A Study on Analysis of Design Variables in Pareto Solutions for Conceptual Design Optimization Problem of Hybrid Rocket Engine

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***Abstract*—Genetic Algorithm (GA)[1] is one of the most effective methods in the application to optimization problems. Recently, Multi-objective Genetic Algorithm (MOGA) is focused on in the engineering design field. In this field, the analysis of design variables in the acquired Pareto solutions, which gives the designers useful knowledge in the applied problem, is important as well as the acquisition of advanced solutions. This paper proposes a new visualization method using Isomap which visualizes the geometric distances of solutions in the design variable space considering their distances in the objective space. The proposed method enables a user to analyze the design variables of the acquired solutions considering their relationship in the objective space. This paper applies the proposed method to the conceptual design optimization problem of hybrid rocket engine and studies the effectiveness of the proposed method. It shows that the visualized result gives some knowledges on the features between design variables and fitness values in the acquired Pareto solutions.**

keywords: Analysis of Design Variables, Visualization, Isomap, Multi-objective Genetic Algorithm, Conceptual Design Optimization Problem of Hybrid Rocket Engine

I. Introduction

Recently, Multi-objective Optimization Problems (MOPs) have been focused on the engineering design field. Generally, it is difficult or impossible to acquire the optimized solution satisfying all objective functions because of their trade-offs. Then in MOPs, it is necessary to acquire Pareto solutions which are superior to other solutions by at least one fitness value. MOGA, which is the application of GA to MOPs, could be effective to solve MOPs because GA is multi-point search and can search various Pareto solutions in one trial[2].

It has been reported in recent years that MOGAs are applied to engineering design problems due to the improvement of computing[3][4][5]. Obayashi[3] worked on design optimization of aircraft configuration problem using MOGA. [3] acquired Pareto solutions by MOGA method and analyzed the design variables of Pareto solutions to grasp the physical features in the problem through a visualization using Self Organizing Map (SOM). Deb[4] studied the method to discover useful information for designers from the Pareto solutions in engineering design problems. In engineering design field, it is important not only to search advanced Pareto solutions using MOGA, but also to grasp useful knowledge for designers and analyze physical relationship between fitness values and design variables[4].

This paper employs the conceptual design optimization problem of hybrid rocket engine (HRE) as an application of MOGA to engineering design problem[6]. This problem has been provided and published on the Web (Japanese)[7] by Japan Aerospace Exploration Agency (JAXA).

HRE is the rocket engine that holds propellant in two different conditions, which has a smaller risk of explosion and a higher safeness. It is also environment-friendly, and it can adjust the thrust by using a slot ring. However, for a design of HRE, it is difficult to acquire general design knowledge because the thrust is generated by burning in the turbulent boundary layer in the reaction chamber, which is different from the conventional rocket engines. In the turbulent boundary layer, the O/F mixture ratio which dictates the thrust is calculated by the length of fuel, the radius of the port and the flow rate of oxidant. Thus, the geometric design of a HRE can be considered as an optimization problem which optimizes the weight of the rocket and the highest reachable altitude. As for the design variables of this problem, there are the flow rate of oxidant$[kg/s]$, length of fuel$[m]$, initial radius of the port$[m]$, burning time$[s]$, pressure in the reaction chamber$[MPa]$ and open area ratio. As for the objective functions, there are the minimization of weight of the rocket$[kg]$ and the maximization of highest reachable altitude$[km]$.

