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Job Destruction and Coordination Failures in Labor Turnover*

Koji Yoneda† and Tadashi Minagawa‡

Abstract

This paper addresses two main subjects. (i) The first is to examine how skill specificity affects the way that negative demand shocks lead to destruction of skilled jobs. Our theoretical model, in which skilled workers are complementary to each other, produces the following results: Once market conditions fall below some critical level, the number of skilled jobs drops discretely, rather than decreasing continuously; facing a large decrease in demand, the percentage decrease in the value of output is greater in an industry with higher skill specificity than that with lower specificity. (ii) The second aim of this paper is to show that, even if the wages in the current firm are greater than the highest wage in the external labor market, workers' turnover can occur due to coordination failures. This makes firms willing to pay wages far above external wages in order to reduce costly labor turnover. This coordination problem arises from a Leontief technology and the timing of workers' decisions on their turnover.

Keywords: firm-specific skills, job destruction, labor turnover, coordination failures.

1 Introduction

Long-term employment with the accumulation of firm-specific skills is, to a greater or lesser extent, a prevailing phenomenon in advanced countries. In Japan long-term employment is highly developed, and this is thought to contribute to the competitiveness of Japanese manufacturing (e.g., Abegglen [1958]).[1] However, in that country, the proportion of regular employees has been decreasing since the 1980s when its economic growth slowed down. In 2010, regular employees constitute 65.6 percent of Japanese workers, down from 83.6

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[1] In 2002, the percentage of employees who had 20 years or more of tenure with their current employer was 21.3 percent in Japan (Ministry of Health, Labor and Welfare, "Basic Survey on Wage Structure") compared to an average of 16.0 percent in 15 European countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom (Eurostat, "Structure of Earning Survey"). However, Japan has never had the lowest percentage of temporary employees as a proportion of total employees among advanced countries (OECD, "Employment Outlook," 2010).

percent in 1985.[2] This can be attributed in part to a slowdown in long-term demand for products.

This paper examines how skill specificity affects the way that decreased demand for products leads to destruction of skilled jobs (i.e., long-term employment). Our theoretical model, where skilled workers are complementary to each other, produces the following results. (i) Once market conditions fall below some critical level, the number of skilled jobs can drop discretely, rather than decreases continuously. (ii) Facing a large decrease in demand, the percentage decrease in the value of output is greater in an industry with higher skill specificity than that with lower specificity. By contrast, facing a slight drop in demand, higher specificity has a smaller relative decline in output.

Another aim of this paper is to illustrate the potential importance of coordination problems for efficiency wages. Some efficiency wage models explain that a profit-maximizing firm pays wages above market-clearing in order to reduce costly labor turnover (e.g., Salop [1979]).[3] In this paper, we will show that, even if the wage in the current firm is greater than the highest wage in the external labor market, workers' turnover can occur due to coordination failure in workers' decisions on turnover. This makes a firm willing to pay wages far above external wages to reduce labor turnover. This coordination problem arises from two key assumptions: productivity falls to the lowest level if one or more workers with firm-specific skills quit the firm (i.e., a Leontief technology); workers have to decide whether to stay or quit (i.e., they have to determine the threshold to cut off the external earning opportunities) before they know the decisions of other workers.

Long-term employment and accumulation of firm-specific skills have been important issues in economics. Becker (1964) explains the mechanism that sharing training costs and returns between firms and employees leads to long-term investment in human capital.[4] Mincer (1974) gives theoretical and empirical analysis of the relation between human capital accumulation and wages. Literatures on contract theory also show what wage and employment contract results from firm-specific training (e.g., Kanemoto and MacLeod [1989]).

Some empirical studies investigate the effect of macroeconomic conditions on investment in firm-specific skills (e.g., Hashimoto and Raisian [1985], Mincer and Higuchi [1988], Ohkusa and Ohta [1994]). Overall, these studies conclude that a decline in long-term de-

[2] The data for 2010 is the yearly average. For 1985, the reference period is the last week of February. The source is Ministry of Internal Affairs and Communications (MIC), "The Special Survey of the Labour Force Survey" for 1985 and MIC, "Labour Force Survey (Detailed Tabulation)" for 2010.

