Theory of a gas laser with a Gaussian field profile

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(Received 12 August 1974; in final form 8 October 1974)

A theory for a gas laser with a Gaussian radial field distribution is considered. Rotating wave approximation is adopted to calculate the atomic polarization of the two-level system. An asymmetry of the shape of the Lamb dip and a shift of its center frequency are numerically obtained in the self-consistent laser field.

I. INTRODUCTION

This paper considers the propagation of optical modes or a Gaussian beam of light inside a laser oscillator. In a laser tube the excitation density of the passive medium supporting the beam is not uniform in the transverse direction. Due to this nonuniform pumping density the gain and the index of refraction vary with the distance from the optical axis. Two main approaches to the problem of the radial profile of the field in a laser have so far been used.

The first approach is to assume that the index of refraction is expressed as a power series of r.[1-3] That is, $n(r)=n_0+n_2r^2+\cdots$, where r is the radial distance from the cylindrical axis. In a medium with this variation of the index of refraction a ray matrix formalism[1] is of practical value. With this tool, often referred to as the ABCD law, the beam radius and the phase-front curvature are described. However, this method neglects saturation effects and does not give a self-consistent solution.

The second approach is often employed to treat the self-focusing effects.[4-6] In this case the index of refraction is assumed to be $n=n_0+n_2E^2+\cdots$. A solution for the field is obtained self-consistently, although the r dependence of n_0 and n_2 is neglected.

In this paper we assume that the radial distribution of the population inversion N is given by[7]

$$N=N_0J_0(a_0r/b), \tag{1}$$

with $a_0=2.405$ and b is the inner radius of the laser tube. The Gaussian beams suspended in the medium of this inversion density propagate in both directions along the optic axis. The macroscopic polarization of the medium is calculated from Lamb's theory[8] by the geometrical representation method.[9] Instead of using exact solutions we compute the polarization with the help of the rotating wave approximation (RWA) which gives a satisfactory agreement with the exact solution. Both sides of the Maxwell wave equation are expanded in the power series of r and the two lowest powers in r are compared on both sides. The saturation and also the r dependence of the index of refraction are considered in a self-consistent way.

II. FLAT MIRROR

The properties of Gaussian beams of light[1] in homogeneous media are well known. Near the optic axis the field distribution $E(r,z)$ of a fundamental complex Gaussian mode is described by

$$E(r,z)=\frac{1}{w}\exp\left[-i(kz+\Phi)-r^2\left(\frac{1}{w^2}+\frac{ik}{2R}\right)\right], \tag{2}$$

where a constant amplitude factor is dropped. k is the wave number, z is the distance along the optic axis, w is the $1/e$ radius of the beam, Φ is the additional phase shift, and R is the radius of curvature of the phase front of the spherical wave.

First we use the following assumptions of the simplified model: (a) the feedback mirrors are flat, (b) the loss is uniformly distributed, and (c) the amplitude and the beam parameters are constant along the cavity length. Under these assumptions we may neglect the additional phase shift Φ, and only the curvature of the phase front changes the sign when a beam is reflected back by the feedback mirror. There are four complex Gaussian beams: A wave propagating in the positive direction, which can be expressed as

$$\exp\left[-\frac{r^2}{w_0^2}+i\left(\omega t-k_0z-\frac{k_0}{2R_0}r^2\right)\right],$$

a wave propagating in the negative direction, which can be expressed as

$$\exp\left[-\frac{r^2}{w_0^2}+i\left(\omega t+k_0z-\frac{k_0}{2R_0}r^2\right)\right],$$

and complex conjugates of these. The subscript 0 denotes the parameters for the flat mirror. We may construct a real Gaussian beam from the linear combination of these complex beams:

$$E(z,r,t)=E_0\exp\left(-\frac{r^2}{w_0^2}\right)\cos\left[\omega t-\left(\frac{k_0}{2R_0}\right)r^2\right]\sin(k_0z), \tag{3}$$

where E_0 is the amplitude. The sign convention for R_0 is taken so that positive and negative R_0 indicate diverging and converging waves, respectively (see Fig. 1).

