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A Hybrid Damper Composed of Elastomer and Piezo Ceramic for Multi-Mode Vibration Control*

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Abstract
This study develops a hybrid damper for multi-modal vibration by combining an elastomer and a piezo ceramic element. The basic idea is to obtain satisfactory damping in the low frequency range, where elastomer does not work well, by introducing a piezoelectric element. In this hybrid damper, the elastomer comprises a dynamic absorber with a metal plate of small mass that deals with higher-frequency vibration, while piezo ceramic comprises a shunt damper with an electric shunt circuit that works for lower-frequency vibration. The effectiveness of this approach is examined through vibration control of a cantilever beam. The aim is to suppress the 1st mode vibration with the shunt damper and the 2nd mode with the dynamic absorber. With appropriate tuning procedures for the control parameters, vibration suppression of more than 10dB is attained in each mode.

Key words: Vibration Control, Elastomer, Piezoelectric Element, Shunt Damper, Dynamic Absorber, Hybrid Damper, Multi Mode Vibration

1. Introduction

Elastomer, also known as vibration-proof rubber, has been widely used for vibration control of mechanical elements such as housing and chassis of consumer electronics and HDD in personal computers, benefiting from advantages such as low cost, light weight, and simple structure. Considerable damping can easily be obtained for a passive mechanism by just putting an elastomer patch on a vibrating surface.

At higher frequencies, its viscoelasticity effectively dissipates vibration energy and blunts resonant peaks without designating the target peak. However, its performance deteriorates with a decrease in frequency because of a mechanical characteristic whereby elastomer becomes a rigid body at lower frequencies.

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While the manufacturers are working to develop an elastomer with low hardness (5-10 Hs in JIS-A) that guarantees sufficient damping in low frequency range, housings and chassis of consumer electronics are getting thinner and thinner. Under these circumstances, it’s very difficult for an elastomer as a single item to attain a satisfactory performance especially below 100Hz.

For example, silicon-oil casing is introduced combining with elastomer for vibration suppression of a mobile disc dive such as a car-mount DVD where the low frequency vibration from around 10Hz is at issue. This kind of countermeasure is, however, not adequate in handiness and productivity. Therefore it is expected to introduce a technique that can compensate the performance of elastomer at low frequencies without spoiling the advantages of elastomer such as passive mechanism and simplicity.

In this study, a hybrid damper is considered that adds a new value to an elastomer from the viewpoint of smart material. The basic concept of a hybrid damper is to share the vibration control between an elastomer and another material according to the frequency range of the vibration to be suppressed. This hybrid damper is expected to have an integrated construction in one united body and to be easily plastered on a vibrating surface.

A piezoelectric element is a representative of smart material and is widely studied for vibration suppression from both viewpoints of active control and passive control. Shunt damping (1) is well-suited for the hybrid damper due to its passive mechanism where a piezoelectric element works with an electrical shunt circuit. A piezoelectric element enables the control system to be very compact compared to a mechanical dynamic absorber and an active mass damper because it has both functions of an actuator and a sensor. This is also advantageous to the hybrid damper to be compact.

A mechanical dynamic absorber (2) is another passive damping method. Ozaki tried to employ an elastomer for a dynamic absorber (3) taking notice that the viscoelasticity of elastomer is replaceable for both components of a spring and a damper of the absorber. Since this usage of elastomer is aiming at damping at the designated resonant frequency, more efficient damping can be attained compared to the conventional way due to low vibration transmissibility. Also, this usage contributes to the absorber to be simple and compact.

So we introduce this approach in order to derive a higher performance from an elastomer. Thus, multi-mode vibration suppression by the hybrid damper is realized by the combination of a shunt damper for lower frequency resonance and a dynamic absorber for higher frequency resonance.

As for the multi-mode vibration control with passive mechanism, several studies have been done for both a dynamic absorber and a shunt damper. Seto et al. developed an excellent design method of multiple dynamic absorbers for a continuous body (4). Since the designed absorbers are put on the vibrating body separately not in one united body, this method is unsuitable for the hybrid damper studied here.

On the other hand, Hollkamp (5) and Wu (6) developed the methods to realize multiple mode shunt damping with use of a single piezoelectric element. Although multiple mode shunt damping might be an alternative plan for the problem mentioned above, weaknesses become conspicuous with an increase in number of mode such that the damping performance for an individual mode deteriorates and that the optimum tuning becomes troublesome due to a complicated electric circuit.

As far as we know, this study is the first attempt to accomplish multi-mode vibration control combining an elastomer with a piezoelectric element. Since the working motions of dynamic absorber and shunt damper are orthogonal each other, as mentioned in chap. 4, control design could be done without worrying about undesirable mutual interference between them.

