A Visualization Method of Third-Order Tensor for Knowledge Extraction from Questionnaire Data

Hiroaki Masai
Dept. of Computational Science and Engineering
Nagoya University
Nagoya, Aichi 464-8603

Tomohiro Yoshikawa
Dept. of Computational Science and Engineering
Nagoya University
Nagoya, Aichi 464-8603
Email: yoshikawa@cse.nagoya-u.ac.jp

Takeshi Furuhashi
Dept. of Computational Science and Engineering
Nagoya University
Nagoya, Aichi 464-8603
Email: furuhashi@cse.nagoya-u.ac.jp

***Abstract*—This paper presents a new visualization method based on Higher Order Singular Value Decomposition(HOSVD). This method enables us to select any pairs of vectors from loading matrices, and visualizes features of loading vectors in the form of biplots of respondents, questions and objects. An experiment is carried out using artificial data. The results shows that the proposed method can visualize the interrelationships between features of selected loading vectors and we can find respondent groups who marked uniquely on particular objects and questions. Some groups are unable to be found by the conventional PCA.**

I. Introduction

Questionnaire researches have been carried out in various occasions. Questionnaires often consist of questions on several objects. The obtained data become third-order tensors with elements of objects × questions × respondents. To these data, tensor factorization and decomposition could be applied to extract knowledge[4][9]. However, there have been few reports on application of tensor analysis to questionnaire data. This might be due to very recent development of computer applications of tensor analysis to data mining[1][7][8]. This paper presents a basic study on visualization of questionnaire data in third-order tensor form. One of the conventional approaches is to unfold or matricize the third-order tensor to second-order tensor, i.e. matrix and apply Principal Component Analysis(PCA). This might lose information contained in the higher-order tensor structure. Three-Mode PCA[5] utilizing the decomposition models such as the CANDECOMP/PARAFAC[2][3] and Tuker model[10] is a tool to preserve the information and visualizes the interrelationships between either objects and questions, or objects and respondents, or questions and respondents.

This paper presents a new visualization method based on Higher Order Singular Value Decomposition(HOSVD)[6]. The proposed method visualizes the interrelationships between respondents and objects and questions in a two-dimensional space. The 2D space is spanned by principal component vectors obtained by SVD of object-mode matrix and question-mode matrix. The proposed method preserves the information contained in third-order tensor more than the conventional PCA does, and is suited for visualizing various respondent groups who evaluated the objects uniquely. This paper is organized as follows: Section 2 describes the conventional three-mode PCA, joint plot and HOSVD. In section 3, we presents our visualization method. Section 4 shows the results of application of the proposed method to artificial data and compares them to those by the conventional PCA. Section 5 is the summary of this paper.

II. Conventional Methods

A. Three-Mode PCA

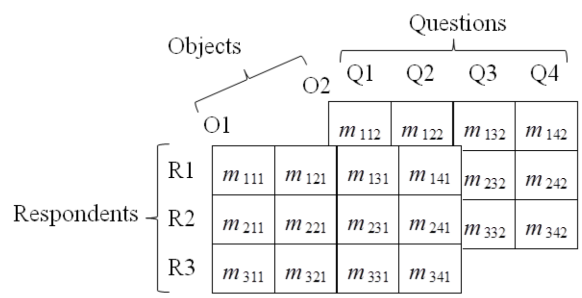


Fig. 1. Example of third-order tensor

Figure 1 shows an example of third-order tensor $\underline{\boldsymbol{X}}$. Its element $\{i, j, k\}$ is denoted by m_{ijk}, $i = 1, \cdots, I$, $j = 1, \cdots, J$, $k = 1, \cdots, K$. In the case of Fig.1, $I = 3$, $J = 4$, $K = 2$. This tensor is a data obtained from three respondents R_1, R_2, R_3 by asking four questions $Q_1, \cdots, Q_4$ on two objects O_1, O_2. A tensor is sliced into matrices and vectors. Frontal slice is a matrix denoted by $\boldsymbol{X_{::k}}$. Figure 2 shows examples of frontal slices obtained from the tensor in Fig.1. The elements are assumed to be given with integer marks. Fig.2(a)(b) are frontal slices $\boldsymbol{X_{::1}}$, $\boldsymbol{X_{::2}}$, respectively. Horizontal slice $\boldsymbol{X_{1::}}$ and lateral slice $\boldsymbol{X_{:1:}}$ are expressed as

$$\boldsymbol{X_{1::}} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 13 & 14 & 15 & 16 \end{pmatrix}, \boldsymbol{X_{:1:}} = \begin{pmatrix} 1 & 13 \\ 5 & 17 \\ 9 & 21 \end{pmatrix}. \quad (1)$$

The tensor in Fig.1 has eight column vectors $\boldsymbol{x_{:jk}}$, six row vectors $\boldsymbol{x_{i:k}}$, and twelve tube vectors $\boldsymbol{x_{ij:}}$. In Fig.2, $\boldsymbol{x_{:11}}$ is given by

$$\boldsymbol{x_{:11}} = (1, 5, 9)^T \quad (2)$$

where T means transpose.

