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A Study on Aggregation of Objective Functions in MaOPs based on Evaluation Criteria

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***Abstract*—Genetic Algorithms (GAs) have been widely applied to Multi-objective Optimization Problems (MOPs), called MOGA. A set of Pareto solutions in MOPs having plural fitness functions are searched, then GA is applied in a multipoint search. MOGA performance decreases with the increasing number of objective functions because solution space spreads exponentially. An effective MOGA search is an important issue in Many-objective Optimization Problems (MaOPs), which are MOPs with four or five objective functions or more. One effective approach is aggregation of objective functions and reducing their number, but appropriate aggregation and the number of objective functions to be aggregated has not been studied sufficiently. Our purpose here is to determine the effects of aggregation of objective functions quantitatively. This paper studies the effects of aggregation with the number of aggregated objective functions based on the evaluation criteria proposed in this paper when MOGA is applied to a Nurse Scheduling Problem (NSP).**

***Index Terms*—Multi-objective Optimization Problem, Many-objective Optimization Problem, Aggregation of Objective Functions, Genetic Algorithm, Nurse Scheduling Problem**

I. Introduction

In applying Genetic Algorithms (GAs)[1] to Multi-objective Optimization Problems (MOPs)[2][3], or MOGA[4], a set of Pareto solutions is searched for that is superior to other solutions in at least one objective function, in MOPs if objective functions are a trade-off. The GA is effective because it features a multipoint search and can search Pareto solutions in one trial. MOGA performance, however, decreases with an increase in the number of objective functions because solution space spreads exponentially[5][6][7]. Effective searching in Many-objective Optimization Problems (MaOPs) – MOPs with four or five objective functions or more – has become an important issue. Approaches such as parallel distributed processing, which improves computation time, or aggregation of objective functions and decreasing evaluation space, can ensure an effective search in such problems. Parallel distributed processing, e.g., the master-slave and island models, reportedly effectively decreases computational time and conserves solutions[8][9][10]. Because appropriately aggregating objective functions has yet to be studied, our purpose here is to determine the effects of aggregation of objective functions quantitatively, and to find appropriate aggregation, i.e., which objective functions should be aggregated, with the number of them in MOPs.

This paper determines the evaluation criteria which enable us to compare the performance of search quantitatively, converged generation which shows when the Pareto solutions were converged and no more effective search could be done, rate of inferior solutions which shows the rate of dominance of Pareto solutions generated in two methods, and cover rate which shows how much a method could keep the diversity of Pareto solutions comparing with the another method. In this paper, Non-Dominated Sorting Genetic Algorithm-II (NSGA-II)[11], which is one of the most famous optimization methods in MOPs, is applied to a Nurse Scheduling Problem (NSP)[12] which has eleven objective functions. This paper investigates the effects of aggregation by comparing two searches, one is the NSP with the eleven objective functions and the other is that with aggregated ones. It studies the performance on the dominance and the diversity of Pareto solutions in accordance with the difference of the number of aggregated objective functions and the combination of them.

II. Problem Establishment

A. Aggregation of Objective Functions

This paper employs the simplest approach to solve the problem in MaOPs, aggregations just by simple summation of objective functions. In the case of eq.(1), among Obj_1 to Obj_5, Obj_1 and Obj_2 are aggregated into F_1 by linear summation and the original number of the five objectives are reduced to four objectives. By the aggregation of objective functions, it is expected that the solution space becomes smaller and the search performance can be improved.

$$\begin{aligned} \min \quad F_1 &= Obj_1 + Obj_2 \\ \min \quad F_2 &= Obj_3 \\ \min \quad F_3 &= Obj_4 \\ \min \quad F_4 &= Obj_5 \end{aligned} \tag{1}$$

B. Nurse Scheduling Problem : NSP

The Nurse Scheduling Problem (NSP) is a combinatorial optimization problem for scheduling monthly nursing work, e.g., day, night, midnight, and holiday shifts, while meeting certain conditions. We combined these four shifts and nurse schedules for twenty six nurses and one month are designed.

Conditions include shift sequence, the number of nurses needed per shift, required nursing skills, and the “fairness” of shifts. Insufficiency in work conditions is used as an objective function and NSGA-II applied. Objective functions of NSP are shown in Table I. All objective functions except for the 4th are degrees of insufficiency for each condition, and the 4th is sufficiency. Obj_4 is multiplied by -1 to minimize objective functions. In actual schedules, no solution satisfies all conditions or is extremely difficult to search for, so the NSP becomes a multi-objective optimization problem.

