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Necessary condition for forward progression in ballistic walking

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Abstract

Ballistic walking mechanism requires an appropriate configuration of posture and velocity at toe-off to avoid backward falling. In this study, we investigate a determinant of the state of the body center of mass (COM) at the toe-off with regard to ballistic walking. We use an inverted pendulum model to represent ballistic trajectories and the necessary condition for forward progression by a simple relationship between the COM states (position and velocity) at toe-off. This condition is validated through a computer simulation of a 7-link musculoskeletal model and measurement experiments of human movements involving stepping and walking. The results of the model simulation were in good agreement with some of the results predicted by the inverted pendulum model. The measurement experiments of walking and stepping movements showed that most COM states at toe-off satisfied the condition for forward progression and the measured trajectories during single support phase were similar to the ballistic trajectories although humans are capable of walking in non-ballistic ways. These results suggest that the necessary condition for forward progression can predict the COM states at toe-off for efficient movement and for avoiding backward falling during single support phase.

Keywords: Ballistic walking, Body center of mass, Inverted pendulum, Phase portrait analysis

1. Introduction

The human walk is characterized by excellent efficiency and stability. It is interesting how the central nervous system (CNS) determines a gait pattern that has an appropriate balance between efficiency and stability. For various combinations of gait velocities and stride length, a human walks using a preferred combination in which the metabolic energy cost is minimum (Elftman, 1966; Zarrugh et al., 1974). The fluctuation in the vertical center of mass (COM) during walking is a functional movement that converts potential energy into kinetic energy and vice versa (Cavagna and Margaria, 1966). The conservation of kinetic and potential energies aids in minimizing the metabolic energy cost. Recent studies showed that minimizing the vertical COM movement leads to an increase in the metabolic cost (Ortega and Farley, 2005; Gordon et al., 2009). Mochon and McMahon (1980) analyzed ballistic walking patterns that simulate movements during single support phase in the absence of active joint torque, under the assumption that muscles act only during the double-support phase (Basmajian, 1976). They showed that the ballistic trajectories using appropriate initial postures and velocities were similar to the trajectories of preferred walking.

Extending the ballistic walking, McGeer (1990) demonstrated a model that could walk down a slight slope without any actuators, which is called passive dynamic walking. The passive dynamic walking pattern is similar to the human walking pattern, which suggests that a determinant of human walking depends on the passive dynamics of the leg mechanism. Although the

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completely passive walker could not walk on level ground because there is no compensation for the energy lost in the foot-floor collision, Collins et al. (2005) demonstrated that biped robots based on passive dynamic walking could walk on level ground using small amounts of power. Based on the passive dynamic walking, Kuo et al. (2005) proposed that the major determinant of the metabolic cost of walking is a redirection of the COM during the double support phase. The inverse dynamics analysis of a musculoskeletal model revealed that there was significant positive power in the ankle joint of the hind leg during the double support phase (Winter, 2005). It appears that the CNS controls the configuration of the posture and its velocity at the toe-off for the following efficient movement.

The ballistic walking mechanism requires an appropriate posture and velocity at the toe-off. When the energy produced during the double support phase is very high, the negative work done during the single support phase detracts from efficiency. On the other hand, when the energy produced is low, additional energy input from the stance hip torque is required (Lewis and Ferris, 2008). Kuo (2002) analyzed a powered walking model and showed that using hip extension moment alone to propel the COM forward is more energetically expensive than using toe-off impulse. Furthermore, a low COM velocity might result in backward falling (Pai and Patton, 1997; Hof et al., 2005). Yang et al. (2007) used a musculoskeletal model with active joint torque to obtain the minimum threshold of the COM state (position and velocity) that was required to avoid falling backwards. Their model could accurately predict the boundary of backward falling of measured slipping movement (Yang et al., 2008). On the other hand, COM states measured during walking at the toe-off showed a large margin for the minimum threshold to avoid backward falling. It can be presumed that COM velocity considerably larger than the threshold is required to achieve the movement using less effort.

The aim of this study is to elucidate the determinant of the COM state at the toe-off for forward progression. We analyze the COM trajectories of human bipedal walking based on the “Linear inverted pendulum mode” theory by Kajita and Tani (1996), which provides prediction and control to bipedal robots on rugged terrain as well as push recovery (Pratt et al., 2006; Rebula et al., 2007). In the present study we show that a necessary condition for forward progression in ballistic walking is given by a simple relationship between the COM position and velocity. First, we examine the necessary condition using a 7-link musculoskeletal model. Second, we compare the measured COM trajectories for natural speed walking and for stepping movements with various step lengths with those estimated by the inverted pendulum model, as shown on the phase portrait. Changing the step length results in a change in the COM position at toe-off; therefore, the dependence of COM velocity on the COM position is evaluated. In addition, we compare the COM trajectories for walking with those for stepping, in order to evaluate the effects of the COM position on different values of COM velocity at toe-off. We expect the necessary conditions for stepping to be more stringent than those for walking.