This paper applies one of the representative multi-objective optimization method NSGA-II (Non-dominated Sorting Genetic Algorithm-II) [8] to the conceptual design optimization problem of HRE. It analyzes the relationship between design variables and fitness values in the acquired Pareto solutions. To consider their relationships, this paper proposes a new visualization method using Isomap which considers relative distance of data in the objective space considering relative distance of data in the design variable space. In the proposed method, it calculates the geodetic distance in the design variable space based on the distance in the objective space and visualizes the relationship among Pareto solutions using Isomap [9][10][11][12]. The experimental result shows that we

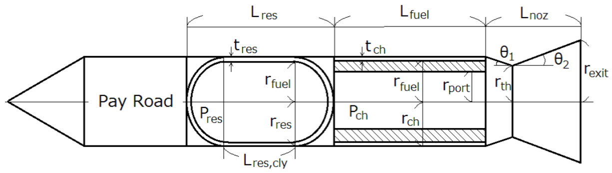


Fig. 1. Hybrid Rocket

TABLE I
DESIGN VARIABLES AND THERE RANGES

V1	$\dot{m}_{oxi}(0)$: Initial flow rate of oxidant $1.0 - 30.0[kg/s]$
V2	L_{fuel} : Length of fuel $1.0 - 10.0[m]$
V3	$r_{port}(0)$: Initial radius of port $0.01 - 0.2[m]$
V4	t_{burn}: Burning time $15.0 - 35.0[s]$
V5	P_{ch}: Pressure in reaction chamber $3.0 - 4.0[MPa]$
V6	ϵ: Open area ratio $5.0 - 7.0$

TABLE II
FITNESS VALUES

Obj1	M_{tot} : Weight of rocket $[kg]$	Min
Obj2	H_{max} : Highest reachable altitude $[km]$	Max
Obj3	M_{pay} : Weight of pay load $[kg]$	Max
Obj4	L_{tot}: Length of rocket $[m]$	Min
Obj5	a_{max} : Maximum acceleration $[km/s^2]$	Min

can see the relationship of the design variables considering that of the fitness values as well, and we can grasp some knowledges on the features between them.

II. PROBLEM ESTABLISHMENT

This section explains the employed conceptual design optimization problem of HRE. Fig.1 shows the conceptual figure of a hybrid rocket, and TABLE I and TABLE II show each design variable and fitness value, respectively. This paper deals with two objective functions Obj1 and Obj2, though this problem can be expanded to 5 objective optimization problem as shown in TABLE II. Weight of the rocket $M_{tot}[kg]$ and highest reachable altitude $H_{max}[km]$ can be calculated by the following equation (1) - (15).

In order to calculate the weight of the rocket $M_{tot}[kg]$, weight of the oxidant $M_{oxi}[kg]$, weight of fuel $M_{fuel}[kg]$, weight of propellant $M_{prop}[kg]$, weight of the oxidizer tank $M_{res}[kg]$ and the weight of the reaction chamber $M_{ch}[kg]$ are calculated by eq. (1) - (5).

$$M_{oxi} = \int_0^{t_{burn}} \dot{m}_{oxi}(t)dt \tag{1}$$

$$M_{fuel} = \int_0^{t_{burn}} \dot{m}_{fuel}(t)dt \tag{2}$$

$$M_{prop}(t) = M_{oxi} + M_{fuel} + \int \dot{m}_{prop}(t)dt \tag{3}$$

$$M_{res} = \rho_{res}V_{res} \tag{4}$$

$$M_{ch} = \rho_{ch}V_{ch} \tag{5}$$

$\dot{m}_{fuel}(t)[kg/s]$ is the time change of the fuel flow, $\dot{m}_{prop}(t)[kg/s]$ is the time change of the propellant flow, $\rho_{res}[kg/m^3]$ is the density of the oxidizer tank, $V_{res}[m^3]$ is the volume of the oxidizer tank, $\rho_{ch}[kg/m^3]$ is the density of the reaction chamber and $V_{ch}[m^3]$ is the volume of the reaction chamber. The weight of the rocket $M_{tot}[kg]$ is calculated by eq. (6) - (9). In this paper, the weight of the pay load $M_{pay}[kg]$ is set to $50[kg]$ (constant).