2. Nomenclature

Table 1 Nomenclature

M_i	Mode mass at ith mode [kg]
K_i	Mode stiffness at ith mode [N/m]
ω_i	Resonant angular frequency of ith mode [rad/s]
L	Inductance of LR shunt circuit [H]
R	Resistance of LR shunt circuit [Ω]
C_p	Capacitance of piezo ceramic [F]
Ψ_1	Electro-mechanical conversion coefficient of piezo ceramic [N/V]
i_p	Current that piezo ceramic generates [A]
i_{su}	Current that runs through shunt circuit [A]
v_a	Voltage applied to piezo ceramic [V]
f_{p1}	Feedback force from piezo ceramic [N]
m	Mass of dynamic absorber [kg]
c	Viscosity of dynamic absorber [Ns/m]
k	Stiffness of dynamic absorber [N/m]
f_{ab2}	Feedback force from dynamic absorber [N]
$\phi_i(x)$	Mode shape function of ith mode
$\eta_i(t)$	Mode displacement of ith mode
f_i^*	ith mode external force [N]
$G_i(s)$	Transfer function from external force to ith mode displacement

3. Hybrid Damper

Figure 1 illustrates the structure of the proposed hybrid damper. The piezo ceramic element, elastomer, and metal plate are layered to make a rectangular patch. This patch is attached to a vibrating object in the same way as is an elastomer patch.

The piezo ceramic element is connected to a shunt circuit consisting of an inductor and a resistor (LR shunt). Together with this non-active circuit, the piezo ceramic element works as a shunt damper. The shunt damper is intended to suppress vibration in the lower frequency range.

The elastomer comprises a dynamic absorber with a small-mass metal plate, and works as the spring and damper of the absorber. Note that the usage of elastomer in this hybrid damper is unconventional. The dynamic absorber is intended to suppress vibration in the higher frequency range.

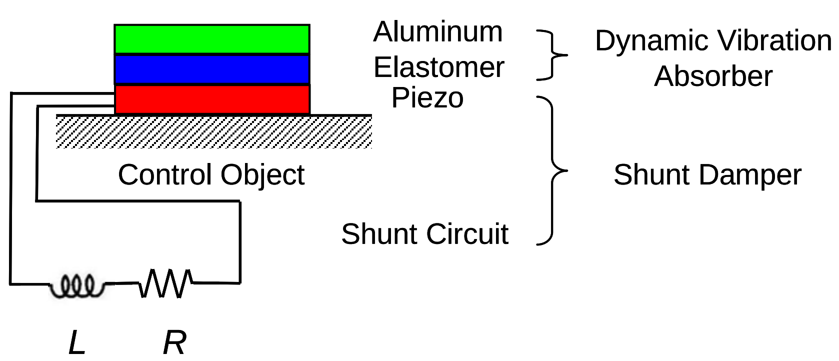

Fig. 1 Structure of Hybrid Damper

4. Control Object

Figure 2 depicts the experiment setup. A cantilever beam made of aluminium (165x22x1mm), the control object in this study, is fixed to the vibrating base at its root. The beam is effectively 140mm long and weighs 8.59g.

Table 2 lists the theoretical values for the beam in the 1st and the 2nd modes, based on simple beam theory (7). They are the mode mass M_i, the mode stiffness K_i, and the resonant angular frequency ω_i, all of which will be required to design optimum parameters for the hybrid damper. The actual values of resonant angular frequency have been confirmed to match the theoretical ones very well.

The hybrid damper is attached to the beam near its fixed end as seen in Fig. 2. In this study, the 1st mode vibration is to be suppressed by the shunt damper and the 2nd mode by the dynamic absorber.

Note that the shunt damper suppresses the vibration by in-plane expansion and contraction of the piezo ceramic element parallel to the vibrating surface, while the dynamic absorber does so by oscillatory motion of the metal plate perpendicular to the vibrating surface. Therefore, the best place for the shunt damper is at the fixed end of the beam where the curvature of the beam is the maximum, while the best place for the dynamic absorber is the other end of the beam where the deflection of the beam is the maximum. The fixed end is thus the best place for the shunt damper but the worst place for the dynamic absorber. In this meaning, cantilever support in this study is an extreme case. In a practical use, the support condition of control object is not always as severe as this example.

A voice-coil actuator is mounted on a vibrating base, supported by four legs at the corners of the rigid base via rubber isolators. The voice-coil moves up and down according to the alternating input voltage to produce the plane bending vibration of the vibrating base. The cantilever beam is excited by this motion of the base. The vibration of the beam at its free end is measured by a laser Doppler vibrometer.