[3] Salop (1979) develops an efficiency wage model in which training costs play an important role. If the current firm pays a higher relative wage or if the unemployment rate is higher, workers will be more reluctant to quit. In one possible equilibrium, all identical firms pay the same wage above market-clearing, and consequent unemployment decreases turnover and thus reduces training costs. In addition, there are some microeconomic foundations for efficiency wage models. Shapiro and Stiglitz (1984) is a famous model where wages above market-clearing reduce workers shirking on the job. Weiss (1980), which is also a well-known model, explains that adverse selection in recruitment of new workers brings higher wages. For details, see the review by Akerlof and Yellen (1986).

[4] Furthermore, Ben-Porath (1967) develops a more detailed model to describe what wage profiles result from firm-specific training. Acemoglu and Pischke (1999) show that, when labor markets are imperfect, firms can pay for investment in general training, which therefore leads to long-term employment.

mand for products decreases investment in firm-specific skills and thus reduces long-term employment.[5] Not only that, it is also important to examine how skill specificity affects the response of employment to macroeconomic conditions. A leading work on this issue, Caballero and Hammour (1998) shows that skill specificity makes the cyclical response of employment elastic in recessions and rigid in expansions.[6]

As with Caballero and Hammour this paper examines how skill specificity affects the response of employment to demand shocks. But different from their analysis which focuses mainly on the effect of specificity on creation of new employment,[7] we will examine its effect on destruction of pre-existing employment. Because our concern here is with job destruction, we do not consider the creation of new employment in this paper. In other words, our analysis pays attention only to the phase when firm-specific skills are lost, and we do not consider problems of investment in those skills.

Our analysis is based to a large extent on Blanchard and Kremer (1997). In the early 1990s, the countries of the former Soviet Union had a large decline in output. Blanchard and Kremer provide sophisticated models to explain how the improvement in private business (employment) opportunities leads to a large decline in output. In their analysis, high specificity that the state firms have no alternative suppliers (workers) plays an important role in the collapse of production in the state firms. Different from Blanchard and Kremer, this paper will examine how decreased demand for products leads to the dissolution of long-term employment in private firms. For this purpose, in this paper, the behavior of profit-maximizing firms is introduced into the model. This makes our analysis more complicated, but permits us to discuss firms' wage setting.

Long-term relations with firm-specific investments are observed not only in labor market but also in transactions of intermediate goods. Minagawa and Yoneda (2011) examine how negative demand shocks affect specific relations between an assembly firm and its suppliers.

This paper provides two models. Section II sets up our basic model with the assumption that productivity falls to the lowest level if one or more workers with firm-specific skills quit the firm. This assumption is the extreme case of complementarity among workers' skills. Section III explains that coordination failures can occur in workers' decisions on their turnover. Section IV examines how a profit-maximizing firm determines the wage for workers with firm-specific skills. In Section V, we provide numerical comparative statics, showing how decreased demand affects worker turnover, the wage for skilled workers,

[5]Hashimoto and Raisian (1985), Mincer and Higuchi (1988), and Ohkusa and Ohta (1994) show that, in periods of high economic growth, training to adapt new technologies makes tenure-wage slopes steeper.

[6]When firms and workers share firm-specific training costs, some of the costs sink into the employment relation. This makes it possible for the party with less sunk costs to appropriate quasi-rents coming from firm-specific skills. This rent appropriation decreases the incentive for the party with greater sunk costs to invest in firm-specific skills (it reduces firms' incentives in recessions and workers' incentives in expansions). Decreased incentive for one side reduces the creation of new employment, given the assumption of fixed coefficients of production. Skill specificity therefore makes recessions excessively sharp and recessions run into bottlenecks.

[7]Caballero and Hammour's model also takes into consideration that difficulties in creating new employment due to the holdup problem lead to rigidities in pre-existing employment.

the number of skilled jobs, and total output of an industry. In addition, we explain that, because of coordination problems, the wage for skilled workers can be far above the highest wage in the external labor market.

