# 熱伝導と接触変成作用
# Heat Conduction and Contact Metamorphism

Simon Wallis, Nagoya University
ウォリス　サイモン

## INTRODUCTION 基本知識

The thermal structure of the earth is fundamental for understanding many geological processes including the locations of volcanoes, earthquakes and hydrothermal fields.

Heat transfer （熱輸送）can be divided into three different types:

- conduction (熱伝導)
- advection（熱移流）
- radiation（熱輻射）

In fluids and gasses advection and radiation are dominant. In rocks, radiation is not an important heat transfer mechanism (or can be treated as part of conduction), but both advection and conduction may be important. Conduction of heat energy (熱エネルギーの拡散) is analogous to diffusion of matter (物質の拡散) and represents the transfer of energy by movement and collisions of atoms, electrons and phonons. Advection is the process of transferring heat by movement of either hot or cold bodies with respect to one another.

One simple way to think of the difference between conduction and advection is to consider a blob of ink in a river (Fig. 1). The concentration of ink is analogous to temperature, and the distribution across the river of the ink concentration (or temperature) at some time and at some distance down stream is a function of how quickly the ink spreads out (conduction) and how quickly the ink is transported down stream (advection).

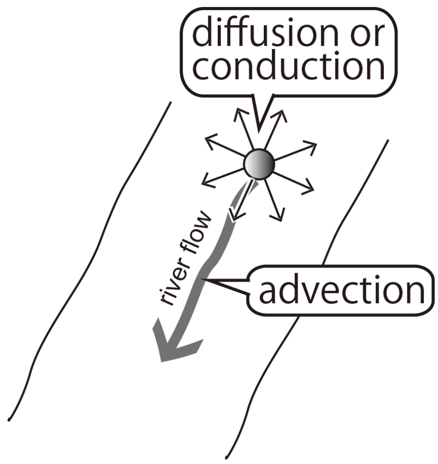


Figure 1. The difference between conduction and advection can be easily thought of in terms of the transport of a spreading blob of ink in a flowing river.

One example of advection of heat is the inflow of cold lithospheric plates in subduction zones. Heat advection is also important in situation where there is movement of hot fluids in rocks, e.g. oceanic ridges （海嶺）. Today we will focus on heat conduction and contact metamorphism (接触変成作用）.

## FOURIER’S LAW AND HEAT FLOW
## フーリェ法則と熱流量

The conductive flow of heat through a substance is described by Fourier's Law (フーリェ法則):

$$Q = -k\frac{dT}{dz} \qquad (1)$$

*Q* is the heat flow (熱流量) in Watts per square meter (W $m^{-2}$)(熱流束-heat flux-ともいう), *k* is the thermal conductivity (熱伝導率) (J $s^{-1}$ $m^{-1}$ $Kelvin^{-1}$), and *dT/dz* is the thermal gradient. The negative sign is there because we define temperature and distance to increase in the same direction and because *Q* flows from high *T* to low *T*. Fourier’s Law is an empirical law but has been well tested and until recently no significant exceptions were known. In recent years it has been suggested that some modern materials exhibit different non-Fourier conductive behaviour.

Typical values for k are 1.5–3.0 J $s^{-1}$ $m^{-1}$ $Kelvin^{-1}$. The average heat flow for continental areas is around 56 mW m$^{-2}$ and for oceanic areas 78 mW m$^{-2}$. The reason for this difference is the young age of the oceanic crust. At the earth's surface, the amount of heat flowing from within the earth is much less than the heat energy derived from the sun (~400 W m$^{-2}$).

## DERIVATION OF THE HEAT CONDUCTION EQUATION
## 熱伝導方程式の導出

The heat conduction equation is derived by considering two relationships: one between heat flow, $Q$, and spatial gradients in $T$ (Fourier's Law) and the other considering the relationship between heat flow, $Q$, and temporal gradients in $T$. By combining these two relationships, we derive an equation that shows how $T$ varies with both space and time.

In this example we will consider a 1-dimensional situation—this is appropriate for narrow elongate intrusions such as dykes and sills. It is also a very good approximation for the cooling of the outermost 100 km of the Earth. Let us consider a small volume of the material we are interested in with thickness $\Delta z$ (Fig. 2).