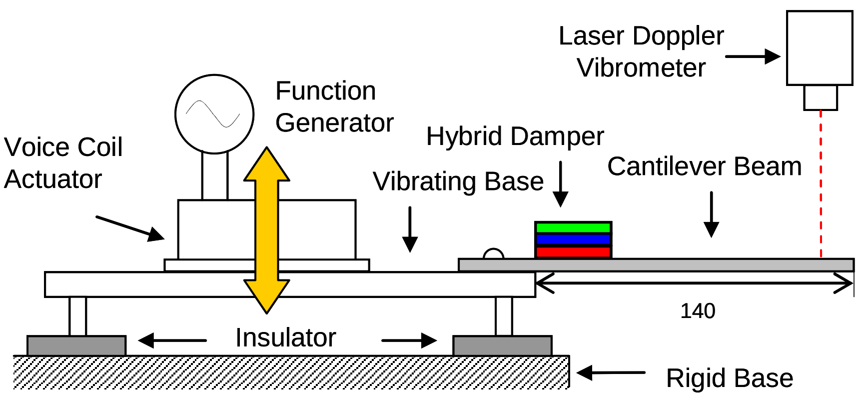

Fig. 2 Experiment Setup

Table 2 Parameters of Cantilever Beam

-	M_i (g)	K_i (Ns/m)	ω_i (rad/s)
1st mode	15.36	1065	263.3
2nd mode	8.28	21573	1614.2

5. Control Design

5.1 Shunt Damper for 1st Mode Vibration

Shunt damping is an electro-mechanical vibration attenuation method utilizing piezoelectricity. Piezoelectric elements convert mechanical vibration energy to electric energy due to the piezoelectric effect, and a resistor within the shunt circuit dissipates vibration energy as heat energy. This passive mechanism of damping is advantageous when constructing a hybrid damper with an elastomer that is also passive.

In this study, an LR shunt circuit is applied to achieve efficient dissipation utilizing the LC resonance realized by the inductance of the shunt circuit and the capacitance of the piezoelectric element. This mechanism of shunt damping is considered to be the electrical realization of a mechanical dynamic absorber.

Figure 3 shows the block diagram of piezoelectric shunt damping system assuming that the 1st mode is dominant. M_1 and K_1 are the mode parameters of the beam at the 1st mode. L and R are the inductance and resistance of the LR shunt circuit. C_p and Ψ_1 are the capacitance and the electro-mechanical conversion coefficient of the piezoelectric element. Ψ_1 depends on the physical properties of piezoelectric element and the mode shape function $\phi_1(x)$. η_1(t) is the 1st mode displacement. i_p, i_{su}, and v_a are the current that a piezoelectric element generates, the current that runs through the shunt circuit, and the voltage applied to a piezoelectric element, respectively. f_{p1} is the feedback force from shunt damper. f_1^* is the 1st mode external force. From this diagram, shunt damping can be considered as velocity feedback control.

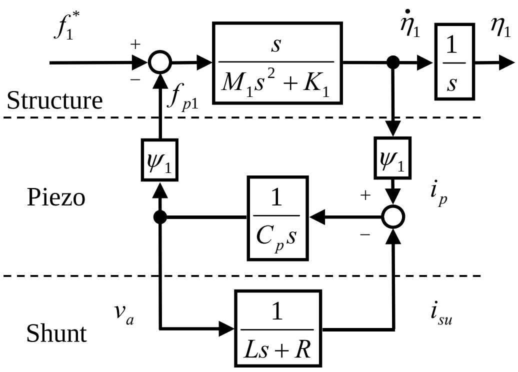

Fig. 3 Block Diagram of Piezoelectric Shunt Damping System

Based on this block diagram, the transfer function from the external force to the displacement of the 1st mode component of the beam becomes

$$G_1(s) = \frac{C_p L s^2 + C_p R s + 1}{M_1 C_p L s^4 + M_1 C_p R s^3 + (M_1 + L(C_p K_1 + \psi_1^2))s^2 + R(C_p K_1 + \psi_1^2)s + K_1} \quad (1)$$

In this system, $\Psi_1^2 L$ and $\Psi_1^2 R$ correspond to mass and damper of a mechanical dynamic absorber, respectively.

Control design of the shunt damper is to determine the values of L and R of the shunt circuit for the given parameters of the piezo ceramic element and the beam used. They should be optimized to minimize the maximum gain of $G_1(s)$ around the 1st mode resonant angular frequency ω_1.