$$M_{tot}(t) = \int \dot{m}_{prop}(t)dt + M_{tot}(0) \tag{6}$$

$$M_{tot}(0) = M_{eng} + M_{pay} + M_{ex} \tag{7}$$

$$M_{eng} = M_{oxi} + M_{fuel} + M_{ch} + M_{res} \tag{8}$$

$$M_{ex} = \frac{2}{3}M_{eng} \tag{9}$$

Next, in order to calculate the highest reachable altitude $H_{max}[km]$, thrust $T[N]$ and acceleration $a[m/s^2]$ are calculated by eq. (10) - (15).

$$a(t) = \begin{cases} \frac{T(t)-D(t)}{M_{tot}(t)} - g \ (V(t) > 0) \\ \frac{T(t)}{M_{tot}(t)} - g \quad (V(t) = 0) \\ \frac{T(t)+D(t)}{M_{tot}(t)} - g \ (V(t) < 0) \end{cases} \tag{10}$$

$$T(t) = \eta_T\big(\lambda\dot{m}_{prop}(t)\mu_e + (P_e - P_a)A_e\big) \tag{11}$$

$$\frac{O}{F} = \frac{\dot{m}_{oxi}(t)}{\dot{m}_{fuel}(t)} \tag{12}$$

$$\lambda = \frac{1}{2}(1 + \cos\theta_2) \tag{13}$$

$$A_e = \epsilon A_{th} \tag{14}$$

$$A_{th} = \frac{\dot{m}_{prop}(0)I_{sp}(0)\eta_{I_{sp}}}{C_F(0)\eta_{C_F}P_{ch}} \tag{15}$$

$D(t)[N]$ is the friction at time t, $g[m/s^2]$ is the acceleration of gravity and $V(t)[km/s]$ is the speed of the rocket. η_T is the loss coefficient of the nozzle ($\eta_T = 1.0$) and $P_a[Pa]$ is the atmosphere pressure. The current velocity of the nozzle exit $u_e[m/s]$ and the pressure of the nozzle exit $P_e[Pa]$ are calculated by the mixture proportion ratio O/F. λ is the revise momentum coefficient to estimate the thrust $T[N]$ and $A_e[m^2]$ is the area of nozzle exit. $A_{th}[m^2]$ is the area of nozzle throat, $I_{sp}(0)[s]$ is the initial specific impulse, $\eta_{I_{sp}}$is the efficient of I_{sp}, $C_F(0)$ is the initial thrust coefficient and η_{C_F} is the efficient of the C_F.

III. PROPOSED METHOD

The proposed method uses the idea of geodetic distance to reflect the similarity of the solutions in the objective space into the geodetic distance in the design variable space. The geodetic distance represents the shortest path along the surface on the manifold configuration. Then, it employs Isomap[9][10][11] to visualize the similarity based on the geodetic distance of the solutions. Isomap visualizes the similarity between data based on their geodetic distance using Multiple Dimensional Scaling (MDS). MDS is one of the multivariate analysis, which is

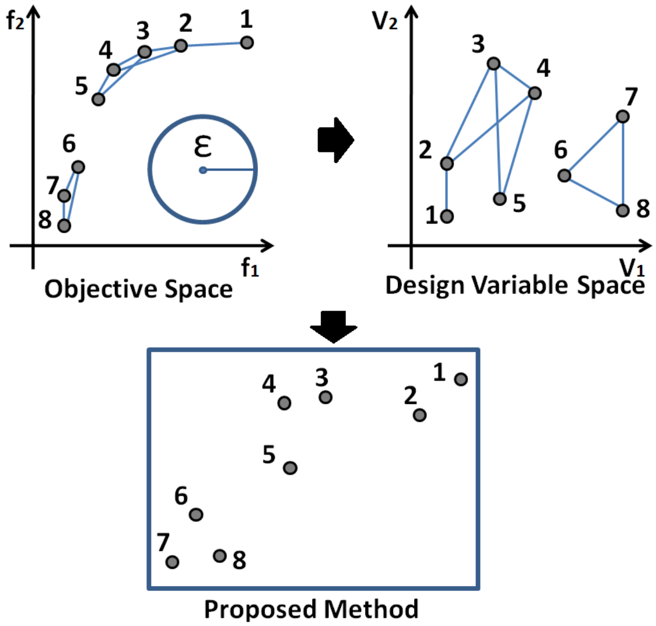


Fig. 2. Image of Proposed Method

often used as a visualization technique to show the similarities or dissimilarities between data. It can reduce the number of dimensions while preserving the distances between data in the original space as much as possible.

