = 10.0$. So that $R_1 = 0.01 I_{2\times2}$

$$T_0 = \begin{pmatrix} I_{2\times2} & 0 \\ 10.0 I_{2\times2} & 0.2 I_{2\times2} \end{pmatrix}.$$

In Fig.6.6-Fig.6.8, the simulation results by using the link and motor information are shown. The simulation results by using only the motor information are shown in Fig.6.9-Fig.6.11. From the simulation results, we know that

1 The system is stable by suitably chosing the control parameters which satisfy the sufficient condition for optimal control.

2 The smaller γ yields the better performance in attenuating the effect of the uncertainties.

3 In spite of using only the motor information, the errors of the system are almost the same because of the robustness of the controllers.

6.6 Conclusion

From this research, it has been shown how H_∞ control can be applied to the manipulators with flexible joints under parameter uncertainties. A transformed control structure can give a solution to the Hamilton-Jacobi equation globally for the link subsystem. The whole system can be guaranteed to be stable by using Lyapunov stability theory. To avoid using the link information, the algorithm using only the motor information has also been proposed. From simulation results with some different prespecified attenuation levels, this control design is effective as we desired.

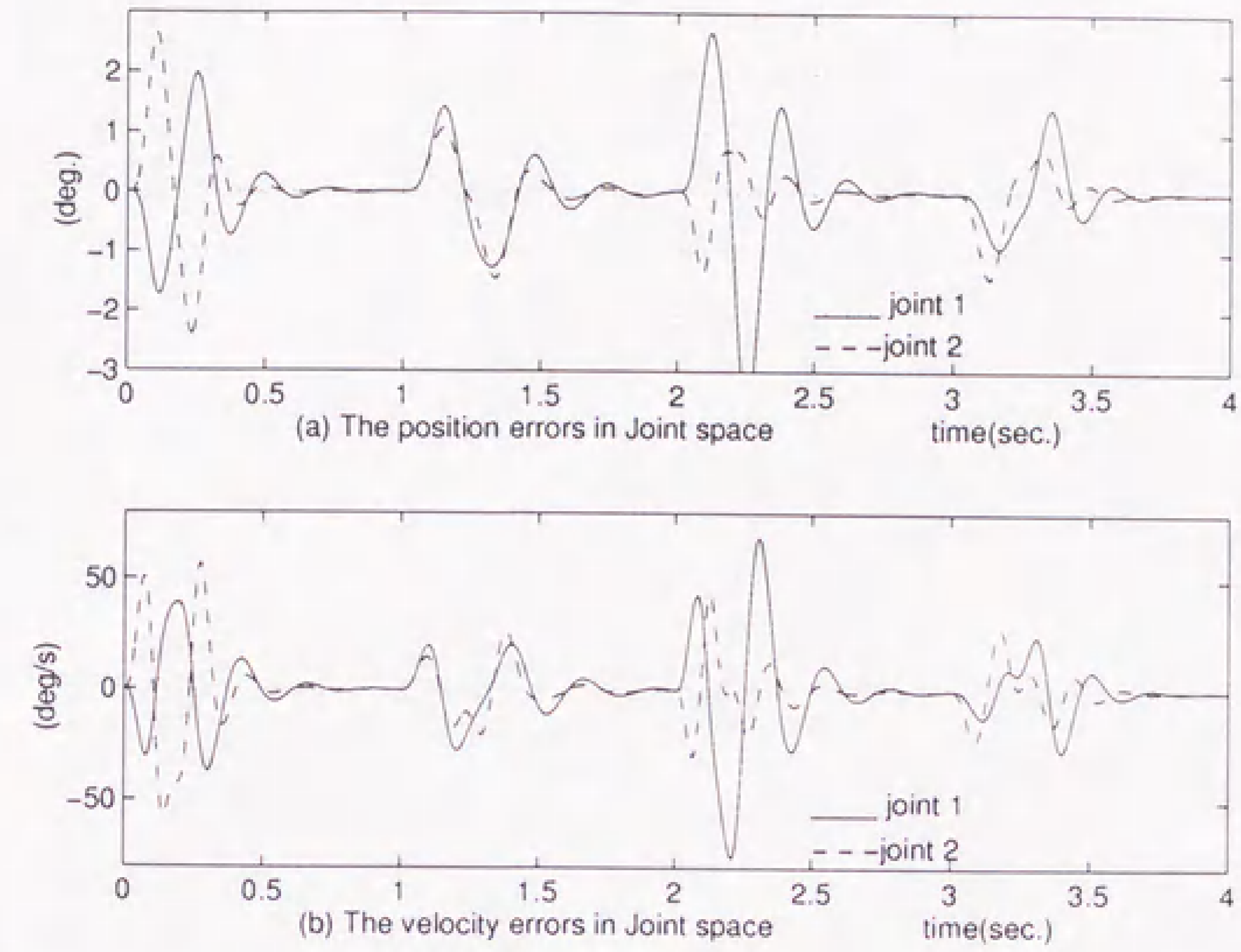


Fig. 6.6 The simulation results by using the link information for Case 1

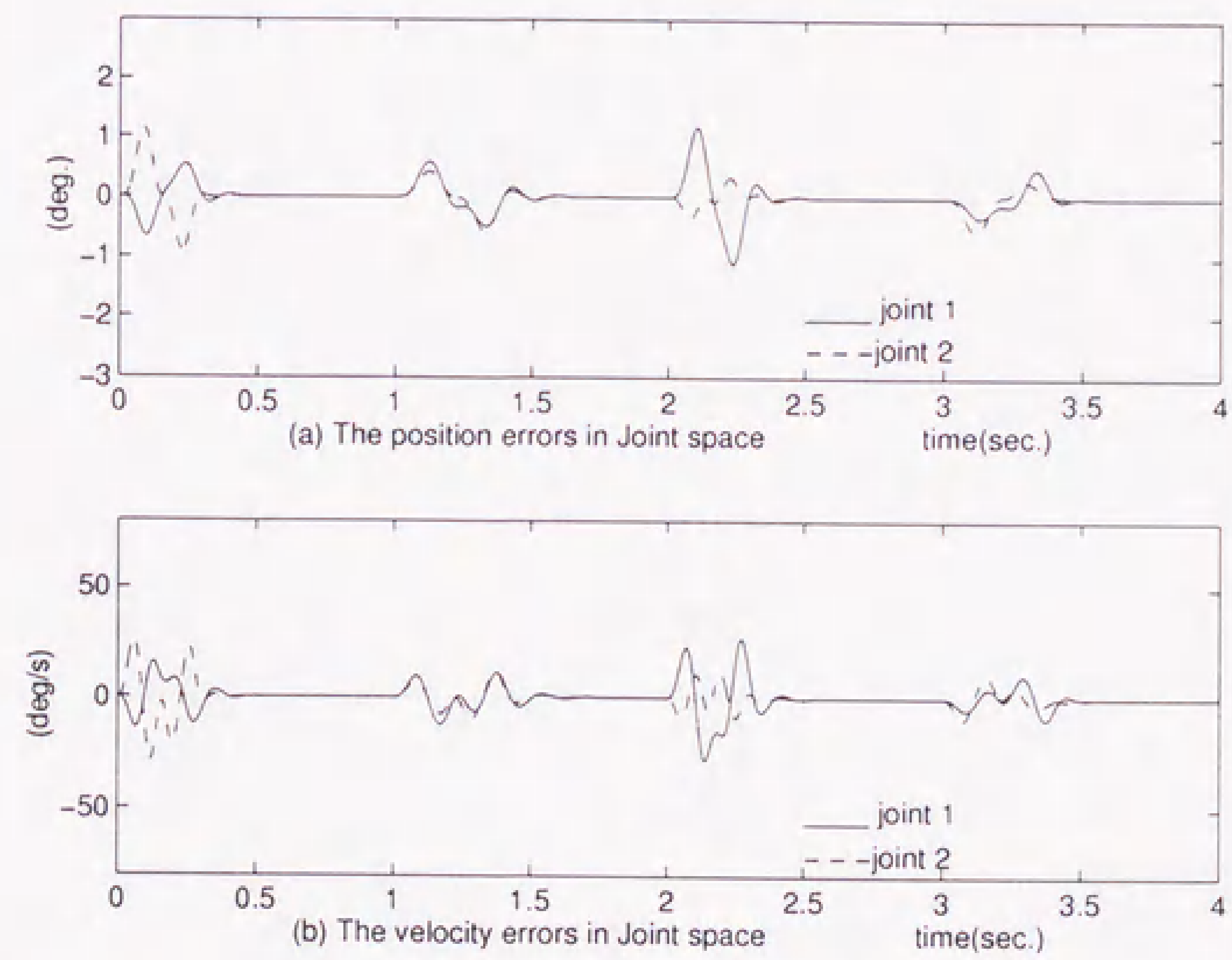


Fig. 6.7 The simulation results by using the link information for Case 2

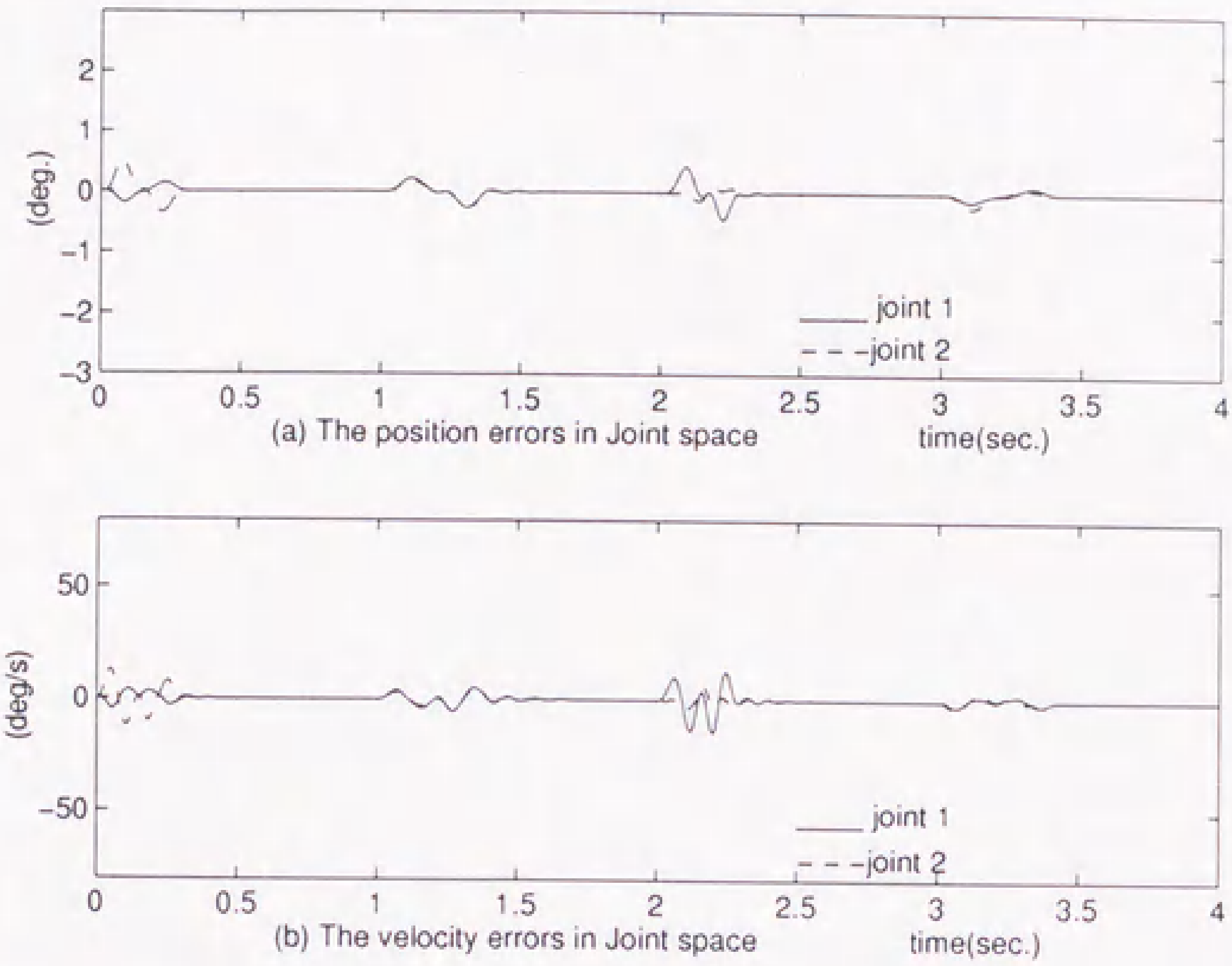


Fig. 6.8 The simulation results by using the link information for Case 3

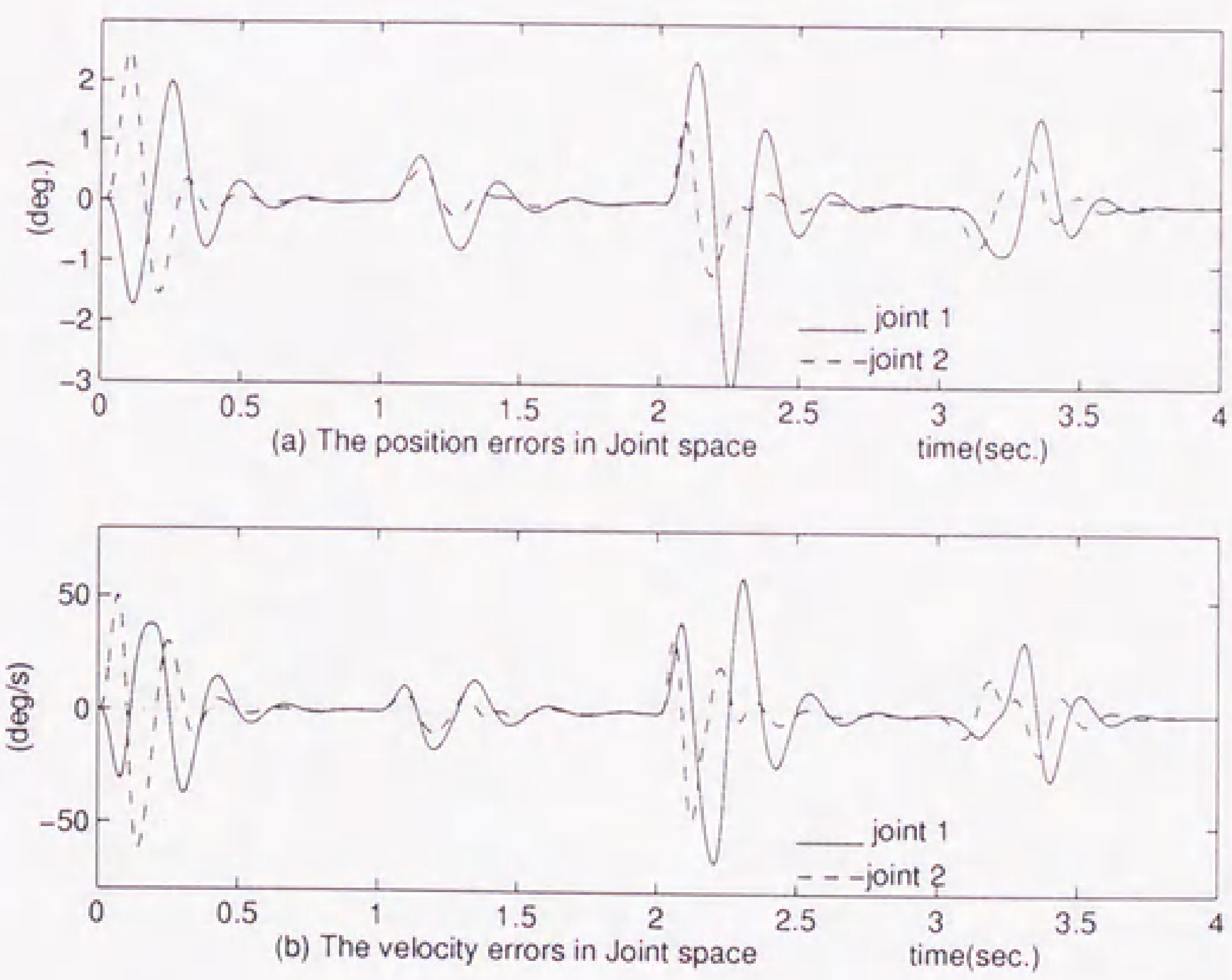