Fixed-point theory is well known for providing an optimum design for this kind of 2nd-order damper system [8]. The optimum values (L_{opt} and R_{opt}) are given by the following equations:

$$L_{opt} = \frac{1}{\omega_1^2 C_p + \psi_1^2} \tag{2}$$

$$R_{opt} = \frac{R_P + R_Q}{2} \tag{3}$$

where

$$\begin{matrix} R_P \\ R_Q \end{matrix} = \sqrt{\frac{3\psi_1^2}{C_P\left(\omega_1^2 C_P + \psi_1^2\right)\left(2\left(\omega_1^2 C_P + \psi_1^2\right) \mp \sqrt{2\psi_1^2\left(\omega_1^2 C_P + \psi_1^2\right)}\right)}} \tag{4}$$

To determine these values, the values of ω_1, C_p, and Ψ_1 are required. The value of C_p is easily measured with an LCR meter. In this study, a 5Z20x30R-S C6 (20x30x0.4mm, Fuji Ceramics) is used. Ψ_1 is estimated by identification (9) because it is very difficult to measure that value. Thus, the optimum values are obtained. Table 3 presents the parameters that define the shunt damper. Instead of a real coil, a synthetic inductor (10) is very useful for realizing such a large value of L_{opt} in a practical application.

Table 3 Parameters of Shunt Damper

C_p (nF)	Ψ_1 (N/V)	L_{opt} (H)	R_{opt} (kΩ)
25.48	$6.03\text{x}10^{-3}$	573.5	26.7

5.2 Dynamic Absorber for 2nd-Mode Vibration

In order to obtain better performance from elastomer, it is used as a dynamic absorber cooperating with an additional small mass. A dynamic absorber is a passive but powerful vibration attenuation method that utilizes the resonant motion of the 2nd-order system attached to the vibrating object. Here, the elastomer is to function as the spring and damper of the absorber.

Figure 4 shows the block diagram of dynamic absorber system assuming that the 2nd mode is dominant. M_2 and K_2 are the mode parameters of the beam at the 2nd mode. m, c, and k correspond to the mass, damper, and spring of the dynamic absorber, respectively. As mentioned in chap. 3, c and k are the viscosity and elasticity of the elastomer and m is the mass of the metal plate. η_2(t) is the 2nd mode displacement. f_{ab2} is the feedback force from dynamic absorber. f_2^* is the 2nd mode external force.

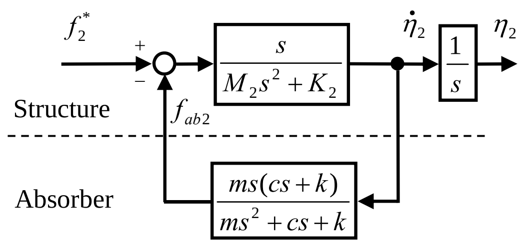

Fig. 4 Block Diagram of Dynamic Absorber System

Based on this block diagram, the transfer function from the external force to the displacement of the 2nd mode component of the beam becomes

$$G_2(s) = \frac{ms^2 + cs + k}{M_2 ms^4 + c(M_2 + m)s^3 + (M_2 k + m(K_2 + k))s^2 + cK_2 s + K_2 k} \tag{5}$$

Control design of the dynamic absorber is to determine the values of m, c, and k for the given parameters of the beam used. They should be optimized to minimize the maximum gain of $G_2(s)$ around the 2nd mode resonant angular frequency ω_2.

Fixed-point theory provides a highly systematic procedure for the control design of a dynamic absorber. In this procedure, m is determined first by considering the mass ratio to the main body M_2. The optimum values (c_{opt} and k_{opt}) are then derived based on m, K_2, and ω_2. This means the viscosity and the elasticity of the elastomer must be adjusted arbitrarily.

However, it is almost impossible to tune these parameters arbitrarily and independently because they are inherent as viscoelasticity in the elastomer used. The only adjustable parameter we have here is m. Thus, fixed-point theory is not applicable in this case.

Therefore, the mini-max approach is introduced in order to find the optimum value of m that provides the best performance for a given viscoelasticity of the elastomer. For some value of m, the maximum gain of $G_2(s)$, denoted by $P(m)$, is

$$P(m) = \max_{\omega} \left| G_2(j\omega) \right| \tag{6}$$

Note that M_2 and K_2 are given in Table 2 and that c and k are given in the specification sheet for the elastomer used (Kitagawa Industries, YMG-55V). The optimum value of m is the one that minimizes the maximum gain.

$$m_{opt} = \arg\min_{m} P(m) \tag{7}$$

Thus, m_{opt} is determined by iterative computation, varying the value of m little by little, and is found to be about 1.6g. This optimum value is realistic because the mass ratio m/M_2 is less than 0.2. Table 4 presents the parameters that construct the dynamic absorber.

Table 4 Parameters of Dynamic Absorber

k (N/m)	c (Ns/m)	m_{opt} (g)
3463	0.333	1.6

6. Simulations

6.1 Shunt Damper for 1st Mode Vibration

The effectiveness of the shunt damper based on fixed-point theory is examined through simulation. Figure 5 plots the simulation result. The red line indicates the gain characteristics $G_1(s)$ with shunt damping; the black line indicates the original gain characteristics of the 1st mode of the beam, assuming the damping factor to be 0.006.