TABLE I
OBJECTIVE FUNCTIONS IN NSP

Obj_1	Level of requisite nurses in each shift per day
Obj_2	Established prohibited working patterns
Obj_3	Established compromised working patterns
Obj_4	Established preferred working patterns
Obj_5	Fairness of the number of working times on night or midnight shifts among nurses
Obj_6	Fairness of the number of holidays among nurses
Obj_7	Fairness of the number of successive holidays among nurses
Obj_8	The prescript number of working times per month on night or midnight shifts in each nurse (within 8 times)
Obj_9	Successive holidays in each nurse (one or more times per month)
Obj_{10}	Successive holidays on Saturday and Sunday in each nurse (one or more times per month)
Obj_{11}	The prescript number of holidays per month in each nurse (2 days per week)

III. EVALUATION CRITERIA

The search performance are compared between the search under the aggregated objective functions and the original ones based on the converged generation, the dominance and the diversity of the generated Pareto solutions. The evaluation criteria for the performance are determined in this section.

A. Converged Generation

T. Hiroyasu et al. suggested that the convergence condition could be calculated by Ratio of Non-dominated Individuals (RNI)[13] between at nth generation and T generations later (at $(n+T)$th generation) given by eq.(2).

$$RNI = \frac{N_n}{N_{(n+T)}} \tag{2}$$

where N_n and $N_{(n+T)}$ denote the number of Pareto solutions at nth and $(n+T)$th generation, respectively, which become the 1st rank when both of nth and $(n+T)$th ones are evaluated at the same time.

In this paper, converged generation is determined based on the RNI above. The closer to 0 the value of RNI becomes, the more advanced the Pareto solutions are. On the other hand, when the value is close to 1, it means that the Pareto solutions did not make progress during T generations. The solid line in Fig.1 shows an example of the transition of RNI in $T = 30$. This paper approximates the the solid line in Fig.1 by eq.(3) with least squares method. The dotted line in Fig.1 shows the approximation of the solid line by eq.(3) when x in eq.(3) corresponds to generations. This paper determines the coefficient G in eq.(3), in which G usually means time constant, as "converged generation", which shows the generation when the Pareto solutions were converged and no more effective progress could be done. In the case of Fig.1, the converged generation G is 246.

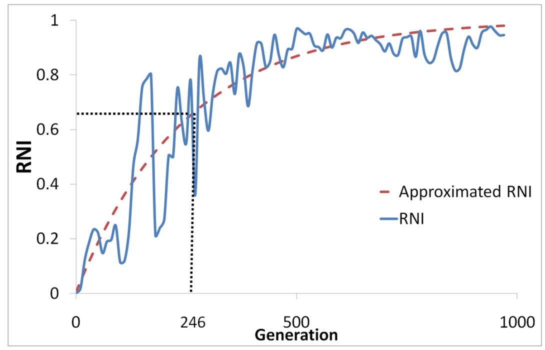


Fig. 1. Transition of RNI

$$f(x) = 1 - \exp\left(-\frac{x}{G}\right) \tag{3}$$

B. Rate of Inferior Solutions

Rate of inferior solutions is a criterion to measure the superiority of Pareto solutions searched under aggregated objective functions. The dominance of all Pareto solutions, searched under aggregated and non-aggregated objective functions, is evaluated based on the original objective functions, and the rate of inferior solutions (RIS) is calculated with eq.(4). In eq.(4), N_o means the number of inferior solutions searched under non-aggregated, i.e., original, objective functions and N_a means that under aggregated ones when the dominance is calculated in the original objective function space based on the both Pareto solutions. The closer to 0 the value as RIS becomes, the more Pareto solutions with aggregation dominated to those with original objective functions. When RIS is 0.5, it means that the dominance was equivalent between them and the aggregation did not work well in terms of acquisition of advanced Pareto solutions.

$$\text{RIS} = \frac{N_a}{N_o + N_a} \tag{4}$$

C. Cover Rate

Cover rate is a criterion to measure the diversity of Pareto solutions searched under aggregated objective functions comparing with the original ones. First, each cover-rate for objective function i, C_{ai} or C_{oi}, which means the diversity of Pareto solutions is calculated. It is the ratio of the number of filled area by the Pareto solutions to that of the divided area. Evaluation value between maximum and minimum one in each objective function obtained under original objective functions is divided into N_d areas. C_{oi} means the cover-rate under original objective functions and C_{ai} means that under aggregated ones. For example, in the case of Fig.2, C_{oi} becomes $6/8 = 0.75$ and C_{ai} is $4/8 = 0.5$ in $N_d = 8$. Then the cover rate (CR) is calculated with eq.(5). In eq.(5), N denotes the number of objective functions. When CR is 1.0, it means that the diversity of Pareto solutions could be keep even if the objective functions were aggregated. When the value of CR is close to 0, the diversity of Pareto solutions is completely lost by the aggregation. In the example of Fig.2, CR is $0.5/0.75 \approx 0.67$.