2. Necessary condition for ballistic walking

We analyze the ballistic COM trajectory by applying a linear inverted pendulum model during the single support phase, as proposed by Kajita and Tani (1996). As shown in Fig. 1A, the COM is regarded as a point mass. Instead of the position of center of pressure, the ankle joint of the stance leg is assumed to be the pivot of the inverted pendulum because the foot pressure sensors described in section 3.2.2 could not measure the center of pressure. The relationship between the horizontal COM position and the ankle joint position is represented by x^{com}. The equation of motion linearized around an equilibrium point ($x^{com} = 0$) is given as

$$\ddot{x}^{com} = \omega^2 x^{com}, \omega = \sqrt{\frac{g}{l}}, \tag{1}$$

where ω is the natural frequency, g is the gravity acceleration, and l is the distance between the ankle joint and the COM position. When the ankle joint moment is zero, i.e., when a ballistic movement is performed, the relationship between the COM position and velocity is given by the law of mechanical energy conservation (Pratt et al., 2006).

$$\frac{1}{2}\{\dot{x}^{com}(t)\}^2 - \frac{1}{2}\omega^2\{x^{com}(t)\}^2 = \frac{E}{M}, \tag{2}$$

where E is the constant mechanical energy and M is body weight. To simplify the analysis, mechanical energy per unit mass is defined as $\tilde{E} = E/M$ (J/kg). The first term of equation (2) represents the kinetic energy, and the second term is the potential energy (see Appendix A for the supplemental description). The gravitational potential energy acts a spring with negative stiffness and causes instability about the equilibrium point. The relationship between the COM position and velocity during ballistic movement is represented by a hyperbolic curve. We assume that the COM position is posterior to the ankle position at the time of toe-off (t_{to}): $x^{com}(t_{to}) < 0$ and that the velocity is in the progressional direction: $\dot{x}^{com}(t_{to}) > 0$. The mechanical energy must be a positive value so that the COM position moves toward the equilibrium point ($x^{com} = 0$). Therefore, the necessary condition for forward progression without energy input is expressed as

$$\dot{x}^{com}(t_{to}) > -\omega \ x^{com}(t_{to}) \tag{3}$$

Equation (3) indicates that the COM velocity must be larger than the product of the natural frequency and the COM position in the posterior direction.

Fig. 1C shows a phase portrait of the COM state space. The hyperbolic curves (dashed lines) indicate the contour lines of the mechanical energy per unit mass (each interval is 0.1 J/kg); further, they correspond to the ballistic COM trajectories. The dashed-thick line is the boundary of the necessary condition for forward progression represented by equation (3). In order to investigate the condition for forward progression, we focus on the second quadrant ($x^{com}(t_{to}) < 0$ and $\dot{x}^{com}(t_{to}) > 0$). In the area where the mechanical energy per unit mass is lower than that on the boundary line, i.e., for $\tilde{E} < 0$, the COM position cannot reach the equilibrium point, and this leads to backward movement. For $\tilde{E} = 0$, the state leads to the equilibrium point along the boundary line. When the COM state is located over the boundary, i.e., when $\tilde{E} > 0$, the trajectory passes through the equilibrium point and this enables forward progression without energy input; in this case, the movement time depends on the mechanical energy.

3. Material and Methods

3.1. Simulation of musculoskeletal system

In the simulation experiments, we compared the ballistic movements predicted by the inverted pendulum model with those calculated by the human musculoskeletal model. Because we examine the predicted COM position and velocity during the single support phase using the inverted pendulum model, the COM trajectories during the single support phase were simulated with various initial toe-off values by the musculoskeletal model. As shown in Fig. 2, the human musculoskeletal system is represented by a 7-link model in the sagittal plane. The body segments include the head-arms-trunk (HAT), thighs, shanks, and feet. The anthropometric parameter values shown in Table 1 were determined by the multiple regression of height and weight (1.75 m and 62 kg) (Zatsiorsky and Seluyanov, 1983). The generalized coordinates *q* are defined as the position of the hip joint in the work space (*x*, *y*) and the body segment angles related to the progressional axis $(\phi_1, \cdots, \phi_7)$. The equation of motion of the 7 -link model is represented as

$$M(q)\ddot{q} + B(q)H(q,\dot{q}) + g(q) = S(q)\gamma + D\{u - \tau(q,\dot{q})\} \tag{4}$$

where $M(q)$ is the inertial term, $B(q)h(q,\dot{q})$ is the coriolis and centrifugal term, $g(q)$ is the gravity term, $S(q)\gamma$ is the ground reaction term, and $D\{u-\tau(q,\dot{q})\}$ is the joint torque term. The ground reaction force γ is calculated by the spring-damper model (Ogihara and Yamazaki, 2001). τ is the passive joint moment of viscosity and elasticity given by Andu and Davy (1985).

u is the active joint torque activated by muscle contraction. Since ballistic walking without active torque could not be performed due to flexion movements of the stance hip and knee, the torque of the hip and knee joint of the stance leg was activated. In addition, the ankle joint torque was applied to ensure sufficient foot-floor clearance. The active joint torque values are given by the following proportional-derivative (PD) control.

