並行分散計算特論 (4)

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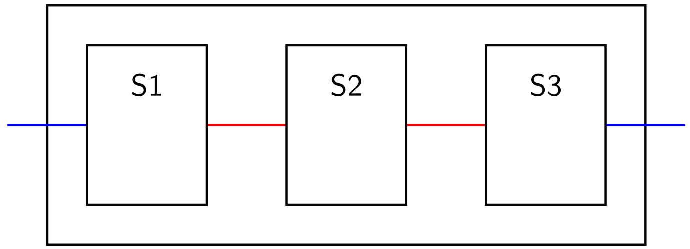
Concurrent Processes and Reaction

A System = Composition of subsystems



——— : Internal communication ——— : External communication

$$S = compose(S1, S2, S3)$$

Compositionarity

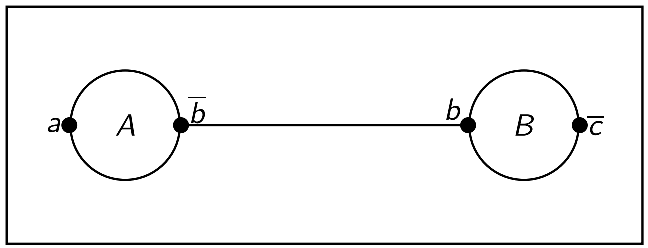
All properties are determined by those of subsystems

Function Composition
$h = g \circ f$ i.e. $h(x) = g(f(x))$
Property = composition of I/O relations.
$h(x)$ is completely determined by f and h.

Behavioral Composition
When P behaves as Q, for all network environment $C[]$, $C[P]$ behaves as $C[Q]$.

When $P = Q$, for all R, $P|R = Q|R$?

Flow Graphs



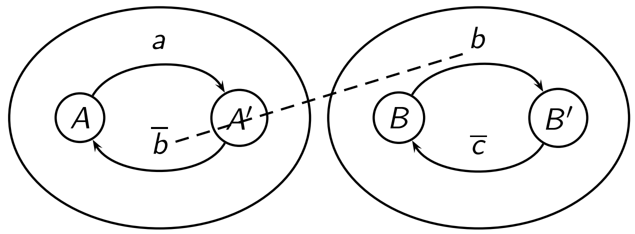
$a,b,\overline{b},\overline{c}$: Ports
$(b, \overline{b})$: complementary ports
Flowgraph = Network Configuration

Observation and Reactions

Flowgraph : Configuration of automaton network
Transition graph: State diagram of an automaton

$A \stackrel{\text{def}}{=} a.A'$ $\qquad$ $B \stackrel{\text{def}}{=} b.B'$

$A' \stackrel{\text{def}}{=} \overline{b}.A'$ $\qquad$ $B' \stackrel{\text{def}}{=} \overline{c}.B$



Concurrent Process Expressions

$$P ::= A\langle a_1, \ldots, a_n\rangle \mid \sum_{i\in I} \alpha_i.P_i \mid P_1|P_2 \mid \text{new } a\ P$$

Prefix operators are stronger than '|'
new a P binds free a in P

$$\text{new } a\ (a.b.0)\{c/a\}\{c/b\} \equiv \text{new } a\ (a.c.0)$$

$P|Q$ Communication composition of P and Q

Transition by Reaction

$$P \rightarrow P'$$

P becomes P' by a reaction in P

$$P = A'|B, P' = A|B'$$

new a P prevents P from synchronization by a with other agents.
(ports named a in P are not bound)
(See: Example 4.3)

Process Congruence

Definition
(Process context)

$$C ::= [\,] \mid \alpha.C + M \mid \text{new } a\ C \mid C|P \mid P|C$$

Definition
(Process congruence)
$\cong$ is an equivalence and satisfies followings. When $P \cong Q$,

$$\begin{array}{rcl} \alpha.P + M & \cong & \alpha.Q + M \\ \text{new } a\ P & \cong & \text{new } a\ Q \\ P|R & \cong & Q|R \\ R|P & \cong & R|Q \end{array}$$

Process Congruence

Proposition

$\cong$ is a process congruence iff
for all contexts C, $P \cong Q$ implies $C[P] \cong C[Q]$

Structural Congruence

Definition

1. *Alpha-conversion (for new operator)*
2. *A-C property (for choice)*
3. *0 is a unit of $|$ and $|$ is AC*
4. *Laws for new operator*
5. *Process definition*

Definition

(Standard form)

$$\text{new } \vec{a}\ (M_1|\cdots|M_n)$$

Theorem

Every process is structually congruent to a standard form

Reaction rules

$$\tau.P + M \to P$$

$$(a.P + M)|(\overline{a}.Q + N) \to P|Q$$

$$\frac{P \to P'}{P|Q \to P'|Q} \qquad \frac{P \to P'}{\text{new } a\ P \to \text{new } a\ Q}$$

$$\frac{P \to P'}{Q \to Q'} \quad \text{where} \quad P \equiv Q \quad \textit{and} \quad P' \equiv Q$$

Transition by inferences

$$\dfrac{\dfrac{\dfrac{}{\overline{b}.\overline{c}|b.0 \to \overline{c}.B|0}}{b.A|\overline{b}.\overline{c}.B|b.0 \to b.A|\overline{c}.B|0}}{b.A|\overline{b}.\overline{c}.B|b.0 \to b.A|\overline{c}.B|0}$$

new prevents unintentional communications
Together with the new rule for $\equiv$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{b.A|\overline{b}.\overline{c}.B \to A|\overline{c}.B}}{\text{new } b\ (b.A|\overline{b}.\overline{c}.B) \to \text{new } b\ (A|\overline{c}.B)}}{\text{new } b(b.A|\overline{b}.\overline{c}.B)|b.0 \to \text{new } b(A|\overline{c}.B)|b.0}}{\text{new } b(b.A|B)|B' \to \text{new } b(A|\overline{c}.B)|B'}$$

Concurrency

Concurrency is modelled in $P|Q$

$$P|Q \to P'|Q \qquad Q|P \to Q|P'$$

when $P \to P'$

Asynchrony between P and Q

Concurrency = interleaving + rendesvouz

Synchronization when P communicates with Q

Example:Lottery

Specifiction

$Lotspec \stackrel{def}{=} \tau.b_1.Lotspec + \cdots + \tau.b_n.Lotspec$

N agents to rotate a token to be selected

$$A(a,b,c) \stackrel{def}{=} \bar{a}.C(a,b,c)$$
$$B(a,b,c) \stackrel{def}{=} b.C$$
$$C(a,b,c) \stackrel{def}{=} \tau.B(a,b,c) + c.A(a,b,c)$$

$$\text{new } \vec{a}(C_1|A_2|\cdots|A_n)$$

Example:Lottery

if n=3

$L_1 \quad = \text{new } a_1a_2a_3(C_1|A_2|A_3)$
$L_2 \quad = \text{new } a_1a_2a_3(A_1|C_2|A_3)$
$L_3 \quad = \text{new } a_1a_2a_3(A_1|A_2|C_3)$