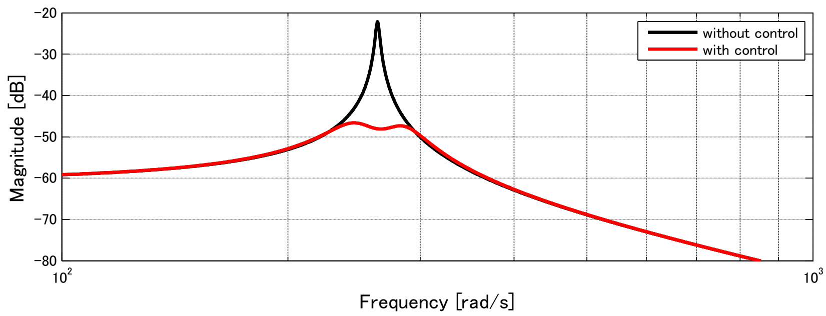

Fig. 5 Simulation Result for Shunt Damper

The resonant peak is divided into two lower peaks, one on either side, with almost the same gain. This is a typical characteristic when using fixed-point theory. As a result, about 25dB of attenuation could be expected with a shunt damper.

6.2 Dynamic Absorber for 2nd Mode Vibration

The effectiveness of the dynamic absorber based on the mini-max approach is examined through simulation. Figure 6 plots the simulation result. There are four lines in the figure. The black line indicates the original gain characteristics of the 2nd mode of the beam, assuming the damping factor to be 0.01.

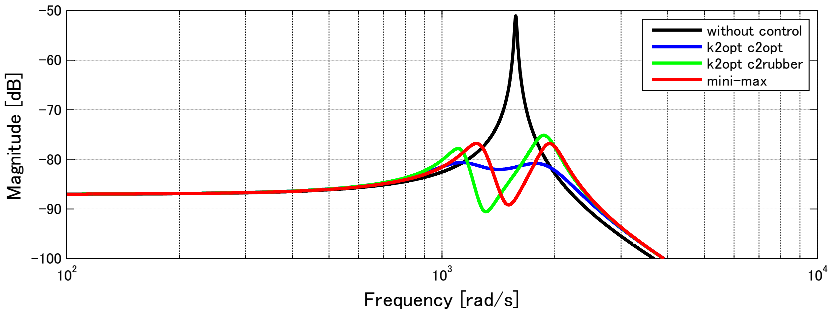

Fig. 6　Simulation Result for Dynamic Absorber

The blue line indicates the gain characteristics $G_2(s)$ when applying fixed-point theory, assuming that the viscoelasticity of the elastomer is controllable. In this case, m is set to 2.0g, a little heavier than m_{opt}. As mentioned in § 6.1, two lower peaks with the same gain can be seen. Thus, the maximum gain around ω_2 could be attenuated by about 30dB.

The green line indicates the result of using the actual values of viscoelasticity instead of those from fixed-point theory. It is clear that the performance deteriorates remarkably because they are not appropriate values for this m.

The red line indicates the result of applying the mini-max approach. Although the performance is not as good as that obtained by fixed-point theory, the best performance is obtained under limited conditions. Similar to fixed-point theory, two peaks of almost the same gain can be seen. As a result, the maximum gain around ω_2 could be attenuated by about 25dB.

6.3 Hybrid Damper for Multi-Modal Vibration

Figure 7 presents the simulation result for multi-modal vibration control by the hybrid damper. The black line indicates the original gain characteristics of the beam, which consists of a linear combination of the 1st and the 2nd modes mentioned above. The red line

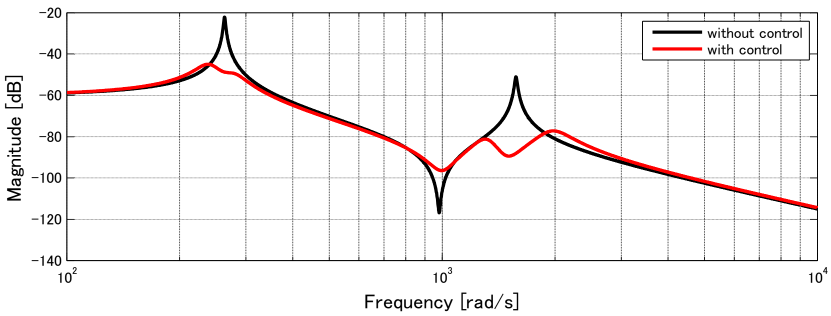

Fig. 7　Simulation Result for Hybrid Damper

indicates the result with the hybrid damper. Both resonant modes are attenuated by more than 25dB, confirming the effectiveness of the hybrid damper. The right side peak that is seen in Fig. 5 and left side one that is seen in Fig. 6 are pulled down due to the anti-resonant trough made by the coupling of the 1st and the 2nd modes.

7. Experiments

The effectiveness of the hybrid damper for multi-modal vibration control is practically assessed by experiment.