The Maxwell wave equation in cylindrical coordinates is given by

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right)-\frac{\partial^2E}{\partial z^2}+\mu_0\sigma\frac{\partial E}{\partial t}+\frac{1}{c_0^2}\frac{\partial^2E}{\partial t^2}=-\mu_0\frac{\partial^2P}{\partial t^2}. \tag{4}$$

By letting

$$C(z,r,t)=\cos[\omega t-(k_0/2R_0)r^2]\sin(k_0z)$$

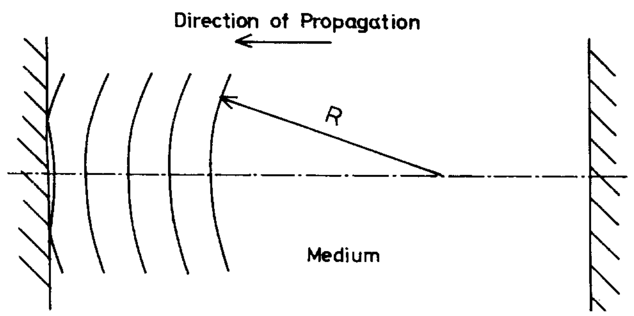

FIG. 1. Gaussian beam with the phase-front curvature R.

and

$$S(z,r,t)=\sin[\omega t-(k_0/2R_0)r^2]\sin(k_0 z),$$

the left-hand side of Eq. (4) becomes

$$E_0\left[\left(k_0^2-\frac{\omega^2}{c_0^2}+\frac{4}{w_0^2}\right)C(z,r,t)-\left(\frac{2k_0}{R_0}+\mu_0\sigma\omega\right)S(z,r,t)\right]$$
$$+r^2E_0\left\{\left[-\frac{8}{w_0^4}-\frac{1}{w_0^2}\left(k_0^2-\frac{w^2}{c_0^2}\right)+\frac{k_0^2}{R_0^2}\right]C(z,r,t)\right.$$
$$\left.+\frac{1}{w_0^2}\left(\frac{6k_0}{R_0}+\frac{\mu_0\sigma\omega}{w_0^2}\right)S(z,r,t)\right\}+\cdots. \qquad (5)$$

The electric field given by Eq. (3) is rewritten as

$$E(z,r,t)=\frac{E_0}{4i}\exp\left(-\frac{r^2}{w_0^2}\right)\left\{\exp\left[i\left(\omega t-\frac{k_0}{2R_0}r^2+k_0z\right)\right]\right.$$
$$-\exp\left[i\left(\omega t-\frac{k_0}{2R_0}r^2-k_0z\right)\right]$$
$$+\exp\left[-i\left(\omega t-\frac{k_0}{2R_0}r^2-k_0z\right)\right]$$
$$\left.-\exp\left[-i\left(\omega t-\frac{k_0}{2R_0}r^2+k_0z\right)\right]\right\}. \qquad (6)$$

In the frame fixed to an atom moving with velocities V_x, V_y, and V_z, the first term on the right-hand side of Eq. (6) is, by a proper Galilean transformation, equal to

$$\frac{E_0}{4i}\exp\left\{i\left[\omega t-[(x+V_xt)^2+(y+V_yt)^2]\right.\right.$$
$$\left.\left.\times\left(\frac{k_0}{2R_0}-\frac{i}{w_0^2}\right)+k_0(z+V_zt)\right]\right\}.$$

The frequency of the field impressed on an atom in this frame is approximated by $\omega+k_0V_z$, where the frequency component proportional to

$$[-2(xV_x+yV_y)+2(V_x^2+V_y^2)t]\left(\frac{k_0}{2R_0}-\frac{i}{w_0^2}\right)$$

is neglected (see Appendix A).

With this approximation for the Doppler-shifted frequency, the microscopic polarization P_v due to a group of two-level atoms moving with velocity V_z is obtained by the method of geometrical representation as

$$P_v=-\frac{2\mu_{ab}^2EN}{i\hbar}L^-(\mathrm{D})E\sum_{m=0}^{\infty}\left(-\frac{\mu_{ab}}{4\hbar}D(\mathrm{D})EL^+(\mathrm{D})E\right)^m, \qquad (7)$$

where μ_{ab} is the matrix element of the dipole moment. D is the Heaviside's operator $\partial/\partial t$:

$$D(\mathrm{D})=\frac{1}{2}\left(\frac{1}{D+\gamma_a}+\frac{1}{D+\gamma_b}\right),$$

and

$$L^{\pm}(D)=\frac{1}{2}\left(\frac{1}{\mathrm{D}+\Gamma_0-i\omega_0}\pm\frac{1}{\mathrm{D}+\Gamma_0+i\omega_0}\right).$$

γ_a and γ_b are the decay constants of the upper and lower levels, respectively, ω_0 is the resonant angular frequency of the atom, and $\Gamma_0=\frac{1}{2}(\gamma_a+\gamma_b)$.

