International Trade and Public Finance with Public Goods

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Dedication

To My Family and My Friends

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Chapter 1

Introduction

One of the government's roles is the provision of public goods. Basically, there are two types of public goods. First is consumption public good. Parks, fire fighting and police services are consumption public goods. Second is production public good. This type of public good is called public intermediate good. Roads, railways, airports and ports belong to public intermediate goods. Meade (1952) recognizes two types of public intermediate goods. One type is pure public intermediate good, which is called 'creation of atmosphere'; the other type is semi-public intermediate good, which is called 'unpaid factors'. Pure public intermediate good is fully available to every firm irrespective of the number of the firms. Free information about technology is a typical example of pure public intermediate good. Semi-public intermediate good suffers from congestion within an industry and thus a reduction of availability to a firm when the number of firms in this industry increases. Roads and airports are examples of semi-public intermediate good. The essential difference between these two type of public intermediate goods is that in the case of pure public intermediate good there are constant returns to scale for each individual industry but not for society as a whole, in the case of semi-public intermediate good there are constant returns to scale as a whole but not for the individual industry. The mathematic formulation of the production function for a private good makes clear distinction between these two types of public intermediate goods. The production functions are linear homogeneous in primary inputs and semi-public intermediate goods but not in pure public intermediate goods.

When there exist public intermediate goods, the traditional trade theorems may be invalid. First, the existence of public intermediate good induces scale economies in private sectors. Based on the analysis of Marshallian externality, when scale economies arise, the production possibility frontier will become convex to the origin, and the price–outputs relationship will be abnormal. Second, the outputs of public intermediate good determine the amount of resources available to the private sectors. The increase of output of public intermediate good brings the same role for the private sectors like the progress of technologies. However, the production of public intermediate good requires input of resources. Hence, the more public intermediate good the government supplies, the less the resources available for private sectors.

In international trade studies, there is a strand of literatures that examine the influences of public intermediate good on traditional trade theorems. Manning and McMillan (1979) introduce the public intermediate good to the Ricardian model and assume that the public intermediate good is optimally provided by the government. They examine the shape of production possibility frontier and the trade patterns. Tawada and Abe (1984) analyze how the existence of public intermediate good affects the result of Heckscher-Ohlin model. They examine the shape of production possibility frontier, the robustness of the Stolper-Samuelson, the Rybczynski, the Heckscher-Ohlin theorems and the factor price equalization theorem. However, there are special assumptions about the production functions in their model. First, the production effects of public intermediate good on two industries are equal. Second, the production effect of public intermediate good is constant irrespective of the amount of public intermediate good. Third, the public intermediate good promotes the production of private sectors by the form of Hicks neutrality. Okamoto (1985) and Ishizawa (1990) show that when the equilibrium is local stable, the price-output ratio may be normal. They also derive the conditions under which the Stolper-Samuelson theorem and Rybczynski theorem hold. Altenburg (1987) finds some conditions which can assure the production possibility frontier is concave to the origin. Ishizawa (1988) shows the existence of Leontief paradox in a model consists of public intermediate good. However, these studies concentrate on a small open economy and thus do not provide any explicit analysis in a two-country model. Moreover, the normative issue of whether an economy will gain from trade was not dealt with in these studies. Ishizawa (1991) shows in a Heckscher-Ohlin economy, despite the presence of increasing returns to scale by a public intermediate good, the PPF is concave to the origin if the market for the public intermediate good is Marshall stable. Suga and Tawada (2007) develop a two-country international trade model, which consists of a one primary factor, two consumer goods and a pure public intermediate good. Each country provides the country-specific pure public intermediate good to get an efficient production. They show that the country with lager factor endowment exports the good whose productivity is more sensitive to the pure public intermediate good. They also find that: First, at least one country gains from trade. Second, if a country incompletely specializes in the trade equilibrium, the country necessarily loses from trade.

In public finance literatures, since the mid-1980s, tax competition has been extensively investigated. Interest in this topic has been stimulated by publicized facts that U.S. states and localities do seem to have engaged in tax competition. They offered large subsidies to foreign and domestic automobile companies in order to influence plant location decisions. One of the most important finds in tax competition is that tax competition induces inefficient low levels of tax rates and under-provision of public goods (accurately speaking, this type of public goods are the goods provided by the government, there are not externalities for these public goods). Hence, tax harmonization is highly required. It is well recognized that cooperation may arise in a repeated game. Then, a repeated tax competition game setting can provide a meaningful insight to discuss tax competition problems. Coates (1993) first analyzes the property tax competition problem in a repeated game setting. He partially examines the open-loop equilibrium of a dynamic game of property tax competition. But he ignores the externalities of one local government's tax change on the tax rates of other governments, and concentrates on the inter-temporal trade-off between current and future consumptions of private and local public goods. He concludes that there may be incentives to subsidize capital. Cardarelli et al. (2002) demonstrate that tax coordination can endogenously arise in a repeated tax competition game. Additionally, they show that tax coordination does not happen when regional asymmetries are too strong. But there are some assumptions which make their model unusual with standard tax competition model. These assumptions are the following: (1) no production activity occurs; (2) the interest rate is exogenously fixed at zero; (3) capital mobility sunk costs are incurred when capital moves across regions. These rather peculiar assumptions of their model make it difficult to have a straightforward comparison of their results with those obtained from the existing literatures of tax competition. Extending this study, Kawachi and Ogawa (2006) incorporate the benefit spillovers of local public goods to show that the cooperative outcome tends to happen as the magnitude of spillover is significant.

Catenaro and Vidal (2006) examine tax competition in a standard tax competition with repeated actions. The only departure from the standard tax competition is that the government's objective in their model is to maximize capital tax revenues rather the well-being of the residents. They conclude that tax coordination is not sustainable if region sizes are too different. Itaya et al. (2008) construct a standard tax competition with repeated actions. The governments in their model maximize the well-being of their residents. The regions are asymmetric with per capita capital endowments and production technologies. The most important contribution of Itaya et al. (2008) is that they show regional asymmetries may be advantageous to tax cooperation.

In the literatures of public finance, there is also another strand of literatures about fiscal equalization scheme. Fiscal equalization scheme is an integral part in the existing federal arrangements. In the US, the state tax sharing is one of the two forms of intergovernmental aid to local governments, the largest element of state expenditure. Within the EU, the Structural Fund and the Cohesion Fund allocate over 40% of the EU budget to under-developed regions and states. Fiscal equalization schemes have also been implemented in Canada, Australia, Denmark and Switzerland, and many developing countries. Then there must be a systematic interaction between tax competition and fiscal equalization. Some public finance literatures pay attention to this issue. They mainly consider tax competition and fiscal equalization in a one-shot game framework. Janeba and Peters (2000) demonstrate that capital tax rates increase if regions are combined into a single tax revenue equalization system; in such a case, fiscal transfers partially internalize fiscal externalities. Kothenburger (2002) shows when horizontal externality arises, a tax base equalization system can eliminate this inefficiency perfectly. Kotsogiannis (2010) proves that when both horizontal externality and vertical externality exist, an efficient level of lower-level government taxation can be achieved when an appropriately adjusted standard tax base equalization formula is introduced.

Now we introduce each chapter in detail. In international trade theory, the specific factors model is a favored vehicle for analyzing the economy-wide effects of changes in commodity prices, factor endowments and other exogenous variables. In Chapter 1, we examine how robust the predictions of the specific factors model are in presence of public intermediate good. Ishizawa (1991) shows in a Heckscher-Ohlin economy, despite the presence of increasing returns to scale by a public intermediate good, the PPF is concave to the origin if the market for the public intermediate good is Marshall stable. We want to examine whether the Marshallian stability condition of public intermediate good market can assure the PPF is concave to the origin in a Specific factors model. We consider an economy with two private goods, two primary factors one of which is industry-specific, a public intermediate good. Following Ishizawa (1991), the public intermediate good is supplied by the government, which follows the Lindal-Samuelson-Kaizuka rule to obtain an efficient production. The government can not respond to the exogenous change in the price ratio of private goods instantly, but adjusts the level of public intermediate good in the course of a Marshallian adjustment process. We find that without the assumption that the factor intensities are different in two private good industries as in the Heckscher-Ohilin model, even the public intermediate good gives rise to increasing returns to scale for the economy, the PPF of the specific factors economy is concave to the origin when the public intermediate good market is stable under a Marshallian quantity adjustment process. The symmetry assumption that the public intermediate good can increase the production of two private good industries equi-proportionally is a sufficiently condition for the Marshallian stability. It is also shows that when the symmetry assumption holds, most but not all of the results of the standard specific factors model, which are about the effect of changes in terms of trade and factor endowments, remain robust in presence of public intermediate good.

Chapter 3 investigates how the semi-public intermediate good affects the trade pattern and gains from trade in a two-country model. This research is supplementary to Suga and Tawada (2007) which examines the effects of pure public intermediate good on trade patterns and trade gains in a two-country trade model. We develop a one primary factor, two consumer goods, one semi-public intermediate good and two-country model. The productivity of private industry 2 is more sensitive to the semi-public intermediate good than that of private industry 1. We assume that the semi-public intermediate good is provided by the government to get an efficient production. From the optimal supply condition, we show that the production possibility frontier is concave to the origin. For any given supply price, the production point is the point where the budget line is tangent with the production possibility frontier. It is demonstrated that when the supply price is given, the relative supply Q_2/Q_1 decreases with the increase of the factor endowment of labor. Since the homothetic preferences are the same in two countries, we can determine that the autarky price of good 1 is lower in the country with larger labor endowment. Hence, the large (small) country exports (imports) the good whose productivity is less sensitive to the semi-public intermediate good. About the normative analysis, when trade opens, both countries will gain from trade.

Chapter 4 examines the relationship between tax competition and fiscal equalization in a repeated tax competition game (Itaya et al., 2008). In particular, it asks the question how the fiscal equalization scheme affects the tax cooperation condition. Itaya et al. (2008) shows the larger the regional asymmetries, the easier the tax coordination is. They also demonstrate the best cooperation tax rate-the one that provides the strongest potential for voluntary cooperation-is zero. However, in practice the cooperative tax rate in Europe is not zero. Countries in Europe set the same positive tax rate as the form of tax cooperation. Hence, Itaya et al. (2008)'s model can not explain the economic facts in reality. It must ignore some things in the federal economy. We introduce the fiscal equalization scheme into Itaya et al. (2008)'s model. It is shown that when the scale of fiscal equalization scheme increases, the capital exporter (importer) has stronger (weaker) incentives to tax coordination. Since the existence of fiscal equalization scheme, the best cooperative tax rate becomes a positive value and increases with the scale of the fiscal equalization scheme.

Chapter 2

International Trade and Public Inputs under the Specific Factors Model in a Small Open Economy

2. 1 Introduction

In recent decades, trade theorists have been increasingly interested in the role of public inputs in international trade models. Manning and McMillan (1979) introduced public inputs into the Ricardian model, and analyzed the shape of the production possibility frontier and the pattern of trade. Tawada and Abe (1984), Altenburg (1987), Ishizawa (1991), etc. extended the analysis to the Heckscher-Ohlin model. They examined the shape of the production possibility frontier, the validity of the Stolper-Samuelson, Rybczynski, Heckscher-Ohlin theorems and the factor price equalization theorem. However, these models assume all production factors are mobile among industries and neglect the role played by those factors that are somehow specific to each industry. Neary (1978) argued that "the intersectoral capital mobility is the source of all the paradoxes which is peculiar to international trade theory, with the exception of those which arise from the failure to adopt first-best policies…." In light of these views in the literature, it is of interest to investigate the role of public inputs under the specific factors model.

This chapter considers an economy with two private goods, two primary factors one of which is industry-specific, and a public input. Following Ishizawa (1991), the public input is provided by the government, which follows the Lindahl-Samuelson-Kaizuka rule to obtain an efficient allocation of resources. The government can not respond to the exogenous change in the price ratio of private goods quickly but revises the level of the public input in the course of a Marshallian adjustment process. It will be shown that although the public input gives rise to increasing returns to scale for the economy as a whole, the production possibility frontier is concave to the origin when the public input market is stable under the Marshallian quantity adjustment process. The symmetry assumption that the public input increases the production of each private good industry equi-proportionately is a sufficient condition for the Marshallian stability. Then, most but not all results of the standard specific factors model, which are about the effect of changes in terms of trade and factor endowments, remain valid under the symmetry assumption.

The chapter is organized as follows. In the next section, the basic model is presented. In section 2. 3, we analyze the stability condition of the equilibrium and its relationship with the shape of the production possibility frontier. Section 2. 4 does comparative static analysis with respect to changes in terms of trade and factor endowments. Section 2. 5 makes some concluding remarks and the appendix provides necessary mathematical details.

2. 2 The Model

Let us consider an economy where there exist three industries: two private good industries and a public input industry. The government supplies the public input which is freely available to each private good industry. Assume X_0 represents the output level of the public input, X_1 and X_2 are the output level of private good 1 and private good 2. Let L_i and K_i denote the labor and capital used in the i th industry and K_i is specific to each industry. The production function of each industry can be written as

$$X_0 = F^0(L_0, K_0) \tag{1}$$

$$X_i = f^i(X_0)F^i(L_i, K_i),\ i = 1,2, \tag{2}$$

where function F^i is assumed to have the following ordinary properties: constant returns to scale; the marginal productivity is positive and is diminishing. f^i is concave that $df^i(X_0)/dX_0 > 0$ and $d^2 f^i(X_0)/dX_0^2 < 0$. If every pair of input (L_i, K_i) in private industry i is doubled, the output of private industry i is more doubled. Hence, the aggregate technology of the economy exhibits increasing returns to scale.

The economy is perfectly competitive. For private good industry i, when they make production decision, wage rate w, rental rate r_i and public input X_0 are exogenously given. Hence, the unit cost function for private good i is $c^i(X_0, w, r_i)$. The zero profit condition under the perfect competition implies the price of the private good i equals to its unit cost

$$p = c^1(X_0, w, r_1), \tag{3}$$

$$1 = c^2(X_0, w, r_2), \tag{4}$$

where p is the relative price of good 1 in terms of good 2.

The unit cost for public input is $c^0(w, r_o)$. Following the Lindahl-Samuelson-Kaizuka rule, the government purchases the public input up to point where the marginal cost of the public input equals its marginal social benefit, which implies

$$c^0(w, r_o) = p\phi^1(X_0)X_1 + \phi^2(X_0)X_2, \tag{5}$$

where $\phi^i(X_0)$ denotes the percentage change in the output of private good i per unit increase in the public input.

We assume the economy is endowed with labor L which is mobile among industries and capital K_i which is specific to i th industry for $i = 0,1,2$. If all resources are fully employed, we can obtain

$$L = l^0(w, r_0)X_0 + \sum_{i=1}^{2} l^i(X_0, w, r_i)X_i, \tag{6}$$

$$K^0 = k^0(w, r_0)X_0, \tag{7}$$

$$K^1 = k^1(X_0, w, r_1)X_1, \tag{8}$$

$$K^2 = k^2(X_0, w, r_2)X_2, \tag{9}$$

where $l^i(X_0, w, r_i)$ and $k^i(X_0, w, r_i)$ represent, respectively, the unit demand for labor and capital in private good industry i. $l^0(w, r_0)$ and $k^0(w, r_0)$ are defined similarly.

Seven equations (3)-(9) constitute a public input economy. In these seven equations there exist seven unknowns $(X_0, X_1, X_2, w, r_1, r_2, r_0)$ and parameters (p, L, K_1, K_2, K_0). For simplicity we take L, K_1, K_2, K_0 as fixed for time being. Each unknown can be solved as a function of p, thus we can derive

$$X_0 = X^{*0}(p), \tag{10}$$

$$X_1 = X^{*1}(p), \tag{11}$$

$$X_2 = X^{*2}(p), \tag{12}$$

$$w = w^{*}(p), \tag{13}$$

$$r_0 = r^{*0}(p), \tag{14}$$

$$r_1 = r^{*1}(p), \tag{15}$$

$$r_2 = r^{*2}(p). \tag{16}$$

If the Lindahl-Samuelson-Kaizuka rule is dropped, which means that the public input X_0 is determined exogenously. We only have six equations for six unknowns. The solution functions for this equation system can be written as

$$X_1 = X^{1}(p, X_0), \tag{17}$$

$$X_2 = X^{2}(p, X_0), \tag{18}$$

$$w = w(p, X_0), \tag{19}$$

$$r_0 = r^{0}(p, X_0), \tag{20}$$

$$r_1 = r^{1}(p, X_0), \tag{21}$$

$$r_2 = r^{2}(p, X_0). \tag{22}$$

2. 3 Stability of the Equilibrium and the Shape of the PPF

The present public input economy is assumed to be a small trading country with the terms of trade p as given. Facing every p, the government chooses the amount of public input X_0; the market forces determine the output of private good X_i and wage rate w and rental rate r_i. Supposing the adjustments in the market are rapid enough to produce an equilibrium before the government can revise the level of the public input X_0. The equilibrium output of private good X_i and wage rate w and rental rate r_i satisfy equations (3), (4) and (6)-(9). They can be expressed by equations (17)-(22).