$$u_{hip}^{st} = K_{hip}\delta_{HAT} - D_{hip}\dot{\delta}_{HAT} \tag{5}$$

$$u_{knee}^{st} = K_{knee}\theta_{knee}^{st} - D_{knee}\dot{\theta}_{knee}^{st} \tag{6}$$

$$u_{ankle}^{sw} = K_{ankle}\theta_{ankle}^{sw} - D_{ankle}\dot{\theta}_{ankle}^{sw} \tag{7}$$

u_{hip}^{st}, u_{knee}^{st}, and u_{ankle}^{sw} are the joint torques of the stance hip, knee, and swing ankle, respectively. δ_{HAT} is the HAT segment angle with respect to the vertical axis. K_{hip} and D_{hip} are the proportional and derivative gains, respectively, required to maintain an upright posture of the trunk; their values are 25000 Nm/rad and 100 Nms/rad, respectively. θ_{knee}^{st} is the angle of the stance knee joint. K_{knee} and D_{knee} are feedback gains to fix the stance knee joint at maximum extension; their values are 400 Nm/rad and 40 Nms/rad, respectively. θ_{ankle}^{sw} is the ankle joint angle of the swing leg, and its feedback gains K_{ankle} and D_{ankle} are 5 Nm/rad and 5 Nms/rad, respectively. It should be noted that the system is not completely passive because the hip joint torque by equation (5) can produce mechanical energy.[1]

In the computer simulation, the initial condition was determined by the state at toe-off. The necessary condition for forward progression was validated by the simulated COM trajectories with various values of mechanical energy per unit mass at toe-off. Due to redundancy between the energy and the generalized coordinates, it was difficult to determine the initial posture and velocity. First, we defined an initial posture based on the measured angles of segments: however, the HAT angle was set to 90 deg, the stance knee angle was 0 deg, and the angle of the stance foot was 90

[1] The positive power of stance hip was produced until the COM reached the equilibrium point, and then the negative power was used. The peak of the positive power ranged from 0.2 to 0.3 W/kg, depending on the initial velocity.

deg (foot-flat position). Second an initial COM velocity was calculated from the COM position and mechanical energy per unit mass $\tilde{\tilde{E}}$.

$$\dot{x}^{com}(t_{to}) = \sqrt{\omega^2\{x^{com}(t_{to})\}^2 + 2\tilde{\tilde{E}}} \tag{8}$$

ω is given by *g/l*, where *l* is the distance between the COM and the ankle in the initial posture. The values of the mechanical energy per unit mass were varied from -0.1 to 0.4 J/kg. Finally an initial velocity was determined by the COM velocity and some assumptions. Since the measured segment velocity implied that the swing leg came in contact with the floor before moving forward, the velocity of swing leg was set by trial and error in order to avoid foot- floor collision. To reduce the number of parameter in the trial and error tuning, the values of angular velocity of the HAT and stance foot were assumed to be 0 deg/s based on the measured data of walking. The horizontal and vertical velocities of the swing ankle were 1.3 and 1.6 m/s, respectively, and the vertical velocity of the swing toe was 1.9 m/s. The integral calculation was terminated when the heel of the swing leg made contact with the ground.

3.2. Measurement experiments

3.2.1. Subjects

Five healthy young subjects participated in this experiment. The mean age of the subjects was 24.6 ± 3.4 years (mean ± SD), the mean body mass was 63 ± 2.3 kg, and the mean height was 170 ± 3.4 cm. The experimental procedure was approved by The Committee of Nagoya University in order to protect human subjects. All subjects gave their informed consent in writing prior to participation in the experiment.

3.2.2. Apparatus and protocol

The experimental setup is shown in Fig. 3. The kinematics data were collected by a three-dimensional position measurement device (Optotrak Certus, Northern Digital Inc.) at 100 Hz. The LED markers were attached to the body as shown on the right side of Fig. 3. In order to acquire foot contact information, pressure sensors (Flexiforce, Tekscan Inc.) were attached to the shoe soles, and pressure data were simultaneously recorded by a 12-bit AD converter (ADI12-8(USB)GY, CONTEC Co., Ltd.). We performed two experiments consisting of stepping and walking movements. In the stepping task, the subjects were asked to move forward from a stepping posture to a quiet standing posture at a self-selected speed and three different step lengths of 0.4, 0.5, and 0.6 m; 10 trials were performed for each step length. In the walking task, subjects were asked to walk on level ground along a line on the floor. Similar to the stepping task, walking trials were performed for different stride lengths. The indices for foot placement for a given step length were attached to the floor, and the subjects were asked to walk with the given step length and at a self-selected speed. The subjects walked approximately 8 m and 10 trials were performed for each step length.

3.2.3. Data analysis

Kinematic data were low-pass filtered with an FIR filter with a cut-off frequency of 10 Hz. For the gait movement, the gait cycle was defined from the instance of one left toe-off to the next left toe-off. Data-sets consisting of the foot pressure and kinematics data during a cycle were obtained using the foot pressure data. The data-sets of the first two cycles and last two cycles in a trial were removed from the analysis, in order to evaluate steady-state walking (Miff et al., 2005).

To avoid the bias of the distribution of the COM states at toe-off, the number of data-sets was aligned among the individual conditions of stride length. Ten data-sets were sampled randomly for each condition. The time at toe-off and that at heel-contact of both feet were detected in both the stepping and walking tasks.

The COM position was calculated from the angle of body segments related to the vertical axis and the segment parameters of mass and the COM, which were specified by the multiple regression of weight and height (Zatsiorsky and Seluyanov, 1983). For the phase portrait analysis, the position of the progressional axis was shifted such that the mean ankle position was at the origin. The natural frequency ω was calculated by the gravity acceleration and by the mean distance between the COM position and the stance ankle from left heel-contact to right toe-off. The mean ω with SD of the subjects was 3.36 ± 0.02 (1/s).