Using the RWA method, the macroscopic polarization is

$$P(z,r,t)=\frac{N_0\mu_{ab}^2J_0(a_0r/b)}{\hbar u\sqrt{\pi}}$$
$$\times\int_{-\infty}^{\infty}\frac{-\Gamma_0S(z,r,t)+(\omega-\omega_0+k_0z)C(z,r,t)}{\Gamma_0^2+(\omega-\omega_0+k_0z)^2}$$
$$\times\left[1+\frac{\mu_{ab}^2\Gamma_0^2}{4\hbar^2\gamma_a\gamma_b}E_0^2\left(\frac{1}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}\right.\right.$$
$$\left.\left.+\frac{1}{\Gamma_0^2+(w-w_0-k_0V_z)^2}\right)\right]\exp\left(-\frac{V_z^2}{u^2}\right)dV_z, \qquad (8)$$

where the velocity profile of atoms is assumed to have the Maxwell-Boltzmann distribution with the mean squared velocity u^2. This polarization and the electric field [Eq. (3)] satisfy Eq. (4). Expanding E and P in power series to the quadratic of r to compare them term by term on both sides of the wave equation [Eq. (4)], we have the following four equations for four unknowns, ω_0, k_0, E_0, and R_0:

$$\frac{4}{w_0^2}+k_0^2-\frac{\omega^2}{c_0^2}$$
$$=\omega^2A\int_{-\infty}^{\infty}\frac{\omega-\omega_0+k_0V_z}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}\frac{\exp(-V_z^2/u^2)}{1+I_0L(\omega,V_z)}dV_z, \qquad (9)$$

$$\frac{2k_0}{R_0}+\mu_0\sigma\omega$$
$$=w^2A\int_{-\infty}^{\infty}\frac{\Gamma_0}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}\frac{\exp(-V_z^2/u^2)}{1+I_0L(\omega,V_z)}dV_z, \qquad (10)$$

$$\frac{8}{w_0^4}+\frac{1}{w_0^2}\left(k_0^2-\frac{\omega^2}{c_0^2}\right)-\frac{k_0^2}{R_0^2}$$
$$=\omega^2A\left(\frac{a_0^2}{4b^2}+\frac{1}{w_0^2}\right)$$
$$\times\int_{-\infty}^{\infty}\frac{\omega-\omega_0+k_0V_z}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}\frac{\exp(-V_z^2/u^2)}{1+I_0L(\omega,V_z)}dV_z$$
$$-\frac{2}{w_0^2}\omega^2A\int_{-\infty}^{\infty}\frac{\omega-\omega_0+k_0V_z}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}$$
$$\times\frac{I_0L(\omega,V_z)\exp(-V_z^2/u^2)}{[1+I_0L(\omega,V_z)]^2}dV_z-\frac{2}{w_0^2}\omega^2A \qquad (11)$$

and

$$\frac{1}{w_0^2}\left(\frac{6k_0}{R_0}+\mu_0\sigma\omega\right)$$
$$=\omega^2A\left(\frac{a_0^2}{4b^2}+\frac{1}{w_0^2}\right)$$
$$\times\int_{-\infty}^{\infty}\frac{\Gamma_0}{\Gamma_0^2+(\omega-\omega_0+k_0V_z)^2}\frac{\exp(-V_z^2/u^2)}{1+I_0L(\omega,V_z)}dV-\frac{2}{w_2^0}\omega^2A$$

$$\times \int_{-\infty}^{\infty} \frac{\Gamma_0}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{I_0 L(\omega, V_z) \exp(-V_z^2/u^2)}{[1 + I_0 L(\omega, V_z)]^2} \, dV_z, \tag{12}$$

where the following substitutions are made:

$$A = \mu_0 N_0 \mu_{ab}^2 / \hbar u \sqrt{\pi},$$

$$I_0 = \mu_{ab}^2 E_0^2 / 4\hbar \gamma_a \gamma_b,$$

and

$$L(\omega, V) = \frac{\Gamma_0^2}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V)^2} + \frac{\Gamma_0^2}{\Gamma_0^2 + (\omega - \omega_0 - k_0 V)^2}.$$

It seems to be almost impossible to find analytical solutions of the above integral equations [Eqs. (9)–(12)]. Thus we resort to numerical calculations.

