並行分散計算特論 (8)

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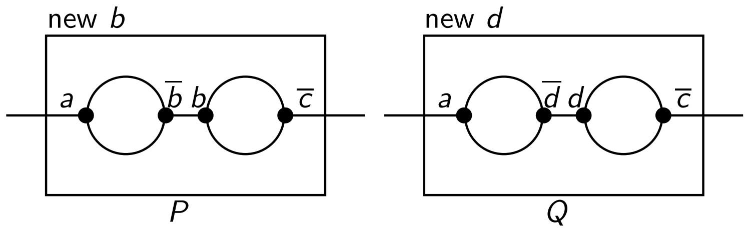
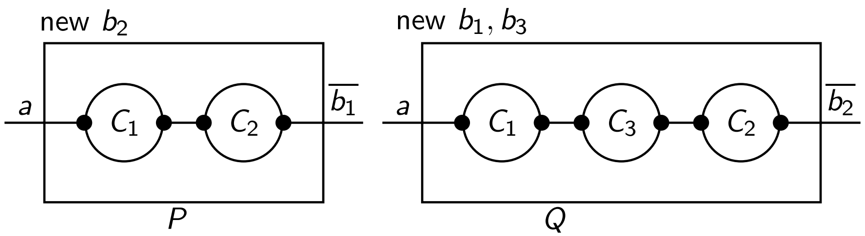
Observation Equivalence



$P \sim Q$

Internal Structure Difference

$C_i \stackrel{\text{def}}{=} a_i.\overline{b_i}.C_i$



Appropriate substitution on b_i's to make the links internal

Both processes are equivalently observed.
Weak semantics.

$P \not\sim Q$

Observable Transitions

$\stackrel{\lambda}{\rightarrow}$ $\lambda \in L$ Observable transitions
$\stackrel{\tau}{\rightarrow}$ Unobservable transitons

$$P \stackrel{a_1}{\rightarrow} \stackrel{\tau}{\rightarrow} \stackrel{c_2}{\rightarrow} P'$$
$$Q \stackrel{a_1}{\rightarrow} \stackrel{\tau}{\rightarrow} \stackrel{\tau}{\rightarrow} \stackrel{c_2}{\rightarrow} Q'$$

P behaves equivalently to Q.

Observations

Definition

(1) $P \Rightarrow Q$ *iff* $P \xrightarrow{\tau}{}^{*} Q$

(2) $s \in Act^{*}. P \stackrel{s}{\Rightarrow} Q$ *iff* $P \Rightarrow \xrightarrow{\alpha_1} \Rightarrow \cdots \xrightarrow{\alpha_n} \Rightarrow Q$

$\xrightarrow{a}$ is observed by $\xrightarrow{\overline{a}}$.
$\xrightarrow{\tau}$ cannot be observed.

Weak bisimulation

Definition
(Weak simulation)

$$(P, Q) \in \mathcal{S} \text{ implies } \begin{array}{ccc} P & \stackrel{e}{\Rightarrow} & \forall P' \\ \mathcal{S} & & \mathcal{S} \\ Q & \stackrel{e}{\Rightarrow} & \exists Q' \end{array}$$

Definition
$\mathcal{S}$ is a weak bisimulation if both $\mathcal{S}$ and $\mathcal{S}^{-1}$ are weak simulations.
$P \approx Q$ if there exists a weak bisimulation containing (P, Q).

Properties

Proposition
$\sim \subsetneq \approx$

Stongly bisimular Processes are weakly bisimular. Not vice versa.
$P \not\sim \tau.P$ but $P \approx \tau.P$

Proposition

1. *$\approx$ is an equivalence.*
2. *$\approx$ is a weak bisimulation. Moreover, it is the largest weak bisimulation.*

An alternative characterization

Problem: $P \stackrel{e}{\Rightarrow} P'$ is an infinite assumption.
You may have to check infinite P' for P

Proposition
$\mathcal{S}$ is a weak simulation iff $(P, Q) \in \mathcal{S}$ implies:

(1) *If* $P \xrightarrow{\tau} P'$ *then* $\exists Q'. Q \Rightarrow Q'$ *and* $P' \mathcal{S} Q$

(2) *If* $P \xrightarrow{\lambda} P'$ *then* $\exists Q'. Q \stackrel{\lambda}{\Rightarrow} Q'$ *and* $P' \mathcal{S} Q$

Proof sketch of the proposition

(⇒): Obvious
(⇐):If $P \Rightarrow P'$, by repeated application of (1). (Formally, induction on $\xrightarrow{\tau}$)
If $P \overset{\lambda_1 \cdots \lambda_n}{\Rightarrow} P'$
When $n = 1$: repeated application of (1) and (2) followed by repeated application of (1), Q' s.t. $P' \mathcal{S} Q'$ exists.
When $n = k + 1$: by the induction hypothesis followed by repeated application of (1) and (2) and again repeated application of (1).

