4.2 銀河の空間分布

2 次元の密度場とゆらぎ

$$\rho_{\rm 2D}(\theta,\varphi) = \int_0^\infty d\chi W(\chi)\rho(\chi,\theta,\varphi) \tag{4.19}$$

$$\delta_{\rm 2D}(\theta,\varphi) = \int_0^\infty d\chi W(\chi)\delta(\chi,\theta,\varphi) \tag{4.20}$$

角度相関関数

$$\begin{aligned} w(\theta) &= \langle \delta_{\rm 2D}(\theta_1,\varphi_1)\delta_{\rm 2D}(\theta_2,\varphi_2)\rangle \\ &= \int d\chi_1 W(\chi_1)\int d\chi_2 W(\chi_2)\left\langle \delta(\chi_1,\theta_1,\varphi_1)\delta(\chi_2,\theta_2,\varphi_2)\right\rangle \end{aligned} \tag{4.21}$$

幾何学的関数

$$S_K(\chi) \equiv \begin{cases} \dfrac{1}{\sqrt{K}}\sin\left(\sqrt{K}\chi\right) & (K>0) \\ \chi & (K=0) \\ \dfrac{1}{\sqrt{-K}}\sinh\left(\sqrt{-K}\chi\right) & (K<0) \end{cases} \tag{4.22}$$

$$C_K(\chi) \equiv \begin{cases} \cos\left(\chi\sqrt{K}\right) & (K>0) \\ 1 & (K=0) \\ \cosh\left(\chi\sqrt{-K}\right) & (K<0) \end{cases} \tag{4.23}$$

恒等式

$$C_K^2(\chi) + K{S_K}^2(\chi) = 1, \quad \frac{dS_K(\chi)}{d\chi} = C_K(\chi), \quad \frac{dC_K(\chi)}{d\chi} = -KS_K(\chi) \tag{4.24}$$

Limber の式 (小角度近似)

$$w(\theta) \simeq \int d\chi W^2(\chi)\int d\chi'\xi\left(\sqrt{{S_K}^2(\chi)\theta^2 + \chi'^2}, \tau_0-\chi\right) \tag{4.25}$$

$$= \int d\chi W^2(\chi)\int_0^\infty \frac{kdk}{2\pi} J_0\left[kS_K(\chi)\theta\right]P(k,\tau_0-\chi) \tag{4.26}$$

角度パワースペクトル

$$\delta_{\mathrm{2D}}(\theta,\varphi) = \sum_{l,m} a_{lm} Y_l^m(\theta,\varphi); \quad a_{lm} = \int \sin\theta d\theta d\varphi \delta_{\mathrm{2D}}(\theta,\varphi) \tag{4.27}$$

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l}\left\langle |a_{lm}|^2 \right\rangle = 2\pi \int_0^\pi \sin\theta d\theta P_l(\cos\theta) w(\theta) \tag{4.28}$$

小角度近似

$$C_l \simeq \int d\chi \frac{W^2(\chi)}{{S_K}^2(\chi)} P\left[\frac{l}{S_K(\chi)}, \tau_0 - \chi\right] \tag{4.29}$$

$$C_l \simeq P_{\mathrm{2D}}(l) \equiv \int d^2\theta\, e^{-i\vec{\theta}\cdot\vec{l}} w(\theta) \tag{4.30}$$

摂動時空における赤方偏移

$$1+z = \frac{1}{a(\tau_{\mathrm{e}})}\left[1 - n^i v_i\Big|_{\mathrm{e}}^{0} + \Phi\Big|_{\mathrm{e}}^{0} - \int_{\tau_{\mathrm{e}}}^{\tau_0} (\Phi' - \Psi')\, d\tau\right] \tag{4.31}$$

ただし、銀河サーベイにおいてはドップラー効果以外は小さい。
視線方向速度成分

$$V \equiv n^i v_i\Big|_{\mathrm{e}}, \quad V_0 \equiv n^i v_i\Big|_0 \tag{4.32}$$

赤方偏移空間共動距離

$$s(\chi) \equiv \int_0^{z(\chi)} \frac{cdz}{H(z)} = \chi + \frac{1}{aH}(V - V_0) \tag{4.33}$$

赤方偏移変形パラメータ

$$\beta \equiv \frac{f}{b} = \frac{1}{b}\frac{a}{D}\frac{dD}{da} \tag{4.34}$$

線形近似、遠観測者近似における赤方偏移空間の銀河数密度

$$\delta^{(\mathrm{s})}(\boldsymbol{x}) = b\left[\delta(\boldsymbol{x}) + \frac{1}{b}\frac{1}{aH}\frac{\partial V}{\partial \chi}\right] = b\left[\delta(\boldsymbol{x}) + \beta\partial_3^2 \triangle^{-1}\delta(\boldsymbol{x})\right] \tag{4.35}$$