III. SPHERICAL MIRROR

To obtain an expression for the electromagnetic field inside a cavity with spherical mirrors, we make two assumptions. The first assumption is that each feedback mirror has the same radius of curvature R_m. Therefore the field is symmetric around the center of the cavity. The second assumption is that R_m is much larger than the length L of the cavity. In these situations the variations of w, k, E, and R along the longitudinal direction are so small that they may be expanded in a power series of z. From the first postulate, $w(z)$, $k(z)$, and $E(z)$ are even functions of z, while $1/R(z)$ has to be odd. Neglecting the terms higher than quadratic, we have

$$w(z) = w_0 + w_2 z^2,$$

$$k(z) = k_0 + k_2 z^2,$$

$$E(z) = E_0 + E_2 z^2,$$

and

$$1/R(z) = (1/R_1) z. \tag{13}$$

By including an offset curvature of the phase front, $1/R_0$ (see Appendix B), the field is assumed to be given by the lowest mode of the resonator as shown by Kogelnik and Li:[10]

$$E(z, r, t) = E(z) \exp\left(-\frac{r^2}{w^2(z)}\right) \cos\left(\omega t - \frac{k(z)}{2R_0} r^2\right) \times \sin\left[k(z)\left(z + \frac{r^2}{2R(z)}\right)\right]. \tag{14}$$

Comparing the zeroth power in z on both sides of the wave equation, one finds the following relation to Eqs. (9)–(12):

$$\frac{4E_0}{w_0^2} + \left(k_0 - \frac{\omega^2}{c_0^2}\right) E_0 - 2E_2 = E_0 \, \omega^2 A \int_{-\infty}^{\infty} \frac{\omega - \omega_0 + k_0 V_z}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{\exp(-V_z^2/u^2)}{1 + I_0 L(\omega, V_z)} \, dV_z \ , \tag{15}$$

$$\left(\frac{2k_0}{R_0} + \mu_0 \sigma \omega\right) E_0 = E_0 = w^2 A \int_{-\infty}^{\infty} \frac{\Gamma_0}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{\exp(-V_z^2/u^2)}{1 + I_0 L(\omega, V_z)} \, dV_z \ , \tag{16}$$

$$\begin{aligned}
&\left[\frac{8}{w_0^4} + \frac{1}{w_0^2}\left(k_0^2 - \frac{w^2}{c_0^2}\right) - \frac{k_0^2}{R_0^2}\right] E_0 + 2\left(\frac{2w_2 E_0}{w_0^3} - \frac{E_2}{w_0^2}\right) - \frac{E_0 k_0^2}{R_1} \\
&= E_0 \left[\omega^2 A \left(\frac{a_0^2}{4b^2} + \frac{1}{w_0^2}\right) \int_{-\infty}^{\infty} \frac{\omega - \omega_0 + k_0 V_z}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{\exp(-V_z^2/u^2)}{1 + I_0 L(\omega, V_z)} \, dV_z \right. \\
&\left. - \frac{2}{w_0^2} \omega^2 A \int_{-\infty}^{\infty} \frac{\omega - \omega_0 + k_0 V_z}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{I_0 L(\omega, V_z) \exp(-V_z^2/u^2)}{[1 + I_0 L(\omega, V_z)]^2} \, dV_z \right],
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
&\frac{E_0}{w_0^2}\left(\frac{6k_0}{R_0} + \mu_0 \sigma \omega\right) - \frac{E_0 k_2}{R_0} \\
&= E_0 \left[\omega^2 A \left(\frac{a_0^2}{4b^2} + \frac{1}{w_0^2}\right) \int_{-\infty}^{\infty} \frac{\Gamma_0}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{\exp(-V_z^2/u^2)}{1 + I_0 L(\omega, V_z)} \, dV_z \right. \\
&\left. - \frac{2}{w_0^2} \omega^2 A \int_{-\infty}^{\infty} \frac{\Gamma_0}{\Gamma_0^2 + (\omega - \omega_0 + k_0 V_z)^2} \frac{I_0 L(\omega, V_z) \exp(-V_z^2/u^2)}{[1 + I_0 L(\omega, V_z)]^2} \, dV_z \right].
\end{aligned} \tag{18}$$