The effects of the tasks (stepping and walking) and the step lengths (0.4, 0.5, and 0.6 m) on the mechanical energy per unit mass given by equation (2), the COM position, and the velocity at toe-off were examined by two-way repeated-measures analysis of variances (ANOVAs). Following significant interaction, one-way ANOVAs and a post hoc test with Tukey's honestly significant difference criterion were carried out in order to elucidate the effects of the step length in each task. A post hoc paired *t* test was used to evaluate the differences between stepping and walking in each step length.

4. Results

4.1. Model simulation

Fig. 4A shows stick diagrams of simulated ballistic movements for mechanical energy values of 0.1, 0.035, and 0.3 J/ kg. Postures shown by solid lines are initial states at toe-off, and postures shown by dashed lines are states at every 100 ms. When the necessary condition for forward progression is not satisfied ($\tilde{E}$ = 0.1 J/kg), the movement pattern indicates backward falling. For $\tilde{E}$ = 0.035 J/kg, the posture at heel strike indicates a standing posture. For larger mechanical energy ($\tilde{E}$ = 0.3 J/kg), the COM position moves forward and the pattern indicates a walking-like movement. Fig. 4B shows trajectories of the COM on the phase portrait. The velocity decreases when the position is negative, and it increases in the positive area. The COM trajectories of the 7-link model roughly correspond to the trajectories of the inverted pendulum model. The difference of predicted boundary of backward falling between the inverted pendulum and 7-link model is 0.035 J/kg.

4.2. Gait measurement experiments

Fig. 5 shows stick diagrams of measured stepping and walking movements. The time interval between postures is 100 ms. The COM position at toe-off is located posterior to the stance ankle position in stepping and walking. For stepping movements, the trunk angle is inclined forward at toe-off, while the postures are upright in walking movements. In addition, the stance ankle angle during stepping is more dorsiflexed position than that during walking at toe-off.

Fig. 6A shows the means and SDs of the mechanical energy (left), the COM position (center), and the velocity (right) at toe-off. For the mechanical energy, results of the two-way repeated-measures ANOVA showed significant effect of the task (stepping and walking), the step length, and interaction ($P < 0.001$). The significant interaction indicates that there was no significant effect of the step length in the stepping movement ($P = 0.086$), while there was a significant effect of the step length in walking ($P < 0.001$). The mechanical energy increased with an increase in the step length in walking ($P < 0.001$). In addition, differences of stepping and walking were significant for all step lengths ($P < 0.001$).

The significant main effects on the COM position at toe-off were found for the task ($P < 0.001$) and step length ($P < 0.001$). The interaction was also significant ($P < 0.001$). The COM position became more posterior with an increase in the step length under both stepping and walking ($P < 0.001$). There was no significant difference in stepping and walking for the step length of 0.4 m ($P = 0.23$), whereas there was a significant difference for step length of 0.5 m ($P < 0.01$) and 0.6 m ($P < 0.001$). For the COM velocity, the main effects of the tasks, step length, and interaction were significant ($P < 0.001$). The one-way ANOVAs and the post-hoc test for the step length showed that the COM velocity increased for a large step length for both stepping and walking ($P < 0.001$). Significant differences were found in the COM velocities for all step lengths ($P < 0.001$).

Fig. 6B shows states at toe-off for stepping and walking for all subjects. The squares and triangles denote the states in stepping and walking, respectively. Although the COM position was located posterior to the ankle joint position, the necessary condition was satisfied for most states. The COM velocity had a higher value when the COM position was located more posterior. Significant correlations between the COM position and velocity were found for stepping ($R^2 = 0.84$) and for walking ($R^2 = 0.65$). The distribution of the states for stepping was located on the asymptotic line that indicates the boundary of forward progression. The distributions of walking were located beyond the boundary. The solid lines represent 95% confidence ellipses of stepping and walking. The major axis of the ellipse in stepping approximately coincided with the asymptotic line, and the slope of the axis increased with velocity. In addition, the minor axis of the ellipse in stepping was distinctly small.

Fig. 7 shows trajectories of the COM state on the phase portraits in the case of a typical subject. Under all conditions, the trajectories during the double support phase (dashed lines) indicate an increase in mechanical energy. The trajectories during the single support phase (solid lines) show the features of the ballistic trajectories.

5. Discussion

A major finding of this study was that the necessary condition for forward progression as derived from the ballistic movement of the inverted pendulum could predict the COM states at toe-off in human ballistic movements. The COM trajectories of the musculoskeletal model showed the same features of hyperbolic curves as the ballistic trajectories of the inverted pendulum; further, there was only a slight difference in the prediction of the boundary for forward progression. Most of the measured COM states at toe-off satisfied the necessary condition at various step lengths even though it is feasible to walk in various non-ballistic ways. Particularly, in stepping movement, the COM states were located on the boundary of the condition and the variance normal to the asymptotic line is distinctly small. In addition, measured COM trajectories in stepping and walking movements during single support phase were similar to the ballistic trajectories. These results suggest that human gait pattern strongly depends on the passive dynamics of the musculoskeletal system.

