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主 論 文 の 要 旨

論文題目 Optimal Trajectory Generation via Double Generating Functions and Application to Biped Robots (二重母関数による最適軌道生成と二足歩行ロボットへの応用)

氏 名 HAO Zhiwei

論 文 内 容 の 要 旨

When a robot is walking in a complex environment, it needs to adjust its step length and walking speed for each step (very probably). This implies that the initial position and/or velocity, the designed terminal position and/or velocity, and the walking time period for each step are very often different. From this view point, the optimal gait generation problem for a biped robot is equivalent to a family of optimal control problems parameterized by the boundary conditions. The finite time optimal control problem with fixed initial and terminal state values can be deduced to a two-point boundary-value problem (TPBVP) for ordinary differential equations (ODEs) with respect to a Hamiltonian system. The shooting method is a conventional method for TPBVP. Since the basic principle of the shooting method is computing the trajectory repeatedly so that the exact one satisfying the boundary values is obtained. It needs to solve the TPBVP (or run the program) again if we change the desired boundary values. Also, the model predictive control (MPC) can be applied to solve finite time optimal control problems. However, since MPC is based on iterative, finite horizon optimization of a dynamic model, the iterative optimization increases the online computational burden. Recently, a method based on single generating function is proposed to solve the optimal control problems with general types of boundary conditions. This method allows one to obtain a family of optimal trajectories for different boundary conditions by integrating the system dynamic equation with a family of optimal control inputs.

In this thesis, for a finite time optimal control problem with fixed boundary condition, the optimal trajectories are given as functions of the boundary condition by using double generating functions. The expressions of the optimal trajectories are given in the form of the Taylor series with respect to the boundary values of the state (the initial value and the terminal value of the state). The coefficients of the Taylor series are time-dependent which can be calculated off-line and are same for any boundary condition. Therefore, the on-demand computation time for different boundary conditions decreases greatly compared with the conventional method, e.g., the shooting method. Since the finite time optimal control problem can be reduced to a TPBVP for a Hamiltonian system, the solution of the optimal control problem is derived from double generating functions by canonical transformations for the Hamiltonian systems. The generating functions are solutions of Hamilton-Jacobi equations (HJEs). This method is called the double generating functions method.

Chapter 1 introduces the history of the optimal control and some typical methods to solve optimal control problems. This thesis considers the finite time optimal control problems with fixed boundary condition and hence the solution of the corresponding HJE is time-dependent. This is different from the infinite time optimal control problems. The generating functions which are well used in classical mechanics are employed to solve the problems considered in this thesis.

Chapter 2 reviews how to render an optimal control problem to a Hamiltonian system, then introduces the canonical transformation and generating functions to generate state transfer for Hamiltonian systems.

For Linear-Quadratic (LQ) optimal control problems, double generating functions generate a parametrization of optimal trajectories for different boundary conditions and different time periods in Chapter 3. It is very convenient to generate optimal trajectories for different boundary conditions. Given a finite time LQ optimal control problem with fixed boundary condition, it is rendered to a TPBVP corresponding to a Hamiltonian system. By adopting generating functions, double ones give the optimal state and input trajectories as functions of the boundary condition. Hence we do not need to integrate the system equation numerically for optimal trajectories required in the conventional methods. Although the method using the double generating functions increases the dimension of the original problem, the generating functions are effective for any boundary conditions and any time intervals for a given system, since they do not depend on fixed boundary values. In addition, they can be calculated off-line. Therefore, the proposed method reduces the online computational effort for different boundary conditions and different time intervals compared with the conventional method. A single generating function also can give the the optimal state and input trajectories directly as double generating functions can. However, they may cause numerical instability. Since there are two types of generating functions well defined for solving optimal control problems and each type of generating function has time forward and time backward forms, there exists many sets of double generating functions. For pairs of generating functions with the same time direction, it is proved that they might causes numerical instability. One pair of double generating functions is given, which is most convenient to be used for optimal trajectory generation for LQ optimal control problems.

Numerical examples illustrate the effectiveness of the proposed method and verify its numerical stability.

