4　摂動宇宙の観測量

4.1　宇宙背景放射のゆらぎ

輝度関数 (brightness function)

$$\Theta(k,\mu,\tau) = \sum_l (-i)^l (2l+1) P_l(\mu) \Theta_l(k,\tau) \tag{4.1}$$

ボルツマン方程式

$$\Theta' + ik\mu\Theta + ik\mu\Theta + \Psi' = {\tau_c}^{-1}\left[\Theta_0 - \Theta + ik\mu v_{\rm b} - \frac{1}{2}P_2(\mu)\Theta_2\right] \tag{4.2}$$

光学的厚み (optical depth)

$$\kappa(\tau) \equiv \int_{t(\tau)}^{t_0} dt n_{\rm e}\sigma_{\rm T}c = \int_\tau^{\tau_0} d\tau a n_{\rm e}\sigma_{\rm T} = \int_\tau^{\tau_0}\frac{d\tau}{\tau_{\rm c}} \tag{4.3}$$

ボルツマン方程式の形式解

$$\Theta(k,\mu,\tau_0) = -\int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)-\kappa}\left\{\Psi' + ik\mu\Phi + \kappa'\left[\Theta_0 + ik\mu v_{\rm b} - \frac{1}{2}P_2(\mu)\Theta_2\right]\right\} \tag{4.4}$$

$$= \int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)}\mathcal{S}(k,\tau) \tag{4.5}$$

源関数 (source function)

$$\mathcal{S}(k,\tau) = e^{-\kappa}\left(\Phi' - \Psi'\right) + g(\tau)\left(\Theta_0 + \frac{1}{4}\Theta_2 + \Phi\right) + (g v_{\rm b})' - \frac{3}{4k^2}\left(g\Theta_2\right)'' \tag{4.6}$$

視程関数 (visibility function)

$$g(\tau) = -\kappa' e^{-\kappa} \tag{4.7}$$

形式解のルジャンドル表示

$$\Theta_l(k,\tau_0) = \int_0^{\tau_0}\mathcal{S}(\tau,k) j_l[k(\tau_0-\tau)] \tag{4.8}$$

近似解 ($g(\tau) \to \delta(\tau-\tau_{\rm dec}), \Theta_2 \simeq 0$)

$$\begin{aligned}\Theta_l(k,\tau_0) \simeq{}& \left[\Theta_0(k,\tau_{\rm dec}) + \Phi(k,\tau_{\rm dec})\right] j_l[k(\tau_0-\tau_{\rm dec})] \\ & - 3\Theta_1(k,\tau_{\rm dec})\left\{j_{l-1}[k(\tau_0-\tau_{\rm dec})] - \frac{l+1}{k(\tau_0-\tau_{\rm dec})} j_l[k(\tau_0-\tau_{\rm dec})]\right\} \\ & + \int_0^{\tau_0} d\tau e^{-\kappa}\left[\Phi'(k,\tau) - \Psi'(k,\tau)\right] j_l[k(\tau_0-\tau_{\rm dec})]\end{aligned} \tag{4.9}$$

温度ゆらぎ

$$\frac{\delta T}{T}(\theta,\phi) = \Theta(\boldsymbol{x}=\boldsymbol{0},\boldsymbol{n},\tau_0) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\phi) \tag{4.10}$$

$$a_{lm} = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, Y_l^{m*}(\theta,\phi)\, \frac{\delta T}{T}(\theta,\phi) \tag{4.11}$$

温度ゆらぎのパワースペクトル

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l}\left\langle |a_{lm}|^2 \right\rangle = 4\pi \int \frac{k^2 dk}{2\pi^2} P_{\Phi_{\mathrm{m}0}}(k) \left|\hat{\Theta}_l(k)\right|^2 \tag{4.12}$$

ここで、

$$\Theta_l(\boldsymbol{k},\tau_0) = \hat{\Theta}_l(k)\Phi_{\mathrm{m}0}(\boldsymbol{k}) \tag{4.13}$$

ザックス・ウォルフェ効果

$$C_l \simeq \frac{4\pi}{9}\int \frac{k^2 dk}{2\pi^2} P_{\Phi_{\mathrm{m}0}}(k) \left[j_l(k\tau_0)\right]^2 \tag{4.14}$$

$$= \frac{1}{2\pi}\left(\frac{{H_0}^2\Omega_{\mathrm{m}0}}{D(\tau_0)}\right)^2 \int \frac{dk}{k^2} P(k) \left[j_l(k\tau_0)\right]^2 \tag{4.15}$$

べき則パワースペクトル $P(k) = Ak^n$ に対して

$$C_l \simeq \frac{A{\tau_0}^{1-n}}{8\sqrt{\pi}}\left(\frac{{H_0}^2\Omega_{\mathrm{m}0}}{D(\tau_0)}\right)^2 \frac{\Gamma\left(\frac{3-n}{2}\right)\Gamma\left(l+\frac{n-1}{2}\right)}{\Gamma\left(2-\frac{n}{2}\right)\Gamma\left(l+\frac{5-n}{2}\right)} \tag{4.16}$$

音響振動

$$C_l \sim \int \frac{dk}{k^2} P(k)\, |\cos[kr_{\mathrm{s}}(\tau_{\mathrm{dec}})]|^2 \left\{j_l[k(\tau_0-\tau_{\mathrm{dec}})]\right\}^2 \tag{4.17}$$

音響ピーク位置

$$l^{(m)} \sim \frac{m\pi(1+z_{\mathrm{dec}})d_{\mathrm{A}}(z_{\mathrm{dec}})}{r_{\mathrm{s}}(\tau_{\mathrm{dec}})} \tag{4.18}$$