Let $H(.)$ be the difference between the marginal social benefit and cost of the public input. According to equation (17)-(22), it is a function of terms of trade p and the output level of public input X_0:

$$H(p, X_0) = [p\phi^1(X_0)X^1(p, X_0) + \phi^2(X_0)X^2(p, X_0)] - c^0[w(p, X_0), r_o(p, X_0)]. \tag{23}$$

Following equation (5), the above function equals to zero when X_0 is evaluated at $X^{*0}(p)$. It implies

$$H(p, X^{*0}(p)) = 0, \quad \text{for all } p. \tag{24}$$

We characterize the adjustment process of the government illustrated previously as follows:

$$\dot{X}_0 = H(p, X_0). \tag{25}$$

The local stability can be derived if $H(.)$ has a negative derivative with respect to X_0 when the value of X_0 is $X^{*0}(p)$:

$$H_0(p, X^{*0}(p)) < 0. \tag{26}$$

This Marshallian stability condition plays a central role in our analysis. Lemma 1 gives an intuitive explanation for this condition.

Lemma 1. *When all three goods are produced in the public input economy for given* p *and* X_0*, and the national income function is defined as:* $m(p,X_0) \equiv pX^1(p,X_0) + X^2(p,X_0)$*. Then for all such* p *and* X_0*, we can obtain* (proved in appendix)

$$H(p,X_0) = m_0(p,X_0) \equiv pX_0^1(p,X_0) + X_0^2(p,X_0). \tag{27}$$

From lemma 1, the Lindahl-Samuelson-Kaizuka rule and the stability condition given in equation (24) and (26) which are expressed in the form of $H(.)$ can be translated into equivalent conditions in terms of $m(.)$ when it is evaluated at $X^{*0}(p)$:

$$m_0(p,X^{*0}(p)) = 0, \tag{28}$$

$$m_{00}(p,X^{*0}(p)) < 0. \tag{29}$$

Equation (28) says that the national income takes an extremal value for given p when X_0 is set equal to $X^{*0}(p)$. Equation (29) implies this extremal is local maximal.

As will be proved that there is a tight relationship between the Marshallian stability condition and the shape of the production possibility frontier.

It is demonstrated that $X^1(.)$ and $X^2(.)$ in equation (17) and (18) satisfy the following properties (see the appendix A):

$$X_p^1 > 0, \tag{30}$$

$$pX_p^1 + X_p^2 = 0, \tag{31}$$

where the subscript of a function refers to partial derivative of that function.

To proceed, consider the following expression:

$$X^{*i}(p) = X^i(p, X^{*0}(p)) \quad \text{for all} \quad p \quad i = 1,2 \tag{32}$$

which is obtained by evaluating equation (17) and (18) at the point where $X^0(.)$ is set equal to $X^{*0}(.)$. Differentiation of (32) with respect to p for private good 1 yields

$$X_p^{*1} = X_p^1 + X_0^1 X_p^{*0}. \tag{33}$$

X_p^1 is the pure price effect, which keeps a constant level of public input. As is shown in equation (30), X_p^1 preserves the nice property established in the traditional Heckscher-Ohlin model. $X_0^1 X_p^{*0}$ denotes the public input effect which measures effect of a change in the public input induced by the change in the price ratio on the production of private good 1. Equation (33) implies that the total price effect can be decomposed into pure price effect and public input effect.

From equation (33), we can find that in order to determine the sign of X_p^{*1}, it is critical to examine the sign of $X_0^1 X_p^{*0}$. For preparation, the following equation is derived (proved in the appendix A):

$$H_p(p, X_0) = X_0^1(p, X_0). \qquad (34)$$

Proposition 1 below shows the supply curve is upward sloping, when the Marshallian stability of the public input market is satisfied.

Proposition 1. *Assume that all three goods are produced in the public input economy. Assume also that the market for the public input is Marshall stable. Then, the supply curve is upward sloping around a neighborhood of a given* p:

$$X_p^{*1} > 0.$$

Proof: Following equation (30) and (33), the proposition can be established if $X_0^1 X_p^{*0}$ is non-negative. Differentiation of (24) yields

$$H_p + H_0 X_p^{*0} = 0. \qquad (35)$$

Combining equation (34) and (35), we can obtain

$$X_0^1 X_p^{*0} = -(X_0^1)^2 / H_0 \geq 0,$$

in which the last inequality follows from the Marshallian stability condition (26).

For convenience, we establish the following assumption:

Assumption 1. *The public input increases the production of each private good industry* *equi-proportionately:*

$$\phi^1(X_0) = \phi^2(X_0) \quad \text{for all} \quad X_0.$$

We call assumption 1 the symmetry assumption.

The following proposition 2 shows the Marshallian stability condition is more general than the symmetry assumption.

Proposition 2. *Assume all three goods are produced in the public input economy under the symmetry assumption for a given p. The market for the public input is Marshall stable around a neighborhood of any given p.*

Proof: With differentiation of $H(.)$ in (23) with respect to X_0, we obtain

$$H_0 = (p\phi_0^1 X^1 + \phi_0^2 X^2) + (p\phi^1 X_0^1 + \phi^2 X_0^2) - c_0^0(w, r_o). \tag{36}$$

Since $\phi_0^i < 0$ (see the appendix A) for all X_0, the first parenthetic term is negative. In view of the symmetry assumption, the second term can be expressed as $\phi^1(pX_0^1 + X_0^2)$ which is zero when it is evaluated in $X^{*0}(p)$ by equation (28). We can prove that $c_0^0(w, r_o) > 0$ (proved in the appendix A). All these implies $H_0 < 0$ at the point where $X^0(p)$ equals . Then we derive the proposition 2.

With intersectoral capital mobility, Ishizawa (1991) also derived that the symmetry assumption is a sufficient condition for the Marshallian stability. But the mechanism behind proposition 2 is different from that of Ishizawa (1991). In order to get the Marshallian stability condition, it is critical that an increase in the output of public input can raise the unit cost of public input. When capital is intersectorally mobile, the unit cost of public input is positively related to the overall wage rate w and overall rental rate r. An increase in the output of public input will raise both the overall wage rate w and overall rental rate r, so an increase in the public input will increase the unit cost of public input. Under specific factors framework, the unit cost of public input is positively related to the overall wage rate w and the rental rate to the specific factor in the public input industry r_0. A rise in the output of public input will increase both overall wage rate w and the rental rate to the specific factor in the public input industry r_0. Then, when the output level of public input increases, the unit cost of public input still goes up.

Following equation (33) and the corresponding equation for private-good 2, we can obtain

$$pX_p^{*1} + X_p^{*2} = (pX_p^1 + X_p^2) + (pX_0^1 + X_0^2)X_p^{*0} = 0. \tag{37}$$

Hence, there exists no distortion in this public input economy.

Following the above analysis, we can get:

Corollary 1. *If all three goods are produced in the public economy, and the market for the public input is Marshall stable, then the production possibility frontier is locally concave to the origin for a given* p. *If all three goods are produced in the public economy, and the production effect of the public input on the two private-good industries is symmetric, the production possibility frontier is globally concave to the origin for any given* p.

Each of the dotted curves in Fig 2. 1 denotes the production possibility frontier of the economy when the public input is given at some constant level. Equation (30) implies this kind of production possibility frontiers is locally concave to the origin for a given p. Equation (31) says the relative price line is tangent to the production possibility frontier at the

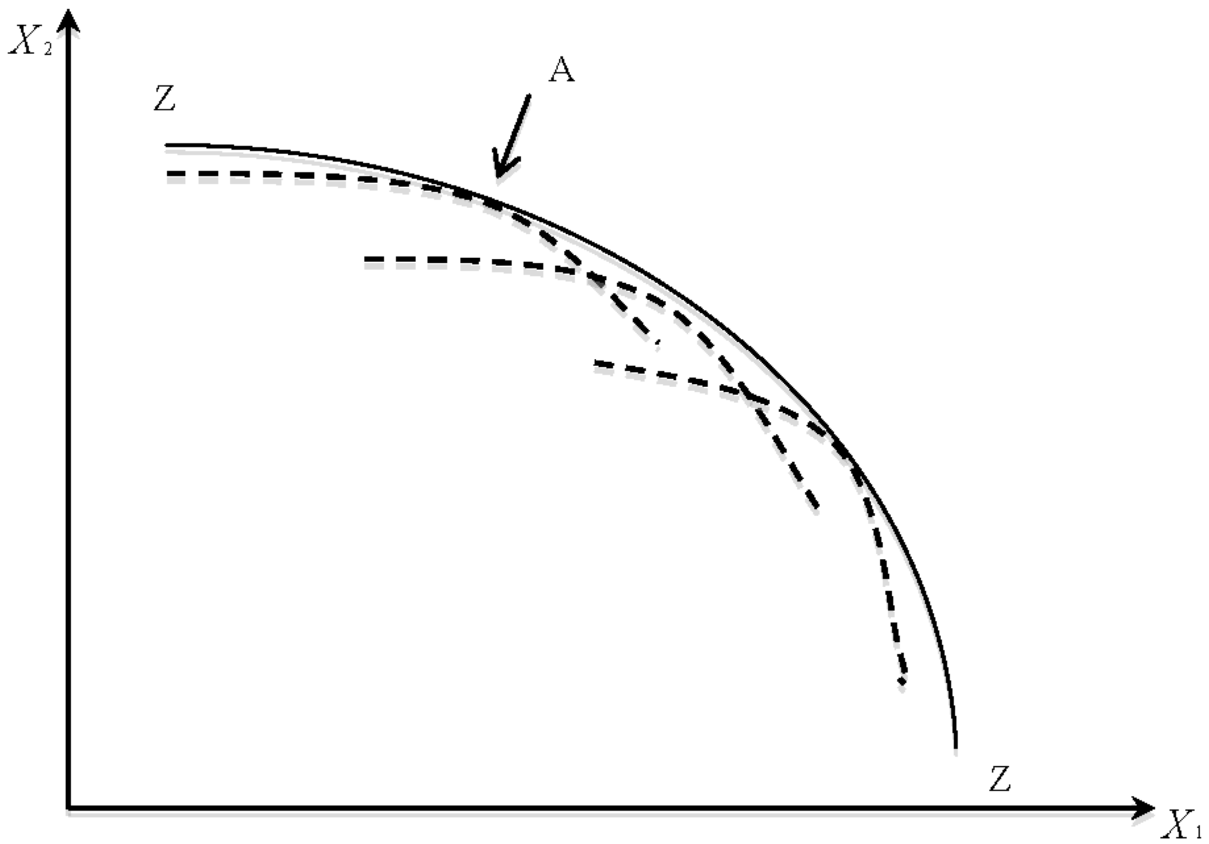

Figure 2. 1 the production possibility frontier

point given by $(X^1(p,X_0),X^2(p,X_0))$. Point A corresponds to the production combination $(X^1(p,X_0),X^2(p,X_0))$ for p where the value of $X^0(p)$ is adjusted to $X^{*0}(p)$. $X^{*0}(p)$ varies as p varies. This variation yields a collection of dotted production possibility frontier. The curve ZZ, which is the envelope of the collection of the dotted production possibility frontier, is the production possibility frontier of the economy when the public input is provided at the optimal level for a given p. Base on the corollary, if all three goods are produced in the economy, the production possibility frontier ZZ is locally concave to the origin for a given p when the market for the public input is Marshall stable; the symmetry assumption is a sufficient condition to ensure that the production possibility frontier ZZ is globally concave to the origin for any given p.

In the standard specific factors model the production possibility frontier is concave to the origin. Proposition 1 and corollary 1 show that even there exists public input which brings increasing returns to scale for the economy, the supply curve of private good is upward sloping and the production possibility frontier is still concave when the public input market is Marshall stable. Under the framework of Heckscher-Ohlin model, Ishizawa (1991) proved that when the factor intensities of two private good industries is different, the supply curve is upward sloping and the production possibility frontier is concave if the market for the public input is Marshall stable. Proposition 1 and corollary 1 make no assumption about the factor intensities in two private good industries. They say that, regardless of the factor intensities in two private good industries, the supply curve is upward sloping and the production possibility frontier is concave to the origin when the Marshallian stability condition is satisfied.

2. 4 The Comparative Static Analysis of a Small Country

Total differentiation of (3)-(9) yields the following equation system

$$\begin{bmatrix} \theta_{L1} & \theta_{K1} & 0 & 0 & 0 & 0 & -e_1 \\ \theta_{L2} & 0 & \theta_{K2} & 0 & 0 & 0 & -e_2 \\ \theta_{L0} & 0 & 0 & \theta_{K0} & -\delta_1 & -\delta_2 & 1 \\ -S_L & \lambda_{L1}\theta_{K1}\sigma_1 & \lambda_{L2}\theta_{K2}\sigma_2 & \lambda_{L0}\theta_{K0}\sigma_0 & \lambda_{L1} & \lambda_{L2} & \lambda_{L0}^{'} \\ \theta_{L0}\sigma_0 & 0 & 0 & -\theta_{L0}\sigma_0 & 0 & 0 & 1 \\ \theta_{L1}\sigma_1 & -\theta_{L1}\sigma_1 & 0 & 0 & 1 & 0 & -e_1 \\ \theta_{L2}\sigma_2 & 0 & -\theta_{L2}\sigma_2 & 0 & 0 & 1 & -e_2 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r}_1 \\ \hat{r}_2 \\ \hat{r}_0 \\ \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_0 \end{bmatrix} = \begin{bmatrix} \hat{p} \\ 0 \\ \delta_1\hat{p} \\ \hat{L} \\ \hat{K}_0 \\ \hat{K}_1 \\ \hat{K}_2 \end{bmatrix}$$

where $\theta_{Li} = wl^i/C^i$, $\theta_{Ki} = r_ik^i/C^i$, $\lambda_{Li} = L^i/L$, $\lambda_{Ki} = K^i/K$ $(i = 1,2,0)$, σ_i is the elasticity of substitution in the i th industry. We also stipulate that:

$$S_L = \lambda_{L0}\theta_{K0}\sigma_0 + \lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2$$

$$\lambda_{L0}^{'} = \lambda_{L0} - \lambda_{L1}e_1(X_0) - \lambda_{L2}e_2(X_0)$$

$$\delta_1 = pe_1(X_0)X_1/[pe_1(X_0)X_1 + e_2(X_0)X_2]$$

$$\delta_2 = e_2(X_0)X_2/[pe_1(X_0)X_1 + e_2(X_0)X_2]$$

$$\varepsilon_i = (de_i/dX_0)(X_0/e_i).$$

We assume the symmetry assumption is satisfied in the following comparative analysis. It is proved that the symmetry assumption can ensure the determinant of the matrix in the LHSS of the above equation system Δ is positive (proved in the appendix A). We also assume that the production effect is constant, hence $\varepsilon_i = 0$.