In Section VI, weakening the assumption of productivity, we provide another model in which productivity decreases continuously with the job separation rate. In our analytical comparative statics, this weaker assumption produces the same results on the destruction of skilled jobs as in the basic model. But under a continuous production function, coordination failures cannot occur in worker turnover.

Finally, Section VII summarizes our findings and provides some implications. An appendix follows.

2 Production Technology and Labor Market

In this model, a firm employs n workers to produce final goods. The firm produces $(1+\theta)n$ units of final goods if all workers have firm-specific skills and n units otherwise (given the assumption of Leontief technology). The parameters $\theta > 0$ and $n \geq 3$ are given by available technologies. We can think of θ as representing the degree of specificity.[8] Final goods are sold at a price p, given by the market conditions.

Each worker in the firm has an alternative earning opportunity v in the external labor market. The external wage v is distributed uniformly on $[0, 1]$. Draws are independent across workers[9] (i.e., if we let v_i denote v for the i th worker, $\mathrm{Cov}(v_j, v_k) = 0$ for all $j \neq k$). The distribution of v is known, but the realization of v is private information to each worker.

The decision sequence is as follows. First, the firm sets the wage for existing workers at w in order to maximize its profit, and announces it.[10] The wage w is the same for all existing workers (this follows from the assumption that the distribution of v is the same for all workers). Second, each existing worker knows his realization of v, and decides whether to take up this alternative opportunity. If he decides not to take it, he loses this opportunity. Workers have to make the decision before they know the decisions of other workers.

If all existing workers stay (i.e., no one takes the alternative opportunity), the firm can produce $(1 + \theta)n$ units of final goods and pays w to all workers. If one or more existing workers quit, the firm has to hire the replacement workers from the external labor market (it always needs n workers to produce). We assume that the firm can hire replacement

[8] Specificity is characterized by a joint surplus from an economic relationship in Blanchard and Kremer (1998) and by irreversible investment in Caballero and Hammour (1996). Our modeling follows the former approach.

[9] We can think that imperfect information about job offers makes v differ among workers (a worker who happens to find a better matching job has greater v).

[10] Throughout this paper, we assume that the firm announces a take-it-or-leave-it wage to workers with firm-specific skills. But we also made the analysis assuming that the wage for workers with firm-specific skills is determined by Nash bargaining between the firm and the typical worker. The result was qualitatively the same as in this paper.

workers at the wage of 1/2, which is equal to the expected wage in the external labor market ($\mathrm{E}(v) = 1/2$). Because the replacement workers have no firm-specific skills, if some existing workers quit, the output of the firm falls to n units. This means that the output is constant at n units regardless of whether someone has firm-specific skills. Therefore, if someone quits, the existing workers who stay are hired at 1/2, not at w.[11] In short, they receive the wage for "unskilled" workers.

3 Workers' Turnover Behavior

At the beginning, we turn to the decision problem facing a typical exisisting worker. (The firm's decision problem will be examined in the following section. It is not considered in this section.) Let v^* be a threshold, such that a worker stays in the current firm if $v \leq v^*$ and quits otherwise. The threshold v^* should be equal to the income that a typical worker can expect to receive if he stays, in order to maximize his expected income. But his income expected if he stays is dependent on whether other workers stay. Given the symmetry build in the assumption, other workers also have the same value of v^* in an equilibrium state, and each of them stays if his realization of v is less than or equal to v^*. Therefore, given v^*, each worker stays with the probability of $s \equiv \min(1, v^*)$,[12] and thereby the probability that all $(n-1)$ other workers stay is equal to s^{n-1}. Suppose that a typical worker decides to stay in the current firm. He receives w if all $(n-1)$ other workers stay, and 1/2 otherwise. Therefore, his income expected if he stays, and thus the threshold v^*, is given by[13]

$$v^* = s^{n-1}w + (1 - s^{n-1})\frac{1}{2}. \tag{1}$$

Rearranging equation (1) gives

$$w = s^{1-n}v^* + (1 - s^{1-n})\frac{1}{2} \tag{2}$$

We will substitute $s = \min(1, v^*)$ into the above equation, but s may have different forms depending on whether v^* is less than 1. Therefore, we have to examine each case separately.