To see if $P \approx Q$, find a relation $\mathcal{R}$ such that $(P, Q) \in \mathcal{R}$ and both $\mathcal{R}$ and $\mathcal{R}^{-1}$ satisfy the conditions (1) and (2) of the proposition.

Weak bisimulation up to ∼

Definition
$\mathcal{S}$ is a weak simulation up to ∼, whenever $(P, Q) \in \mathcal{S}$,

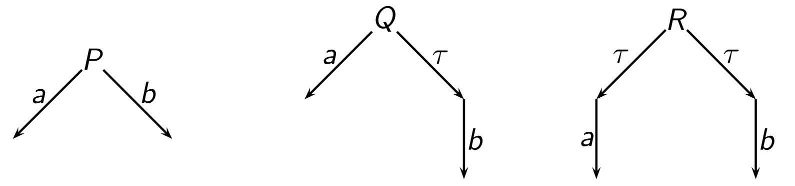
(1) $P \xrightarrow{\tau} P'$ *implies* $Q \Rightarrow Q'$ *for some* Q' *such that* $Q' \sim \mathcal{S} \sim Q'$

(2) $P \xrightarrow{\lambda} P'$ *implies* $Q \overset{\lambda}{\Rightarrow} Q'$ *for some* Q' *such that* $Q' \sim \mathcal{S} \sim Q'$

If both $\mathcal{S}$ and $\mathcal{S}^{-1}$ are weak simulation up to ∼, $\mathcal{S}$ is a weak bisimulation up to ∼.

Proposition
If $\mathcal{S}$ is a wak bisimulation up to ∼, then $\mathcal{S} \subseteq \approx$

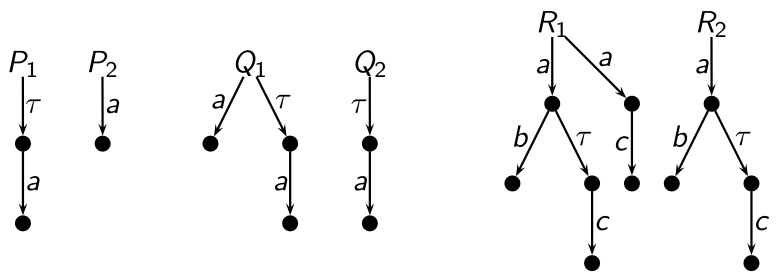
Examples



$P = a.0 + b.0$ $Q = a.0 + \tau.b.0$ $R = \tau.a.0 + \tau.b.0$

$P \neq Q$, $Q \neq R$, $P \neq R$

Examples2



$a \approx \tau.a$ $a + \tau.a \approx \tau.a$ $a.(b + \tau.c) + a.c \approx a.(b + \tau.c)$

τ laws

Theorem

(1) $P \approx \tau.P$

(2) $M + N + \tau.N \approx M + \tau.N$

(3) $M + \alpha.P + \alpha.(\tau.P + N) \approx M + \alpha.(\tau.P + N)$

Weak Process Concruence

Proposition

If $P \approx Q$, then

(1) $\alpha.P + M \approx \alpha.Q + M$

(2) new a $P \approx$ new a Q

(3) $P|R \approx Q|R$

(3) $R|P \approx R|Q$

Unique solutions of Equations

Well-definedness of process identifiers:

$$\vec{X} \stackrel{\text{def}}{=} P(\vec{X})$$

How equation(s) is characterized by $\approx$

$$X \stackrel{\text{def}}{=} \tau.X \qquad X \stackrel{\text{def}}{=} a.P + \tau.X$$

Guarded Equation

Theorem

$$\begin{array}{lcl} X_1 & \approx & \alpha_{11}.X_{k(11)} + \cdots + \alpha_{1n_1}.X_{k(1n_1)} \\ X_2 & \approx & \alpha_{21}.X_{k(11)} + \cdots + \alpha_{2n_2}.X_{k(2n_2)} \\ \cdots & & \cdots \\ X_m & \approx & \alpha_{m1}.X_{k(m1)} + \cdots + \alpha_{mn_2}.X_{k(mn_m)} \end{array}$$

where $\alpha_{ij} \neq \tau$. Then, up to $\approx$, there is a unique sequnce $P_1, P_2, \cdots, P_m$ of processes which satisfies the equations.