	Q1	Q2	Q3	Q4
R1	1	2	3	4
R2	5	6	7	8
R3	9	10	11	12

(a) $X_{::1}$

	Q1	Q2	Q3	Q4
R1	13	14	15	16
R2	17	18	19	20
R3	21	22	23	24

(b) $X_{::2}$

Fig. 2. Example of frontal slices with hypothetical marks

	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
R1	1	2	3	4	13	14	15	16
R2	5	6	7	8	17	18	19	20
R3	9	10	11	12	21	22	23	24

(a) mode-r matrix $X_{(r)}$

	R1	R2	R3	R1	R2	R3
Q1	1	5	9	13	17	21
Q2	2	6	10	14	18	22
Q3	3	7	11	15	19	23
Q4	4	8	12	16	20	24

(b) mode-q matrix $X_{(q)}$

	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
O1	1	5	9	2	6	10	3	7	11	4	8	12
O2	13	17	21	14	18	22	15	19	23	16	20	24

(c) mode-o matrix $X_{(o)}$

Fig. 3. Examples of matricization of third-order tensor with hypothetical marks

Figure 3 shows examples of matricization, also called unfolding or flattening, of the tensor $\underline{\boldsymbol{X}} \in \mathbb{R}^{3\times4\times2}$ in Fig.1. Marks are hypothetical ones as given in Fig.2. Mode-r matrix $\boldsymbol{X}_{(r)}$ is obtained as

$$\boldsymbol{X}_{(r)} = \begin{pmatrix} 1 & 2 & 3 & 4 & 13 & 14 & 15 & 16 \\ 5 & 6 & 7 & 8 & 17 & 18 & 19 & 20 \\ 9 & 10 & 11 & 12 & 21 & 22 & 23 & 24 \end{pmatrix}. \quad (3)$$

Three-Mode PCA[4][5] is a method of data compression that applies Tucker 3 decomposition[10]. A third-order tensor $\underline{\boldsymbol{X}} \in \mathbb{R}^{I\times J\times K}$ is decomposed into a core tensor multiplied by three factor matrices $\boldsymbol{A} \in \mathbb{R}^{I\times L}, \boldsymbol{B} \in \mathbb{R}^{J\times M}, \boldsymbol{C} \in \mathbb{R}^{K\times N}$. It is an approximation expressed as

$$\underline{\boldsymbol{X}} \approx \underline{\boldsymbol{G}} \times_l \boldsymbol{A} \times_m \boldsymbol{B} \times_n \boldsymbol{C} \quad (4)$$

where $\underline{\boldsymbol{G}} \times_l \boldsymbol{A}$ is l-mode product. Figure4 shows the obtained core tensor $\underline{\boldsymbol{G}}$ and three factor matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ from the tensor with the hypothetical data in Fig.2. Tucker 3 decomposition is easily carried out by using tucker_als algorithm available in MATLAB toolbox. The factor matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ are orthogonal and the principal components in mode-r, mode-q, and mode-o matrices, respectively.

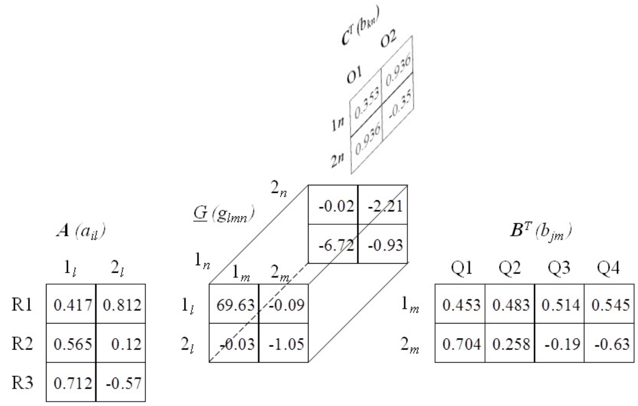


Fig. 4. Tucker 3 decomposition

$l-$mode product $\underline{\boldsymbol{G}} \times_l \boldsymbol{A}$ in Fig.4 is done by, first, multiplying matrx $\boldsymbol{A}$ to mode-l matrix $\boldsymbol{G}_{(l)}$as