From the lowest power in z, one obtains

$$\frac{E_0 k_0}{R_1} + 3E_0 k_2 + 2E_2 k_0 = 0, \tag{19}$$

$$\frac{6E_0 k_0}{w_0^2 R_1} + \frac{6E_0 k_2}{w_0^2} - \frac{3E_0 k_2}{R_1} - \frac{2E_2 k_0}{R_1} + 4\left(\frac{E_2}{w_0^2} - \frac{2w_2}{w_0^3} E_0\right) k_0 = 0, \tag{20}$$

and

$$\frac{k_0}{R_1} + k_2 = 0. \tag{21}$$

Besides Eqs. (15)–(21), we have a boundary condition on the spherical mirrors:

$$R_1 = \tfrac{1}{2} L R_m. \tag{22}$$

We have to solve Eqs. (16)–(22) simultaneously.

IV. NUMERICAL COMPUTATIONS

The integrals appearing in Eqs. (9)–(12) are somewhat difficult to evaluate. Thus we use a computer to find the solutions.

The calculated laser intensity for the 3.51-μm Xe transitions[11] is shown in Fig. 2 for various pumping densities $\bar{n}$ normalized to the cavity with flat mirrors. The solid line indicates the values computed for the flat mirror with $LR_m = \infty$ and the broken line is for the spherical mirror with $LR_m = 2.5 \times 10^4$ cm^2.

In Fig. 3 the radius of curvature of the phase front at the center of the cavity is plotted. The parameters are $\gamma_a/\Gamma_0 = 6.4 \times 10^{-2}$, $\gamma_b/\Gamma_0 = 1.94$, $ku/\Gamma_0 = 37.0$, Q

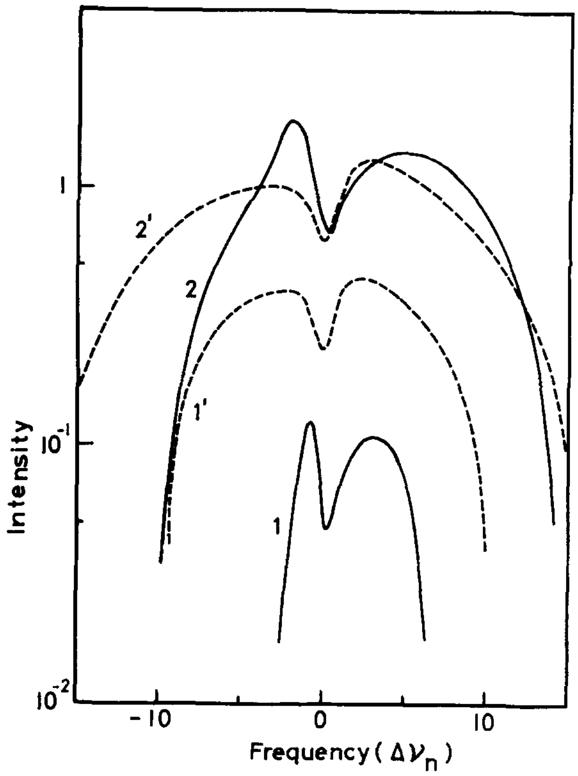

FIG. 2. Output spectrum. The solid line and the broken line show the calculated intensity $\mu_{ab}^2 E_0^2/4\hbar^2\gamma_a\gamma_b$ as a function of frequency for the flat and spherical mirror, respectively. The abscissa is in the units of natural line width $\Delta\nu_n$ (3.73 MHz) of the transition. Curves 1 and 1′ indicate the values for the pumping density 5% above threshold. Curves 2 and 2′ indicate that for 50%.

($=\epsilon_0\omega/\sigma$) $= 10^8$, and the inner diameter of the laser tube is 0.5 cm.

The calculated frequency shift of the Lamb dip in the unit of the natural line width $\Delta\nu_n$ (3.73 MHz) vs normalized pumping density $\bar{n}$ is given in Fig. 4. The solid line and the broken line give the values for the flat and the spherical mirrors, respectively.