5.1. Necessary condition for forward progression

From the linear inverted pendulum model by Kajita and Tani (1996), the necessary condition for forward progression in ballistic walking is represented by a simple relationship between the COM states at toe-off. They demonstrated that the body trajectories of a biped robot were predicted by the model. In addition, they proposed a control method of a biped robot to walk on rugged terrain based on the model. The linear inverted pendulum mode provides a closed form solution of the COM trajectory which is useful to predict and control the bipedal mechanism. Pratt et al. (2006) proposed a “capture point” which is an appropriate foot placement for push recovery step predicted by the relationship between the COM position and velocity. The capture point is determined such that a COM state at foot contact is on the boundary line of forward progression, and the COM state converges to the equilibrium point, which corresponds to a quiet standing posture, along the asymptotic line. Using the linear inverted pendulum mode, they addressed the stabilization for external force by the appropriate foot placement, whereas we analyzed the necessary condition to achieve ballistic walking without backward falling.

We found a slight difference between the simulated trajectories obtained by the musculoskeletal model and the hyperbolic curves indicating ballistic trajectories of the inverted pendulum (Fig. 4B). For the forward progression boundary, the difference in mechanical energy per unit mass between the inverted pendulum model and the 7-link model was 0.035 J/kg, and the COM trajectories were identical to the hyperbolic curves on the phase portrait. Therefore, it can be inferred that the COM state at toe-off is a dominant factor in determining the boundary of forward progression during the ballistic movement.

The necessary condition is similar to the stability assessments for backward balance loss reported in previous researches (Pai and Patton, 1997; Hof et al., 2005). An essential difference is that in our method, the necessary condition is derived from ballistic movements, whereas in previous researches, the necessary condition was derived by considering active joint torque for balance control. Yang et al. (2007, 2008) analyzed the minimum requirements for the state of the COM at toe-off for initiating backward falling using a musculoskeletal model with actuated joints. Although the experimental data of Yang et al. (2007) were beyond the minimum COM state predicted by their model, there was a large margin between the experimental data and the boundary. In the necessary condition derived by us, the area representing backward falling includes the states that can recover the balance with sufficient joint torque. Therefore, we consider the necessary condition in the present study to be more stringent than the boundary suggested by Yang et al. Nevertheless, most of the measured data satisfied our condition, suggesting that the determinants of the COM state at toe-off are not only the stability for backward falling but also the efficiency of the subsequent movement.

5.2. COM trajectories during single support phase

The relationship between the COM position and the velocity at toe-off showed asymmetrical interactions. The COM velocity at toe-off depended on the movement velocity and step length. The dependence of the COM velocity on the step length was explained as a feature of ballistic movement, by which higher COM velocity is required for a more posterior COM position. The COM position at toe-off showed lesser dependence on the COM velocity. The variability in the COM position at toe-off can be severely limited by the kinematic constraints of the step length. However, the COM positions for stepping were located significantly anterior than the COM positions for walking movements for the step lengths of 0.5 and 0.6 m; this suggests the existence of an arrangement in which the COM position is located anteriorly by trunk inclination, ankle dorsiflexion, and knee flexion, in order to compensate for the insufficient COM velocity, and therefore, enable the COM state to satisfy the necessary condition for forward progression.

The phase portrait analysis shows that the simulated and measured COM trajectories in the single support phase have features of ballistic trajectories of the inverted pendulum model. The measured trajectories for all step lengths for stepping correspond to the asymptotic line with a mechanical energy per unit mass of 0 J/kg. These results suggest that the COM state at toe-off is coordinated such that the ballistic movement with a desired gait velocity and stride length could be achieved. Push-off and additional movements during the double support phase, such as trunk inclination and stance ankle dorsiflexion, might play a role in the configuration of the COM state at toe-off. Several researches have hypothesized that the COM position and velocity are utilized for balance control during quiet standing (Masani et al., 2003) and sit-to-stand movement control (Scholz and Shöner, 1999). The analysis of slipping during walking showed that the COM states at toe-off changed significantly to avoid backward falling; this occurred due to adaptation as a result of exposure to slipping (Bhatt and Pai, 2005, 2009). Furthermore, a control model utilizing the ballistic COM movement by Kooij et al. (2003) showed robust behaviors of walking under various constraints with respect to the skeletal system and environments. It is plausible that COM states are associated with the postural control and trajectory organization of full body movements.

5.3. Limitations of our model

In the model simulation of musculoskeletal system, some assumptions were required because completely ballistic walking was not feasible. As mention in section 3.1, the musculoskeletal model is not passive due to the stance hip torque by PD control which maintained the upright trunk orientation. The profiles of the power by the torque were similar to data of human gait (Winter, 2005). Extension moment at the stance hip was activated when the COM approached the equilibrium point. The trunk inclined anteriorly without the hip extension moment because the horizontal velocity of hip position was decelerated by gravity. The peak of the positive power was less than 20 % of the value of human walking. Due to the simplification of the linear inverted pendulum model, it is not clear how the power of the stance hip affects the results of the phase portrait analysis. Other assumptions were about the initial velocity. We determined the velocity of the swing ankle and toe by trial and error because a swing movement was not feasible using the measured data due to a collision between the toe and the ground. This difference might be caused by lack of active torque of the swing hip and knee joints in our model. In addition, it was observed that the trunk inclined anteriorly in the stepping movement, which is inconsistent with our assumptions of the trunk posture and velocity. Although our model simulation roughly predicts the COM trajectories during single support phase, it is insufficient to predict the movements of each segment due to the assumptions of the states at toe-off and the control of joints.