$$\begin{aligned} \boldsymbol{A}\boldsymbol{G}_{(l)} &= \begin{pmatrix} 0.42 & 0.81 \\ 0.56 & 0.12 \\ 0.71 & -0.57 \end{pmatrix} \\ &\quad \times \begin{pmatrix} 69.63 & -0.09 & -0.02 & -2.21 \\ -0.03 & -0.10 & -6.72 & -0.93 \end{pmatrix} \\ &= \begin{pmatrix} 29.03 & -0.89 & -5.47 & -1.68 \\ 39.31 & -0.18 & -0.82 & -1.36 \\ 49.6 & 0.53 & 3.83 & -1.04 \end{pmatrix} \quad (5) \end{aligned}$$

and, then, transforming this matrix into third-mode tensor. m-mode product is then carried out by multiplying matrix $\boldsymbol{B}$ to mode-m matrix $(\underline{\boldsymbol{G}} \times_l \boldsymbol{A})_{(m)}$

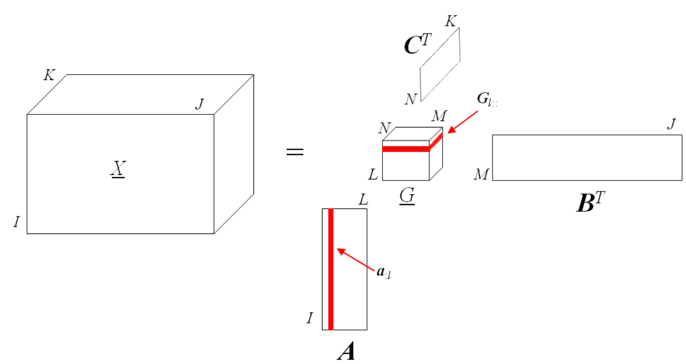


Fig. 5. Joint plot

B. Joint Plot

Joint plot[5][9] is a way to visualize information contained in the decomposed core tensor and factor matrices. Figure 5 shows the Tucker 3 decomposition. A component l of the matrix $\boldsymbol{A}$ is selected and for this component a joint plot is to be made. The corresponding matrix $\boldsymbol{G}_{l::}$ is calculated by multiplying the vector $\boldsymbol{a}_{:l}$ to the core tensor $\underline{\boldsymbol{G}}$. The content of $\boldsymbol{G}_{l::}$ is distributed between matrix $\boldsymbol{B}$ and $\boldsymbol{C}$ as follows:

$$\begin{aligned} \boldsymbol{E}_l &= \boldsymbol{B}\boldsymbol{G}_{l::}\boldsymbol{C}^T = \boldsymbol{B}[\boldsymbol{S}\boldsymbol{V}^{1/2}\boldsymbol{D}]\boldsymbol{C}^T \\ &= [\boldsymbol{B}\boldsymbol{S}\boldsymbol{V}^{1/4}][\boldsymbol{V}^{1/4}\boldsymbol{D}\boldsymbol{C}^T] = \boldsymbol{B}_l\boldsymbol{C}_l^T \end{aligned} \quad (6)$$

$\boldsymbol{G}_{l::}$ is decomposed into $\boldsymbol{S}\boldsymbol{V}^{1/2}\boldsymbol{D}$ by singular value decomposition and the eigenvalues are distributed equally between $\boldsymbol{B}$ and $\boldsymbol{C}$. From the matrices $\boldsymbol{B}_l, \boldsymbol{C}_l$, a biplot of two variables, i.e. questions and objects, is made. There are L possible biplots by selecting the component l.

C. HOSVD

Higher-order SVD (HOSVD)[6] is a method to obtain truncated Tucker decomposition. In this paper, the procedure of HOSVD is explained by using the hypothetical data in Fig.2. First, SVD is applied to mode-r, mode-q, and mode-o matrices $\boldsymbol{X}_{(r)}, \boldsymbol{X}_{(q)}, \boldsymbol{X}_{(o)}$ as follows:

$$\boldsymbol{X}_{(r)} = \boldsymbol{S}_r\boldsymbol{V}_r\boldsymbol{D}_r,\ \ \boldsymbol{X}_{(q)} = \boldsymbol{S}_q\boldsymbol{V}_q\boldsymbol{D}_q,\ \ \boldsymbol{X}_{(r)} = \boldsymbol{S}_o\boldsymbol{V}_o\boldsymbol{D}_o \quad (7)$$

The loading matrices $\boldsymbol{S}_r, \boldsymbol{S}_q, \boldsymbol{S}_o$ are shown in Fig.6. The core tensor $\underline{\boldsymbol{G}}$ is given by

$$\underline{\boldsymbol{G}} = \underline{\boldsymbol{X}} \times_r \boldsymbol{S}_r^T \times_q \boldsymbol{S}_q^T \times_o \boldsymbol{S}_o^T. \quad (8)$$

Figure 7 shows an example of this calculation where zero vectors in $\boldsymbol{S}_r, \boldsymbol{S}_q$ are neglected.