V. CONCLUSION

With a model of a Gaussian radial profile of the laser field a self-consistent calculation has been carried out for both flat and spherical feedback mirrors. The output shows an asymmetrical Lamb dip in contrast to the symmetrical profile for the plane-wave model. Dispersion of the nonlinear refractive index of the medium with population inversion (see Fig. 5) is responsible for the distortion. The self-focusing effect, the transverse distribution of the laser field, and also the geometry of the resonator are found to significantly influence the shape of the Lamb dip. The distorted shape brings about a frequency shift of the apparent center of the laser line. Therefore, when the Lamb dip is used as a frequency standard these factors should be taken into consideration.[12]

ACKNOWLEDGMENT

The present work was started during the stay of one of the authors (H. M.) at the California Institute of

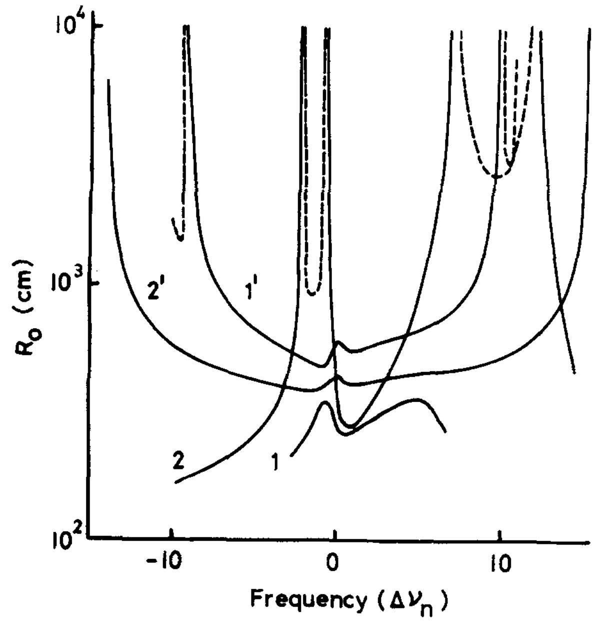

FIG. 3. Radius of curvature of the phase front. Curves 1 and 2 are the plots for flat mirror, while curves 1′ and 2′ are for spherical mirror systems. Curves 1 and 1′ indicate R_0 (in units of cm) for $\bar{n}=1.05$. Curves 2 and 2′ are for $\bar{n}=1.50$. Solid and broken lines give positive and negative values of R_0, respectively.

Technology. He is thankful to Professor Amnon Yariv. The assistance of Dr. T. Ohnishi in the computations is also acknowledged.

APPENDIX A: CONTRIBUTIONS OF THE TRANSVERSE VELOCITY COMPONENTS

We may assume that the inner diameter of the tube is so much smaller than R_0 that the transverse com-

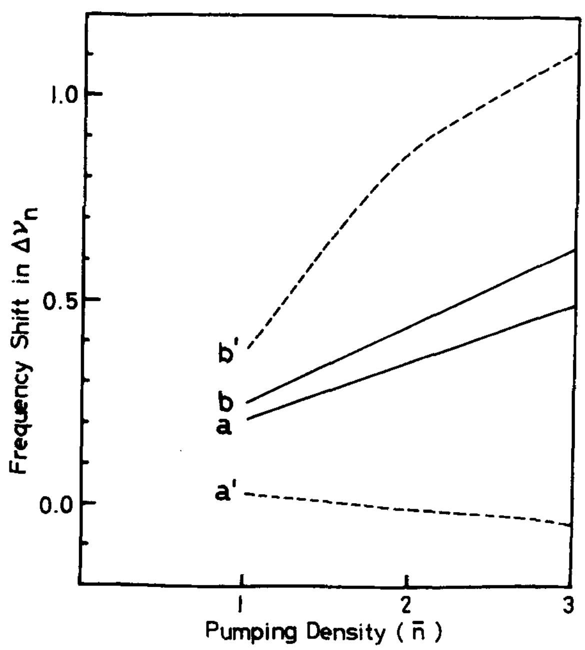

FIG. 4. Frequency shift of the Lamb dip. Curves a and a' give the shift for the 0.50-cm-i.d. tube. Curves b and b' show that for the 0.25-cm-i.d. tube. The solid line corresponds to $LR_m=\infty$. The broken line corresponds to $LR_m = 2.5\times10^4$ cm^2. The unit of the vertical scale is $\Delta\nu_n = 3.73$ MHz.