First, it defines "neighborhood" of each data in the objective space, and links the defined neighborhood data. There are two ways to define the neighborhood, one is to use the Euclidean distance in the objective space and the other is to define the number of neighborhoods around each data. This paper used the former. Next, the geodetic distance is calculated. In geodetic distance, the distance between linked data is simply measured by Euclidean distance and that between not linked data is calculated by the sum of Euclidean distance of the data which give shortest way to reach the target data along the linked ones. The distance at the data which are not able to reach is given as D_{ue}. Usually, D_{ue} is set to relatively large value[12]. The feature of this method is to define the neighborhood in the objective space and calculate the geodetic distance in the design variable space.

Fig.2 shows the image of the proposed method. In Fig.2, the radius ϵ is the parameter to determine the "neighborhood" in the objective space. For example, data 1 and data 2 become the neighborhood because the distance between data 1 and 2 is smaller than the radius ϵ, then they are linked each other. In the same way, data 2, 3 and 4 and data 3, 4 and 5 are the neighborhood, respectively, but data 2 and 5 are not.

This information of their links is applied to the design variable space. In the right figure of Fig.2, data 2 and 4 have a direct link so that the distance between them is simply defined as the Euclidean distance in the design variable space. Data 2 and 5 do not have a link so that the distance between them is defined as the minimum of $(d_{24}+d_{45})$, the sum of the distance of data 2-4 and that of data 4-5, and $(d_{23}+d_{35})$. The data

TABLE III
EXPERIMENTAL PARAMETER

Trial	10
Generation	32
Population Size	64
Crossover Rate	1.0
Mutation Rate	0.033

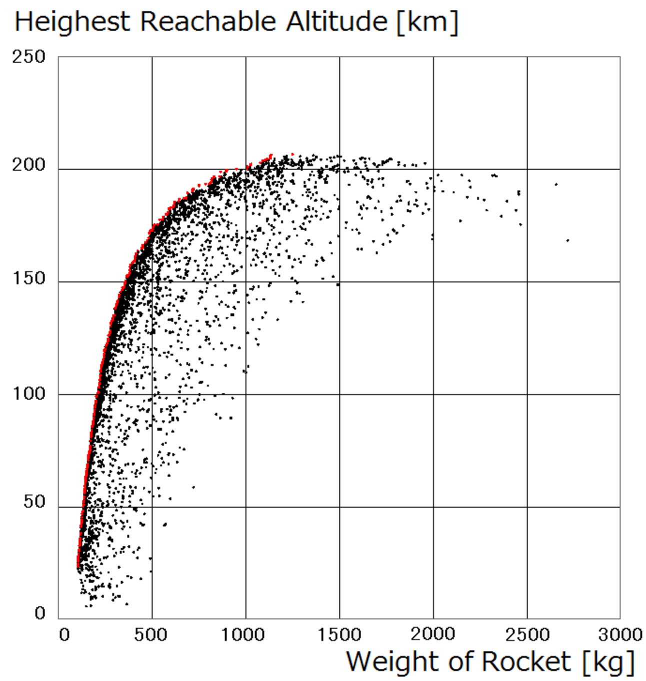


Fig. 3. Acquired Pareto Solutions

1,2,3,4,5 and 6,7,8 are separated each other because they are not connected. It is expected that we can grasp the similarity of data in the design variable space considering those in the objective space by the proposed method.