The required mechanical energy for forward progression in the 7-link model is slightly larger than that in the inverted pendulum model, and the difference in mechanical energy per unit mass of the predicted boundary is 0.035 J/kg. This difference might be due to the influence of the swing leg movement. The swing movement is primarily dominated by the dynamics of a single pendulum, and its trajectory contradicts the ballistic movement of the inverted pendulum. The mechanical work done by the stance hip torque might also relate to the inconsistency. Quantitative prediction using a simple model is difficult (Selles et al., 2001; Buczek et al., 2006), while accurate prediction using a detailed model requires precise identification of model parameters. However, we believe that a simple model is useful for elucidating the dominant factors that influence the trajectories of walking, and for estimating the effects of other factors that are not included in the simple model.

We observe a significant interaction between the task (stepping and walking) and step length with respect to the mechanical energy at toe-off. This interaction indicates that the effects of step length are different for stepping and walking. For walking movement, the mechanical energy at toe-off is dependent on the step length. Since the effects of the increase in the mechanical energy cannot be interpreted by the inverted pendulum model, it appears that there are other effects associated with the COM state at toe-off. The coupling of stance and swing leg movements is important for carrying out ballistic walking with the desired stride length. Since pendulum-like swing movement might restrict the range of the swing time, the possible range of swing time might be related to the dependence of the mechanical energy at toe-off on the step length. In addition, our simulation lacks the movements during the double support phase that should be taken account for energy input (Kuo et al., 2005). A limit cycle model including a double support phase might provide a possible explanation with respect to the observed combinations of COM position and velocity at toe-off.

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Appendix A. Potential energy of a linear inverted pendulum

The complement of the term representing the potential energy in equation (2) from the formula Mgh is presented, where M is the body mass, g is gravity acceleration, and h is the vertical position from a reference position of zero potential energy. As shown in Figure 1B, we assumed the potential energy P to be zero when the COM position is at the equilibrium point. For a COM position x^{com} and a link angle with the vertical axis θ, the potential energy is given by

$$P = Mgh = -Mgl(1-\cos\theta) = -\frac{Mgl\sin^2\theta}{1+\cos\theta} \cong -\frac{1}{2}M\omega^2(x^{com})^2, \qquad (9)$$

where we applied an approximation $\cos\theta = 1$ when $\theta \cong 0$.

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Table 1: Anthropometric parameter values of 7-link model.

Segment	Length (m)	Mass (kg)	Center of mass (m)	Inertia (kg m^2)
HAT	0.61	38.2	0.35	5.4
Thigh	0.36	8.4	0.13	0.30
Shank	0.37	2.6	0.21	0.14
Foot	0.09	0.9	0.04	0.005

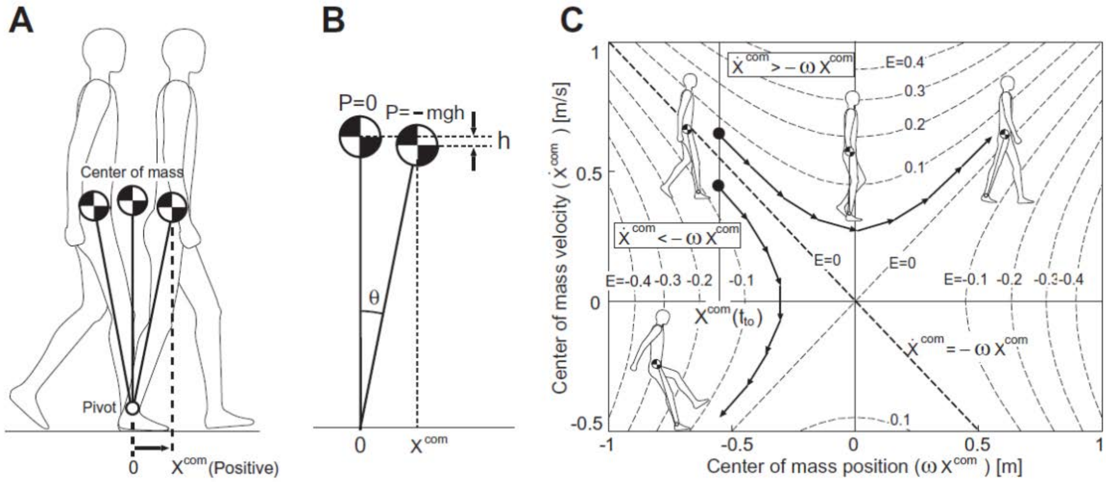

Figure 1: Inverted pendulum model that simplifies dynamics of COM movement during single support phase. A: Definition of inverted pendulum model. The point mass of the inverted pendulum corresponds to the body center of mass, and the pivot corresponds to the ankle joint of the support leg. B: Potential energy of inverted pendulum model. C: Phase portrait of position and velocity of the body center of mass. The position represents the product of the center of mass position and natural frequency ($\omega\ x^{com}$). The relationship between the COM position and velocity is represented by a hyperbolic function described in equation (2). The interval between the hyperbolic curves is 0.1 J/kg. The thick line shows the critical velocity of the position required for the necessary condition to be satisfied.