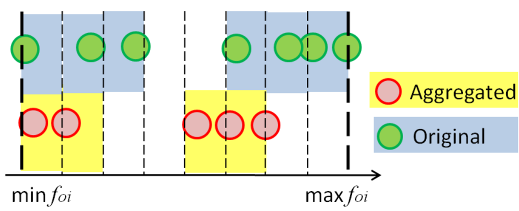


Fig. 2. Cover Rate

$$\mathrm{CR} = \frac{\sum_{i=1}^{N} C_{ai}}{\sum_{i=1}^{N} C_{oi}} \tag{5}$$

IV. Experiment

NSGA-II was applied to NSP with eleven objective functions as shown in II. and the performance compared, with one in which objective functions were aggregated and not aggregated based on the evaluation criteria above. For GA parameters, solutions numbered 1000, the crossover-rate was 1.0, and the mutation-rate was (1 / gene length). Each result bellow comes from ten trials.

A. Experiment 1: Effects of Aggregation

The effects of the aggregation of objective functions were studied here.

Fig.3 show the transition of RNI and the approximated one in a trial of search until 1000th generation under eleven objective functions. The converged generation G described in III.-A. was 83 in this case. It shows strong convergence in the early generations, which would cause no more advance of Pareto solutions when it had eleven objective functions.

Fig.4 shows the converged generations when the number of objective functions was reduced by aggregating them. The objective functions to be aggregated were selected randomly and ten combinations (ten trials in each combination) for the searches under ten to six objective functions were carried out, respectively, and the converged generations were averaged with the 100 trials in total. In Fig.4, though obvious effects of the aggregations for the early convergence can not be found until the reduction to nine objective functions, the converged generation becomes large from the reduction to eight objectives. When the genetic parameter for the number of generations is 1000, the appropriate number of the reduction of objective functions would be eight in this NSP.

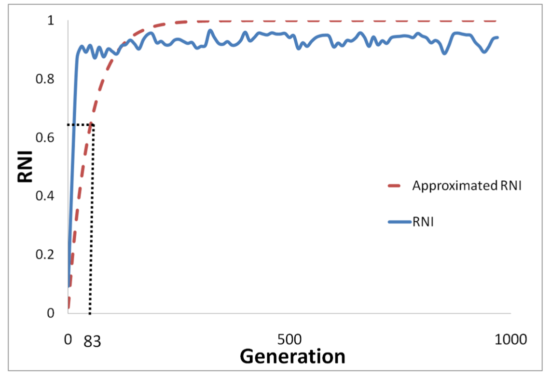


Fig. 3. Transition of RNI (11 objectives)

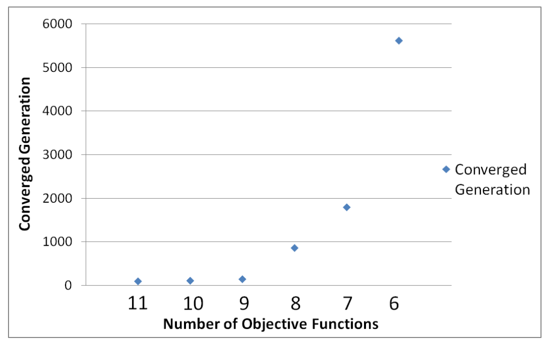


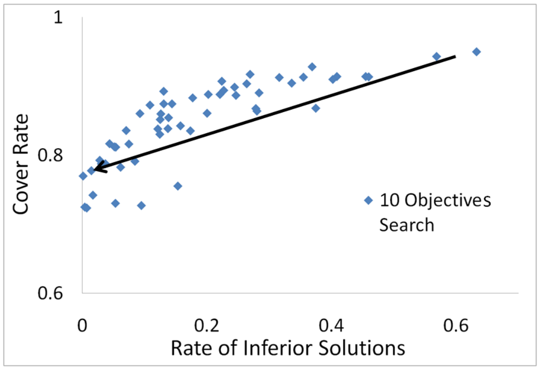
Fig. 4. Converged Generation

B. Experiment 2: Relationship between Dominance and Diversity

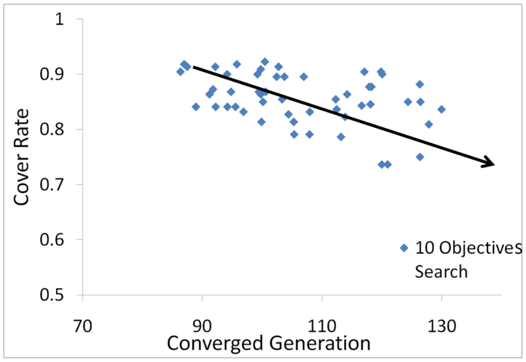
The relationship between the dominance and the diversity of Pareto solutions , i.e., between RIS and CR, were studied here.