R1	0.417	0.812	0
R2	0.565	0.12	0
R3	0.712	-0.57	0

(a) Loading matrix S_r

Q1	0.453	0.704	0	0
Q2	0.483	0.258	0	0
Q3	0.514	-0.19	0	0
Q4	0.545	-0.63	0	0

(b) Loading matrix S_q

O1	0.353	0.936
O2	0.936	-0.35

(c) Loading matrix S_o

Fig. 6. Loading matrices after SVD

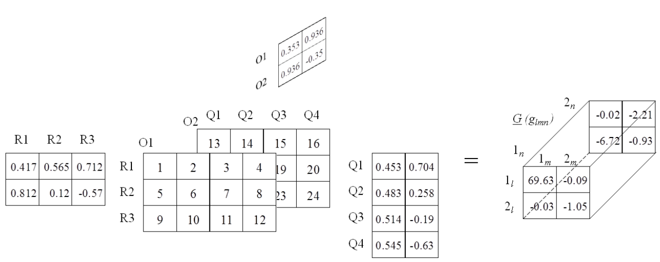


Fig. 7. Transformation of tensor $\underline{\boldsymbol{X}}$ and loading matrices $\boldsymbol{S}_r, \boldsymbol{S}_q, \boldsymbol{S}_o$ to core tensor $\underline{\boldsymbol{G}}$

III. New Visualization Method based on HOSVD

This paper presents a visualization method based on HOSVD. Different from the joint plot in II-B, this method makes biplots of respondents and "questions×objects". The key idea is to select a pair of vectors from loading matrices $\boldsymbol{S}_q$ and $\boldsymbol{S}_o$, and define a score vector and a principal vector of the third-order tensor. A component m of $\boldsymbol{S}_q$ and another component n of $\boldsymbol{S}_o$ is assumed to be selected. These vectors are denoted by $\boldsymbol{s}_{q:m}$ and $\boldsymbol{s}_{o:n}$. The score vector of third-order tensor $\boldsymbol{d}_{mn}$ is expressed as

$$\boldsymbol{d}_{mn} = \underline{\boldsymbol{X}} \times_q \boldsymbol{s}_{q:m} \times_o \boldsymbol{s}_{o:n} \quad (9)$$

Figure 8 shows an example of calculation of score vector of third-order tensor $\boldsymbol{d}_{11}$. Here, $\boldsymbol{s}_{q:1}$ and $\boldsymbol{s}_{o:1}$ are selected and calculation of mode product is carried out. The result $\boldsymbol{d}_{11}$ is shown in Fig.9.

The principal component vector $\boldsymbol{p}_{mn}$ is defined by using Kronecker product as

$$\boldsymbol{p}_{mn} = \boldsymbol{s}_{q:m} \otimes \boldsymbol{s}_{o:n}. \quad (10)$$

The obtained principal component vector $\boldsymbol{p}_{11}$ is shown in Fig.9. In order to explain how the Kronecker product is calculated, let us use a simple case where $\boldsymbol{s}_{q:m} = (1,2,3)^T, \boldsymbol{s}_{o:n} = (4,5)^T$. It is calculated as follows:

$$\boldsymbol{s}_{q:m} \otimes \boldsymbol{s}_{o:n} = (1\times4, 2\times4, 3\times4, 1\times5, 2\times5, 3\times5)^T \quad (11)$$

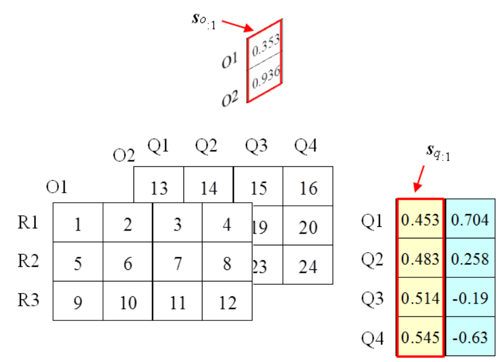
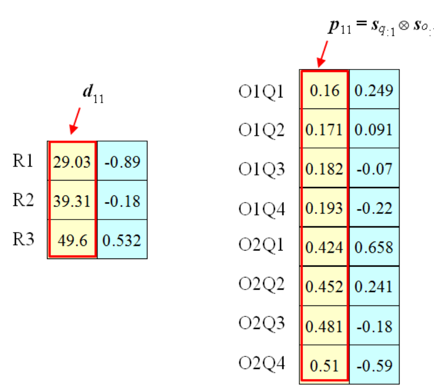


Fig. 8. Calculation of scores of third-order tensor



(a) Score vectors (b) Principal vectors

Fig. 9. Score vectors and principal component vectors of third-order tensor

The eigen value of principal component vector $\boldsymbol{p}_{mn}$ is given by variance of the scores of third-order tensor $\boldsymbol{d}_{mn}$ as

$$\lambda_{mn} = \frac{1}{I-1}\boldsymbol{d}_{mn}^T\boldsymbol{d}_{mn} \quad (12)$$