A 1mm-thick aluminium plate is cut to 20x22mm so that the mass is equal to m_{opt}. A 3mm-thick elastomer sheet is cut into 5mm squares. Four squares are adhered to the aluminium plate at its four corners to form a dynamic absorber. Then it is adhered to a unimorph piezo ceramic element (20x30x0.4mm) to form a hybrid damper as seen in Fig.1. This hybrid damper is attached to the beam near its fixed end.

The piezo ceramic element is connected to the shunt circuit. In the experiment, a synthetic inductor is applied to achieve the optimum inductance L_{opt}. Figure 8 shows the beam with the hybrid damper thus obtained.

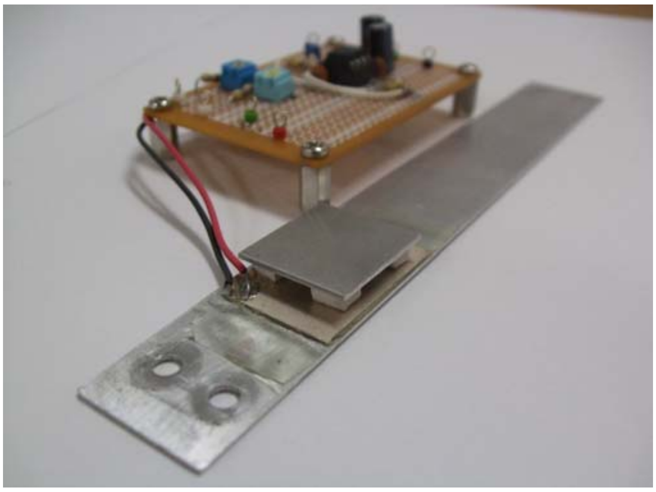

Fig. 8 Beam with the Hybrid Damper

Figure 9 presents the experiment results for the gain characteristics of the beam obtained by a sinusoidal sweep test, which corresponds to the simulation result shown in Fig. 7. The black line indicates the original gain characteristics from the voltage impressed on a voice-coil actuator to the amplitude of the vibration at the tip of the beam. The red line indicates the result with the hybrid damper. Attenuation of 10dB around the 1st mode and of 16dB around the 2nd mode is attained.

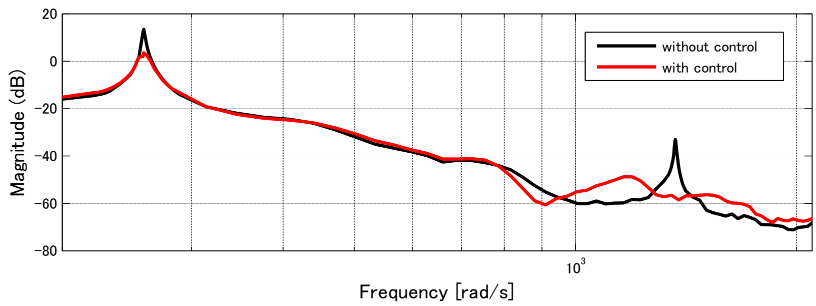

Fig. 9 Gain Characteristics of Control

In contrast to the simulation, the resonant peak of the 1st mode is blunted as though it were suppressed by direct-velocity feedback. This difference and the undulating shape of the gain are thought to be due to the dynamics of the vibrating base as mentioned in chap. 4, which is not taken into account in the simulation.

Although the input quantities are different between simulation and experiment, the impressed voltage can be considered to be equivalent to the applied force since the impedance of the voice-coil actuator is almost constant along the frequency range used in this study.

Figure 10 presents the experiment results for multi-modal vibration control by the hybrid damper. Figure (a) shows the time-series response of vibration of the beam without the hybrid damper when the beam is excited simultaneously at the 1st and the 2nd mode resonant frequencies. The wave form clearly indicates the superposition of the 1st mode and the 2nd mode vibrations according to the impressed voltage.

Figure (d) shows the result with the hybrid damper. Compared to fig. (a), it is apparent that both components of the resonant vibrations are damped effectively with use of the hybrid damper. Attenuation of 15dB at the 1st mode and of 24dB at the 2nd mode is attained.

Figure (b) shows the result when the shunt circuit is electrically turned off. Compared to fig. (a), the 2nd mode vibration is almost suppressed by the dynamic absorber but the 1st mode remains. Figure (c) shows the result when the dynamic absorber (aluminium plate + elastomer sheet) is removed from the hybrid damper, leaving the piezo ceramic element on the beam. In contrast to fig. (b), the 1st mode vibration is almost suppressed by the shunt damper but the 2nd mode remains.

These results indicate that the shunt damper contributes only to the 1st mode attenuation and the dynamic absorber does only to the 2nd mode and consequently that they do not interfere each other.