Case 1. $v^* \geq 1$. Substituting $s = 1$ into equation (2), we get

$$w = v^*.$$

[11] A worker who stays expects to receive 1/2 in the external labor market if he quits. On the other hand, if the firm fires him, it has to pay 1/2 to hire a replacement worker. Therefore, if someone quits, the existing workers who stay are hired at 1/2.

[12] The random variable v has the density $f(v) = 1$ for $v \in [0, 1]$ and $f(v) = 0$ otherwise. Thus, $\Pr(v \leq v^*) = \int_0^{v^*} f(v)dv = \min(1, v^*)$.

[13] One may think that $w = 1$ always produces $v^* = 1$. But equation (1) is an $(n-1)$th degree equation of v^*, since $s = \min(1, v^*)$. Thus, v^* can have multiple solutions. For example, given $w = 1$ and $n = 5$, the solution for equation (1) contains $v^* = 1$ and $v^* \approx 0.54$.

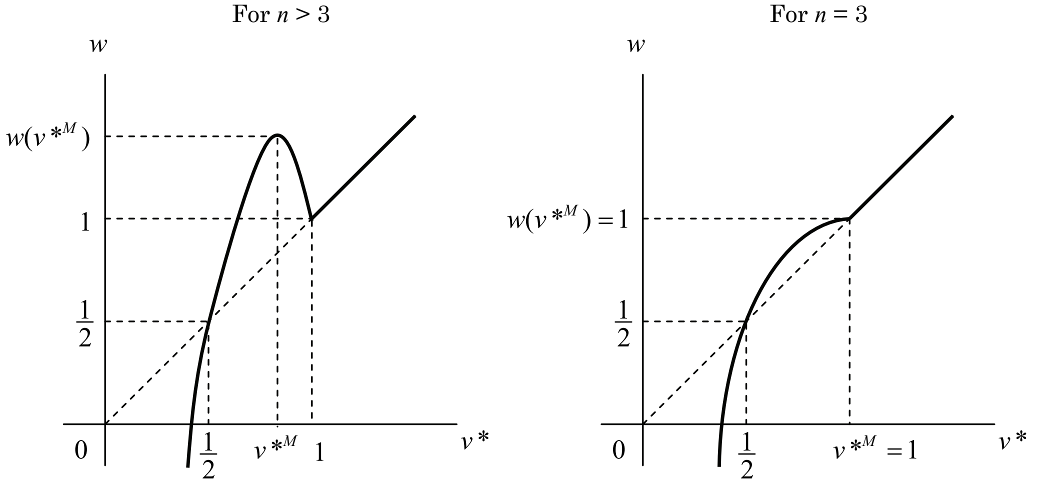

Figure I: Relation between w and v^*.

Case 2. $v^* < 1$. Substituting $s = v^*$ into equation (2) yields[14]

$$w = w(v^*) \equiv (v^*)^{2-n} - \frac{1}{2}(v^*)^{1-n} + \frac{1}{2}.$$

The function $w(v^*)$ defined in the above equation is concave with a maximum at[15]

$$v^{*M} \equiv \frac{n-1}{2n-4}.$$

We have now obtaind w as the function of v^* for $v^* \geq 1$ and $v^* < 1$. Figure I shows the graphical representation of the functions for both intervals together. For $n = 3$, w is represented by a monotonically increaseing function of v^*, since $v^{*M} = 1$. Inverting the axes of the figure for $n > 3$, we get Figure II. The figure shows the reaction of workers to the wage that the firm annouces. At any point on the curve, no worker has an incentive to change his own v^*, unless others change their decisions. From the figure, we can see that there are multiple equilibria[16] for $1 < w < w(v^{*M})$. For instance, if the firm sets $w = w_1$, there are two equilibria at point A with higher v^* and B with lower v^* (the equilbrium in-between is unstable in standard arguments). The equilibrium at the point A is Pareto superior to at B where coordination failure occurs among workers' decisions (for further

[14]Hereafter, $(v^*)^a$ denotes the a th power of v^*.