Two objective functions out of the eleven were aggregated so that searches with ten objectives were carried out in 55 (${}_{11}\mathrm{C}_2$) ways, and the Pareto solutions obtained by each search were evaluated based on the evaluation criteria described in III. Note that the values of the evaluation criteria were averaged with 10 trials. The relationships between the rate of inferior solutions and the cover rate is shown in Fig.5(a), and that between the converged generation and the cover rate is also shown in Fig.5(b). Each dot corresponds to an aggregation out of the 55 ways.

In Fig.5(a), the rate of inferior solutions and the cover rate have a proportional relation with the coefficient of determination 0.665 in a regression analysis. Making the search area narrower and less diversity is needed in order to obtain advanced Pareto solutions. In addition, it is also found that the converged generation and the cover rate are a trade-off in Fig.5(b) with the coefficient of determination 0.499 in a



(a) RIS and CR



(b) G and CR

Fig. 5. Relationship between Evaluation Criteria

regression analysis. Later convergence caused the advanced Pareto solutions described above.

C. Experiment 3: Effective Aggregation

The relationship between the evaluation criteria and the number of the reduction of objective functions by aggregation was studied here.

The number of objective functions was reduced from ten to seven by the same aggregation with IV.-A. Fig.6 shows the relationship between the rate of inferior solutions and the cover rate in each trial, 100 trials in total for each number of objectives.

In the case that the number of objectives is ten or nine, the rate of inferior solutions runs a wide value from 0 to 1.0 and the cover rate runs only from 0.7 to 1.0. In the reduction of the number of objective functions from eleven to ten or nine, the diversity of the obtained Pareto solutions could be kept, then the aggregation to let the rate of inferior solutions low is better way to employ.

On the other hand, in the case that the number of objectives is eight or seven, the rate of inferior solutions in most of the searches is almost 0 and the cover rate runs a wide value from 0.1 to 0.7. Fig.4 also showed that the early convergence could be avoid by reducing the number of objective functions to less than or equal to eight objectives. Fig.6 shows that the Pareto solutions obtained under eight objectives were enough advanced comparing with those under eleven objectives, then it is more effective for the aggregation to consider the cover rate in this case. In the NSP employed in this paper, the characteristics of search is drastically changed when the number of objective functions is reduced to less than or equal to eight by the simple aggregation.

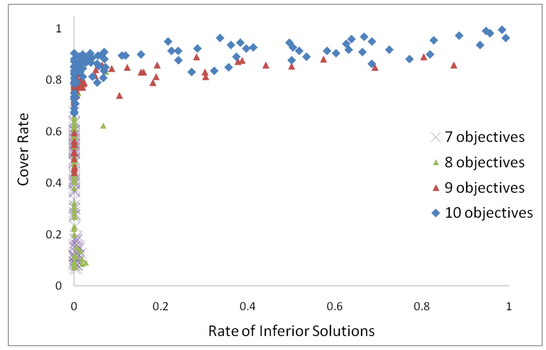


Fig. 6. Relationship between Evaluation Criteria in Each Number of Aggregated Objective Functions

V. Conclusion

This paper determined the evaluation criteria, converged generation, rate of inferior solutions and cover rate, which enable us to compare the performance of search quantitatively, to determine the effects of aggregation of objective functions in Many-objective Optimization Problems. NSGA-II was applied to Nurse Scheduling Problem, and the effects of the aggregation of objective functions were studied by comparing the search under aggregated objective functions with the original ones. First, it was confirmed that the search under eleven objectives cased early convergence and the reduction by an aggregation to less than or equal to eight objectives could help it. Second, it was found that the converged generation and the cover rate were a trade-off while the rate of inferior solutions and the cover rate had a proportional relation. Finally, it was found that the aggregation to let the rate of inferior solutions low was better in the reduction to ten or nine objectives and that to let the cover rate high was better in the reduction to eight or less objectives. As the future work, we plan to investigate the concrete guideline for effective aggregations which provide low rate of inferior solutions or high cover rate, and we will also study other aggregation methods and the effects of the aggregations in other problems.

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