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報告番号	※　　　　第　　　　号

主　論　文　の　要　旨

論文題目　On τ-tilting theory and higher Auslander–Reiten theory

（τ 傾理論と高次元 Auslander-Reiten 理論について）

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論　文　内　容　の　要　旨

The purpose of this thesis is to report on recent results obtained by the author in τ-tilting theory and higher Auslander-Reiten theory. These theories, introduced by Adachi-Iyama-Reiten in [1] respectively Iyama in [7], are generalizations of tilting theory and Auslander-Reiten theory.

When studying finitely generated modules over a finite dimensional algebra A, an important problem is to classify the indecomposable A-modules and the morphisms between them. Auslander-Reiten theory, which is concerned with the study of certain short exact sequences called almost-split sequences, plays an important role in these classification problems. On the other hand, tilting theory is concerned with the construction and investigation of derived equivalences involving categories of modules over a finite dimensional algebra. In these theories, one may replace finite dimensional algebras for larger classes of algebras or rings, for example Artin algebras.

Higher Auslander-Reiten theory, as the name suggests, is a generalization of Auslander-Reiten theory from the point of view of the length of the sequences involved. An important class of algebras in higher Auslander-Reiten theory is that of 2-representation-finite algebras [8]. These are the algebras of global dimension 2 whose module category contains a 2-cluster-tilting subcategory. In Chapter 1 we classify the 2-representation-finite algebras which are derived-equivalent to the category $\mathsf{coh}\,\mathbb{X}$ of coherent sheaves of a so-called weighted projective line. This classification is a by-product of a more general result: For $X \in \mathsf{coh}\,\mathbb{X}$ let τX denote the sheaf obtained by tensoring X with the dualizing sheaf of $\mathbb{X}$. We classify the tilting complexes in $T \in \mathsf{coh}\,\mathbb{X}$ which satisfy $\tau^2 T \cong T$. As another consequence of this more general result, we obtain a classification of the selfinjective cluster-tilted algebras of canonical type. These results are available in preprint form in [10].

From a more general perspective, in Chapter 2 we introduce n-abelian and n-exact categories to serve as a a categorical framework for the investigation of the intrinsic homological properties of n-cluster-tilting subcategories. These are higher generalizations of abelian respectively exact categories, again with regard to the length of exact sequences. It is then important to observe that the 2-cluster-tilting subcategories mentioned above are examples of 2-abelian categories. We also relate these new concepts to Geiß-Keller-Oppermann's $(n+2)$-angulated categories [5]. More precisely, we introduce Frobenius n-exact categories and show that their stable categories have a natural structure of a $(n+2)$-angulated category. We provide a collection of examples at the end of the chapter which suggest that our theory is far from abstract nonsense. These results are available in preprint form in [11].

We now move from higher Auslander-Reiten theory to τ-tilting theory. We remind the reader that an A-module T with $\mathsf{p.dim.}\, T \leq 1$ is called a *tilting module* if $\mathsf{Ext}^1_A(T, T) = 0$ and the number of pairwise non-isomorphic indecomposable direct summands of T equals the number of simple A-modules (which is finite for A is a finite dimensional algebra). The importance of tilting modules is expressed by the existence of an equivalence of triangulated categories

$$\mathrm{D}^{\mathrm{b}}(\mathsf{mod}\, A) \xrightarrow{\sim} \mathrm{D}^{\mathrm{b}}(\mathsf{mod}\, \mathsf{End}_A(T)).$$

This allows us to relate the representation theory of A to that of $\mathsf{End}_A(T)$, which is useful when the latter algebra is well-understood, see [2, Chap. VI, VII and VIII] for further details. When the algebra A is hereditary, *i.e.* when $\mathsf{gl.dim.}\, A \leq 1$, tilting modules have nice combinatorial properties. In particular, if we enlarge slightly the class of tilting A-modules to include also the so called *support tilting A-modules*, then a certain "mutation pheonomenon" appears: Let T be a basic tilting A-module and M an indecomposable direct summand of T. Then, there exist *exactly two* support tilting A-modules which have T/M as a direct summand. This combinatorics are closely related to the combinatorics of cluster algebras [3, 6].

Adachi-Iyama-Reiten introduced τ-titling theory in [1] in order to obtain similar mutation combinatorics when A is not a hereditary algebra. We say that an A-module M is τ-tilting if $\mathsf{Hom}_A(M, \tau M) = 0$ and the number of pairwise non-isomorphic indecomposable direct summands of M equals the number of simple A-modules. Then, the same mutation phenomenon described above holds for an arbitrary finite dimensional algebra if one replaces "tilting" by "τ-tilting".

In Chapter 3 we study general version of this mutation. More precisely, we parametrize the support τ-tilting modules having a fixed τ-rigid A-module U as a direct summand. Such A-modules are parametrized by the support τ-tilting modules over a different algebra C, constructed explicitly from U. It is also worth mentioning that we introduce *τ-perpendicular categories*, which are a generalization of Geigle-Lenzing's perpendicular categories [4]. The formation of τ-perpendicular categories is one of our main tools in this chapter. These results are available in preprint form in [9].

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