[15]Taking the derivative of the above equation with respect to v^*, we get

$$w'(v^*) = (2-n)(v^*)^{1-n} - (1-n)\frac{1}{2}(v^*)^{-n}.$$

Solving $w'(v^*) \geq 0$ and $w'(v^*) < 0$, we can see that $v^* \leq v^{*M} \Leftrightarrow w'(v^*) \geq 0$ and $v^* > v^{*M} \Leftrightarrow w'(v^*) < 0$.

[16]If $n \leq 3$, there is only one equilibrium, as shown in Figure I.

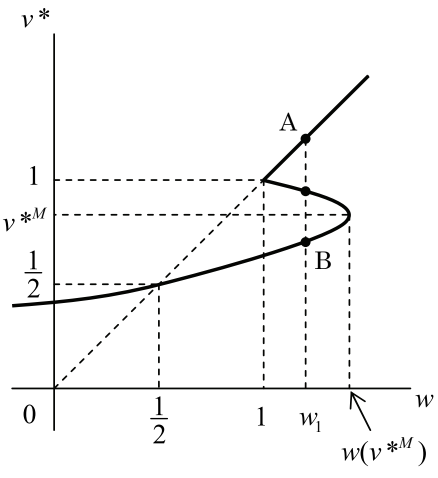

Figure II: Multiple Equilibria in Workers' Decisions on Turnover.

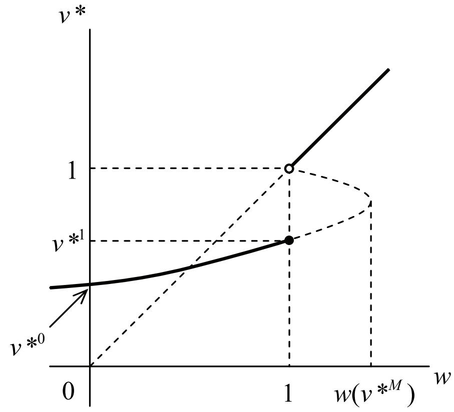

Figure III: Response of v^* to w in Optimistic Equilibrium.

discussion, see Appendix). Throughout this paper, we call the equilibrium at the point A and B in this figure respectively the *optimistic* and the *pessimistic equilibrium*.

Figure III shows the behavior of v^* for w in the optimistic equilibrium. Here and hereafter, v^{*0} and v^{*1} are respectively defined as the solution to $w(v^*) = 0$ and $w(v^*) = 1$ with respect to v^*. From that figure, we can see that v^* takes values only in the ranges $v^{*0} \leq v^* \leq v^{*1}$ and $v^* > 1$ for any $w \geq 0$ (note that the equilibrium with $v^* = 1$ for $w = 1$ is unstable).[17]

Turning to the pessimistic equilibrium, Figure IV shows the behaviour of v^* for w.

[17]The income that a typical worker can expect to receive if he stays is given by the right-hand side of equation (1). Substituting $s = \min(1, v^*)$, we have

$$\frac{1}{2} + \min\left[1, (v^*)^{n-1}\right]\left(w - \frac{1}{2}\right).$$

At the equilibrium with $v^* = 1$ for $w = 1$, a slight drop in v^* makes the income in the above equation lower than v^*, and consequaently v^* falls further.

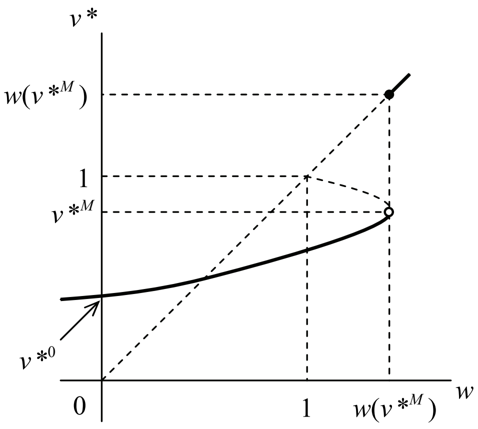

Figure IV: Response of v^* to w in Pessimistic Equilibrium.