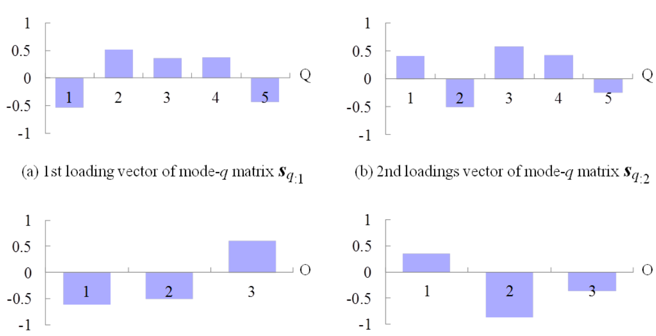
After selecting two pairs of vectors from $\boldsymbol{S}_q$ and $\boldsymbol{S}_o$, we can make a biplot. The significant contribution of this method is that **we can select any pairs of vectors, which have features of respondents' markings, from the loading matrices S_q and S_o, and grasp the features in biplot**.

IV. Experiment

In order to demonstrate the performance of the proposed method, we carried out an experiment using artificial data in Table I. This data is assumed to be obtained from twenty respondents by asking five questions on three objects. The respondents marked from -2 to 2. From this table, a third order tensor $\underline{\boldsymbol{X}} \in \mathbb{R}^{20\times5\times3}$ is made. The element of $\underline{\boldsymbol{X}}$ is denoted by $x_{ijk},\ i = 1, \cdots, 20,\ j = 1, \cdots, 5,\ k = 1, 2, 3$. Figure 10 shows loading vectors of mode-q and mode-o matrices after applying SVD to each mode matrix. The horizontal axis is the question number(top figures) and object number(bottom figures), and the vertical axes are the amount of loadings. First, we selected the first and second loading vectors of mode-q matrix$\boldsymbol{s}_{q:1}$ and $\boldsymbol{s}_{q:2}$, and the first one of mode-o matrix $\boldsymbol{s}_{o:1}$. We calculated the score vectors of third-order tensor $\boldsymbol{d}_{11}, \boldsymbol{d}_{21}$, and the principal component vectors of third-order tensor $\boldsymbol{p}_{11}, \boldsymbol{p}_{21}$.

TABLE I
ARTIFICIAL DATA

	O1					O2					O3				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
R1	-2	2	1	1	-2	-2	2	0	0	-1	2	-2	-1	-1	1
R2	-2	2	0	0	-1	-2	2	1	1	-2	2	-2	-2	-2	2
R3	-2	2	0	1	-2	-2	2	0	1	-2	2	-2	-1	-2	1
R4	-2	2	1	0	-1	-2	2	1	0	-2	2	-2	-2	-1	2
R5	-2	2	-1	0	-2	-2	2	-1	0	-1	2	-2	-1	-1	2
R6	-1	0	2	1	-1	-1	0	1	1	-2	1	-1	-1	-2	1
R7	-1	0	1	2	-2	-1	0	2	2	-1	1	-1	-2	-2	1
R8	-1	0	2	2	-1	-1	0	1	1	-2	1	-1	-1	-1	1
R9	2	-2	-1	-1	0	2	-2	-2	-2	1	-2	2	0	0	-1
R10	2	-2	-2	-2	1	2	-2	-1	-1	0	-2	2	1	0	-2
R11	2	-2	-1	-2	1	2	-2	-2	-1	0	-2	2	0	1	-1
R12	2	-2	-1	-1	0	2	-2	-1	-2	1	-2	2	1	0	-1
R13	2	-2	0	-1	0	2	-2	0	-1	0	-2	2	1	1	-2
R14	0	-1	-2	-1	2	0	-1	-2	-2	1	-1	0	1	1	-1
R15	0	-1	-1	-2	1	0	-1	-1	-2	1	-1	0	2	1	-1
R16	0	-1	-2	-1	2	0	-1	-2	-1	2	-1	0	2	2	-1
R17	-2	2	1	2	-2	2	-2	-1	-1	1	2	-2	-1	-1	2
R18	-2	2	2	1	-1	2	-2	-1	-1	1	2	-2	-1	-2	1
R19	-2	2	1	1	-1	1	-1	-2	-2	2	2	-2	-2	-1	2
R20	-1	1	1	1	-2	1	-1	-2	-2	2	1	-1	-1	-1	1