First, let us examine the effect of an increase in the factor supply on the output level of two private good industries. We can derive

$$\frac{\hat{X}_1}{\hat{L}} = [\theta_{K1}\theta_{L2}\sigma_2\theta_{L0}\sigma_0\delta_2 e_1 + \theta_{K1}\theta_{K2}\theta_{L0}\sigma_0 e_1 + \theta_{L1}\sigma_1\theta_{K2}\theta_{L0}\sigma_0(1-\varepsilon) + \theta_{L1}\sigma_1\theta_{K2}\theta_{K0} + \theta_{L1}\sigma_1 \theta_{K2}\theta_{L0}\sigma_0 e_1(1-\delta_2)]/\Delta > 0 \quad (38)$$

$$\frac{\hat{X}_2}{\hat{L}} = [\theta_{K1}\theta_{L2}\sigma_2\theta_{L0}\sigma_0 + \theta_{K1}\theta_{L2}\sigma_2\theta_{K0} - \theta_{K1}\theta_{L2}\sigma_2\theta_{L0}\sigma_0\delta_1 e_1 + \theta_{K1}\theta_{K2}\theta_{L0}\sigma_0 e_2 + \theta_{K1}\theta_{L2}\sigma_2 e_2 \theta_{L0}\sigma_0 + \theta_{L1}\sigma_1\delta_1\theta_{K2}\theta_{L0}\sigma_0 e_2 + \theta_{L1}\sigma_1\delta_1\theta_{L2}\sigma_2\theta_{L0}\sigma_0 e_2 - \theta_{L1}\sigma_1 e_1\delta_1\theta_{L2}\sigma_2\theta_{L0}\sigma_0]/\Delta > 0 \quad (39)$$

$$\frac{\hat{X}_1}{\hat{K}_1} = [\theta_{L0}\sigma_0\theta_{K1}\lambda_{L1}\sigma_2 - \theta_{L0}\sigma_0\theta_{K1}\delta_2\lambda_{L1}e_1\theta_{L2}\sigma_2 - \theta_{L0}\sigma_0\theta_{K1}\lambda_{L2}\sigma_2\delta_2 e_2 + \theta_{L0}\sigma_0\theta_{K1}e_2\lambda_{L2}\sigma_2 - \theta_{L0}\sigma_0\theta_{K1}\theta_{K2}\lambda_{L1}e_1 - \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1\theta_{K2}\delta_2 e_2 - \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1 e_2\delta_2\theta_{L2}\sigma_2 + \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1} \sigma_1(1-\varepsilon)\theta_{K2} + \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1 e_1\delta_2\theta_{L2}\sigma_2 + \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1 e_1\theta_{K2} + \theta_{K2}\theta_{K0}\lambda_{L1}\theta_{K1}\sigma_1 + \theta_{K1}\delta_2\lambda_{L0}\sigma_0\theta_{L2}\sigma_2 + \theta_{K1}\theta_{K0}\lambda_{L2}\sigma_2 + \theta_{K1}\theta_{K2}\lambda_{L0}\sigma_0]/\Delta > 0 \quad (40)$$

$$\frac{\hat{X}_2}{\hat{K}_1} = [\theta_{L0}\sigma_0\theta_{K1}\delta_1\lambda_{L1}e_1\theta_{L2}\sigma_2 - \theta_{L0}\sigma_0\theta_{K1}\lambda_{L1}\theta_{L2}\sigma_2 + \theta_{L0}\sigma_0\theta_{K1}\lambda_{L2}\sigma_2\delta_1 e_2 - \theta_{L0}\sigma_0\theta_{K1}\theta_{K2}\lambda_{L1} e_2 - \theta_{L0}\sigma_0\theta_{K1}e_2\lambda_{L1}\theta_{L2}\sigma_2 + \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1\theta_{K2}\delta_1 e_2 + \theta_{L0}\sigma_0\lambda_{L1}\theta_{K1}\sigma_1 e_2\delta_1\theta_{L2}\sigma_2 - \theta_{L0} \sigma_0\lambda_{L1}\theta_{K1}\sigma_1 e_1\delta_1\theta_{L2}\sigma_2 - \theta_{K1}\theta_{L2}\sigma_2\theta_{K0}\lambda_{L1} - \theta_{K1}\theta_{L2}\sigma_2\lambda_{L0}\sigma_0\delta_1]/\Delta < 0 \quad (if\, \sigma_1 < 1, \theta_{K1} < \theta_{L1}) \quad (41)$$

$$\frac{\hat{X}_1}{\hat{K}_2} = [\theta_{L0}\sigma_0\theta_{K2}\delta_2\lambda_{L2}e_1\theta_{L1}\sigma_1 - \theta_{L0}\sigma_0\theta_{K2}\lambda_{L2}\theta_{L1}\sigma_1 + \theta_{L0}\sigma_0\theta_{K2}\lambda_{L1}\sigma_1\delta_2 e_1 - \theta_{L0}\sigma_0\theta_{K1}\theta_{K2}\lambda_{L2} e_1 - \theta_{L0}\sigma_0\theta_{K2}e_1\lambda_{L2}\theta_{L1}\sigma_1 + \theta_{L0}\sigma_0\lambda_{L2}\theta_{K2}\sigma_2\theta_{K1}\delta_2 e_1 + \theta_{L0}\sigma_0\lambda_{L2}\theta_{K2}\sigma_2 e_1\delta_2\theta_{L1}\sigma_1 - \theta_{L0} \sigma_0\lambda_{L2}\theta_{K2}\sigma_2 e_2\delta_2\theta_{L1}\sigma_1 - \theta_{K2}\theta_{L1}\sigma_1\theta_{K0}\lambda_{L2} - \theta_{K2}\theta_{L1}\sigma_1\lambda_{L0}\sigma_0\delta_2]/\Delta < 0 \quad (if\, \sigma_2 < 1, \theta_{K2} < \theta_{L2}) \quad (42)$$

$$\frac{\hat{X}_2}{\hat{K}_2} = [\theta_{L0}\sigma_0\theta_{K2}\lambda_{L2}\sigma_1 - \theta_{L0}\sigma_0\theta_{K2}\delta_1\lambda_{L2}e_2\theta_{L1}\sigma_1 - \theta_{L0}\sigma_0\theta_{K2}\lambda_{L1}\sigma_1\delta_1 e_1 + \theta_{L0}\sigma_0\theta_{K2}e_1\lambda_{L1}\sigma_1 - \theta_{L0}\sigma_0\theta_{K1}\theta_{K2}\lambda_{L2}e_2 - \theta_{L0}\sigma_0\lambda_{L2}\theta_{K1}\sigma_2\theta_{K2}\delta_1 e_1 - \theta_{L0}\sigma_0\lambda_{L2}\theta_{K2}\sigma_1 e_1\delta_1\theta_{L1}\sigma_2 + \theta_{L0}\sigma_0\lambda_{L2}\theta_{K1} \sigma_2\theta_{K2} + \theta_{L0}\sigma_0\lambda_{L2}\theta_{K2}\sigma_2 e_1\delta_1\theta_{L1}\sigma_1 + \theta_{L0}\sigma_0\lambda_{L2}\theta_{K2}\sigma_2 e_2\theta_{K1} + \theta_{K2}\theta_{K0}\lambda_{L2}\theta_{K1}\sigma_2 + \theta_{K2}\delta_1\lambda_{L0} \sigma_0\theta_{L1}\sigma_1 + \theta_{K2}\theta_{K0}\lambda_{L1}\sigma_1 + \theta_{K1}\theta_{K2}\lambda_{L0}\sigma_0]/\Delta > 0. \quad (43)$$

These inequalities can be summarized as

Proposition 3. *When the relative commodity price is constant, an increase in the supply of labor will expand the output level of two private good industries. Provided that* $\sigma_i < 1, \theta_{Ki} < \theta_{Li}$*, when the commodity price is constant, an increase in the endowment of one* *specific factor will raise the output level of the private good industry using that specific factor, and lower that of the other private good industry.*

The output-factor supply relationship of proposition 3 is the same as that of the standard specific factors model. However, in this public input economy, to ensure a rise in the supply of one specific factor K_1 (K_2) will lower the output of the private good X_2 (X_1), we have to assume the elasticity of substitution σ_1(σ_2) is less than 1, and the income share of capital θ_{K1} (θ_{K2}) is less than the income share of labor θ_{L1} (θ_{L2}). Both assumptions are not necessary in the standard specific factors model.

Now we consider the effect of the change in terms of trade. We have

$$\begin{aligned}\frac{\hat{w}-\hat{p}}{\hat{p}} = [&\lambda_{L1}\sigma_1\theta_{K2}\theta_{K0} + \lambda_{L1}\sigma_1\theta_{K2}\theta_{L0}\sigma_0(1-\varepsilon-e_1\delta_1-e_2\delta_2) - \lambda_{L1}\sigma_1\theta_{L2}\sigma_2 e_2\delta_2\theta_{L0}\sigma_0 + \\ &\theta_{L1}\sigma_1\theta_{K2}\delta_1\lambda_{L0}\sigma_0 + \theta_{L1}\sigma_1 e_2\delta_1\lambda_{L2}\sigma_2\theta_{L0}\sigma_0 + \theta_{K1}\theta_{K2}\lambda_{L0}\sigma_0\delta_1 + \lambda_{L2}\sigma_2\theta_{K1}\theta_{L0}\sigma_0 e_2\delta_1 + \\ &\lambda_{L1}\sigma_1\theta_{K2}\theta_{L0}\sigma_0 e_1\delta_1 - \theta_{K1}\lambda_{L2}\sigma_2\theta_{K0} - \theta_{K1}\lambda_{L2}\sigma_2\theta_{L0}\sigma_0(1-\varepsilon) - \theta_{K1}\theta_{L2}\sigma_2\lambda_{L0}\sigma_0\delta_2 - \theta_{K1} \\ &\theta_{K2}\lambda_{L0}\sigma_0 - \lambda_{L1}\sigma_1\theta_{K2}\theta_{K0} - \theta_{K2}\lambda_{L1}\sigma_1\theta_{L0}\sigma_0(1-\varepsilon) - \theta_{K2}\theta_{L1}\sigma_1\delta_1\lambda_{L0}\sigma_0]/\Delta < 0\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\hat{r}_1-\hat{p}}{\hat{p}} = [&\theta_{L0}\sigma_0\lambda_{L1}\sigma_1\theta_{K2}(1-e_1\delta_1-e_2\delta_2) - \theta_{L0}\sigma_0\lambda_{L1}\sigma_1 e_2\delta_2\theta_{L2}\sigma_2 + \theta_{L0}\sigma_0\lambda_{L2}\sigma_2(1-e_1) + \\ &\theta_{L0}\sigma_0\lambda_{L2}\sigma_2 e_2 + \theta_{L0}\sigma_0\lambda_{L2}\sigma_2 e_2\delta_1\theta_{L1}\sigma_1 + \delta_2\lambda_{L0}\sigma_0\theta_{L2}\sigma_2 + \lambda_{L2}\sigma_2\theta_{K0} + \lambda_{L0}\sigma_0\theta_{K2} + \\ &\lambda_{L0}\sigma_0\theta_{K2}\delta_1\theta_{L1}\sigma_1 + \theta_{K2}\lambda_{L1}\sigma_1\theta_{K0} - \lambda_{L0}\sigma_0\delta_1\theta_{L1}\theta_{K2} - \delta_1\theta_{L1}e_2\lambda_{L2}\sigma_2\theta_{L0}\sigma_0 + \delta_1 e_1\lambda_{L2}\sigma_2 \\ &\theta_{L0}\sigma_0 + \delta_1 e_1\theta_{K2}\lambda_{L1}\sigma_1\theta_{L0}\sigma_0 - \theta_{K1}\lambda_{L2}\sigma_2\theta_{K0} - \theta_{K1}\lambda_{L2}\sigma_2\theta_{L0}\sigma_0 - \theta_{K1}\theta_{L2}\sigma_2\lambda_{L0}\sigma_0\delta_2 - \\ &\theta_{K1}\theta_{K2}\lambda_{L0}\sigma_0 - \theta_{K1}\lambda_{L1}\sigma_1\theta_{K0} - \theta_{K2}\lambda_{L1}\sigma_1\theta_{L0}\sigma_0 - \theta_{K2}\theta_{L1}\sigma_1\delta_1\lambda_{L0}\sigma_0]/\Delta > 0\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{\hat{r}_2-\hat{p}}{\hat{p}} = [&-\theta_{L2}\lambda_{L1}\sigma_1\theta_{K0} - \theta_{L2}\lambda_{L1}\sigma_1\theta_{L0}\sigma_0(1-\varepsilon-e_1\delta_1-e_2\delta_2) - \theta_{L2}\theta_{L1}\sigma_1\lambda_{L0}\sigma_0\delta_1 + \theta_{L2}\delta_1 \\ &\theta_{K1}\lambda_{L0}\sigma_0 + \theta_{L2}\delta_1 e_1\lambda_{L1}\sigma_1\theta_{L0}\sigma_0 - e_2\theta_{L2}\sigma_2\delta_2\lambda_{L1}\sigma_1\theta_{L0}\sigma_0 - e_2\theta_{L0}\sigma_0\lambda_{L1}\sigma_1 + e_2\theta_{L0}\sigma_0 \\ &\delta_1\lambda_{L2}\sigma_2\theta_{L1}\sigma_1 - \theta_{K1}\lambda_{L2}\sigma_2\theta_{K0} - \theta_{K1}\lambda_{L2}\sigma_2\theta_{L0}\sigma_0 - \theta_{K1}\theta_{L2}\sigma_2\lambda_{L0}\sigma_0\delta_2 - \theta_{K1}\theta_{K2}\lambda_{L0}\sigma_0 \\ &- \theta_{K2}\lambda_{L1}\sigma_1\theta_{K0} - \theta_{K2}\lambda_{L1}\sigma_1\theta_{L0}\sigma_0 - \theta_{K2}\theta_{L1}\sigma_1\lambda_{L0}\sigma_0\delta_1]/\Delta < 0.\end{aligned} \tag{46}$$

These results can be summarized in

Proposition 4. *With fixed factor supplies, the real return to the mobile factor will decrease in terms of the good experiencing price increase. A rise in the relative price of a good raises the real return to the factor specific to that good and lowers the real return to the specific factor of the other private good industry.*

As is shown is proposition 4, the factor return rate-commodity price relationship is the same as that of the standard specific factors model.

Finally, we analyze the effect of an increase in the factor supplies on the return to each factor. We can get

$$\frac{\hat{r}_1}{\hat{L}} = [\theta_{L1}\theta_{K2}\theta_{K0} + \theta_{L1}\theta_{K2}\theta_{L0}\sigma_0(1-e_1) - \theta_{L1}e_2\theta_{L2}\sigma_2\delta_2\theta_{L0}\sigma_0 + e_1\theta_{L2}\sigma_2\delta_2\theta_{L0}\sigma_0 + e_1\theta_{K2}\theta_{L0} \sigma_0 + e_1\theta_{K2}\theta_{L0}\sigma_0\theta_{L1}\sigma_1\delta_1]/\Delta > 0 \tag{47}$$

$$\frac{\hat{r}_2}{\hat{L}} = [\theta_{K1}\theta_{L2}\theta_{K0} + \theta_{K1}\theta_{L2}\theta_{L0}\sigma_0(1-e_1) + \theta_{K1}e_2\theta_{L0}\sigma_0 + \theta_{K1}e_2\theta_{L2}\sigma_2\delta_2\theta_{L0}\sigma_0 + \theta_{L1}\sigma_1\delta_1\theta_{L0}\sigma_0 e_2 - \theta_{L1}\sigma_1\delta_1 e_1\theta_{L2}\theta_{L0}\sigma_0]/\Delta > 0 \tag{48}$$

$$\frac{\hat{w}}{\hat{K}_1} = [\theta_{K1}\theta_{K2}\theta_{K0}\lambda_{L1} - \theta_{K1}\theta_{K2}\delta_1\lambda_{L1}e_1\theta_{L0}\sigma_0 + \theta_{K1}\theta_{K2}(1-\varepsilon)\lambda_{L1}\theta_{L0}\sigma_0 + \theta_{K1}\theta_{K2}\delta_1\lambda_{L0}\sigma_0 - \theta_{K1}\theta_{K2} e_2\delta_2\lambda_{L1}\theta_{L0}\sigma_0 - \theta_{K1}e_2\theta_{L0}\sigma_0\delta_2\lambda_{L1}\theta_{L2}\sigma_2 + \theta_{K1}e_2\theta_{L0}\sigma_0\delta_1\lambda_{L2}\sigma_2 + e_1\theta_{K2}\lambda_{L1}\theta_{K1}\sigma_1\theta_{L0}\sigma_0 \delta_1]/\Delta > 0 \tag{49}$$

$$\frac{\hat{w}}{\hat{K}_2} = [\theta_{K1}\theta_{K2}\theta_{K0}\lambda_{L2} - \theta_{K1}\theta_{K2}\delta_2\lambda_{L2}e_2\theta_{L0}\sigma_0 + \theta_{K1}\theta_{K2}\lambda_{L2}\theta_{L0}\sigma_0 + \theta_{K1}\theta_{K2}\delta_1\lambda_{L0}\sigma_0 - \theta_{K1}\theta_{K2}e_1\delta_1\lambda_{L2} \theta_{L0}\sigma_0 - \theta_{K2}e_1\theta_{L0}\sigma_0\delta_1\lambda_{L2}\theta_{L1}\sigma_1 + \theta_{K2}e_1\theta_{L0}\sigma_0\delta_2\lambda_{L1}\sigma_1 + e_2\theta_{K1}\lambda_{L2}\theta_{K2}\sigma_2\theta_{L0}\sigma_0\delta_2]/\Delta > 0. \tag{50}$$

These inequalities can be summarized as

Proposition 5. *When relative commodity price is constant, an increase in the supply of* *labor will raise the return to the specific factors in two private good industries. An increase in either specific factor will raise the return to labor.*

The results of proposition 5 are the same as that of the standard specific factors model. In the standard specific factors economy, an increase in the supply of labor will lower the wage rate to labor, and a rise in the supply of one specific factor will decrease the rental rate to both specific factors. However, these effects are ambiguous in the present public input economy.