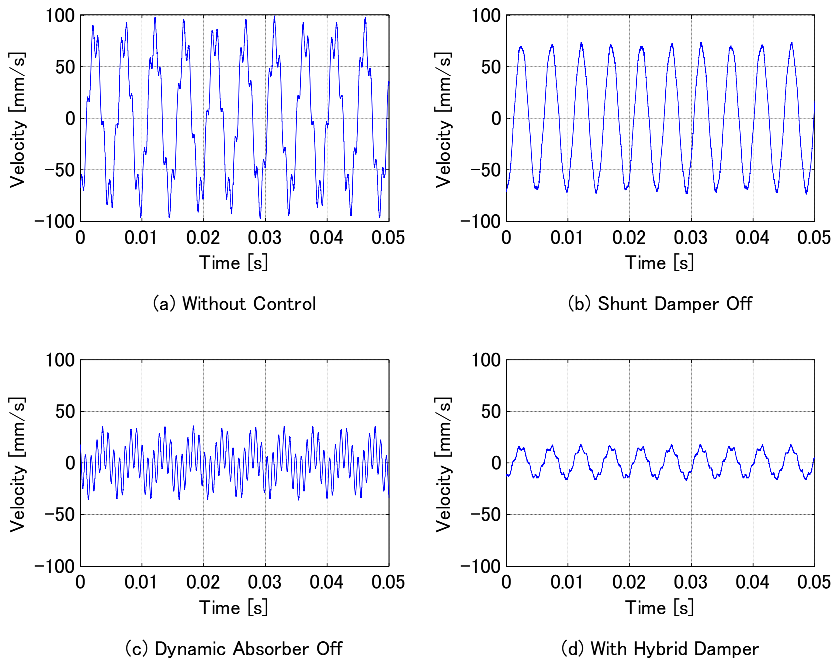

Fig. 10 Time-series Data of Hybrid Damper

In order to verify the effectiveness of the dynamic absorber for the 2nd mode, two comparative experiments were carried out. Because one might think that the damping performance at the 2nd mode shown in Fig. 9 and Fig. 10 is due to the addition of mass of dynamic absorber or to the shearing of elastomer sheet, not to the mechanism of dynamic absorber.

Fig. 11(a) shows the result when an aluminium plate or an elastomer sheet of the same mass as the absorber is attached to the beam instead of the absorber. In either case, no significant attenuation is found only showing the shift of the resonant peak to the higher frequency due to the increase in bending rigidity. Fig. 11(b) shows the result when an aluminium plate is removed from the absorber. Attenuation as shown in Fig.9 can not be seen. It thus turned out that the attenuation at the 2nd mode is due to neither the mass of the dynamic absorber nor the elastomer's viscoelasticity alone.

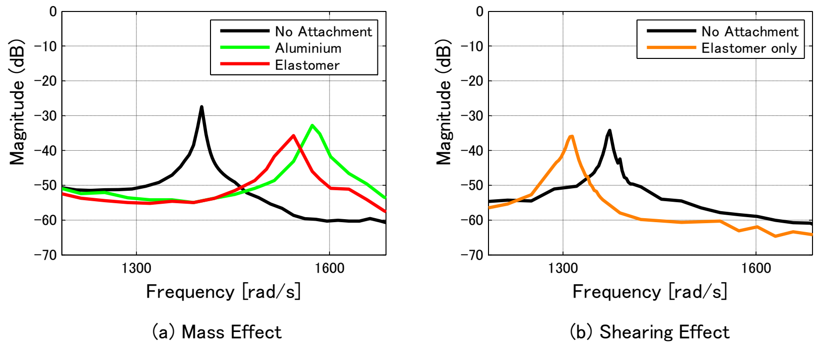

Fig. 11 Comparative Experiments for Dynamic Absorber

In order to verify the robustness against the parameter fluctuation in the shunt damper for the 1st mode, comparative experiments were carried out with different values of *L*. The beam is excited at the 1st mode resonant frequency. As seen in Fig. 12, the best performance is obtained under the designed optimum value L_{opt} and the performance deteriorates when the value of *L* varies 25% from L_{opt}. Note that the value of *R* of the shunt circuit is held to R_{opt}. No significant attenuation is found when the fluctuation is 50%. Thus, the performance is very sensitive to the parameter fluctuation. This would be the problem when the resonant frequency of the control object changes during operation.