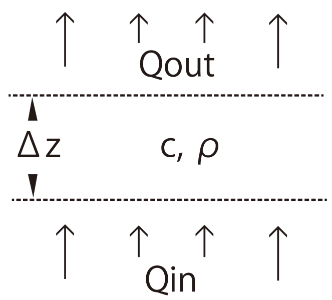


Figure. 2. To derive the heat conduction equation, we first consider a one-dimensional region with thickness $\Delta z$, heat capacity $c$, density $\rho$ and with a difference in the heat flow in ($Q_{in}$) and the heat flow out ($Q_{out}$).

The Law of conservation of energy tells us:

$$\frac{\partial T}{\partial t} = \frac{Q_{in} - Q_{out}}{c\rho\Delta z} \qquad (2).$$

Where $c$ is the specific heat of the material (比熱) (J/K/kg) and $\rho$ is the density. That is, if the heat flow out ($Q_{out}$) of the area is less than the heat flow in ($Q_{in}$) then the material is heating up at a rate ($dT/dt > 0$) related to its specific heat and density. Different rocks will change temperature by different amounts for the same input of heat energy. Heat and temperature are not the same! Let us consider changes in unit time.

-The temperature rise for c joules is 1° for 1 kg of material.
-The temperature rise for Q joules is (J/c)° for 1 kg of material.
-The temperature rise for Q joules and $\rho\Delta z$ kg of material is $\left(T/c\rho\Delta z\right)$°.

Secondly we can say that

$$Q_{out} = Q_{in} + \Delta z \frac{\partial Q}{\partial z}. \qquad (3)$$

We can now rewrite the first equation to relate the spatial variation in temperature with the rate of temperature change.

$$\frac{\partial T}{\partial t} = \frac{Q_{in} - Q_{in} - \Delta z \frac{\partial Q}{\partial z}}{c\rho\Delta z}$$

$$= {-\frac{\partial Q}{\partial z}}\Big/{c\rho}$$

We can now include the substitute for Q using Fourier's Law,

$$\frac{\partial T}{\partial t} = {\frac{\partial\left(k\,\partial T/\partial z\right)}{\partial z}}\Big/{c\rho}$$

$$= \frac{k}{c\rho}\frac{\partial^2 T}{\partial z^2}$$

$$= \frac{k}{c\rho}\frac{\partial^2 T}{\partial z^2}.$$

Now we let $k/c\rho = \kappa$ where $\kappa$ (m$^2$/s) is the thermal diffusivity and so

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \qquad (4).$$

This is the fundamental equation of heat conduction in one dimension.

## QUALITATIVE UNDERSTANDING OF HEAT CONDUCTION
## 熱伝導の定性的な理解

Heat conduction is controlled by a second order differential equation relating temperature, time and position. Changes in temperature by conduction only occur if there are curves in the heat profile. If there is a linear gradient then no change in temperature takes place. The greater the curvature is, the greater the rate of change of temperature. This means that if sharp changes are introduced in the thermal structure, for instance by intruding a hot magma into a cold country rock, then the temperature at the boundaries will change much faster than elsewhere. Positive curvature causes a rise in *T*, while negative curvature causes a decrease in *T*—protrusions of the thermal structure get flattened out and depressions get filled in.

A second important point is that if two conducting solid objects are brought together, the temperature at the boundary is the average of the two temperatures. This means that if a 1000°C magma is intruded into rock with a temperature of 200°C, then the initial temperature at the boundary will be (1000+200)/2 = 600°C.

## APPLICATION TO MAGMATIC INTRUSIONS
## 貫入岩体への適応例

We can now use our understanding of heat conduction to predict changes in temperature around an igneous intrusion intruded into cold country rock.