Fig. 6.9 The simulation results by no using the link information for Case 1

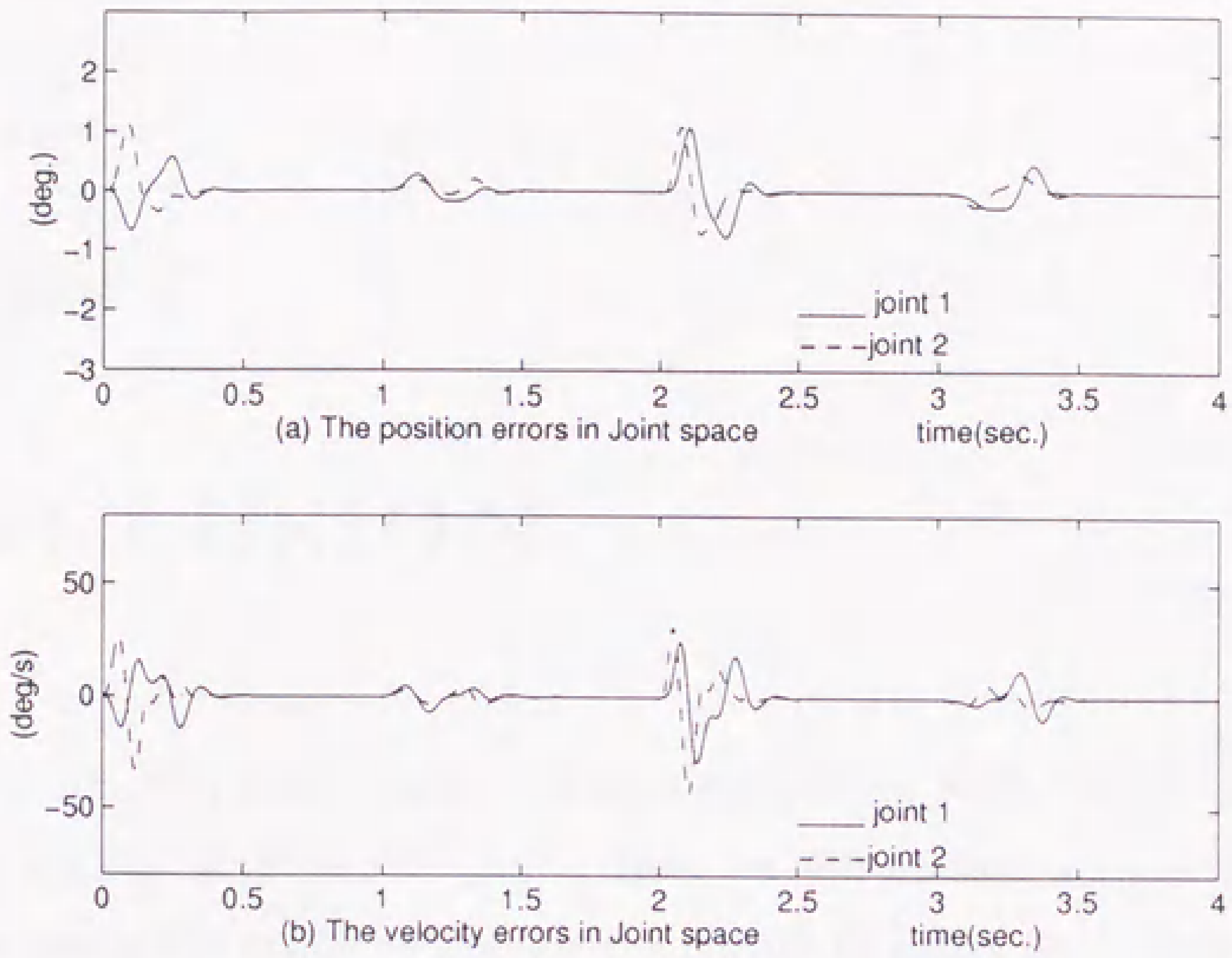


Fig. 6.10 The simulation results by no using the link information for Case 2

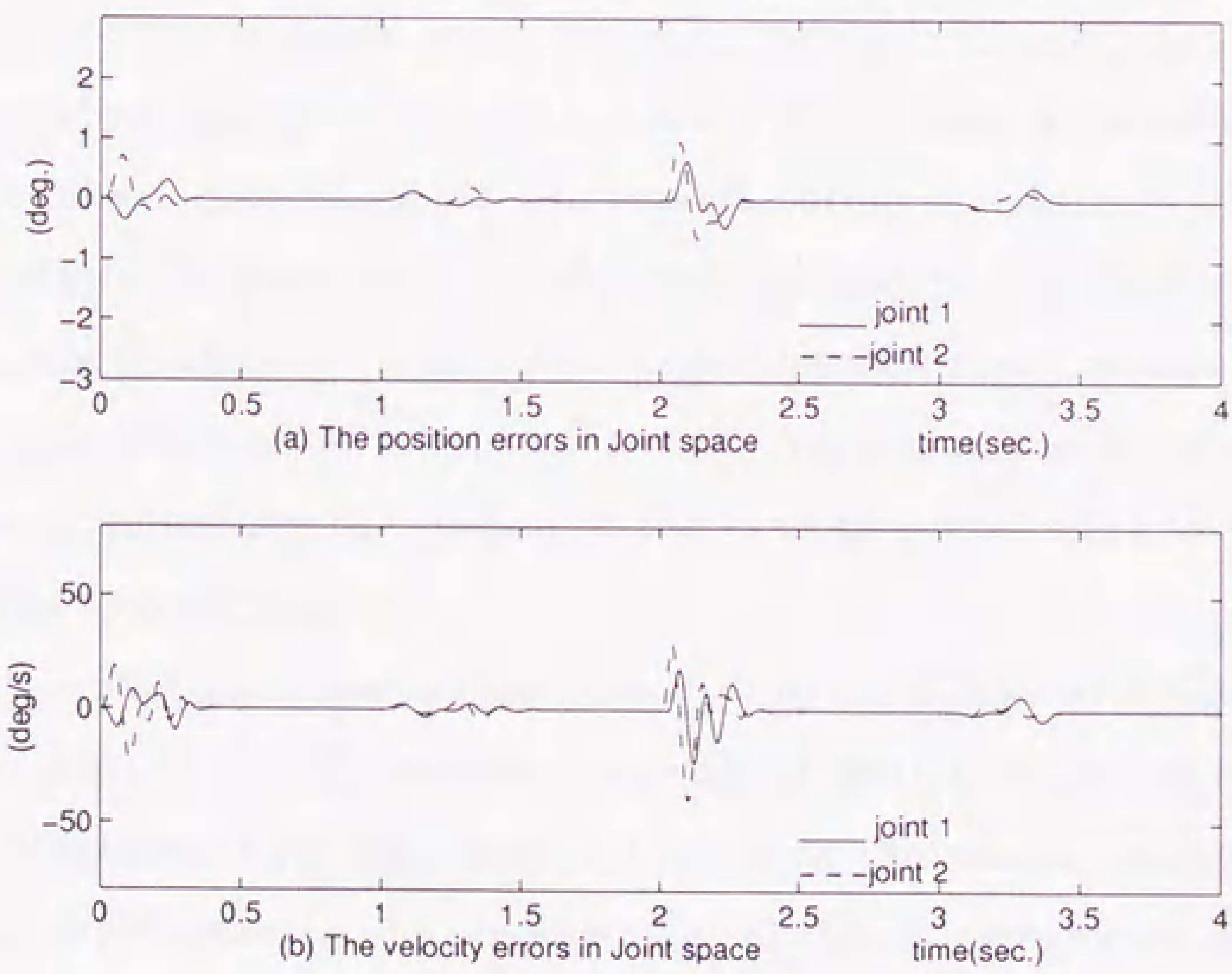


Fig. 6.11 The simulation results by no using the link information for Case 3

Chapter 7

CONCLUSION

The previous chapters have presented some contributions to the control methods for robot manipulators with flexible joints. Now we will present a summary of these results as well as a discussion concerning the remaining issues for further researches.

7.1 Summary

In the field of manufacturing, owing to recent advance in computer technology, mankind is at the threshold of a second Industrial Revolution. Automation has come to mean the introduction of robots into manufacturing environment. A successful application of robot is dependent upon their control system. This thesis is concerned with the control problem in the executive control level for robot manipulator. Since the requirement imposed on robot manipulator concerning the speed and quality of operation is increasing, the control at the executive level must take the robot dynamics into account precisely.

Traditional robot manipulators were modeled by rigid links with rigid transmission units, in order to simplify analysis and design of control law. In last two decades, many control schemes have been proposed based on this model. In fact, the flexibility in the manipulator can be observed. In this thesis we explored the dynamic control of manipulator with flexible joints. For control system design, in Chapter 2, we give the full-order dynamic model of MFJ under some reasonable assumptions, as well as some useful properties of the model.