2. 5 Conclusion

Ishizawa (1991) assumed that the public input is supplied under a Marshallian quantity adjustment process in a Heckscher-Ohlin economy. It was shown that the supply curve of a private good is upward sloping and the production possibility frontier is concave to the origin if the public input market is Marshall stable, and the symmetry assumption is a sufficient condition for the Marshallian stability.

To examine the role of public inputs in the specific factors model, this chapter considers a specific factors economy in which the public input is provided by the same rule as that of Ishizawa (1991). Proposition 1 and corollary 1 have demonstrated that even though the aggregate technology of the economy exhibits increasing returns to scale, the supply curve of a private good is upward sloping and the production possibility frontier is concave to the origin when the Marshallian stability for the public market is satisfied. This is proved without assumption that factor intensities in two private good industries are different as Ishizawa (1991). Proposition 2 establishes that the symmetry assumption is still a sufficient condition for the Marshallian stability even in the specific factors economy.

The comparative static analysis demonstrates that with the symmetry assumption, most but not all of the results of the standard specific factors model remain valid in presence of the public input. In this public input economy, in order to ensure a rise in the supply of specific factor K_1 (K_2) will lower the output of private good X_2 (X_1), we have to make the assumption that the elasticity of substitution σ_1 (σ_2) is less than 1, and the income share of capital θ_{K1} (θ_{K2}) is less than the income share of labor θ_{L1} (θ_{L2}). The standard specific factors model makes no such assumption. And in the standard specific factors economy, a rise in the supply of labor will decrease the wage rate to labor, and an increase in the endowment of one specific factor will lower the rental rate to the specific factors in two private good industries. However, these effects are ambiguous when a public input arises in the economy.

Appendix A:

This appendix will prove lemma 1, (30), (31), (34), (36).

For notational simplicity, le $L^D(X_0, X_1, X_2, w, r_1, r_2, r_0)$, $K^{D0}(X_0, w, r_0)$, $K^{D1}(X_0, X_1, w, r_1)$, and $K^{D2}(X_0, X_2, w, r_2)$ denote the demands for labor and capital representing the RHSS of (6)-(9) respectively. Then given parameters, the solution functions in (17)-(22) satisfy the following equations:

$$p = c^1(X_0, w, r_1) \tag{A1}$$

$$1 = c^2(X_0, w, r_2) \tag{A2}$$

$$L = L^D(X_0, X_1, X_2, w, r_1, r_2, r_0) \tag{A3}$$

$$K^0 = K^{D0}(X_0, w, r_0) \tag{A4}$$

$$K^1 = K^{D1}(X_0, X_1, w, r_1) \tag{A5}$$

$$K^2 = K^{D2}(X_0, X_2, w, r_2) \tag{A6}$$

$l^i(\cdot)$ and $k^i(\cdot)$ are homogeneous of degree zero in factor prices and symmetric, $l^i_{r_i} = k^i_w > 0, i = 0,1,2$. It follows from the definitions of L^D , K^{D0} , K^{D1} , K^{D2} that

$$L^D_w = l^0_w X_0 + l^1_w X_1 + l^2_w X_2 \tag{A7}$$

$$L^D_{r_0} = l^0_{r_0} X_0 \tag{A8}$$

$$L^D_{r_1} = l^1_{r_1} X_1 \tag{A9}$$

$$L^D_{r_2} = l^2_{r_2} X_2 \tag{A10}$$

$$K^{D0}_w = k^0_w X_0 \tag{A11}$$

$$K^{D0}_{r_0} = k^0_{r_0} X_0 \tag{A12}$$

$$K^{D1}_w = k^1_w X_1 \tag{A13}$$

$$K^{D1}_{r_1} = k^1_{r_1} X_1 \tag{A14}$$

$$K^{D2}_w = k^2_w X_2 \tag{A15}$$

$$K^{D2}_{r_2} = k^2_{r_2} X_2 \tag{A16}$$

thus

$$L^D_w w + L^D_{r_0} r_0 + L^D_{r_1} r_1 + L^D_{r_2} r_2 = K^{D0}_w w + K^{D0}_{r_0} r_0 = K^{D1}_w w + K^{D1}_{r_1} r_1 = K^{D2}_w w + K^{D2}_{r_2} r_2 = 0 \tag{A17}$$

$$L^D_{r_1} = K^{Di}_w > 0 \tag{A18}$$

Now, differentiate (A1)-(A6) with respect to P , X_0 ; then use Shephard's lemma to obtain

$$\begin{bmatrix} l^1 & 0 & k^1 & 0 & 0 & 0 \\ l^2 & 0 & 0 & k^2 & 0 & 0 \\ L^D_w & l^0_{r_0} X_0 & l^1_{r_1} X_1 & l^2_{r_2} X_2 & l^1 & l^2 \\ k^0_w \cdot X_0 & k^0_{r_0} X_0 & 0 & 0 & 0 & 0 \\ k^1_w \cdot X_1 & 0 & k^1_{r_1} X_1 & 0 & k^1 & 0 \\ k^2_w \cdot X_2 & 0 & 0 & k^2_{r_2} X_2 & 0 & k^2 \end{bmatrix} \begin{bmatrix} \partial w \\ \partial r_0 \\ \partial r_1 \\ \partial r_2 \\ \partial X_1 \\ \partial X_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \partial p + \begin{bmatrix} -c^1_0 \\ -c^2_0 \\ -L^D_0 \\ -k^0 \\ -k^1_0 X_1 \\ -k^2_0 X_2 \end{bmatrix} \partial X_0 \tag{A19}$$

Let Δ denote the determinant of the matrix on the LHS of the above equation system, we have

$$\Delta = -l^1 k^2 k^0_{r_0} X_0 l^1_{r_1} X_1 k^1 k^2 + l^1 k^2 k^0_{r_0} X_0 k^1_{r_1} X_1 l^1 k^2 - k^1 l^2 k^0_{r_0} X_0 l^2_{r_2} X_2 k^1 k^2 + k^1 l^2 k^0_{r_0} X_0 k^2_{r_2} X_2 k^1 l^2 - k^1 k^2 l^2 k^0_{r_0} X_0 k^1 k^2_w X_2 + k^1 \left(k^2\right)^2 L^D_w k^0_{r_0} X_0 k^1 - k^1 \left(k^2\right)^2 l^1 k^0_{r_0} X_0 k^1_w X_1 - k^1 \left(k^2\right)^2 l^0_{r_0} X_0 k^0_w X_0 k^1$$

Since $k^0_{r_0}, k^1_{r_1}, k^2_{r_2} < 0$, $l^1_{r_1}, l^2_{r_2}, k^1_w, k^0_w > 0$, $L^D_w < 0$, we can determine $\Delta > 0$

Proof of equations (30) and (31). Using the Cramer's rule, we can derive equations (30) and (31).

Proof of lemma 1. To prove lemma 1, the effect of X_0 on the cost function $c^i(\cdot)$ and demand functions for labour $L^D(\cdot)$ and capital K^{D0}, K^{D1}, K^{D2} must be examined first. For these, we consider the 'unit-cost' function $\hat{c}^i(w, r_i)$ associated with the 'production function' $F^i(l_i, k_i)$ for private good industry i, and denote the 'unit-demand' function for labour and capital associated with that cost function by $l^i(w, r_i)$ and $k^i(w, r_i)$, respectively. From the homogeneity property of $F^i(\cdot)$, these functions are related with ordinary functions used in the text:

$$f^i(X_0)c^i(X_0, w, r_i) = \hat{c}^i(w, r_i)$$
$$f^i(X_0)l^i(X_0, w, r_i) = \hat{l}^i(w, r_i)$$
$$f^i(X_0)k^i(X_0, w, r_i) = \hat{k}^i(w, r_i) \qquad i = 1, 2 \tag{A20}$$

Partial differentiation of the first equation with respect to X_0 yields

$$c_0^i(X_0, w, r_i) = -\phi^i(X_0)c^i(X_0, w, r_i) \qquad i = 1, 2 \tag{A21}$$

Similarly, from the last two equations of (A20) and the definition of $L^D(\cdot)$ and K^{D0}, K^{D1}, K^{D2}, differentiation of (A3)-(A6) with respect to X_0 yields

$$L_0^D(X_0, X_1, X_2, w, r_1, r_2, r_0) = l^0(w, r_0) - \sum_{i=1}^{2}\phi^i(X_0)l^i(X_0, w, r_i)X_i \tag{A22}$$
$$K_0^{D0}(X_0, w, r_0) = k^0(w, r_0) \tag{A23}$$
$$K_0^{D1}(X_0, X_1, w, r_1) = -\phi^1(X_0)k^1(X_0, w, r_1)X_1 \tag{A24}$$
$$K_0^{D2}(X_0, X_2, w, r_1) = -\phi^2(X_0)k^2(X_0, w, r_2)X_2 \tag{A25}$$

From this and the definition of cost function $c^i = wl^i + r_ik^i$, we have

$$L_0^D w + K_0^{D0}r_0 + K_0^{D1}r_1 + K_0^{D2}r_2 = c^0 - \sum_{i=1}^{2}\phi^i c^i X_i \tag{A26}$$

Now apply the Cramer's rule to the equation system, we derive

$$\Delta(pX_0^1 + X_0^2) = \begin{vmatrix} l^1 & 0 & k^1 & 0 & -c_0^1 & 0 \\ l^2 & 0 & 0 & k^2 & -c_0^2 & 0 \\ L_w^D & l_{r_0}^0 X_0 & l_{r_1}^1 X_1 & l_{r_2}^2 X_2 & -L_0^D & c^1l^2 - c^2l^1 \\ k_w^0 X_0 & k_{r_0}^0 X_0 & 0 & 0 & -k^0 & 0 \\ k_w^1 X_1 & 0 & k_{r_1}^1 X_1 & 0 & -k_0^1 X_1 & -c^2k^1 \\ k_w^2 X_2 & 0 & 0 & k_{r_2}^2 X_2 & -k_0^2 X_2 & c^1k^2 \end{vmatrix}$$

multiply the fourth, fifth and sixth row of the above matrix by $r_0, \phi^1 r_1, \phi^2 r_2$ respectively, add to the third row multiplied by w, and use (A17), (A18) and (A26) to obtain

$$\Delta(pX_0^1 + X_0^2) = \frac{\sum_{i=1}^{2}\phi^i c^i X_i - c^0}{w}\begin{vmatrix} l^1 & 0 & k^1 & 0 & 0 \\ l^2 & 0 & 0 & k^2 & 0 \\ k_w^0 X_0 & k_{r_0}^0 X_0 & 0 & 0 & 0 \\ k_w^1 X_1 & 0 & k_{r_1}^1 X_1 & 0 & -c^2k^1 \\ k_w^2 X_2 & 0 & 0 & k_{r_2}^2 X_2 & c^1k^2 \end{vmatrix} = \Delta\left(\sum_{i=1}^{2}\phi^i c^i X_i - c^0\right)$$

this equation, (A1), (A2) and (23) establish lemma 1.

Proof of (34). Differentiation of the national income function $m(p, X_0) \equiv pX^1(p, X_0) + X^2(p, X_0)$ with respect to p yields

$$m_p(p, X_0) = X^1(p, X_0) + pX_p^1(p, X_0) + X_p^2(p, X_0) \tag{A27}$$

From (31)

$$m_p(p, X_0) = X^1(p, X_0) \tag{A28}$$

Differentiation with (A28) with respect to X_0, we can get

$$m_{p0}(p, X_0) = X_0^1(p, X_0) \tag{A29}$$

Using the Young's theorem and lemma 1, we have

$$H_p(p, X_0) = X_0^1(p, X_0)$$

Proof of (36). Since $\phi^i \equiv \dfrac{d \ln f(X_0)}{dX_0} = \dfrac{1}{f(X_0)} \dfrac{df(X_0)}{dX_0}$, then $\phi_0^i = -\dfrac{1}{[f(X_0)]^2}[\dfrac{df(X_0)}{dX_0}]^2 + \dfrac{1}{f(X_0)} \dfrac{d^2 f(X_0)}{dX_0{}^2}$. Because $f(\cdot)$ is concave, we get $\dfrac{d^2 f(X_0)}{dX_0{}^2} < 0$. Hence, $\phi_0^i < 0$.

In view of $c^0 = l^0(w, r_0)w(p, X_0) + k^0(w, r_0)r_0(p, X_0)$, we must have $c_0^0 = l^0(w, r_0)\dfrac{\partial w}{\partial X_0} + k^0(w, r_0)\dfrac{\partial r_0}{\partial X_0}$.

From (A19), we derive

$$\frac{\partial w}{\partial X_0} = \frac{1}{\Delta}\frac{1}{w}[k_{r_0}^0 X_0 \phi^1 (c^1)^2 (k^2)^2 k_{r_1}^1 X_1 + k_{r_0}^0 X_0 \phi^2 (c^2)^2 (k^1)^2 k_{r_2}^2 X_2 + k_{r_0}^0 X_0 c^2 k^2 (k^1)^2 k_0^2 X_2]$$

$$\frac{\partial r_0}{\partial X_0} = \frac{1}{\Delta}\frac{1}{w}[c^1 (k^2)^2 (-l^1 k^0 k_{r_1}^1 X_1 - k_0^1 X_1 k_w^0 X_0 k^1 + k^1 k^0 k_w^1 X_2 - k_w^0 X_0 \phi^1 c^1 k_{r_1}^1 X_1) + c^2 (k^1)^2 (-l^2 k^0 k_{r_2}^2 X_2 - k_0^2 X_2 k_w^0 X_0 k^2 + k^2 k^0 k_w^2 X_2 - k_w^0 X_0 \phi^2 c^2 k_{r_2}^2 X_2)]$$

From (A20), we obtain

$$k_0^i(X_0, w, r_i) = -\phi^i(X_0) k^i(X_0, w, r_i) < 0$$

This inequality and $\Delta > 0$, $k_{r_0}^0, k_{r_1}^1, k_{r_2}^2 < 0$, $k_w^1, k_w^0 > 0$ give

$$\frac{\partial w}{\partial X_0} > 0, \quad \frac{\partial r_0}{\partial X_0} > 0$$

Hence, $c_0^0 = l^0(w, r_0)\dfrac{\partial w}{\partial X_0} + k^0(w, r_0)\dfrac{\partial r_0}{\partial X_0} > 0$ is established.

Chapter 3

Semi-public Intermediate Goods and International Trade in a Two-country Model

3. 1 Introduction

The existence of public intermediate goods induces externalities in an economy. Then the production possibility frontier may become convex to the origin. The traditional trade theorems which are based on a concave production possibility frontier may not be robust. Hence, many literatures-e.g. Manning and McMillan (1979), Khan (1980), Tawada and Okamoto (1983), Tawada and Abe (1984), Okamoto (1985), Altenburg (1987), and Ishizawa (1988)-have examined the influences of public intermediate goods on the fundamental theorems in tradition trade theories. However, these studies mainly are analyzed in a small open economy model, which means the trade patterns and terms of trade are exogenous. But both of them are critical to discuss the gains from trade. It is better to discuss in a two-country model. Suga and Tawada (2007) analyzed the role of pure public intermediate goods on trade patterns and gains from trade in a two-country model. They showed that the country with large factor endowment exports the good whose productivity is more sensitive to pure public intermediate goods. They also find that at least one country gains from trade, and if a country incompletely specializes in the trade equilibrium, the country necessarily loses from trade.