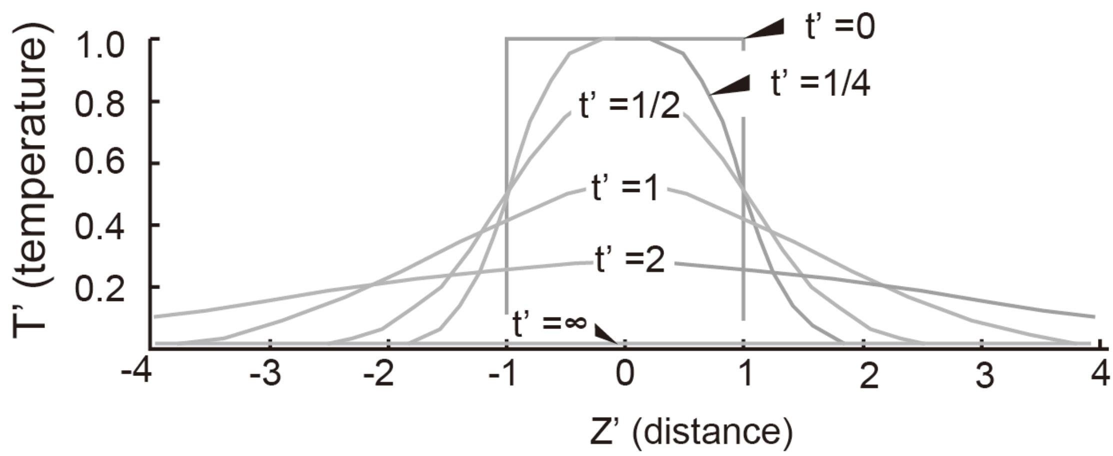


Figure. 3 The change in the temperature around an intrusion with time. Time, *t'*, temperature, *T'*, and distance, *Z'*, are expressed as dimensionless quantities with definitions as follows:

$t' = \sqrt{\frac{\kappa t}{h^2}}$ (*t* is time and kappa is the thermal diffusivity) , $T' = \frac{T - T_0}{T_s - T_0}$ (*T* is the temperature, $T_s$ is the original temperature of the intrusion and $T_0$ is the original temperature of the country rock and the temperature at infinity), $Z' = z/h$ (*z* is the distance and h is the half width of the intrusion)

A common boundary condition is to assume initial uniform temperature extending to infinity (Fig. 3). This means that after an infinite amount of time the thermal anomaly created by the dyke will disappear. If we assume the intrusion was instantaneous then the initial thermal profile will be a square wave. The temperature along the contact will be the average of the intrusion and country rock. So we know the initial conditions and final conditions. The heat conduction equation here tells us what happens in between.

We know from our result above that the most rapid changes in temperature will occur where the gradients are steepest, i.e. near the contacts, which are the corners in the thermal structure shown in figure 3. These corners will become rounded and the heat will move outwards.

Within the body we will see a gradual cooling. Outside we will see a very different behaviour. First the temperature will rise and then fall. Also the peak of temperature will not occur at the same time in all places. We can see that even for a simple situation there will be a complex variation in the relationships between temperature-time history of rocks and their position.

## PARAMETERS OF THERMAL CONDUCTION IN ROCKS
## 岩石における熱伝導のパラメター

Something very fortunate is that although *c* and *k* vary considerably $\kappa$ only varies very little for most rock types. It is usually considered to be $10^{-6}$ $m^2\ s^{-1}$. However, it is about half this for calcite-rich rocks. Changing $\kappa$ affects the time scale for heating but does not affect the peak temperature attained.

The most common problem is to determine how the temperature varies with time and space for a given intrusion geometry, magma temperature and pre-intrusion temperature. Different magmas vary in temperature by several 100s of degrees. Intrusion sizes can vary from mm to 10s kilometers.

We will look in more detail at the modeling during the practical. To get some idea, the time *t* taken for a body with half width *h* to reach approximately half its original temperature in the core of the intrusion is related to its size and $\kappa$ by the equation:

$$t = \frac{h^2}{\kappa} \text{ (Table 1).}$$

| h (meters) | T(seconds) | t(years) |
|---|---|---|
| **1** | $10^6$ | 0.0317 |
| **10** | $10^8$ | 3.17 |
| **100** | $10^{10}$ | 317 |
| **1000** | $10^{12}$ | 31700 |
| **10000** | $10^{14}$ | $3.17 \times 10^6$ |

Table 1. Cooling time scales for intrusions of different sizes.

So we can see that 1m dykes may take only weeks to cool, but large plutons may take millions of years. An example of cooling around a pluton is given in figure 4. These data show that cooling has taken several millions of years implying that the pluton must have a radius of the order of 10km.