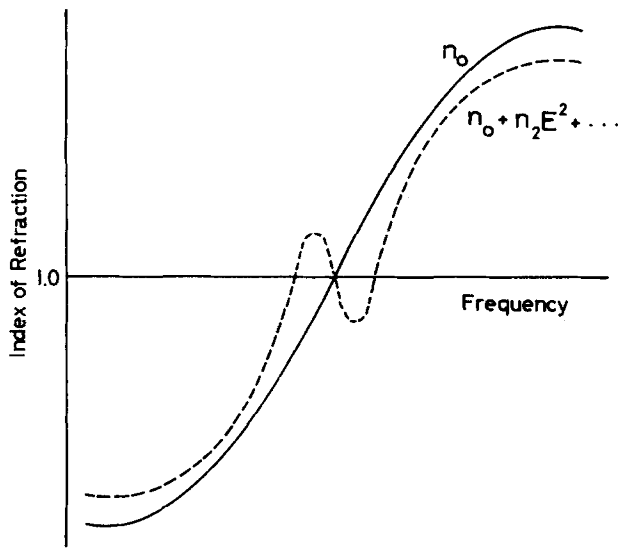

FIG. 5. Dispersion around the frequency of transition for a gaseous medium with population inversion. The solid line shows the index of refraction under a weak field. The broken line shows the nonlinear effect under a strong laser field E.

ponents of the motion do not cause an atom to see any phase shift of the field due to the finite curvature or the gradient of the amplitude of the field. Thus, we may neglect the first term,

$$-2(xV_x+yV_y)\left(\frac{k_0}{2R_0}-\frac{i}{w_0^2}\right).$$

If we bring V_x and V_y into our numerical calculations the contribution of these transverse velocity components to the macroscopic polarization is less than one part in 10^3, and it is negligibly small. However, this term may become important when the laser tube has a very small inner diameter.

For the second term, the time interval Δt between excitation and deexcitation of an atom is the order of $1/\Gamma_0$. We may assume that the component of the Doppler shift satisfies the condition

$$|kV_z| \gg 2(V_x^2+V_y^2)\Delta t\left|\frac{k_0}{2R_0}-\frac{i}{w_0^2}\right|$$

for most atoms.

APPENDIX B: GAUSSIAN BEAM IN A RESONATOR WITH SPHERICAL MIRRORS

In free space the complex Gaussian beam is given by

$$\frac{w_0}{w}\exp\left[-i(kz-\Phi)-r^2\left(\frac{1}{w^2}+\frac{ik}{2R}\right)\right].$$

The evolutions of the beam parameters are

$$w^2=w_0^2\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right],$$

$$R=z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right],$$

and

$$\Phi=\tan^{-1}\left(\frac{\lambda z}{\pi w_0^2}\right),$$

where λ is the wavelength and w_0 is the spot size at the beam waist. For small z we may expand the above parameters as

$$w=w_0\left[1+\frac{1}{2}\left(\frac{\lambda z}{\pi w_0^2}\right)^2+\cdots\right],$$

$$1/R=(\lambda/\pi w_0^2)^2 z+\cdots,$$

and

$$\Phi=\lambda z/\pi w_0^2+\cdots.$$

Since $|\Phi| \ll |kz|$ we may neglect Φ. Similar to these, we have the expressions for $w(z)$ and $1/R(z)$ given in Eq. (13).

As found in Sec. II, there is a constant offset component of the curvature of the phase front in the medium of a gas laser where the inversion density varies with the distance from the axis of the laser tube. Thus, the complex Gaussian beam propagating in the positive z direction is proportional to

$$\exp\left[i\omega t-ik(z)z-r^2\left(\frac{1}{w^2(z)}+\frac{ik(z)}{2R(z)}\right)-\frac{ik(z)}{2R_0}r^2\right].$$

The beam symmetric to this may be given by

$$\exp\left[i\omega t+ik(z)-r^2\left(\frac{1}{w^2(z)}-\frac{ik(z)}{2R(z)}\right)-\frac{ik(z)}{2R_0}r^2\right].$$

Expression (14) will be found from the linear combination of the above two beams and the complex conjugates of them.

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