Meade (1952) recognized two types of public intermediate goods. One type is pure public intermediate goods, which are called 'creation of atmosphere'. Pure public intermediate goods are fully available to every firm irrespective of the number of firms. Free information about technology is pure public intermediate good. Another type is semi-public intermediate goods, which are called ‘unpaid factors’. Semi-public intermediate goods suffer from congestion within an industry and thus a reduction of availability to a firm when the number of firms in this industry increases. Railways, airports are semi-public intermediate goods.

In this chapter, we develop a two-country trade model in which one primary factor, two consumer goods, and one semi-public intermediate good exist. We also assume the semi-public intermediate goods are efficiently supplied by the government. It is shown that the country with large (small) factor endowment exports (imports) the good whose productivity is less sensitive to the semi-public intermediate goods. When trade opens, both of the countries gain from trade.

3. 2 The Model

We present an economy with two consumer goods called good 1 and good 2, and one semi-public intermediate good. Now we only consider an economy with L units of labor, which is the primary factor of production.

Production Technologies

The production functions of two consumer goods and semi-public intermediate good take the following forms:

$$Q_i = R^{\alpha_i} L_i^{1-\alpha_i}, \quad 0 < \alpha_i < 1, \quad i = 1, 2, \tag{1}$$

$$R = f(L_R), f'(\bullet) > 0, f''(\bullet) < 0, \tag{2}$$

where Q_i and R are the outputs of good i and the semi-public intermediate good, L_i and L_R are the labor used in relative industries. We impose the following assumption:

Assumption 1. *For all* $R > 0$ *, about the elasticity of output* Q_i *with respect to the semi-public intermediate good,* $(\partial Q_i / \partial R) \bullet (R / Q_i) = \alpha_i$ *, satisfies* $\alpha_1 < \alpha_2$.

This assumption implies that productivity of the first industry is less sensitive to the semi-public intermediate good than that of the second industry.

The Semi-public Intermediate Good Supply

We assume that the government supplies the semi-public intermediate good to get an efficient production of the economy. The production of the semi-public intermediate good is assumed to financed by the lump-sum income tax.

The production possibility frontier is the locus of pairs Q_1 and Q_2. We define $Q_2 = \Gamma(Q_1, L)$, where $\Gamma(Q_1, L)$ is

$$\Gamma(Q_1, L) \equiv \max_{L_1, L_2, L_R R} R^{\alpha_2} L_2^{1-\alpha_2}$$

$$s.t \quad R^{\alpha_1} L_1^{1-\alpha_1} = Q_1 \quad R = f(L_R) \quad L_1 + L_2 + L_R = L .$$

The optimal conditions of the above problem are

$$\frac{\alpha_1}{1-\alpha_1} Q_1^{\frac{1}{1-\alpha_1}} R^{\frac{1}{\alpha_1-1}} + \frac{\alpha_1}{1-\alpha_1} Q_1^{\frac{1}{1-\alpha_1}} R^{\frac{1}{\alpha_1-1}} = \frac{1}{f^{'}(L_R)}, \tag{3}$$

$$Q_1^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_1}{\alpha_1-1}} + Q_2^{\frac{1}{1-\alpha_2}} R^{\frac{\alpha_2}{\alpha_2-1}} + f^{-1}(R) = L . \tag{4}$$

Equation (3) represents the Samuelson-Kaizuka condition for the efficient supply of the semi-public intermediate good. Equation (4) is the budget constraint of labor endowment.

Based on (3) and (4), we can derive how the efficient supply of the semi-public intermediate good and the output of good 2 change to increases in Q_1 and L. We have the following equations

$$\frac{\partial R}{\partial Q_1} < 0 , \tag{5}$$

$$\frac{\partial Q_2}{\partial Q_1} = -\frac{(1-\partial_2) R^{\frac{\alpha_1}{\alpha_1-1}} Q_1^{\frac{\alpha_1}{1-\alpha_1}}}{(1-\partial_1) R^{\frac{\alpha_2}{\alpha_2-1}} Q_2^{\frac{\alpha_2}{1-\alpha_2}}} < 0 , \tag{6}$$

$$\frac{\partial R}{\partial L} > 0 , \tag{7}$$

$$\frac{\partial Q_2}{\partial L} > 0 . \tag{8}$$

Partial differentiation of (6) with respect to Q_1 and L yields

$$\frac{\partial^2 Q_2}{\partial Q_1^2} < 0 , \tag{9}$$

$$\frac{\partial^2 Q_2}{\partial Q_1 \partial L} > 0 . \tag{10}$$

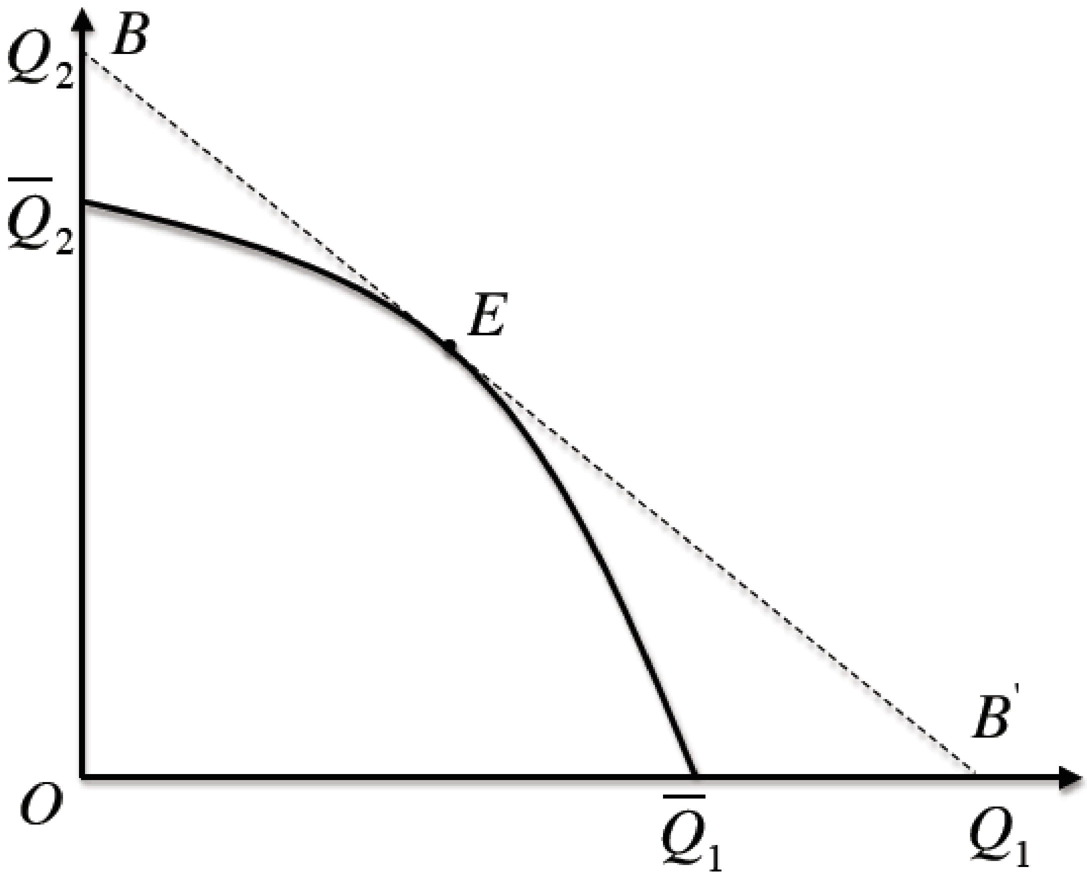

Figure 3. 1 Production Possibility Frontier

Equation (9) implies the PPF is strictly concave to the origin. (This concavity property have been proved by Tawada, 1980, Proposition 2). The PPF is depicted in Figure 3. 1 as the curve $\overline{Q}_2\overline{Q}_1$.

Assume that all firms within each industry take the quantity of semi-public intermediate good as given and use it as 'unpaid factors'. Perfect competition and profit maximization in all private industries, together with (6), induces

$$\frac{p_1}{p_2} = -\frac{(1-\partial_2)R^{\frac{\alpha_1}{\alpha_1-1}} Q_1^{\frac{\alpha_1}{1-\alpha_1}}}{(1-\partial_1)R^{\frac{\alpha_2}{\alpha_2-1}} Q_2^{\frac{\alpha_2}{1-\alpha_2}}} = -\frac{\partial Q_2}{\partial Q_1}, \qquad (11)$$

where p_i is the price of good i. Hence, production take places at the point on the PPF where the budget line is tangent. In Figure 3. 1, the line BB' is the budget line, and E is the production point.

Preferences

We assume that preferences are homothetic. Denoting the price of good 1 relative to good 2 by P, the expenditure function with the level of utility u is

$$E(P,\ u) = e(P) \bullet \phi(u),\ \ e'(\bullet) > 0,\ \ e''(\bullet) < 0,\ \ \phi'(\bullet) > 0,$$

where $e(\bullet)$ is the unit expenditure function. Then the relative demand is

$$C_1/C_2 = \gamma/(1-\gamma)P \equiv Z(P), \tag{12}$$

where $\gamma = e'(P)P / e(P)$ is the share of income spent on good 1.

We have the relative demand C_1/C_2 is declining in P, because

$$dZ/dP = -\varepsilon \bullet Z/(1-\gamma)P < 0, \tag{13}$$

where $\varepsilon = -e''(P)P / e'(P)$ is the price elasticity of compensated demand for good 1.

3. 3 Trade Pattern

We now examine the trade pattern between two countries. The two countries are identical in both production technologies and preferences but not in labor endowments. The large country's labor endowment is L_L, the small country's labor endowment is L_S, and $L_L > L_S$. The semi-public intermediate good has no international spillovers. The footnote (L) denotes the large country's variables, the footnote (S) denotes the small country's variables.

Total differentiation with equations (1)-(4) and (11) yields

$$\frac{\partial(Q_2/Q_1)}{\partial L} < 0 . \tag{14}$$

We can summarize the result as

Lemma 1. *Under assumption 1, given the supply price* P*, when the labor endowment* L *increases, the relative supply* Q_2/Q_1 *decreases.*

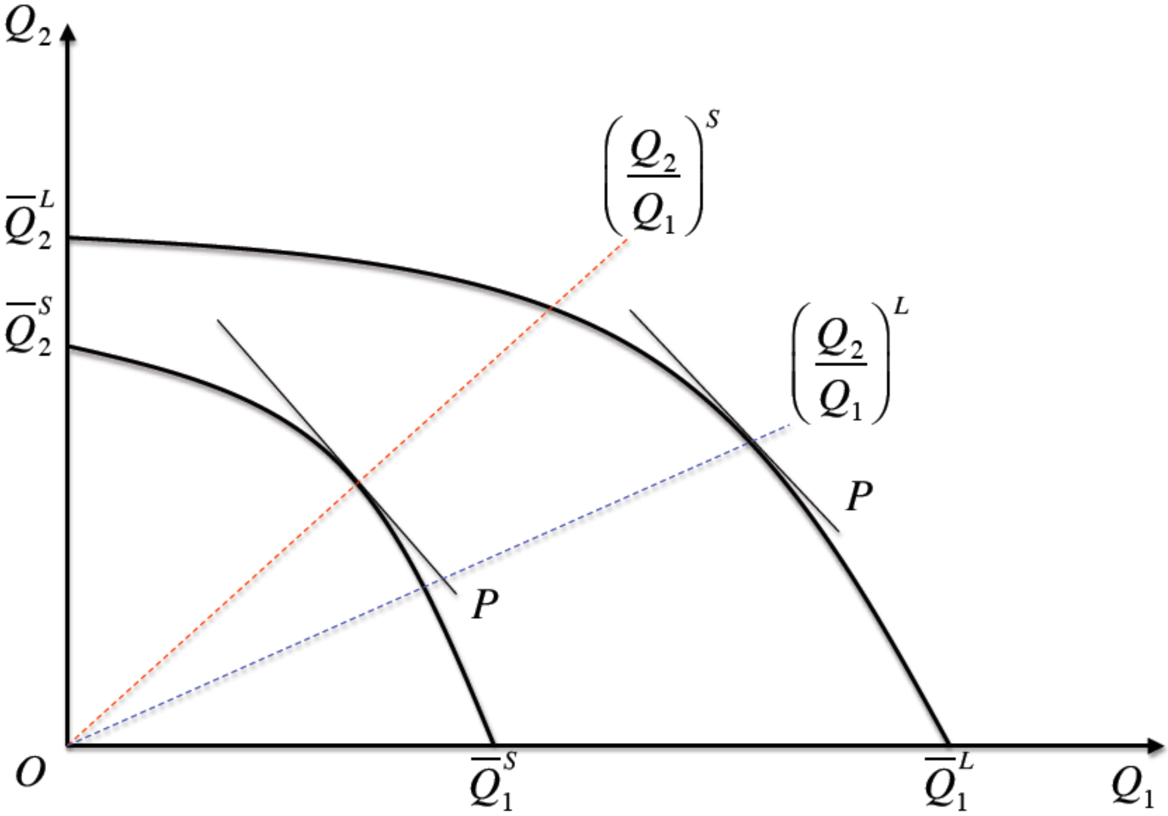

Figure 3. 2 The relative supply with different factor endowment

In Figure 3. 2, Given the supply price P, the relative supply of the small country is $(Q_2/Q_1)^S$ and the relative supply of the large country is $(Q_2/Q_1)^L$. We have $(Q_2/Q_1)^L < (Q_2/Q_1)^S$, which means that for the same supply price P, the relative supply decreases when the labor endowment increases. From Lemma 1, we can derive

Proposition 1. *Under assumption 1, the relative price of good 1 is lower (higher) in the autarky equilibrium in the large (small) country. Then the large (small) country becomes an exporter (importer) of the good whose productivity is less sensitive to the semi-public intermediate goods.*

Proof: From Lemma 1, the increase in labor endowment in the large country leads to a decrease of the relative supply Q_2/Q_1 for the same supply price. Given the homothetic preferences, the output changes leads to an excess supply of the good 1 and an excess demand of good 2. A new autarky supply equilibrium in the large country is formed, with a lower price ratio of good 1 than that in the small country.

The result of proposition 1 is opposite to that of Suga and Tawada (2007), in which the large (small) country is an exporter (importer) of the good whose productivity is more sensitive to the pure public intermediate goods. Hence, the two types of public intermediate goods affect the trade patterns quite differently.

In Figure 3. 3, the autarky price in the small country is P_A^S and the autarky price in the large country is P_A^L. P_A^L is less than P_A^S. Hence the large (small) country is an exporter (importer) of good 1 whose productivity is less sensitive to the semi-public intermediate good.