(a) 1st loading vector of mode-q matrix $\boldsymbol{s}_{q:1}$

(b) 2nd loadings vector of mode-q matrix $\boldsymbol{s}_{q:2}$

(c) 1st loading vector of mode-o matrix $\boldsymbol{s}_{o:1}$

(d) 2nd loading vector of mode-o matrix $\boldsymbol{s}_{o:2}$

Fig. 10. Loading vectors of mode-q and mode-o matrices

Figure 11 shows the biplot. In this paper, the score vectors and the principal component vectors were normalized so that the norms of vectors were 1. The eigen values were omitted, i.e. set to be 1. The circle dots are principal component coefficients of {objects} × {questions} and the diamond dots are scores of respondents. It is known from the loadings $\boldsymbol{s}_{q:1}$ and $\boldsymbol{s}_{o:1}$ in Fig.10 that the coefficients {O1, O2} × {Q1, Q5} and {O3} × {Q2, Q3, Q4} are on the right hand side, and those {O3} × {Q1,Q5} and {O1, O2} × {Q2, Q3, Q4} are on the left hand side. from the loadings $\boldsymbol{s}_{q:2}$ and $\boldsymbol{s}_{o:1}$, {O1, O2} × {Q2, Q5} and {O3} × {Q1, Q3, Q4} are on the upper side, and those {O3} × {Q2,Q5} and {O1, O2} × {Q1, Q3, Q4} are on the lower side. Respondents can be divided into four major groups circled by solid, dashed, one-dot chain, and two-dot chain lines. The corresponding marks are circled in the Table II. The same types of lines are corresponding to each other in Fig.11 and Table II. In the first quadrant, respondents 14, 15, 16 are plotted because they marked high on {Object1} × {Qestion5} (O1Q5 for short), O2Q5, O3Q3, and O4Q4.

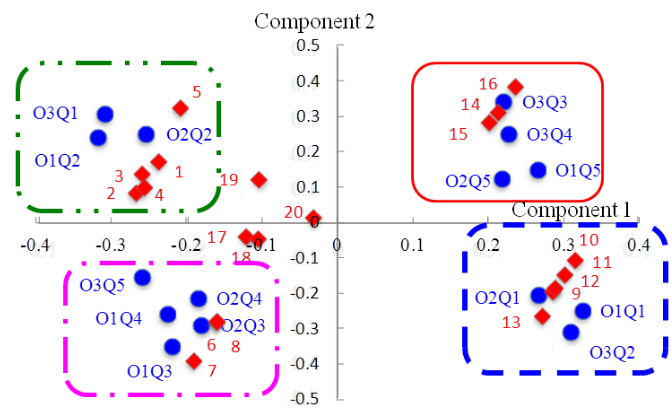


Fig. 11. Biplot obtained by the proposed method($\boldsymbol{p}_{11}, \boldsymbol{p}_{21}, \boldsymbol{d}_{11}, \boldsymbol{d}_{21}$)

For comparison, we applied the conventional PCA to mode-r matrix and made a biplot of the first and the second principal components. Figure 12 shows the result. Three major groups are found. Table III shows the corresponding marks. Respondents encircled in the solid line are found in the solid and dashed lines in Fig.11. Respondents in the dashed line in Fig.12 are in one-dot, and two-dot chain lines in Fig.11. A new group consisting of respondents 17, 18, 19, 20 was found in Fig.12. These respondents are near the origin in Fig.11.

Then we selected the first and second loading vectors of mode-q matrix$\boldsymbol{s}_{q:1}$ and $\boldsymbol{s}_{q:2}$, and the second one of mode-o matrix $\boldsymbol{s}_{o:2}$. The obtained biplot is shown in Fig.13. From the loadings $\boldsymbol{s}_{q:1}$ and $\boldsymbol{s}_{o:2}$ in Fig.10, the coefficients {O2} × {Q1, Q5} are on the right hand side, and those {O2} × {Q2, Q3, Q4} are on the left hand side. The loadings $\boldsymbol{s}_{q:2}$ and $\boldsymbol{s}_{o:2}$ in Fig.10 tell us that the coefficients {O2} × {Q2, Q5} are on the upper side, and those {O2} × {Q1, Q3, Q4} are on the lower side. The group consisting of respondents 17, 18, 19, 20 found by the conventional method in Fig.12 were extracted, and divided into two smaller groups in Fig.13. The

TABLE II
FEATURES EXTRACTED BY THE PROPOSED METHOD$(\boldsymbol{p}_{11}, \boldsymbol{p}_{21}, \boldsymbol{d}_{11}, \boldsymbol{d}_{21})$