IV. EXPERIMENT

A. Acquisition of Pareto Solutions

First, NSGA-II was applied to the conceptual design optimization problem of HRE explained in II. and Pareto solutions were acquired. TABLE III shows the genetic parameters used in NSGA-II.

As for the coding, it divided the range of each design variable V1 - V6 shown in TABLE I into 1024 and they were expressed by 10 - bit binary number. Then, the length of the gene became 60 bit. As for the crossover, uniform crossover which treated one design variable as a unit was used. The weight of the pay load $M_{pay}[kg]$ was set to $50[kg]$, the loss coefficient of the nozzle η_T was set to 100% and the flow rate of oxidant $\dot{m}_{oxi}[kg/s]$ was set to a constant.

Fig.3 shows the solutions obtained in 10 trials in all generations. In Fig.3, horizontal axis is the weight of the rocket $M_{tot}[kg]$ and vertical axis is the highest reachable altitude $H_{max}[km]$. The red dots in Fig.3 are the Pareto solutions which were non-dominated in all solutions. The analysis bellow is done to these Pareto solutions.

B. Application of Proposed Method

Fig.4(c) shows the result of the proposed method to the Pareto solutions above.f Fig.4(a) and Fig.4(b) are the distribution of the Pareto solutions in the objective space and in the design variable space visualized by MDS, respectively. The radius ϵ to define the neighborhood in the objective space was calculated by eq. (16).

$$\epsilon = \frac{f_{max} - f_{min}}{10} \tag{16}$$

Each data in Fig.4(a)-(c) is correspond to same color. The blue data had higher M_{tot} and H_{max}, i.e., upper-right in Fig.4(a) and the red data had lower M_{tot} and H_{max}, i.e., lower-left in Fig.4(a). Each variable in the data in Fig.4(a)(b) was standardized to average 0 and variance 1.

V. CONSIDERATION

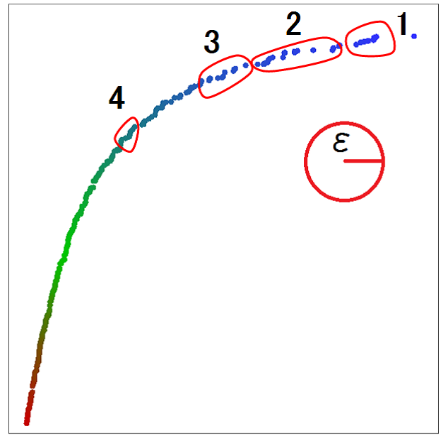
In this paper, the proposed method visualized the similarity of the solutions in the design variable space based on the links defined in the objective space, in which the Pareto solutions were uniformly distributed. Therefore, the result might be strongly influenced by the relationship in the objective space. However, we can see that there are crowding area and sparse area from the visualized result.

First, it considers this result from the physical side. The solutions in the bottom-left part in Fig.4(c) are crowding, which means the geodetic distances are small and the design variables are similar one another, and correspond to low M_{tot} and low H_{max} area in the objective space. When we want to reduce the weight of the rocket rather than getting higher reachable altitude, there are few variations for the design variables. Because it is obvious that small amount of fuel makes the weight of the rocket low but it can not reach high altitude. On the other hand, The solutions in the upper part in Fig.4(c) are sparse and have some groups, and correspond to high M_{tot} and high H_{max} area in the objective space. It means that there are several variations for design variables to design rockets reaching high altitude.

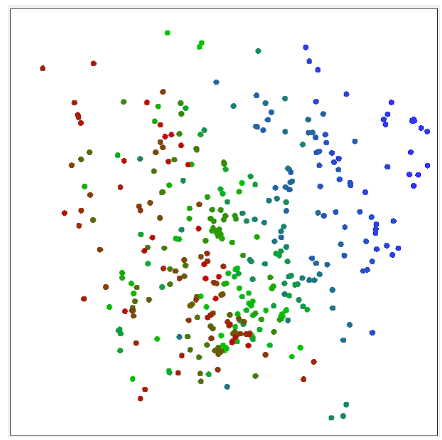
TABLE IV
DESIGN VARIABLES OF PARETO SOLUTIONS