3. 4 Gains from Trade

By Proposition 1, the Autarky price of good 1 is lower in the large country than in the

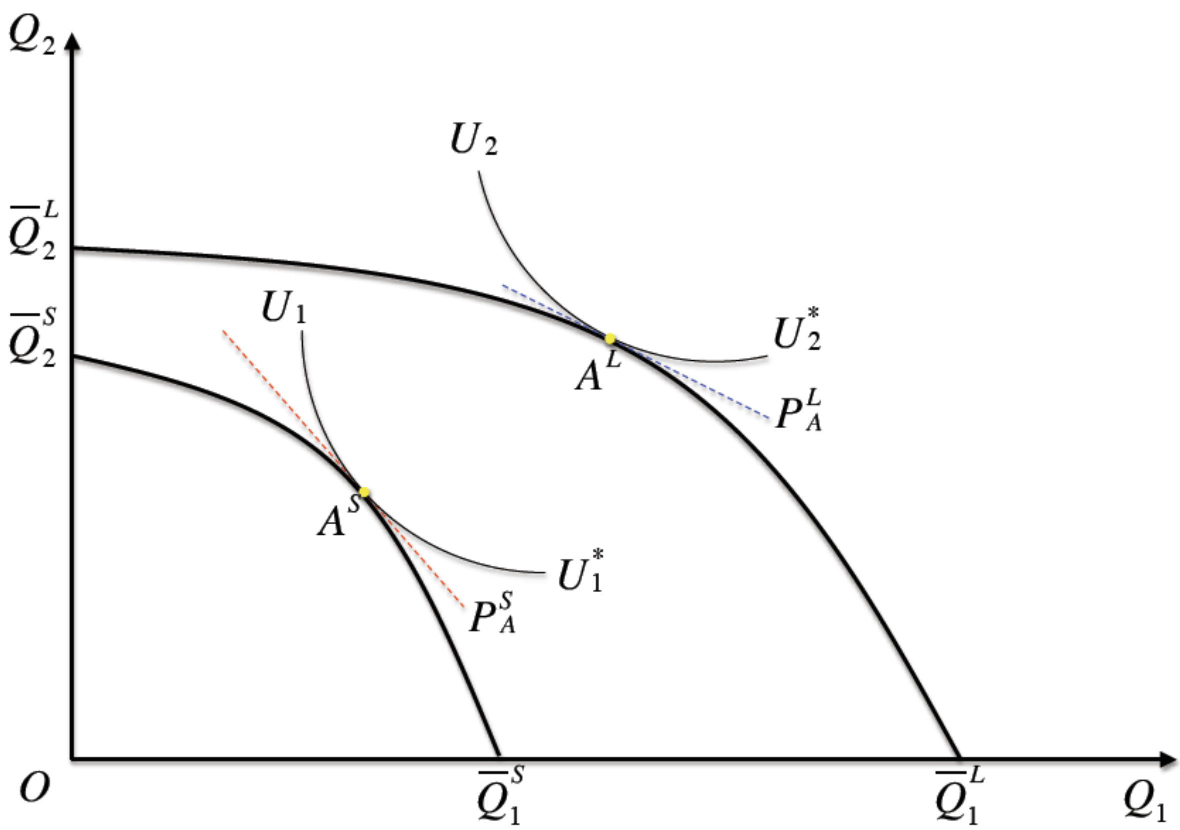

Figure 3. 3 The Autarky Equilibrium with different factor endowments

small country. Hence, we have

$$\frac{Q_{1A}^{S}}{\Gamma\,(Q_{1A,}^{S}L\;)}<\frac{Q_{1A}^{S}+Q_{1A}^{L}}{\Gamma\,(Q_{1A,}^{S}L\;)+\Gamma\,(Q_{1A,}^{L}L\;)}<\frac{Q_{1A}^{L}}{\Gamma\,(Q_{1A,}^{L}L\;)}, \tag{15}$$

where $Q_{1A}^{S}(Q_{1A}^{L})$ is the small (large) country's output level of good 1 in the autarky equilibrium.

Proof of (15): Since the autarky price can clear the market, we have

$$\frac{Q_{1A}^{S}}{\Gamma\,(Q_{1A,}^{S}L\;)}=Z(P_{A}^{S}) \text{ and } \frac{Q_{1A}^{L}}{\Gamma\,(Q_{1A,}^{L}L\;)}=Z(P_{A}^{L}).$$

From $P_{A}^{S}>P_{A}^{L}$ and $Z'(\bullet)<0$, we can derive

$$\frac{Q_{1A}^{S}}{\Gamma\,(Q_{1A,}^{S}L\;)}<\frac{Q_{1A}^{L}}{\Gamma\,(Q_{1A,}^{L}L\;)}.$$

Then we can get equation (15).

From (14), we can derive

$$P_{S}^{S}>P_{D}>P_{S}^{L}. \tag{16}$$

When trade opens, the demand price in the world market is lower (higher) compared to the small (large) country' supply price. This means that $\dot{Q}_{1}^{S}<0(\dot{Q}_{1}^{L}>0)$.

We can summarize the following result

Proposition 2. *When trade happens, both countries gain from trade.*

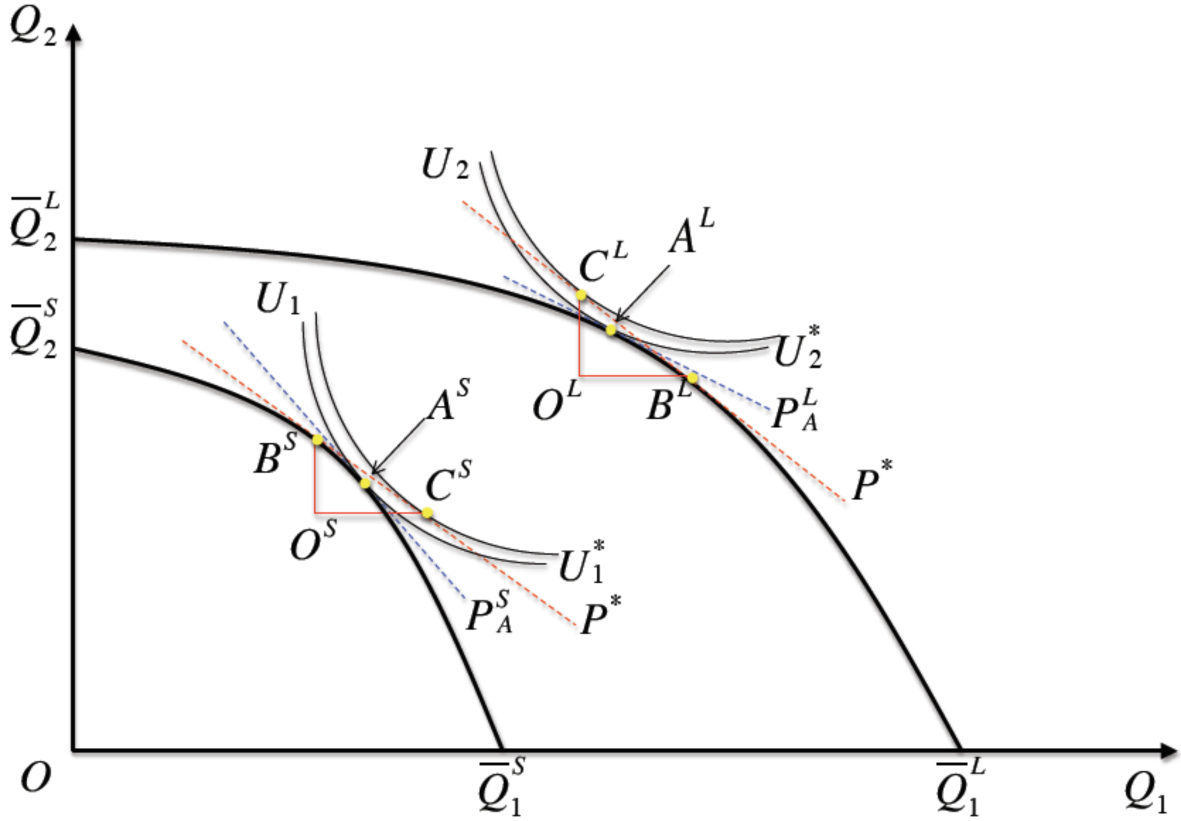

Figure 3. 4 The Gains from Trade in Two Countries

In Figure 3. 4, for the small country the autarky production point is A^S. In the trade equilibrium, the new production point is B^S, the new consumption point is C^S. Hence, the small country imports good 1 by $O^S C^S$, and exports good 2 by $O^S B^S$. For the large country A^L is the autarky production point. After trade, the new production point becomes B^L, the new consumption point becomes C^L. Then the large country exports good 1 by $O^L B^L$, and imports good 2 by $O^L C^L$. We can conclude that both countries gain from trade.

3. 5 Conclusion

We analyzed how semi-public intermediate goods affect the trade pattern and gains from trade in a two-country model. When semi-public intermediate good is sufficiently supplied, the large (small) country is an exporter (importer) of the good whose production is less sensitive in the semi-public intermediate goods. When trade opens, both of the two countries will gain from trade.

Chapter 4

Tax Competition and Fiscal Equalization in a Repeated Game Setting

4. 1 Introduction

The efficiency and redistributive effects of tax competition have been extensively investigated in the literature of public finance. When a region increases its tax rate, the outflow of the tax base generates a positive fiscal externality. Tax competition thus induces an inefficient low tax rate and low public service level, implying that tax harmonization is needed to eliminate this inefficiency.

The potential for cooperation to arise in a repeated interaction setting is well known. Hence, a repeated interactions model would provide a better perspective on how to obtain sustained and efficient tax coordination among local governments. Only recently have a few studies confirmed this motivation. Coates (1993) investigates the open-loop equilibrium of a dynamic game of property tax competition. He pays atention to the intertemporal trade-off between current and future consumptions of private and local public goods, finding that there may be incentives to subsidize capital. Cardarelli et al. (2002) prove that tax coordination can endogenously arise in a conventional repeated game, and that tax coordination does not prevail when regional asymmetries are too strong. Their study assumes that: (1) production activity does not occur; (2) the interest rate is exogenously determined at zero; and (3) when capital is invested abroad, a sunk cost occurs. Extending this study, Kawachi and Ogawa (2006) incorporate the benefit spillovers of local public goods to show that the cooperative outcome tends to take as the magnitude of spillover is significant. Catenaro and Vidal (2006), using the standard tax competition model with repeated interactions, demonstrate that tax coordination is not sustainable when region sizes are too different. In their model, however, the government's objective is to maximize the revenue from capital income tax rather than the welfare of its citizens. Itaya et al. (2008) construct a repeated game model of tax competition, wherein regions are asymmetrical in per capita capital endowments and production technologies. They conclude that: the larger the differences in per capita capital endowments, the easier the tax coordination; the larger the differences in production technologies, the more difficult the tax coordination; they also find that the larger the differences among net capital exporting positions, the more likely is tax coordination across regions. Their study further indicates that the best cooperative tax rate, which conduces to the highest possibility of cooperation, is zero; this finding justifies the rule of no-tax on mobile capital.

However, the theoretical analyses of tax competition within a repeated game model have not yet taken the fiscal equalization scheme into account. The fiscal equalization scheme is an integral part of existing federal arrangements. In the US, the state tax sharing is one of the two forms of intergovernmental aid to local governments, the largest element of state expenditure. Within the EU, the Structural Fund and the Cohesion Fund allocate over 40% of the EU budget to under-developed regions and states. Fiscal equalization schemes have also been implemented in Canada, Australia, Denmark and Switzerland, and many developing countries. These facts suggest that the existence of a systematic interaction between tax competition and fiscal equalization.

This issue has recently been formally addressed in literature. Janeba and Peters (2000) demonstrate that capital tax rates increase if regions are combined into a single tax revenue equalization system; in such a case, fiscal transfers partially internalize fiscal externalities. Kothenburger (2002) analyzes the relationship between fiscal equalization and tax competition in a standard model of capital tax competition among regions that are allowed to have differing labor endowments (Wilson, 1991). The study shows that fiscal equalization may eliminate the externalities induced by tax competition. Especially, when the tax base equalization is introduced, efficient tax rates arise. Kotsogiannis (2010) extends the analysis to a standard capital tax competition model wherein there are horizontal tax externalities between regions and vertical tax externalities between the levels of government. He shows that an efficient level of lower-level government taxation can be achieved with an appropriately adjusted standard tax base equalization formula.

Within the framework of a repeated tax competition game (Itaya et al., 2008), in which regions differ in per capita capital endowments and production technologies, this paper investigates the relationship between tax competition and fiscal equalization. In particular, it focuses on the question of how the fiscal equalization scheme affects the cooperation condition of tax competition with repeated interactions.

The main results of this chapter are the following: (i) when the scale of fiscal equalization scheme increases, the capital exporter is more likely to cooperate, but the capital importer is less likely to cooperate; (ii) while the best cooperative tax rate without fiscal transfers is zero, as argued by Itaya et al. (2008), it becomes 2α in the presence of fiscal transfers, in which α is the scale of the fiscal equalization scheme. Interestingly, when the cooperative tax rate is set at 2α, even the scale of fiscal equalization α changes, the willingness of both regions to cooperate in tax coordination keeps the same level; (iii) in the cooperation phase, the introduction of the fiscal equalization scheme lowers the welfare of the large region, raises the welfare of the small region, and has no effect on the total welfare of the federal economy.

This chapter is organized as follows. Section 4. 2 describes a basic model of tax competition; the introduction of a fiscal equalization transfers is a central feature of this model. Section 4. 3 presents the relative results in a repeated game. Section 4. 4 briefly concludes the chapter.

4. 2 The Model

The structure and parameters of the model are identical to those of Itaya et al. (2008), the only difference being the introduction of a fiscal equalization scheme into the federation.

The model considers a federal economy consisting of two regions, that are asymmetric in their per capita capital endowments and production technologies. Both regions have the same populations. The per capita capital endowment of region S (small) and region L (large) are $\bar{k}_S \equiv \bar{k} - \varepsilon$ and $\bar{k}_L \equiv \bar{k} + \varepsilon$ respectively, where $\varepsilon \in (0, \bar{k}]$. We can see that the average per capita capital in the economy is $\bar{k} = (\bar{k}_S + \bar{k}_L)/2$. Capital can costlessly and freely flow across regions; however, workers are fixed in each region. A homogeneous consumption good is produced. The production function in per capita terms is $f^i(k_i) \equiv (A_i - k_i)k_i$, i=S, L; A_i is the technology parameter of each region. $A_i > 2k_i$, i=S, L, then the marginal productivity of capital is positive but diminishing. k_i denotes the capital employed in region i. Firms in each region maximize profits. Given a source-based capital tax τ_i, the profit maximizing input equilibrium can be characterized as the interest rate $r = f_k^i(k_i) - \tau_i = A_i - 2k_i - \tau_i$, and the wage rates $w_i = f^i(k_i) - k_i f_k^i(k_i) = k_i^2$.

The federation implements a fiscal equalization transfer system. Here, we consider a tax base equalization scheme*, which is conditioned by the difference in tax base capacity between two regions. This means that

$$\beta_S = \alpha(k_L - k_s),$$

$$\beta_L = \alpha(k_s - k_L),$$

where β_i is the fiscal transfer component allocated to region i, and α is the scale of the fiscal equalization system. We assume that the federal government can freely adjust α and $0 \le \alpha < 1/2$ †. It should be noted that this fiscal equalization scheme is budget-balancing, (i.e., $\beta_s + \beta_L = 0$).

In the capital market equilibrium, we obtain the interest rate and the capital allocation as

$$r^* = \frac{1}{2}[A_S + A_L - (\tau_S + \tau_L)] - 2\bar{k}, \tag{1}$$

$$k_S^* = \bar{k} + \frac{1}{4}[(\tau_L - \tau_S) - (A_L - A_S)], \tag{2}$$

$$k_L^* = \bar{k} + \frac{1}{4}[(\tau_S - \tau_L) + (A_L - A_S)]. \tag{3}$$

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* Basically, there are two kinds of fiscal equalization schemes: tax revenue equalization scheme and tax base equalization scheme. Kothenburger (2002) proves that while a tax base equalization scheme can completely eliminate the fiscal externality, a tax revenue equalization scheme may deteriorate the fiscal externality. Tax base equalization schemes are adopted , e.g., in Canada, Denmark, Switzerland, and Australia.

† $0 \le \alpha < 1/2$ can ensure that the rich region gets a big share of the difference of tax base, the poor region gets a small share of the difference of tax base. $1/2 \le \alpha < 1$ induces that the rich region gets a small share of the difference of tax base, the poor region gets a big share of the difference of tax base. Then the incentive of the fiscal equalization scheme will be bad.

We assume $A_L - A_S = \theta$ in the following.

In addition to the capital endowment, each resident inelastically provides one unit of labor. Thus, the total income constitutes wage, w_i^*, and the interest income, $r^*\overline{k_i}$. The residents use these incomes to consume private goods, c_i. The public good in each region is financed by the capital tax revenue and the fiscal equalization transfer from the federation. Therefore, the budget constraint on the regional government is $g_i = \tau_i k_i^* + \beta_i$. The regional governments maximize the utility of a representative resident by choosing an optimal tax rate, then

$$\tau_i^* \in \underset{\tau_i}{\arg\max} \quad u_i(c_i, g_i) \equiv c_i + g_i = f^i(k_i^*) - r^*\left(k_i^* - \overline{k_i}\right) + \beta_i, i = S, L. \tag{4}$$

we can obtain the reaction functions as

$$\tau_S = \frac{\tau_L}{3} + \frac{4\varepsilon}{3} + \frac{4\alpha}{3} - \frac{\theta}{3}, \tag{5}$$

$$\tau_L = \frac{\tau_S}{3} - \frac{4\varepsilon}{3} + \frac{4\alpha}{3} + \frac{\theta}{3}. \tag{6}$$

Since the slope of the reaction functions is positive and less than one, there exists a Nash equilibrium. We derive the Nash equilibrium in a one-shot game as follows:

$$\tau_S^N = 2\alpha + (\varepsilon - \frac{\theta}{4}), \quad \tau_L^N = 2\alpha - (\varepsilon - \frac{\theta}{4}). \tag{7}$$

Since the fiscal equalization can partially internalize the fiscal externality, the tax rates in both regions increase by 2α. Based on Eqs. (1), (2), (3), and (7), the Nash equilibrium interest rate, r^N, and the per capita capital demand in each region, k_i^N are

$$r^N = \frac{1}{2}(A_S + A_L) - 2\alpha - 2\bar{k}\,, \tag{8}$$

$$k_S^N = \bar{k}_S + \frac{1}{2}(\varepsilon - \frac{\theta}{4})\,, \tag{9}$$

$$k_L^N = \bar{k}_L - \frac{1}{2}(\varepsilon - \frac{\theta}{4})\,. \tag{10}$$

When $\theta = 0$, a small region imposes high taxation to import capital, and a large region imposes low taxation to export capital. This result is induced by the pecuniary externality or the terms-of-trade effect, which is the same as that in Depater and Myers (1994), Peralta and van Ypersele (2005), and Itaya et al. (2008). However, the effects on the terms of trade, manipulated by both regions, cancel each other out. The interest rate r^N is then unchanged as Eq. (8).