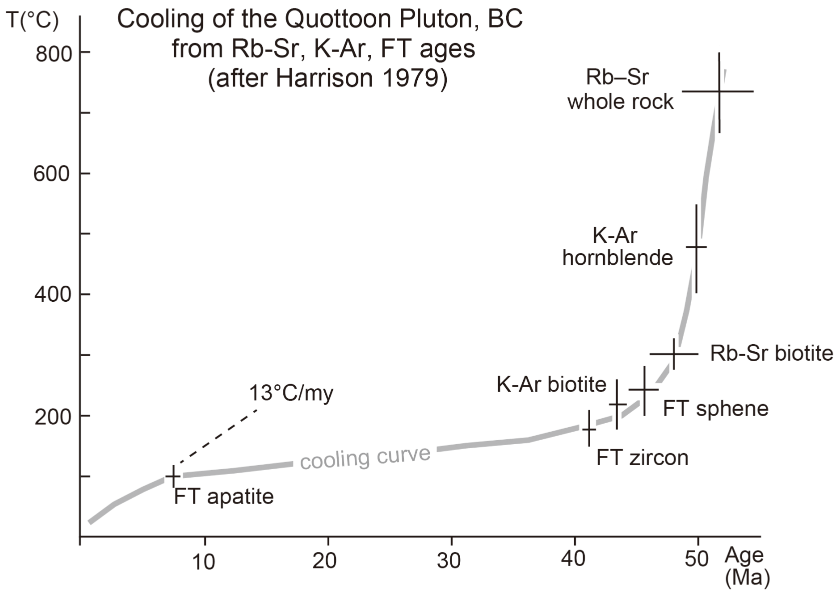


Figure 4. Cooling of the Quottoon Pluton in British Columbia (Canada) shown by radiometric dating. The dates record the time at which the rock reached different temperatures. It took several millions of years to cool significantly implying the presence of a large pluton.

## COMPLICATING FACTORS
## その他の影響

A more complete analysis of the temperature distribution around an intrusion requires consideration of other factors. More complete modeling requires numerical solutions to the conduction equation. This approach can include other complications such as temperature dependence of thermal conductivity (熱伝導の温度依存性), 2-d and 3-d geometries of the intrusions, an initial thermal gradient in the region with the intrusion (初期地温勾配), latent heat (潜熱), convection of fluids near the intrusion (貫入岩体周囲の熱水循環) etc.

There are ways to incorporate some of these features in the 1-d modeling we have looked at. We will look at two simple ways to treat latent heat and advection.

Latent heat (潜熱) is released as the liquid magma crystallizes. The extra heat energy released maintains the temperature at a high value until all the magma is crystallized. The contribution of latent heat can be approximated by adding an amount equivalent to releasing all the energy at the same time and treating it as an equivalent temperature rise. For example some approximate values for basaltic magma are as follows.

Latent heat of crystallization (潜熱) $4 \times 10^5$ J / kg
Specific heat (比熱) $10^3$ J /Kg / ℃

If the latent heat were instantaneously released and all used to increase the temperature of the magma the rise would be

$(4 \times 10^5)/10^3 = 400$℃

This number can be added to the initial temperature for the magma. This does not correspond to a physically meaningful process, but the result does give a good approximation for the peak T around the intrusion. Numerical methods are required for accurate estimates of T within and very close to the intrusion.

Advection of heat by the convection of hot fluids (熱水循環による熱移流) is quite common around intrusions. Convection transports heat more effectively than conduction and results in a lower temperature close to the intrusion and a higher temperature at a greater distance. In many cases the zone of convection is restricted close to the intrusion. An example where both domains can be identified is the Kakkonda (葛根田) hydrothermal area in NE Japan (Fig. 5), where a young and hot granite body has intruded to depths of about 3km beneath the surface. The zone of convection can be modeled using a high value for $\kappa$, which has the same effect.