	M_{tot}	H_{max}	$\dot{m}_{oxi}(0)$	L_{fuel}	$r_{port}(0)$	t_{burn}	P_{ch}	ϵ
Group 1	1095	205.5	27.0	4.05	0.041	15.4	36.0	6.77
Group 2	874	199.3	20.6	3.56	0.037	15.8	36.9	6.73
Group 3	728	191.8	15.9	3.28	0.037	16.8	37.2	6.89
Group 4	442	167.4	9.1	2.76	0.041	16.9	37.1	6.81
Max in Pareto	1124	207.5	28.0	4.51	0.054	18.0	39.9	6.99
Min in Pareto	90	24.7	1.03	1.04	0.014	15.1	30.2	5.06

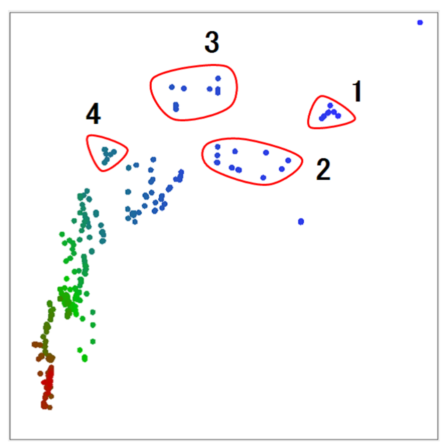
TABLE IV shows the average of design variables of the group 1-4 in Fig.4(c). In the objective space, these four groups are in the high M_{tot} and H_{max} area and comparatively have similar M_{tot} and H_{max}. However, group 1 has large amount



(a) Objective Space



(b) Design Variable Space (using MDS)



(c) Proposed Method

Fig. 4. Visualization Result

of $\dot{m}_{oxi}(0)$ and L_{fuel} and low P_{ch}, which makes a lot of difference from group 4. Moreover, though group 2 and 3 are very close in the objective space, they have different $\dot{m}_{oxi}(0)$ and P_{ch} in the design variable space.

Next, it considers the feedback into genetic operations from the visualized result. In the bottom-left crowded area and in the upper circled area in Fig.4(c), the solutions have similar fitness values and similar design variables one another. It means that the area around a gene in the design variable space is corresponding well to that around the gene in the objective apace. In these areas, it is expected that Local Search (LS) methods[13][14][15], which searches around genes with high fitness values but actually searches in the design variable space, and NSGA-II, which considers the ranking of genes by the degree of crowding rate in the objective space, will work well. On the other hand, in the sparse area in Fig.4(c), the solutions have similar fitness values but various design variables. In this area, LS method or the operation of NSGA-II, which preferentially selects the genes with low density as parents for genetic operations with expecting that generated offsprings fill in the sparse area in the objective space, will not work well. Because the objective space and the design variable space are not corresponding each other, then the search around a gene in the design variable space does not mean the search of the target area in the objective space, and *vice versa*.

VI. Conclusion

This paper applied NSGA-II to the conceptual design optimization problem of HRE to analyze the design variables in the acquired Pareto solutions. This paper proposed a new visualization method using Isomap which visualized the geometric distances of solutions in the design variable space considering their distances in the objective space. The proposed method enabled a use to analyze the design variables of the acquired solutions considering their relationships in the objective space. The experimental result showed that the solutions with low weight of the rocket and low highest reachable altitude did not have variations on the design variables while those with high weight of the rocket and high highest reachable altitude had several variations on them. The feedback into genetic operations from the visualized result was also described in this paper.

For the future work, we will study more on the proposed method by applying not only to Pareto solutions but also to all solutions or visualizing the geodetic distance of the objective space based on the similarity in the design variable space. The concrete feedback based on the obtained knowledge is also needed in the future work.

VII. Acknowledgment

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