It is completely consistently Itaya et al. (2008) that: since the higher marginal capital product can induce greater capital demand in the large region, exceeding its large capital endowment. The technology difference ($\theta > 0$) can thus reverse the above net capital exporter position and tax policy. In order to summarize, we state the following proposition:

Proposition 1. *The sign of* $\Phi \equiv \varepsilon - (\theta / 4)$ *determines the net capital positions of the two regions; when* $\Phi > (<) 0,$ *the large (small) region is a capital exporter.*

Using Eqs. (4), (8), (9), and (10), we derive the Nash equilibrium utility level of each regions, u_i^N as

$$u_S^N = [\bar{k} + \frac{1}{2}(\varepsilon - \frac{3}{4}\theta)][\bar{k} - \frac{1}{2}(\varepsilon + \frac{3}{4}\theta)] + r^N(\bar{k} - \varepsilon) + 2\alpha\bar{k}\ , \tag{11}$$

$$u_L^N = [\bar{k} - \frac{1}{2}(\varepsilon - \frac{3}{4}\theta)][\bar{k} + \frac{1}{2}(\varepsilon + \frac{3}{4}\theta)] + r^N(\bar{k} + \varepsilon) + 2\alpha\bar{k}\ . \tag{12}$$

From Eqs. (11) and (12), we can obtain $u_L^N - u_L^N = \theta\bar{k} + 2\varepsilon r^N > 0$, meaning that the large region's payoff outweighs that of the small region. When α increases, the interest rate r^N decreases, as implied by Eq. (8). We can conclude that a fiscal equalization scheme can reduce the payoff differences between two regions.

4. 3 A Repeated Game

We now consider a repeated game between the two regions. The discount factor of each region is $\delta_i \in [0,1)$. We assume that each region cooperates in tax competition on the current stage, if the other region cooperated in the last stage; if a region defects, cooperation between two regions collapses, triggering the punishment stage which means Nash equilibrium persists forever. The conditions of sustained cooperation in region i=S, L, are

$$\frac{1}{1-\delta_i}u_i^C \geq u_i^D + \frac{\delta_i}{1-\delta_i}u_i^N, i = S, L, \tag{13}$$

where u_i^j for j=C, D, and N, denote the utility levels of the cooperation, deviation, and punishment phases, respectively. The left side of Eq. (13) indexes the total discounted utility of the residents in region i, when both regions cooperate infinitely in taxation. The right side of Eq. (13) indexes the sum of the current period's utility of tax deviation and the total discounted utility of Nash equilibrium in the following periods.

The cooperative tax rate τ^C maximizes the federation's utilitarian welfare, which is given by $u_F = u_S + u_L = f^S(k_S) + f^L(k_L)$. We can derive the cooperative tax rate as

$$\tau^C = \tau_S = \tau_L \,. \tag{14}$$

Although the cooperative tax rate is indeterminate, the capital allocation of the cooperative phase is unique:

$$k_S^C = \bar{k}_S + \Phi \;, \tag{15}$$

$$k_L^C = \bar{k}_L - \Phi \;. \tag{16}$$

Based on Eqs. (1), (4), (14), (15), and (16), we obtain the following utility levels in the cooperative phase:

$$u_S^C = (\bar{k} + \tau^C - \frac{\theta}{4})(\bar{k} - \frac{\theta}{4}) + r^C(\bar{k} - \varepsilon) + \frac{\alpha\theta}{2}, \tag{17}$$

$$u_L^C = (\bar{k} + \tau^C + \frac{\theta}{4})(\bar{k} + \frac{\theta}{4}) + r^C(\bar{k} + \varepsilon) - \frac{\alpha\theta}{2}, \tag{18}$$

$$u_F^C = u_S^C + u_L^C = A_S k_S^C - k_S^{C2} + A_L k_L^C - k_L^{C2}, \tag{19}$$

where $r^C = [(A_S + A_L)/2] - \tau^C - 2\bar{k}$, u_i^C for $i =$ S, L, and F, represent the utility levels of the small region, the large region, and the federation, respectively. Thus, we can state the following proposition:

Proposition 2. *The introduction of a fiscal equalization scheme increases the utility level of the small region, decreases the utility level of the large region and has no effect on the total utility level of the federation in the cooperation phase.*

Following Eqs. (11), (12), (17) and (18), the participation constraint for each region, (i.e., $u_i^C \geq u_i^N$, $i = S, L,$) is as follows:

$$u_S^C - u_S^N = \frac{1}{4}\Phi^2 + \tau^C\Phi - 2\alpha\Phi \geq 0, \qquad (20)$$

$$u_L^C - u_L^N = \frac{1}{4}\Phi^2 - \tau^C\Phi + 2\alpha\Phi \geq 0, \qquad (21)$$

which implies that the necessary condition to sustain cooperation is

$$\left|\tau^C - 2\alpha\right| \leq \frac{1}{4}\left|\Phi\right|. \qquad (22)$$

Assuming the rival region's tax rate is τ^C, the best-deviation tax rates τ_i^D maximize the utility of region i's residents. It can then be derived that

$$\tau_S^D = \frac{\tau^C}{3} + \frac{4}{3}\Phi + \frac{4}{3}\alpha, \qquad (23)$$

$$\tau_L^D = \frac{\tau^C}{3} - \frac{4}{3}\Phi + \frac{4}{3}\alpha. \qquad (24)$$

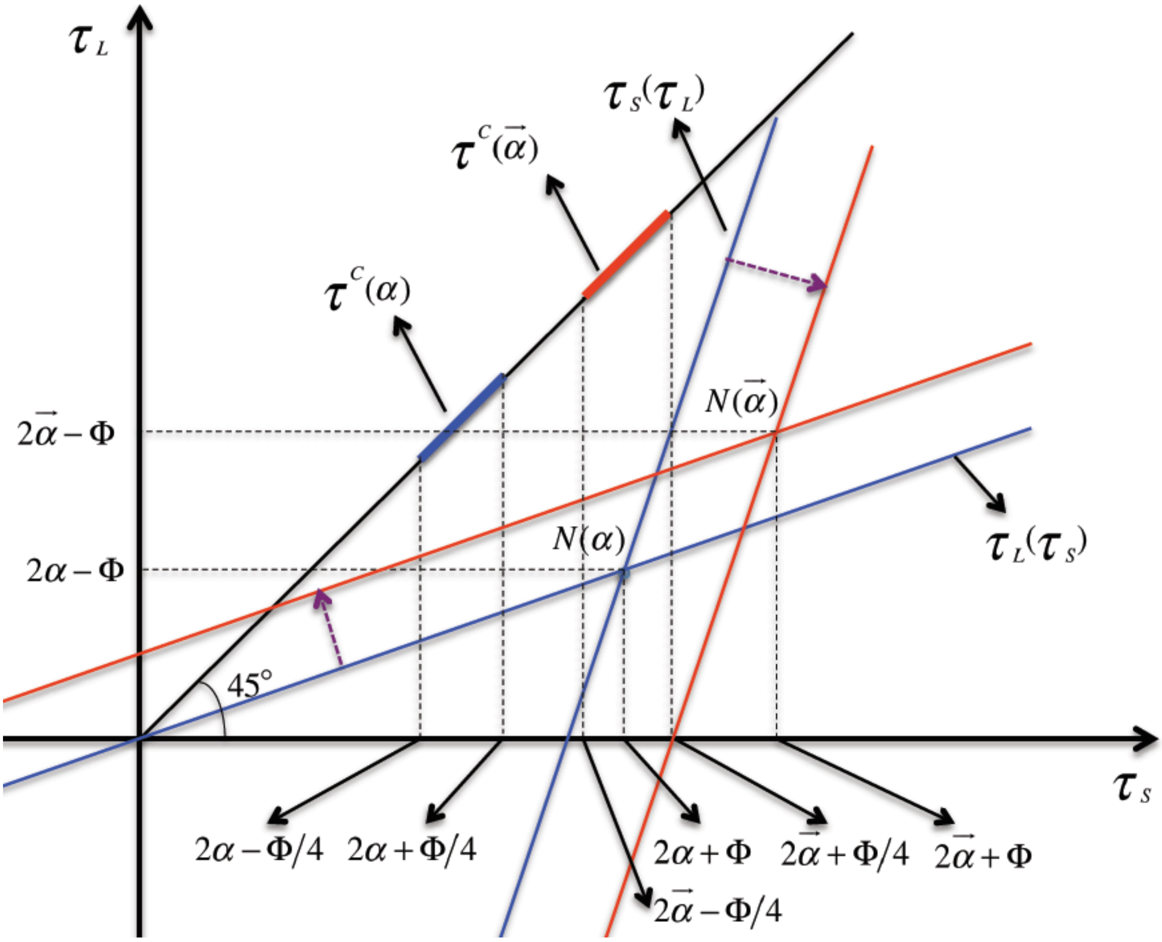

Figure 4. 1 The reaction curves of two regions, when the large (small) region is an exporter (importer); that is $\Phi \equiv \varepsilon - (\theta / 4) > 0$

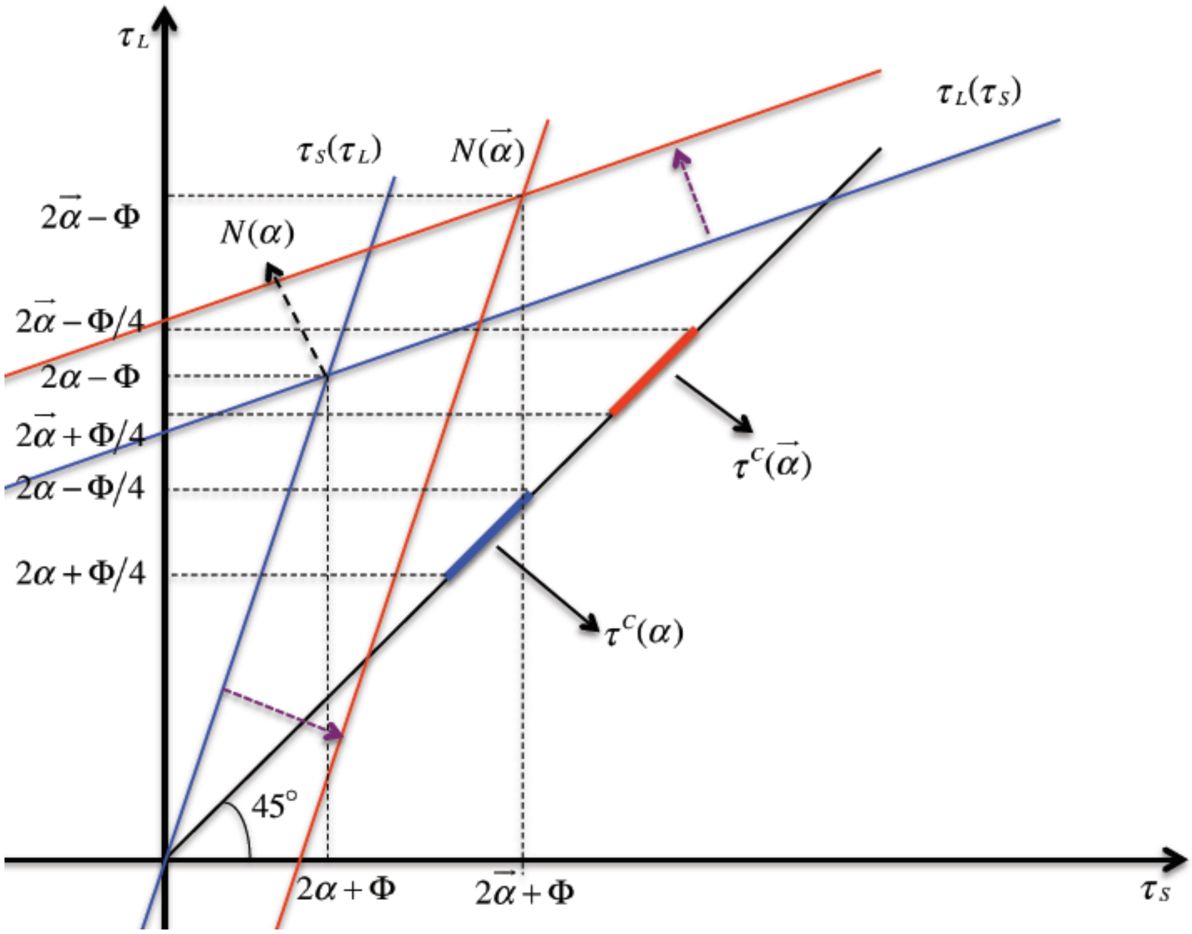

Figure 4. 2 The reaction curves of two regions, when the small (large) region is an exporter (importer); that is $\Phi \equiv \varepsilon - (\theta/4) < 0$

The utility levels of deviation u_i^D, i=S, and L, are respectively

$$u_S^D = [\bar{k} - \frac{1}{2}(\theta - \tau^C - 2\varepsilon)][\bar{k} - \frac{1}{6}(\theta - \tau^C + 2\varepsilon)] + r_S^D(\bar{k} - \varepsilon) + \frac{2\alpha\bar{k}}{3} + \frac{\alpha\theta}{3} + \frac{\alpha^2}{3} - \frac{\alpha\tau^C}{3}, \quad (25)$$

$$u_L^D = [\bar{k} + \frac{1}{2}(\theta + \tau^C - 2\varepsilon)][\bar{k} + \frac{1}{6}(\theta + \tau^C + 2\varepsilon)] + r_L^D(\bar{k} + \varepsilon) + \frac{2\alpha\bar{k}}{3} - \frac{\alpha\theta}{3} - \frac{\alpha^2}{3} + \frac{\alpha\tau^C}{3}. \quad (26)$$

Substituting (11), (12), (17), (18), (25) and (26) for (13), we can obtain the threshold of the discount factor in each region i as

$$\underline{\delta}_S = \frac{u_S^D - u_S^C}{u_S^D - u_S^N} = \frac{4(2\tau^C - 4\alpha + \theta - 4\varepsilon)^2}{(4\tau^C - 8\alpha - 7\theta + 28\varepsilon)(4\tau^C - 8\alpha - \theta + 4\varepsilon)}, \quad (27)$$

$$\underline{\delta}_L = \frac{u_L^D - u_L^C}{u_L^D - u_L^N} = \frac{4(2\tau^C - 4\alpha - \theta + 4\varepsilon)^2}{(4\tau^C - 8\alpha + 7\theta - 28\varepsilon)(4\tau^C - 8\alpha + \theta - 4\varepsilon)}. \quad (28)$$

When the actual discount factor exceeds both discount factor thresholds $\underline{\delta}_i$, i=S, L, in two regions, then tax cooperation can be a sub-game perfect Nash equilibrium of the repeated tax interaction.