Advanced robot application often requires effective controller in order to achieve a accurate tracking of fast desired motion. If the physical parameters of a manipulator are known, many control approaches can be used for this purpose. If the physical parameters are not exactly known, however, we have to consider some advanced control schemes, e.g., adaptive control, robust control, and etc. In Chapter 3, we have proposed an adaptive controller for MFJ with position and velocity feedback, which is different from many other adaptive schemes in the following way: arbitrary joint stiffness is allowed and all the system parameter is set unknown. The proposed control scheme guarantees that the tracking error converges to zero and all the signals in the system remain bounded. Some experiments, without and with adaptive mechanism, have been made. The experimental results have shown the effectiveness of the proposed adaptive control scheme.

To handle uncertain parameters and time-varying loads in MFJ, in Chapter 4, we also proposed a robust controller with position and velocity feedback. In other words, the robust controller is suitable to the case where the ranges of uncertain parameters are known *a priori* without the assumption that the parameters are time-invariant or slowly time-varying in the design of adaptive one. The convergency of the tracking error and boundedness of all the signals are also obtained. Especially, it is different from many other robust control schemes in a way of obtaining the continuous control output. To illustrate the effectiveness of the proposed scheme, some computer simulations and experiments have been made in the cases of without and with robust compensation term under uncertain parameters and time-varying load. As an alternative to the adaptive controller, the proposed robust controller seems to be most attractive in the environment where the range of uncertainty is not too large, with a time-varying load. The choice of the control parameter is quite simple.

In Chapters 3 and 4, the adaptive and robust controllers have been derived respectively. The stability of systems is guaranteed for both controllers by using Lyapunov stability theory. The robustness with respect to the unmodeled dynamics has also been analyzed. In these two approaches, as many previous methods proposed by other researchers, not only the information of motor but also that of link are required in feedback.

In many present industrial robot manipulators, we can have only the measurement of the motor information. In Chapter 5, we have presented nonlinear tracking controllers with observer. Roughly speaking, the design of the controllers can be divided into two steps. The first is to design the desired "fictional motor position" which would make the link subsystem asymptotically stable. The second is to design an input of motor subsystem which could realize the desired "fictional motor position" and guarantee the stability of the whole system. Before giving the controller with observer, we have proposed a nonadaptive tracking controller without observer under the conditions that the physical parameters are known and the motor and link informations are available. Then we give a nonadaptive tracking controller with sliding observer. By using the output of the observer, the desired "fictional motor position" is defined. The stability of the whole system is guaranteed under some condition of the controller. However, this control scheme can be viewed as a feedforward computed torque control with respect to the coefficient matrices of the link subsystem. The effectiveness of the controller is thus dependent on the adequate knowledge of the physical parameters of the link subsystem. In practice, these parameters are uncertain or time-varying. Therefore, we have also proposed an adaptive controller with robust sliding observer with respect to the uncertainty of the link subsystem. In the design of the controllers, we give only sufficient conditions of stability of the whole system including the dynamics of MFJ, the observer and the controller. Fortunately, these sufficient conditions can be satisfied easily in most applications.

In the proposed tracking controllers using sliding observer, the measurement of motor acceleration is required. However, in practice, this request is difficult to be met. The acceleration can only be obtained by numerical differentiation from the motor velocity. To decrease high frequency disturbance, some low-pass filter can also be used actually. Because of the numerical differentiation and filter, some noise or error will be introduced. We have analyzed the stability of the system under some measurement noise or error. The simulation results have shown the effectiveness of the controller.

Linear H_∞ optimization is now very popular. It is therefore interesting to see if the H_∞ optimization can also be applied to nonlinear manipulator systems with the same benefits of robustness carried over from the linear case. Referring to the H_2 optimal control for MRJ proposed by Johansson (1990), in Chapter 6, we have given a H_∞ optimal control scheme for MRJ. The nominal model of the manipulator is time-varying along the desired trajectory. The explicit solution corresponding to Hamilton-Jacobi equation has been got by an elegant definition of the value function. Further, we have tried to modify the H_∞ optimal control scheme from MRJ to MFJ. Similar to that in Chapter 5, we have separated the model of MFJ into link and motor subsystems. The desired "fiction input torque" is designed according to H_∞ optimal control for MRJ. The desired motor trajectory is synthesized by nominal parameters of the MFJ and desired link trajectory. With robust compensation terms, the input for motor subsystem can realize the desired "fiction input torque" and guarantee the stability of the whole system. The most attractive property of this control is that the controller can use only the motor position and velocity in feedback when all the parameters are not exactly known. The simulation results have shown the effectiveness of the controller. The performance of the system, as we expected, is dependent on the attenuating level γ.