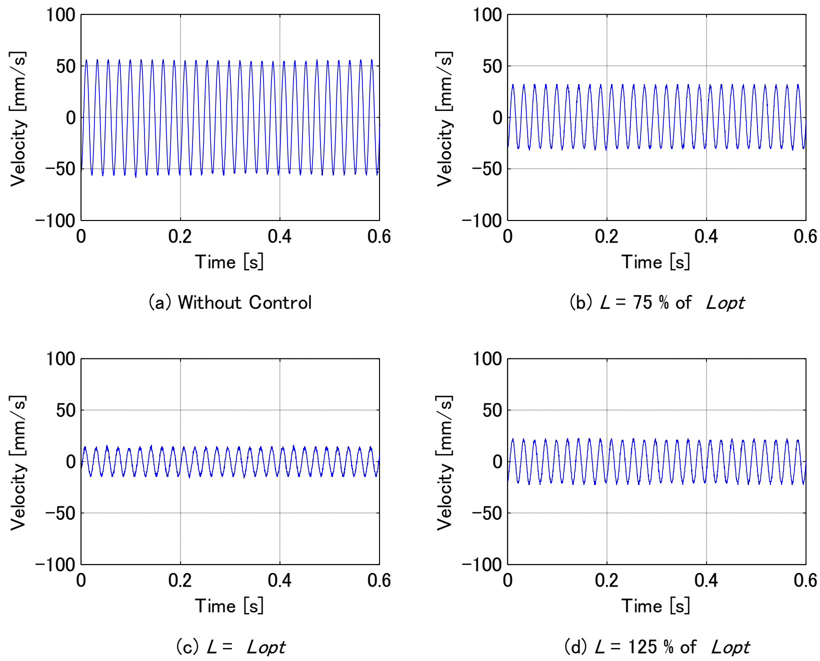

Fig. 12 Comparative Experiments for Different Values of L

8. Conclusion

To achieve vibration control over a wide frequency range, a hybrid damper combining a piezoelectric element with an elastomer is proposed.

The key idea is to compensate for the weakness of the elastomer in the lower frequency range by using a piezoelectric element. In order to make the most of the passive mechanism of vibration control by the elastomer, a piezo ceramic element is used as a form of shunt damper. The control parameters are easily obtained from fixed-point theory. The elastomer is used as a dynamic absorber, ensuring more efficient damping. Control parameters were successfully obtained using the mini-max approach, despite the fact that the viscoelasticity of the elastomer is inherent.

The effectiveness of this hybrid damper was examined by multi-modal vibration control of a cantilever beam. The resonant vibrations of the beam are effectively suppressed by the shunt damper for the 1st mode and by the dynamic absorber for the 2nd mode without unfavorable interference.

For practical applications, the next step in this research is to develop a technique for controlling three or more modes and to design a scheme to deal with fluctuations in the resonant frequency. The optimum location of the hybrid damper also should be examined that depends on the support form of the vibrating object.

A precise comparison would be a great academic interest between the proposed method and the multi-mode shunt damping with use of the same experiment setup, although this study is motivated by a demand to add a new value to an elastomer.

References

(1) N W Hagood, A von Flotow, Damping of Structural Vibrations with Piezoelectric Materials and Passive Electrical Networks, *Journal of Sound and Vibration* Vol.142, No.6 (1991), pp. 243-268.

(2) A Nagamatsu, K Seto et al., *Dynamics Handbook* (in Japanese), (1993), pp. 353-355, Asakura Publishing Co., Ltd.

(3) K Ozaki, Characteristics of Dynamic Absorber Using Rubber Vibration Isolator (in Japanese), *Transactions of the Japan Society of Mechanical Engineers C* Vol.60, No.580 (1994), pp. 4116-4122.

(4) K Seto, K Iwanami, Y Takita, Vibration Control of Multi-Degree of Freedom Systems by Dynamic Absorbers (in Japanese), *Transactions of the Japan Society of Mechanical Engineers C* Vol.50, No.458 (1984), pp. 1962-1969.

(5) J J Hollkamp, Multimodal Passive Vibration Suppression with Piezoelectric Materials and Resonant Shunt, *Journal of Intelligent Material Systems and Structures* Vol.5 (1994), pp. 49-57.

(6) S Y Wu, Method for Multiple Mode Shunt Damping of Structural Vibration Using a Single PZT Transducer, *Proceedings SPIE, Smart Structures and Materials, Damping and Isolation* Vol.3327 (1998), pp. 159-168.

(7) L Meirovitch, *Elements of Vibration Analysis*, 2nd ed., (1986), McGraw-Hill Science/Engineering/Math.

(8) K Takagi, K Nagase et al., Parameter Identification Method for Piezoelectric Shunt Damping, *Proceedings of Smart Materials*, (2004), Nano, and Micro-Smart Systems: 5648-15.

(9) K Takagi, Z W Luo, Modeling and Parameter Identification for Passive Vibration Control Using Piezoelectric Element (in Japanese), *Proceedings of the 49th Japan Joint Automatic Control Conference* (2006): SU8-3-6.

(10) R Schaumann, M E Van Valkenburg, *Design of Analog Filters*, (2001), pp. 533-540, Oxfordshire: Oxford University Press.