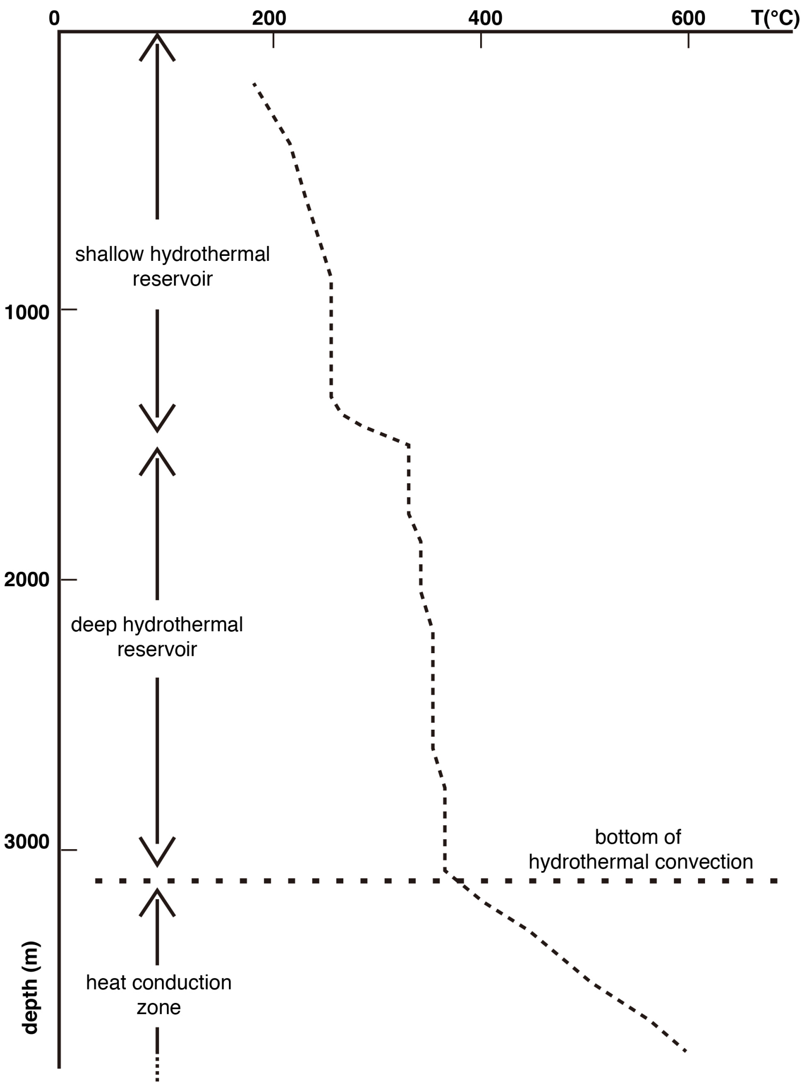


Figure 5. Thermal structure in the Kakkonda hydrothermal region showing the shallow regions with circulating water and small differences in temperature compared to the deeper region with steep thermal gradients due to conduction (after Ikeuchi et al., 1998). Greater depth acts against free flow of fluid both due to pressure closing gaps in the rock and the rapid deposition of minerals filling in fractures.

## REFERENCES

### Key References

Carslaw, H. S. & Jaeger, J. C. 2000. *Conduction of Heat in Solids*. Oxford University Press, Oxford, pp. 510.

- A comprehensive treatise on exact solutions to the heat conduction equation for a range of geometries and boundary conditions. First published in 1946 and still highly relevant. A classic textbook!

Spear, F. S. 1994. *Metamorphic Phase Equilibria and Pressure-Temperature-Time Paths.* Monograph of the Mineralogical Society of America, Ch. 3.

- A simplified approach to understanding the heat equation and some applications to problems of importance in metamorphic geology.

Other References

Harrison, T. M., Armstrong, R. L., Naeser, C. W. & Harakal, J. E. 1979. Geochronology and thermal history of the Coast Plutonic complex near Prince Rupert, British Columbia. *Canadian Journal of Earth Science*, 16, 400-410.

Ikeuchi, K., Doi, N., Sakagawa, Y., Kamenosono, H., Uchida, T. 1998. High temperature measurement in well WD-1A and the thermal structure of the Kakkonda geothermal system Japan. *Geothermics* 27, 591-607.

Roetzel et al. 2003. Experiment and analysis for non-Fourier conduction in materials with non-homogenous inner structure. *International Journal of Thermal Sciences*, 42 541-552

- Discussion of the reality of non-Fourier heat conduction.