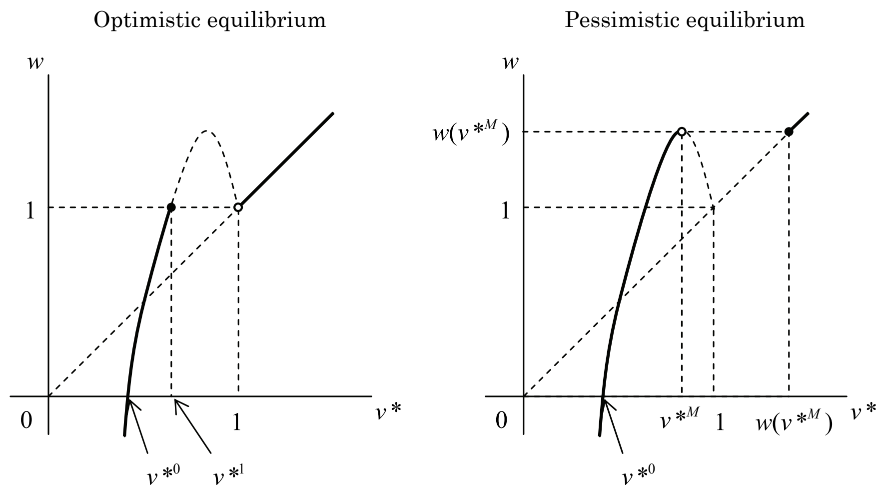

Figure V: Relation between w and v^* in Optimistic and Pessimistic Equilibrium.

In this figure, v^* takes values only in the ranges $v^{*0} \leq v^* < v^{*M}$ and $v^* \geq w(v^{*M})$ for any $w \geq 0$ (note that the equilibrium with $v^* = v^{*M}$ for $w = w(v^{*M})$ is unstable).[18] In additon, it is remarkable that v^* is less than 1 for $1 < w < w(v^{*M})$. This means that, even if the wage in the current firm is greater than 1, which is the highest wage in the external labor market, workers' turnover can occur (the probability that a worker quits is given by $\Pr(v > v^*) = \max(0, 1 - v^*)$). It results from coordination failure, where a worker (we'll call him John) decides to quit on the suspicion that someone decides to quit on the suspicion that John decides to quit on the suspicion... (this information structure is similar to that of "common knowledge" called by Aumann [1976]).

Inverting the axes of Figure III and IV again, we can draw Figure V. As obvious from

[18]At the equilibrium with $v^* = v^{*M}$, a slight increase in v^* makes v^* lower relative to the income that a typical worker can expect to receive if he stays, and thereby v^* rises further.

the figures, the relation between w and v^* in the optimistic equilibrium can be finally written as

$$w = \begin{cases} (v^*)^{2-n} - \frac{1}{2}(v^*)^{1-n} + \frac{1}{2}, & \text{if} \quad v^{*0} \le v^* \le v^{*1} \\ v^*, & \text{if} \quad v^* > 1, \end{cases} \tag{3}$$

and in the pessimistic equilibrium as

$$w = \begin{cases} (v^*)^{2-n} - \frac{1}{2}(v^*)^{1-n} + \frac{1}{2}, & \text{if} \quad v^{*0} \le v^* < v^{*M} \\ v^*, & \text{if} \quad v^* \ge w(v^{*M}). \end{cases} \tag{4}$$

4 Firm's Profit Maximization

Now we turn our attention to firm's expected profit. If all n workers with firm-specific skills stay, the firm makes revenue of $(1+\theta)np$ and pays nw. Otherwise, it makes np and pays $n/2$. Given s, the probability that all n existing workers stay is equal to s^n, so the expected profit π is thus

$$\pi = s^n[(1+\theta)np - nw] + (1-s^n)\left(np - \frac{n}{2}\right). \tag{5}$$

In the above equation, s is dependent on w, because s is determined by v^* which depends on w. Taking that into account, the firm sets w in a way to maximize its expected profit.

