並行分散計算特論 (7)

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Properties of transitions

Proposition

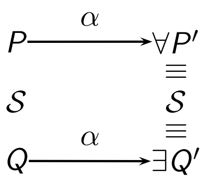
(1) *Finite branching*
Given P, there are only finitely many transtions from P.

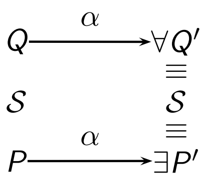
(2) *No free name produced by transitions*
If $P \stackrel{\alpha}{\to} P'$, then $\mathrm{fn}(P', \alpha) \subseteq \mathrm{fn}(P)$

(3) *substitution stability*
If $p \stackrel{\alpha}{\to} P'$ and σ is any substitution over names, then $\sigma P \stackrel{\sigma\alpha}{\to} \sigma P'$.

Strong Bisimularity for CPE

Expression is usually too fine
Bisimulation up to $\equiv$





up-to bisimulation is also a bisimulation

up-to Bisimulation

Proposition

If $\mathcal{S}$ is a strong bisimulation up to $\equiv$ and $P\mathcal{S}Q$, then $P \sim Q$.

Proof: Show $\equiv \circ \mathcal{S} \circ \equiv$ is a strong bisimulation. Clearly, $P \equiv \circ \mathcal{S} \circ \equiv Q$ since $P\mathcal{S}Q$. □

Syntactic differences wrt $\equiv$ can be ignored in bisimulation equivalence.

Algebraic Properties

Proposition

$\forall P \in \mathcal{P}. P \sim \sum\{\beta.Q \mid P \xrightarrow{\beta} Q\}$

Proposition

$P_i \in \mathcal{P}$ *for* $i \leq n$.

$P_1|\cdots|P_n \sim$

$$\begin{cases} \sum\{\alpha.(P_1\cdots|P_i'|\cdots|P_n)\ ;\ 1 \leq i \leq n, P_i \xrightarrow{\alpha} P_i')\} \\ +\sum\{\tau.(P_1\cdots|P_i'|\cdots|P_j'|\cdots|P_n)\ ;\ 1 \leq i < j \leq n, P_i \xrightarrow{\lambda} P_i', P_j \xrightarrow{\overline{\lambda}} P_j'\} \end{cases}$$

Algebraic Properties

Expansion Theorem

Proposition

$P_i \in \mathcal{P}$ *for* $i \leq n$.

new $\vec{a}(P_1|\cdots|P_n) \sim$

$$\begin{cases} \sum\{\alpha.\text{new } \vec{a}(P_1\cdots|P_i'|\cdots|P_n) \\ \quad ;\ 1 \leq i \leq n, P_i \xrightarrow{\alpha} P_i')\text{ and } \alpha, \overline{\alpha} \notin \vec{a}\} \\ +\sum\{\tau.\text{new } \vec{a}(P_1\cdots|P_i'|\cdots|P_j'|\cdots|P_n) \\ \quad ;\ 1 \leq i < j \leq n, P_i \xrightarrow{\lambda} P_i', P_j \xrightarrow{\overline{\lambda}} P_j'\} \end{cases}$$

There exists a (finite) sequential process for a (finite) process.

; as a derived operator

$P; Q \stackrel{\text{def}}{=}$ new $start(P\{start/done\}|start.Q)$

where P emits $\overline{done}$ when finished.

$(P;Q);R \sim P;(Q;R)$ but $(P;Q);R \not\equiv P;(Q;R)$

Congruence property

Proposition

$\sim$ *is a process congruence. Namely, if* $P \sim Q$,

$$\begin{array}{rcl} \alpha.P + R & \sim & \alpha.Q + R \\ \text{new } a.P & \sim & \text{new } a.Q \\ P|R & \sim & Q|R \\ R|P & \sim & R|P \end{array}$$

By proposition 4.6, for all context $C[]$,

$$P \sim Q \text{ implies } C[P] \sim C[Q]$$