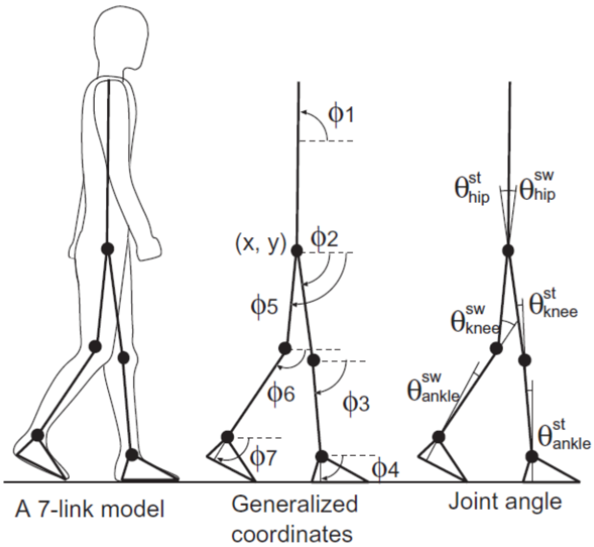

Figure 2: Seven-link model of a human musculoskeletal system consisting of trunk, thighs, shanks, and feet in the sagittal plane.

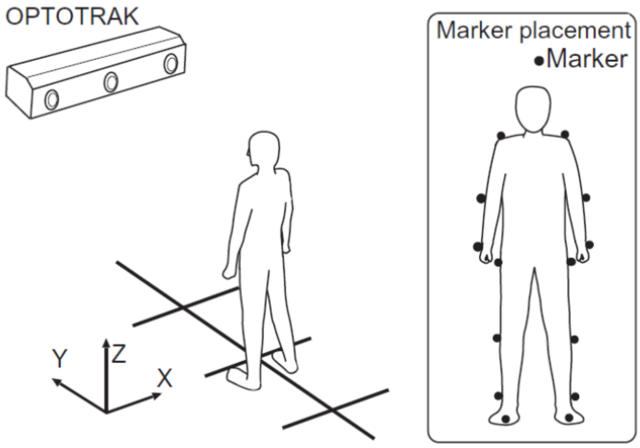

Figure 3: Experimental setup for gait movement measurements. Fourteen markers were attached to the bodies of subjects.

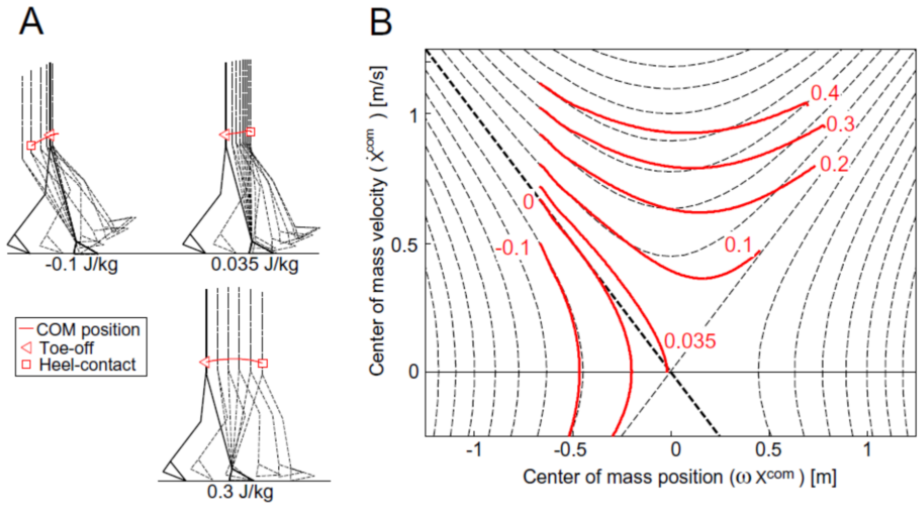

Figure 4: Simulation results of musculoskeletal model. A: Stick diagrams of simulated ballistic movements for mechanical energies of -0.1, 0.035, and 0.3 J/kg. The solid and dashed lines denote the postures at toe-off and the postures at 100 ms time intervals, respectively. The solid lines around the hip joint denote COM trajectories. The triangles and squares denote the COM position at toe-off and heel contact, respectively. B: Phase portrait of position and velocity of body center of mass for ballistic movements simulated by 7-link model. Trajectories of the COM are shown for mechanical energy per unit mass $\tilde{E}$ of -0.1, 0, 0.035, 0.1, 0.2, 0.3, and 0.4 J/kg.