フーリエ表示とパワースペクトル ($\mu_{\boldsymbol{k}} \equiv \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{z}} = k_3/k$)

$$\tilde{\delta}^{(\mathrm{s})}(\boldsymbol{k}) = b\left(1 + \beta\mu_{\boldsymbol{k}}^2\right)\tilde{\delta}(\boldsymbol{k}), \tag{4.36}$$

$$P^{(\mathrm{s})}(\boldsymbol{k}) = b^2\left(1 + \beta\mu_{\boldsymbol{k}}^2\right)^2 P(k) \tag{4.37}$$

相関関数 ($\mu = \hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{z}} = x_3/x$)

$$\xi^{(\mathrm{s})}(\boldsymbol{x}) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)\xi_0(x)P_0(\mu) - \left(\frac{4}{3}\beta + \frac{4}{7}\beta^2\right)\xi_2(x)P_2(\mu) + \frac{8}{35}\beta^2\xi_4(x)P_4(\mu) \tag{4.38}$$

4.3 弱重力レンズ場

角度方向の測地線方程式 (ω_{ab}: 2 次元球面計量)

$$y^a \equiv S_K(\chi)\delta x^a, \qquad y_a = \omega_{ab} y^b \tag{4.39}$$

$$\frac{d^2 y_a}{d\chi^2} + K y_a = -\frac{2}{S_K(\chi)}\partial_a \Phi \tag{4.40}$$

形式解 (平坦球面近似)

$$\vec{\theta}^{\mathrm{S}}(\chi) = \vec{\theta} - 2\int_0^{\chi} d\chi' \frac{S_K(\chi-\chi')}{S_K(\chi)S_K(\chi')}\vec{\nabla}_\theta \Phi\left[S_K(\chi')\vec{\theta}^{\mathrm{S}}, \chi', \tau_0 - \chi'\right] \tag{4.41}$$

レンズポテンシャル

$$\psi(\vec{\theta}, \chi) \equiv 2\int_0^{\chi} d\chi' \frac{S_K(\chi-\chi')}{S_K(\chi)S_K(\chi')}\Phi\left[S_K(\chi')\vec{\theta}, \chi', \tau_0 - \chi'\right] \tag{4.42}$$

ボルン近似解

$$\vec{\theta}^{\mathrm{S}}(\vec{\theta}, \chi) = \vec{\theta} - \vec{\nabla}_\theta \psi(\vec{\theta}, \chi) \tag{4.43}$$

変換行列

$$A_{ab} = \frac{\partial \theta_a^{\mathrm{S}}}{\partial \theta_b} = \delta_{ab} - \partial_a \partial_b \psi \tag{4.44}$$

$$(A_{ab}) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \tag{4.45}$$

$$\kappa = \frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\psi, \quad \gamma_1 = \frac{1}{2}\left(\partial_1^2 - \partial_2^2\right)\psi, \quad \gamma_2 = \partial_1\partial_2\psi \tag{4.46}$$

平均レンズポテンシャル

$$\psi(\vec{\theta}) \equiv \int_0^{\infty} d\chi W(\chi)\psi(\vec{\theta}, \chi) = 2\int_0^{\infty} d\chi \frac{g(\chi)}{S_K(\chi)}\Phi\left[S_K(\chi')\vec{\theta}, \chi', \tau_0 - \chi'\right] \tag{4.47}$$

$$g(\chi) \equiv \int_{\chi}^{\infty} d\chi' \frac{S_K(\chi'-\chi)}{S_K(\chi')}W(\chi') \tag{4.48}$$

平均収斂場 (小角度近似)

$$\kappa(\theta) = \frac{3}{2}H_0^2\Omega_{m0}\int_0^{\infty} d\chi S_K(\chi) g(\chi) \frac{\delta\left[S_K(\chi')\vec{\theta}, \chi', \tau_0 - \chi'\right]}{a(\tau_0 - \chi)} \tag{4.49}$$

パワースペクトル

$$P_\kappa(l) = \frac{9}{4} H_0^4 \Omega_{\mathrm{m0}}^2 \int_0^\infty d\chi \frac{g^2(\chi)}{a^2(\tau_0 - \chi)} P\left[\frac{l}{S_K(\chi)}, \tau_0 - \chi\right] \tag{4.50}$$

$$P_\psi(l) = \frac{4}{l^4} P_\kappa(l) \tag{4.51}$$

$$P_{\gamma_1}(l) = \frac{(l_1^2 - l_2^2)^2}{4} P_\psi(l) = \frac{(l_1^2 - l_2^2)^2}{l^4} P_\kappa(l) \tag{4.52}$$

$$P_{\gamma_2}(l) = l_1^2 l_2^2 P_\psi(l) = \frac{4 l_1^2 l_2^2}{l^4} P_\kappa(l) \tag{4.53}$$