	O1					O2					O3				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
R1	-2	2	1	1	-2	-2	2	0	0	-1	2	-2	-1	-1	1
R2	-2	2	0	0	-1	-2	2	1	1	-2	2	-2	-2	-2	2
R3	-2	2	0	1	-2	-2	2	0	1	-2	2	-2	-1	-2	1
R4	-2	2	1	0	-1	-2	2	1	0	-2	2	-2	-2	-1	2
R5	-2	2	-1	0	-2	-2	2	-1	0	-1	2	-2	-1	-1	2
R6	-1	0	2	1	-1	-1	0	1	1	-2	1	-1	-1	-2	1
R7	-1	0	1	2	-2	-1	0	2	2	-1	1	-1	-2	-2	1
R8	-1	0	2	2	-1	-1	0	1	1	-2	1	-1	-1	-1	1
R9	2	-2	-1	-1	0	2	-2	-2	-2	1	-2	2	0	0	-1
R10	2	-2	-2	-2	1	2	-2	-1	-1	0	-2	2	1	0	-2
R11	2	-2	-1	-2	1	2	-2	-2	-1	0	-2	2	0	1	-1
R12	2	-2	-1	-1	0	2	-2	-1	-2	1	-2	2	1	0	-1
R13	2	-2	0	-1	0	2	-2	0	-1	0	-2	2	1	1	-2
R14	0	-1	-2	-1	2	0	-1	-2	-2	1	-1	0	1	1	-1
R15	0	-1	-1	-2	1	0	-1	-1	-2	1	-1	0	2	1	-1
R16	0	-1	-2	-1	2	0	-1	-2	-1	2	-1	0	2	2	-1
R17	-2	2	1	2	-2	2	-2	-1	-1	1	2	-2	-1	-1	2
R18	-2	2	2	1	-1	2	-2	-1	-1	1	2	-2	-1	-2	1
R19	-2	2	1	1	-1	1	-1	-2	-2	2	2	-2	-2	-1	2
R20	-1	1	1	1	-2	1	-1	-2	-2	2	1	-1	-1	-1	1

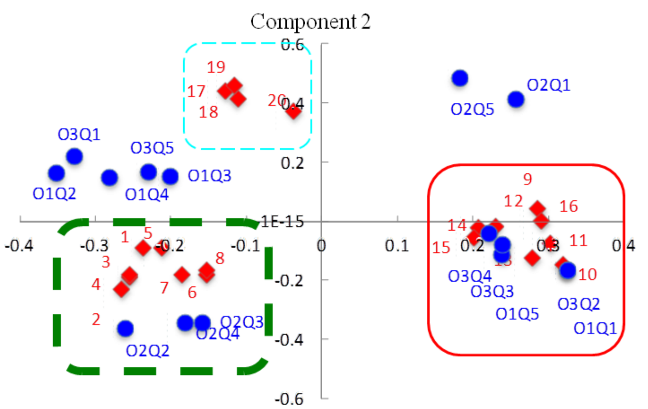


Fig. 12. Biplot of 1st and 2nd principal components obtained by the conventional PCA on mode-r matrix

corresponding marks are shown in Table.IV. Respondents 19, 20 evaluated O2Q5 higher than respondents 17,18 did, and on O2Q1, their marks were reversed. Respondents 19, 20 marked negatively to O2Q3 and O2Q4 more than respondents 17, 18 did. On the other hand, they marked reversely on O2Q2. These markings made them be separated.

Figure 14 shows the biplot of the second and the third principal components obtained by the conventional PCA. Corresponding marks are shown in Table V. Respondents 17, $\cdots$, 20 were plotted away from other respondents, but they were not separated unlike the case in Fig.13.

V. CONCLUSIONS

This paper presented the new visualization method based on HOSVD. We were able to select any pairs of vectors from loading matrices, obtained by SVD on mode matrices, and grasp features of loading vectors in biplots of respondents and questions$\times$objects. The experiment was carried out by using the artificial data. The results showed that the proposed method visualized the interrelationships between the features of selected loading vectors and we could find respondent groups who marked uniquely on some particular objects and questions. Some groups were not found by the conventional PCA. We are currently applying the proposed method to actual questionnaire data and is going to publish the results soon.