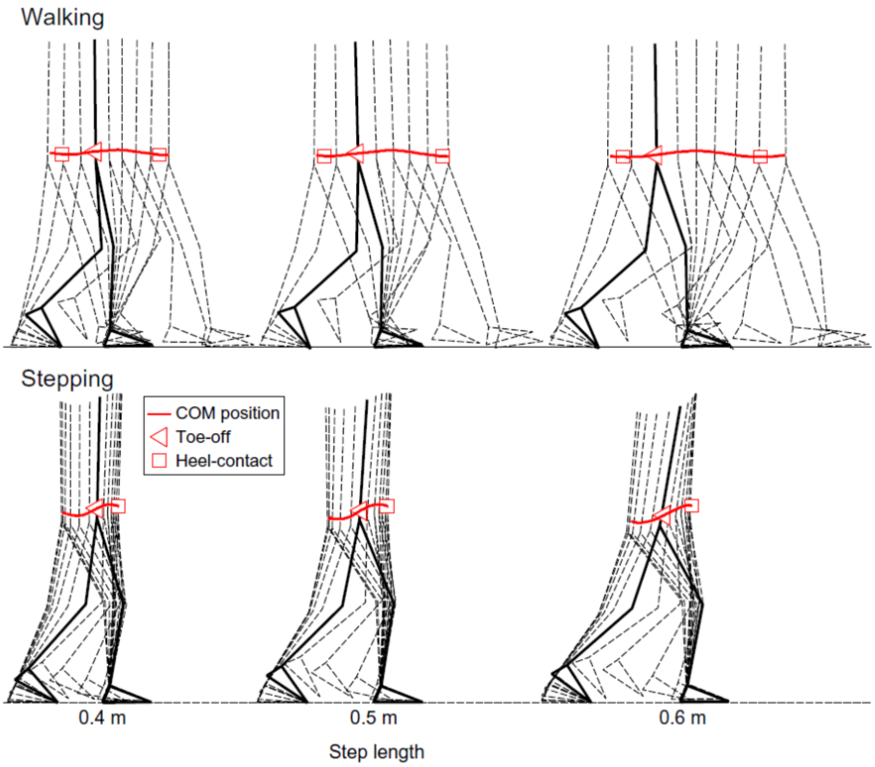

Figure 5: Stick diagrams of measured walking movements (upper images) and stepping movements (lower images). Step lengths are 0.4 m (left), 0.5 m (center), and 0.6 m (right). The solid lines denote the postures at toe-off. The time interval between consecutive stick diagrams is 100 ms. The dashed and solid lines around hip joint show the COM trajectories during the double support phase and single support phase, respectively. The triangles and squares denote COM positions at toe-off and heel contact, respectively.

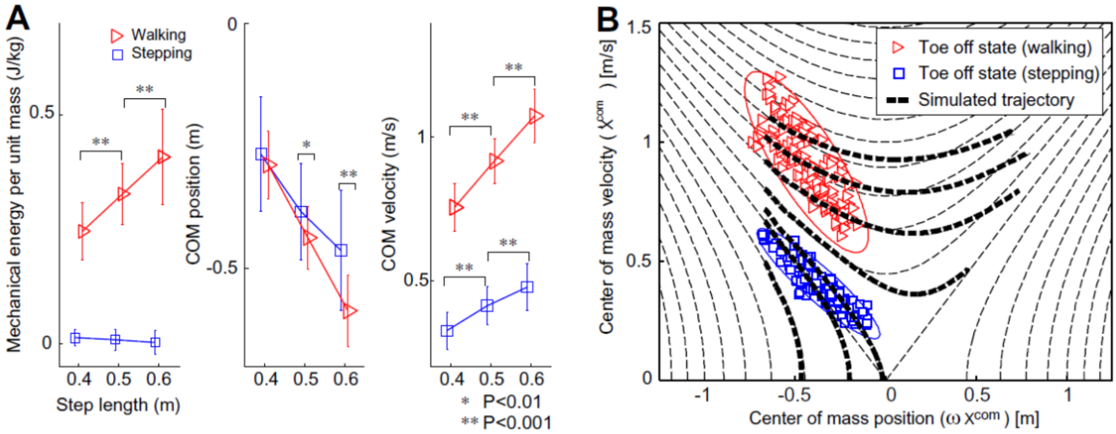

Figure 6: Measured COM position, velocity and mechanical energy per unit mass based on inverted pendulum model. A: Means and SDs of mechanical energy, COM position, and COM velocity at toe-off for different tasks of walking and stepping with various step lengths. B: Measured COM states of all subjects at toe-off. The triangles and squares denote the states of walking and stepping, respectively. The solid lines show the 95% confidence ellipse. The lines shown in Fig. 4B are also shown by dashed thick lines.

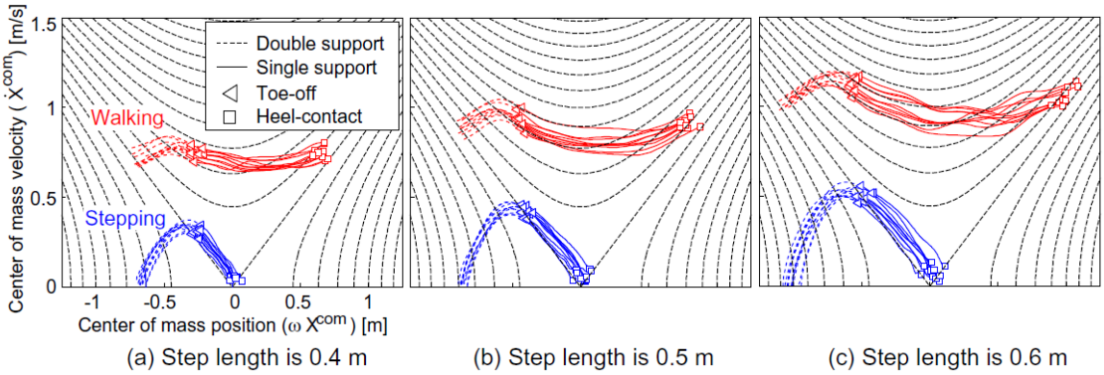

Figure 7: Phase portrait of measured COM trajectories of walking and stepping for a typical subject. (a), (b), and (c) show the results for step lengths of 0.4 m, 0.5 m, and 0.6 m, respectively. The solid and dashed lines show the trajectories during the single and double support phases, respectively. The triangular and square markers denote the states at toe-off and heel contact, respectively.