7.2 Further Research

In this thesis, we have focused on the dynamic control of the robot manipulators with flexible joints. In industrial applications, a demand for lighter robots operating with the same precision and speed can be recognized. These requirements suggest to consider the dynamic effects of the distributed elasticity of the flexible links. As we know, the joint flexibility complicates the manipulator dynamic model. The complexity of a dynamic model becomes greater if the link elasticity is further considered. As our further research, the control of the manipulator with flexible joints and links is challenging.

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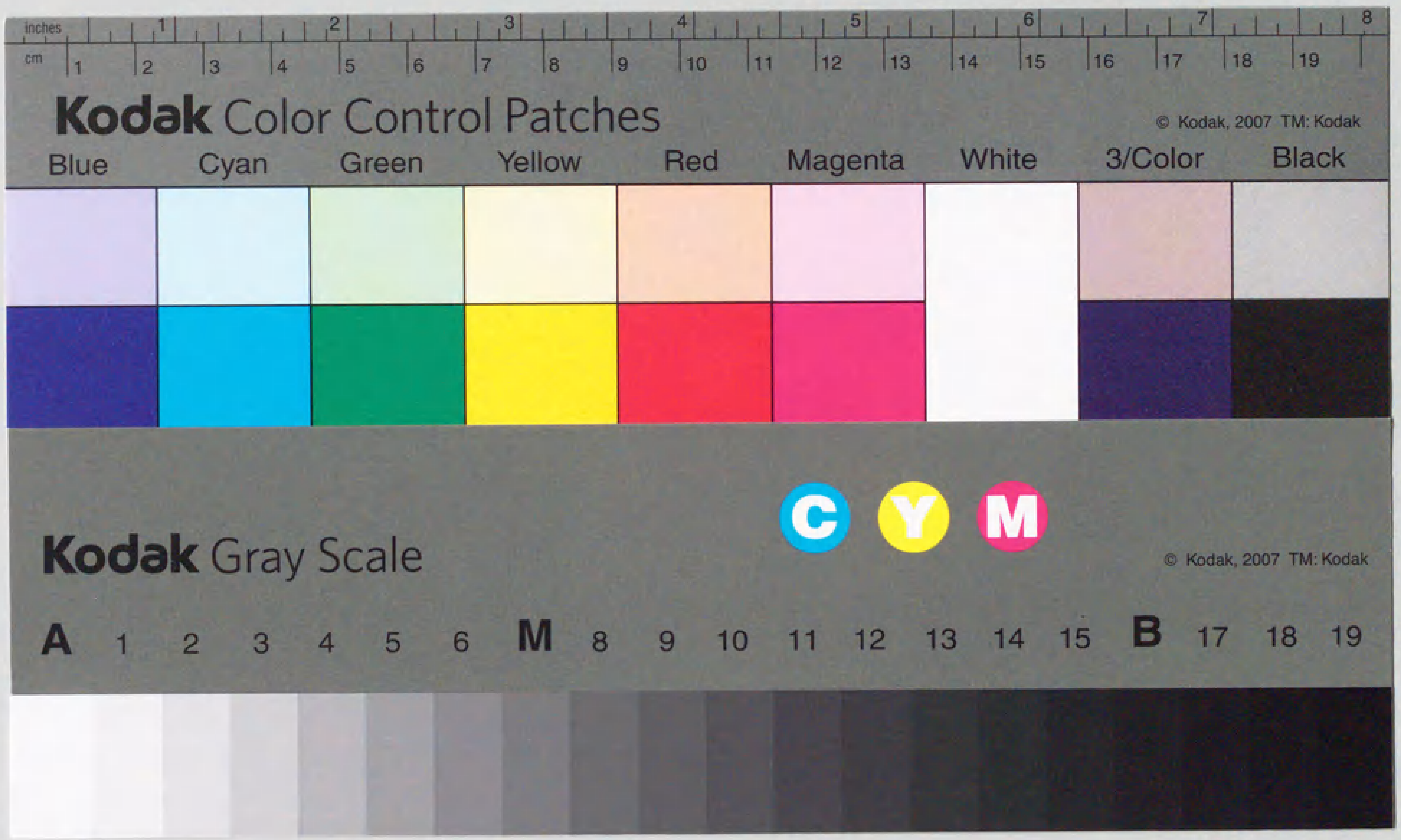

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