Most of nonlinear optimal control problems is reduced to solving a Hamilton-Jacobi equation (HJE) for a value function, which is a nonlinear partial differential equation. In general, it is very difficult to solve a HJE for an exact solution. There exist many methods to solve it for a numerical solution, e.g., the Galerkin's spectral method. These methods for value function use iterative algorithms. We can use the shooting method to solve finite time optimal control problems with fixed boundary condition. However, we need to solve the problem separately for each set of boundary condition. All of these methods cause that on-demand computation effort for the different boundary condition will become heavier as the number of boundary conditions increases. The double generating functions method is proposed for LQ optimal control problems in Chapter 3, which give a parametrization of optimal trajectories for different boundary conditions. The generating function is quadratic with respect to its variables for LQ optimal problems. However, the generating function is more complicated in the nonlinear case and it is difficult to solve HJE for generating functions.

Chapter 4 propose a systematic algorithm to obtain an approximate solution to HJE for generating functions. An approximate generating function is given in the Taylor series form with respect to its variables. In the proposed algorithm, the coefficients of the Taylor series of the generating function are calculated numerically by solving a sequence of first order ODEs recursively up to any prescribed order. Particularly, the exact coefficients of the Taylor expansion of the generating function are obtained under a certain technical condition. The single generating function method for optimal trajectory generation is employed to show the effectiveness of the obtained generating function.

The double generating functions method is extended to the nonlinear optimal control problems in Chapter 5. Firstly, it is proved that the optimal trajectory can be given as a Taylor series with respect to the boundary value of the state and the coefficients of the Taylor series are time-dependent. Secondly, an algorithm with appropriate data structure is developed to calculate the Taylor series of the optimal trajectories based on the Kronecker production. This algorithm furnishes a systematic procedure to compute the coefficients of the Taylor series solution recursively up to any order off-line. The optimal state and input trajectories are given in the form of the Taylor series with respect to the initial value and the terminal value of the state. Therefore, it is easy to generate a family of optimal state and input trajectories for different boundary conditions using this algorithm. The on-demand computational effort for different boundary conditions decreases greatly compared with the conventional method, and this effect increases as the number of the boundary conditions increases. Numerical examples shows this advantage.

In Chapter 6, the double generating functions method for LQ problems is applied to the on-demand optimal gaits generation of a compass biped robot walking on the level ground. For a robot walking in a complex environment, the double generating function method gives the optimal state and input trajectories as functions of the boundary values of the state, the initial time, and the terminal time directly. Since the generating functions can be calculated in advance for a given finite time LQ optimal control problem, the on-demand computation time of adjusting the step length and the time period is almost none. The dynamic equation of a compass biped walking robot is approximated by the Jacobian linearization to satisfy the precondition of the double generating function method. The optimal state and input trajectory generated by the double generating function method for the linear model is taken as the reference one. A conventional PD controller is designed to track the reference state trajectory with the reference input for the original nonlinear model. Therefore, the modeling error by the linearization can be shown by comparing the state and input trajectories generated by the simulation of the nonlinear control system with the reference ones. Based on the simulation result, we choose the design parameters appropriately such as the weighting matrix of the cost function with respect to the input and the time period to make the input small enough. The simulation result shows that the state and the input trajectories for the original nonlinear system are almost same with the ones for the linearized system when the robot walks in a low speed with a reasonable size of the step length. At the same time, the ankle input is about half of the hip one and both of them are not so large. This means that the optimal state and input trajectories for the linearized system can be used as the optimal ones for the original nonlinear system in practice. Because of the advantage of the double generating function method, both the non-periodic gait and the periodic gait are easy to be generated on demand. Therefore, it would be very useful for controlling the real biped robots, especially, when they are walking in a complex environment.

This thesis considers non-constraint optimal control problems. However, there exits constraint condition on the state and the input for many practical optimal control problem. The constraint optimal control problems should be studied in the future. Chapter 6 is just theoretical analysis. The experiments of the real robot control should be considered in the future. Also, from the practical point of view, what kind of the boundary condition can achieve a high energy efficiency is still a problem yet.