We can regard $\underline{\delta}_i$ as a function of τ^C. Substituting the upper- and lower-bound val-ues of τ^C, given by Eq. (22), in Eqs. (27) and (28), produces $\underline{\delta}_S(2\alpha - \Phi/4) = \underline{\delta}_L(2\alpha + \Phi/4) = 1$, $\underline{\delta}_L(2\alpha - \Phi/4) = \underline{\delta}_S(2\alpha + \Phi/4) = 49/145$. Differentiating Eqs. (27) and (28) with respect to τ^C yields

$$\frac{\partial \underline{\delta}_S}{\partial \tau^C} = \frac{1536\Phi(\tau^C - 2\alpha - 2\Phi)(2\tau^C - 4\alpha + 5\Phi)}{\left[(4\tau^C - 8\alpha - 7\theta + 28\varepsilon)(4\tau^C - 8\alpha - \theta + 4\varepsilon)\right]^2}, \tag{29}$$

$$\frac{\partial \underline{\delta}_L}{\partial \tau^C} = \frac{1536\Phi(2\Phi + \tau^C - 2\alpha)(5\Phi - 2\tau^C + 4\alpha)}{\left[(4\tau^C - 8\alpha - 7\theta + 28\varepsilon)(4\tau^C - 8\alpha - \theta + 4\varepsilon)\right]^2}, \tag{30}$$

implying the following results:

$$\frac{\partial \underline{\delta}_S}{\partial \tau^C} < 0, \frac{\partial \underline{\delta}_L}{\partial \tau^C} > 0 \text{ if } \Phi > 0, \tag{31}$$

$$\frac{\partial \underline{\delta}_S}{\partial \tau^C} > 0, \frac{\partial \underline{\delta}_L}{\partial \tau^C} < 0 \text{ if } \Phi < 0. \tag{32}$$

It is shown that $\underline{\delta}_S$ ($\underline{\delta}_L$) is a decreasing (increasing) function of τ^C, if $\Phi > 0$ and vice versa if $\Phi < 0$, and $\underline{\delta}_S(2\alpha) = \underline{\delta}_L(2\alpha) = 4/7$. We define $\delta^* \equiv \max\{\underline{\delta}_S, \underline{\delta}_L\}$. In Fig 4. 3, $\delta^* = \underline{\delta}_L$ for $\tau^C \in [2\alpha, 2\alpha + \Phi/4]$, whereas $\delta^* = \underline{\delta}_S$ for $\tau^C \in [2\alpha - \Phi/4, 2\alpha]$ if $\Phi > 0$. Fig 4. 2 shows the case $\Phi < 0$. However, the following two features are independent of the sign of Φ: $\delta^* \in [4/7, 1)$; the closer the value of τ^C is to 2α, the easier it is for the two regions to enter tax cooperation. We can summarize the above results in the following proposition:

Proposition 3. *A capital exporter (importer) has a relatively stronger incentive to deviate from the cooperative tax rate, when the cooperative tax rate is higher (lower) than* 2α. *The closer the value of the cooperative tax rate is to* 2α, *the easier it is to cooperate in tax coordination.*

Note that $sign[\partial(u_L^C - u_L^N)/\partial\tau^C] = sign(-\Phi)$ and $sign[\partial(u_L^D - u_L^C)/\partial\tau^C] = sign\Phi$ (proved in appendix B). When $\Phi > 0$, since $\partial r^C/\partial\tau^C < 0$, the large region-capital exporter-prefers a lower cooperative tax rate, which is coordinated with $\partial u_L^C / \partial \tau^C < 0$, and u_L^N is independent with τ^C. Hence $\partial(u_L^C - u_L^N) / \partial \tau^C < 0$, which indicates when τ^C increases, the deviation cost for the large region decreases. $\partial(u_L^D - u_L^C) / \partial \tau^C > 0$ is easy to understand. In Fig 4. 1, when cooperative tax rate τ^C increases, the width between the reaction curve of the large region and the cooperative tax rate line becomes broader. This means when the cooperative tax rate τ^C becomes higher, the deviation ability of the large region becomes stronger. This indicates when τ^C increases, the deviation gain for the large region increases. We can conclude that when the cooperative tax rate τ^C increases, the incentive to deviate for the large region becomes stronger. Then the large region's minimum discount rate $\underline{\delta}_L$ increases with the cooperative tax rate. Since the conditions faced by the small region are opposite with that of the large region, the small region's minimum discount rate $\underline{\delta}_S$ decreases, when the cooperative tax rate τ^C increases. About the case $\Phi < 0$, we can analyze it with the same way as the case $\Phi > 0$.

It is important that the best cooperative tax rate is 2α in Proposition 3. By introducing the fiscal equalization scheme, our model is a more general case of Itaya et al. (2008). When $\alpha = 0$, our model reduces to their model. And the best cooperative tax rate becomes zero. But the cooperative tax rate is not zero in the EU. Itaya et al. (2008)'s model cannot completely explain the economic reality. Our model implies that since the existence of fiscal equalization scheme in every federal economy, the best cooperative tax rate should take a positive value and increase with the scale of equalization transfers.

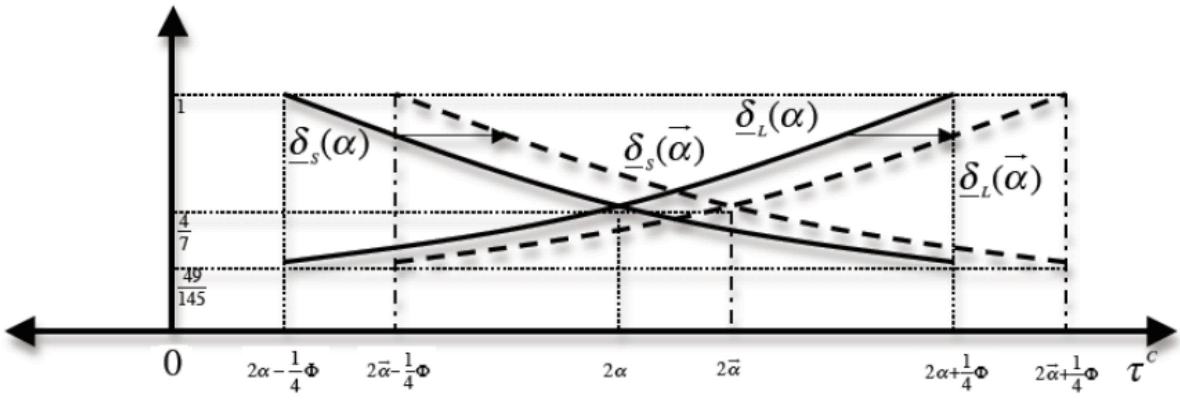

Figure 4. 3 The effects of an increase in α on $\underline{\delta}_i$, i =S, L, when the large (small) region is an exporter (importer); that is $\Phi \equiv \varepsilon - (\theta / 4) > 0$

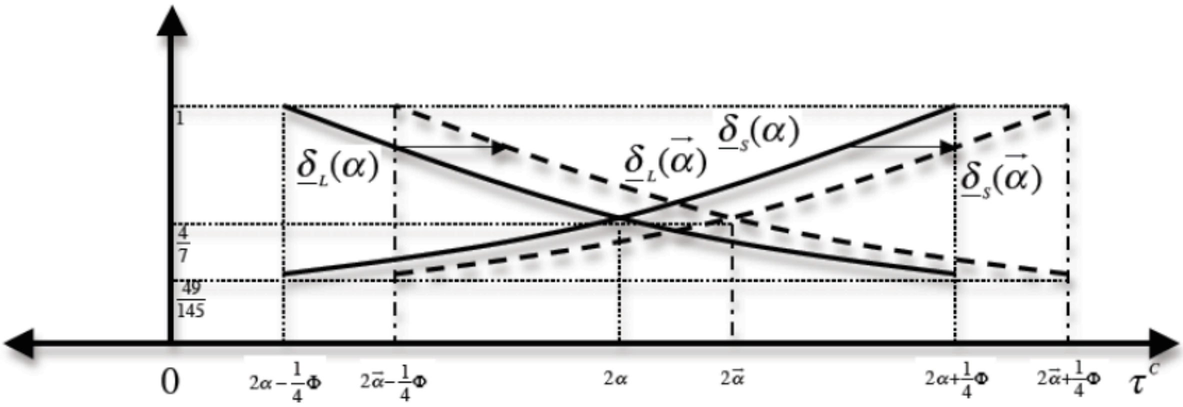

Figure 4. 4 The effects of an increase in α on $\underline{\delta}_i$, i =S, L, when the small (large) region is an exporter (importer); that is $\Phi \equiv \varepsilon - (\theta / 4) < 0$

We now examine the effect of increasing the scale of fiscal equalization α on the willingness of each region i to engage in tax cooperation. We differentiate $\underline{\delta}_i$, i=S, L, in Eqs. (27) and (28) with respect to α, then

$$\frac{\partial \underline{\delta}_S}{\partial \alpha} = \frac{3072\Phi(2\Phi - \tau^C + 2\alpha)(5\Phi + 2\tau^C - 4\alpha)}{\left[(4\tau^C - 8\alpha - 7\theta + 28\varepsilon)(4\tau^C - 8\alpha - \theta + 4\varepsilon)\right]^2}, \tag{33}$$

$$\frac{\partial \underline{\delta}_L}{\partial \alpha} = \frac{3072\Phi(\tau^C - 2\alpha + 2\Phi)(2\tau^C - 4\alpha - 5\Phi)}{\left[(4\tau^C - 8\alpha - 7\theta + 28\varepsilon)(4\tau^C - 8\alpha - \theta + 4\varepsilon)\right]^2}, \tag{34}$$

implying that

$$\frac{\partial \underline{\delta}_S}{\partial \alpha} > 0, \frac{\partial \underline{\delta}_L}{\partial \alpha} < 0 \text{ if } \Phi > 0, \tag{35}$$

$$\frac{\partial \underline{\delta}_S}{\partial \alpha} < 0, \frac{\partial \underline{\delta}_L}{\partial \alpha} > 0 \text{ if } \Phi < 0. \tag{36}$$

The above analysis allows us to state the following proposition:

Proposition 4. *When the scale of fiscal equalization α increases, the capital exporter (importer) is more (less) cooperative in implementing tax coordination.*

It holds that $sign[\partial(u_L^C - u_L^N)/\partial\alpha] = sign\Phi$ and $sign[\partial(u_L^D - u_L^C)/\partial\alpha] = sign(-\Phi)$ (proved in appendix B). In the case of $\Phi > 0$, for the large region (capital exporter), when the scale of fiscal equalization scheme α increases, the deviation cost increases, the deviation gain decreases, then the incentive to cooperate becomes stronger. It also holds that $sign[\partial(u_S^C - u_S^N)/\partial\alpha] = sign(-\Phi)$ and $sign[\partial(u_S^D - u_S^C)/\partial\alpha] = sign\Phi$ (proved in appendix B).

In the case of $\Phi > 0$, for the small region (capital importer), when the scale of fiscal equalization scheme α increases, the deviation cost becomes smaller, the deviation gain becomes larger, then incentive to cooperate becomes weaker. Obviously, we can proceed the analysis of the case $\Phi < 0$ with the same root as above.

In Figure 4. 3 and Figure 4. 4, when α increases, the locus $\underline{\delta}_i$ of a capital exporter will parallel shift right, the locus $\underline{\delta}_i$ of a capital importer will also parallel shift right. And it should be noted that $\underline{\delta}_S(2\alpha) = \underline{\delta}_L(2\alpha) = 4/7$. Thus we have the following result:

Proposition 5. *When the cooperative tax rate is set at* 2α*, even* α *changes, the willingness of both regions to cooperate remains at the same level.*

In Fig 4. 1 and Fig 4. 2, when the scale of fiscal equalization scheme α increases, the triangle which consists of the cooperative tax rate line, two reaction curves parallel shifts. Then the relationships among cooperation, deviation and punishment do not change.

4. 4 Conclusion

Itaya et al. (2008) analyze how the increased regional differences in per capita capital endowment and /or production technologies affect the willingness of each region to engage in tax cooperation in a repeated game model of capital tax coordination. They show that regional asymmetries in net capital exporting position may be advanntageous to the achievement of tax coordination. However, the best cooperative tax rate is zero in their model. Motivated by this finding, this chapter introduces a fiscal equalization scheme into Itaya et al. (2008)'s model. Its purpose is to examine how the fiscal equalization scheme affects the tax cooperation condition. It has shown that: with an unchanged federal utility level, the best cooperative tax rate becomes 2α; when the scale of fiscal equalization α increases, a capital exporter is more cooperative in tax coordination, a capital importer is less cooperative in tax coordination; interestingly, when the cooperative tax rate is set at 2α, even α changes, whereas the willingness of both regions to implement tax coordination remains constant.

Appendix B

$$\frac{\partial(u_S^C - u_S^N)}{\partial\tau^c} = \Phi$$

$$\frac{\partial(u_S^D - u_S^C)}{\partial\tau^C} = \frac{1}{6}(\tau^C - 2\Phi - 2\alpha)$$

$$\frac{\partial(u_L^C - u_L^N)}{\partial\tau^C} = -\Phi$$

$$\frac{\partial(u_L^D - u_L^C)}{\partial\tau^c} = \frac{1}{6}(\tau^C + 2\Phi - 2\alpha)$$

$$\frac{\partial(u_S^C - u_S^N)}{\partial\alpha} = -2\Phi$$

$$\frac{\partial(u_S^D - u_S^C)}{\partial\alpha} = -\frac{1}{3}(\tau^C - 2\Phi - 2\alpha)$$

$$\frac{\partial(u_L^C - u_L^N)}{\partial\alpha} = 2\Phi$$

$$\frac{\partial(u_L^D - u_L^C)}{\partial\alpha} = -\frac{1}{3}(\tau^C + 2\Phi - 2\alpha)$$

Chapter 5

Conclusion

This paper consists of two small topics related with public goods. One is international trade and public intermediate goods, the other is tax competition.

Chapter 2 and Chapter 3 are about international trade and public intermediate goods. Chapter 2 analyzes how the existence of public intermediate goods affects the result of the specific factors model. We introduce the public intermediate goods into the standard specific factors model. We assume the public intermediate goods are supplied by the government by the Lindal-Samuelson-Kaizuka rule. However, the government can not respond the exogenous change in the price ratio of the private goods instantly. It can only revise the level of public intermediate goods by a Marshallian quantity adjustment process. It is shown that even the public intermediate goods give rise to increasing returns to scale in the specific factors economy, the production possibility frontier is concave to the origin, when the market of public intermediate goods is Marshall stable. The symmetry assumption that the public intermediate goods promote the production of the two private industries equi-proportionally is a sufficient condition of the Marshallian stability. Under the symmetry assumption most but not all of the results of the standard specific factors model are still robust even in presence of the public intermediate goods.

Suga and Tawada (2007) investigate the influence of pure public intermediate goods on the trade patterns and gains from trade in a two-country model. Chapter 3 examines how the semi-public intermediate goods affect the trade pattern and trade gains in a two-country trade model. We construct a one primary factor, two consumer goods, one semi-public intermediate good and two-country model. We assume that the productivity of good 1 is less sensitive to the semi-public intermediate goods that that of good 2. It is also assumed that the semi-public intermediate goods are supplied by the government to get an efficient production. We show that the production possibility frontier is concave to the origin. The production take places in the point where the budget line is tangent with the production possibility frontier. Given the relative price of good 1, when the labor endowment increases, the relative supply of good 2 with respect good 1 decreases. Since the same homothetic preferences in two countries, the autarky price of good 1 is lower in the large country than that of the small country. Then the large (small) country is an exporter (importer) of the good whose productivity is less sensitive to the semi-public intermediate goods. This result is opposite with that of Suga and Tawada (2007). We also find that when trade opens, both countries gain from trade.

In Chapter 4, we introduce a fiscal equalization scheme into the model of Itaya et al. (2008). We ask the question that how the fiscal equalization scheme affects the tax cooperation condition. It is show that the introduction of the fiscal equalization transfer increases the utility of the small region, decreases the utility of the large region, has no effect on the federal utility level. When the scale of the fiscal equalization scheme increases, the capital exporter (importer) has more (less) incentives to tax cooperation. Since the existence of the fiscal equalization transfer, the best cooperative tax rate takes a positive value and increases with the scale of the fiscal equalization scheme.

About international trade and public intermediate goods, we can consider follow directions to extend the analysis. First, most of the existing studies concentrate on national public intermediate good, however, railways, highways are constructed across national boards recently. Hence, we can consider international public intermediate good in trade models. Second, public intermediate goods are considered only as production factors, we can accommodate them as tradable goods in the trade models. The international market of public goods like high-speed rail, is flourishing in reality.

Basically, there are two kinds of fiscal equalization scheme: tax base equalization scheme and tax revenue equalization scheme. In Chapter 4, we analyze how the tax base equalization scheme affects tax cooperation condition. We may consider how the tax revenue equalization scheme affects the tax cooperation condition in the further research.

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