TABLE III
FEATURES OF 1ST AND 2ND PRINCIPAL COMPONENTS EXTRACTED BY THE CONVENTIONAL PCA ON MODE-r MATRIX

	O1					O2					O3				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
R1	-2	2	1	1	-2	-2	2	0	0	-1	2	-2	-1	-1	1
R2	-2	2	0	0	-1	-2	2	1	1	-2	2	-2	-2	-2	2
R3	-2	2	0	1	-2	-2	2	0	1	-2	2	-2	-1	-2	1
R4	-2	2	1	0	-1	-2	2	1	0	-2	2	-2	-2	-1	2
R5	-2	2	-1	0	-2	-2	2	-1	0	-1	2	-2	-1	-1	2
R6	-1	0	2	1	-1	-1	0	1	1	-2	1	-1	-1	-2	1
R7	-1	0	1	2	-2	-1	0	2	2	-1	1	-1	-2	-2	1
R8	-1	0	2	2	-1	-1	0	1	1	-2	1	-1	-1	-1	1
R9	2	-2	-1	-1	0	2	-2	-2	-2	1	-2	2	0	0	-1
R10	2	-2	-2	-2	1	2	-2	-1	-1	0	-2	2	1	0	-2
R11	2	-2	-1	-2	1	2	-2	-2	-1	0	-2	2	0	1	-1
R12	2	-2	-1	-1	0	2	-2	-1	-2	1	-2	2	1	0	-1
R13	2	-2	0	-1	0	2	-2	0	-1	0	-2	2	1	1	-2
R14	0	-1	-2	-1	2	0	-1	-2	-2	1	-1	0	1	1	-1
R15	0	-1	-1	-2	1	0	-1	-1	-2	1	-1	0	2	1	-1
R16	0	-1	-2	-1	2	0	-1	-2	-1	2	-1	0	2	2	-1
R17	-2	2	1	2	-2	2	-2	-1	-1	1	2	-2	-1	-1	2
R18	-2	2	2	1	-1	2	-2	-1	-1	1	2	-2	-1	-2	1
R19	-2	2	1	1	-1	1	-1	-2	-2	2	2	-2	-2	-1	2
R20	-1	1	1	1	-2	1	-1	-2	-2	2	1	-1	-1	-1	1

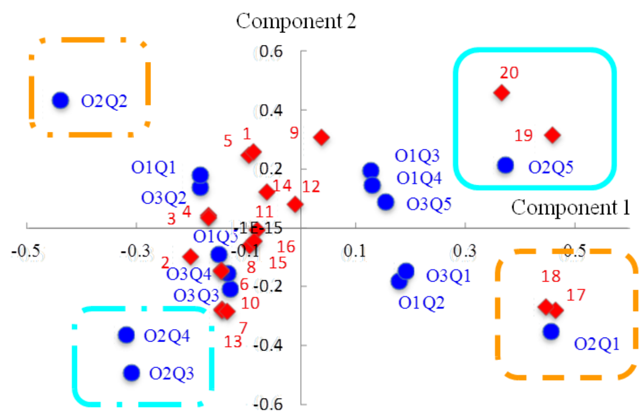


Fig. 13. Biplot obtained by the proposed method$(\boldsymbol{p}_{12}, \boldsymbol{p}_{22}, \boldsymbol{d}_{12}, \boldsymbol{d}_{22})$

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TABLE IV

FEATURES EXTRACTED BY THE PROPOSED METHOD($\boldsymbol{p}_{12}, \boldsymbol{p}_{22}, \boldsymbol{d}_{12}, \boldsymbol{d}_{22}$)

	O1					O2					O3				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
R17	-2	2	1	2	-2	2	-2	-1	-1	1	2	-2	-1	-1	2
R18	-2	2	2	1	-1	2	-2	-1	-1	1	2	-2	-1	-2	1
R19	-2	2	1	1	-1	1	-1	-2	-2	2	2	-2	-2	-1	2
R20	-1	1	1	1	-2	1	-1	-2	-2	2	1	-1	-1	-1	1

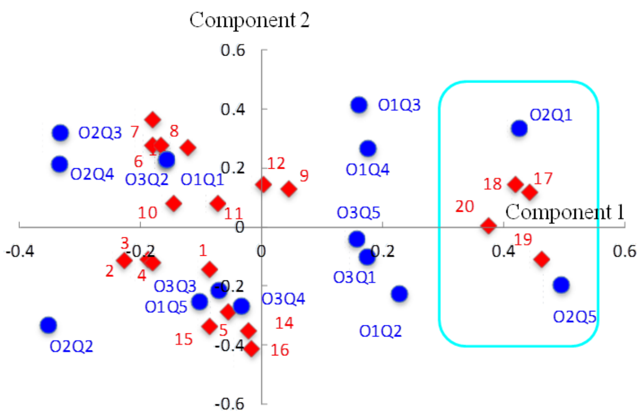

Fig. 14. Biplot obtained by the conventional PCA on respondent-mode matrix(011)

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TABLE V

FEATURES OF 2ND AND 3RD PRINCIPAL COMPONENTS EXTRACTED BY THE CONVENTIONAL PCA ON MODE-r MATRIX

	O1					O2					O3				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
R17	-2	2	1	2	-2	2	-2	-1	-1	1	2	-2	-1	-1	2
R18	-2	2	2	1	-1	2	-2	-1	-1	1	2	-2	-1	-2	1
R19	-2	2	1	1	-1	1	-1	-2	-2	2	2	-2	-2	-1	2
R20	-1	1	1	1	-2	1	-1	-2	-2	2	1	-1	-1	-1	1