For the moment, we suppose that the firm knows which equilibrium will occur.[19] The relation between w and v^* for the optimistic (pessimistic) equilbrium are given by equation (3) (equation (4)). Thus, if we substisute equation (3) (equation (4)) into equation (5) and maxmize with respect to w, we obtain the profit-maximizing wage.

However, it is not easy to maximize π with respect to w, because $v^*(w)$, the inverse function of $w(v^*)$, is extremely complicated. For this reason, we will maximize π with respect to v^*, instead of to w. Having obtained the profit-maximizing value of v^*, finding the value of w at which π is maximized is straight forward, since v^* corresponds to a unique value of w within each equilibrium (we can think that the firm controls v^* by setting w).

4.1 Optimistic Equilibrium

At the beginning, we will substitute $s = \min(1, v^*)$ into equation (5), but s may have different forms depending on whether v^* is less than 1. Therefore, we have to examine each case separately.

Case 1. $v^* \ge 1$. Substituting $s = 1$ into equation (5) gives

$$\pi = (1+\theta)np - nw. \tag{6}$$

[19] It is not easy to find which equilbrium is more likely to occur, so we do not address the problem of equilibrium selection. However, if the firm makes a decision in the way to maximize the possible profit for a worst case (i.e., to maximize the minimum gain), then in the wage setting, the firm may supposes that the pessimistic equilibrium will occur.

The relation between w and v^* for $v^* \geq 1$ is given by $w = v^*$ in equation (3). Substituting $w = v^*$ into equation (6), we get

$$\pi = (1+\theta)np - nv^*. \tag{7}$$

The expected profit π in equation (7) is a linear function of v^* with the slope of $-n$.

Case 2. $v^* < 1$. Substituting $s = v^*$ into equation (5) yields[20]

$$\pi = n\left[-(v^*)^n w + (v^*)^n \left(\theta p + \frac{1}{2}\right) + p - \frac{1}{2}\right]. \tag{8}$$

The relaton between w and v^* for $v^* < 1$ is given by $w = (v^*)^{2-n} - (1/2)(v^*)^{1-n} + 1/2$ in equation (3). Note, however, that an equilibrium exists only for $v^{*0} \leq v^* \leq v^{*1}$. Substituting this equation into equation (8), we obtain[21]

$$\pi = n\left[\theta p (v^*)^n - (v^*)^2 + \frac{1}{2}v^* + p - \frac{1}{2}\right]. \tag{9}$$

In equation (9), π is represented by a polynomial function of v^* with degree n.

We have now obtained π as the function of v^* for $v^* \geq 1$ and $v^* < 1$. Figure VI shows examples of the function for both intervals (given the other parameters $n = 5, \theta = 0.5, p = 0.45$). Note here that an equilibrium exists only for $v^{*0} \leq v^* \leq v^{*1}$ and $v^* > 1$. From the figure, we can see that the profit-maximizing value of v^* is in $v^{*0} \leq v^* \leq v^{*1}$ or equal to 1 (a value greater than but sufficiently close to 1). We cannot find the general solution for the profit-maximizing value of v^*, since equation (9) is an n order polynomial. But in our simulation results, π is maximized only at $v^* = v^{*0}$ or $v^* = 1$, as shown in this figure and those in the following section.

Note here that, as shown in the Figure VI, the value of v^* at which π is maximized does not always make non-negative expected profits. Moreover, the figure suggests that the maximum value of π is more likely to be negative as p or θ is lower (the same goes for the pessimistic equilibrium).

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[20] Rewriting equation (5) gives

$$\pi = n\left[-s^n w + s^n \left(\theta p + \frac{1}{2}\right) + p - \frac{1}{2}\right].$$

Substituting $s = v^*$, we get equation (8).

[21] Substituting $w = (v^*)^{2-n} - (1/2)(v^*)^{1-n} + 1/2$ into equation (8) yields

$$\pi = n\left\{-(v^*)^n \left[(v^*)^{2-n} - \frac{1}{2}(v^*)^{1-n} + \frac{1}{2}\right] + (v^*)^n \left(\theta p + \frac{1}{2}\right) + p - \frac{1}{2}\right\}.$$

Rewriting, we obtain equation (9).

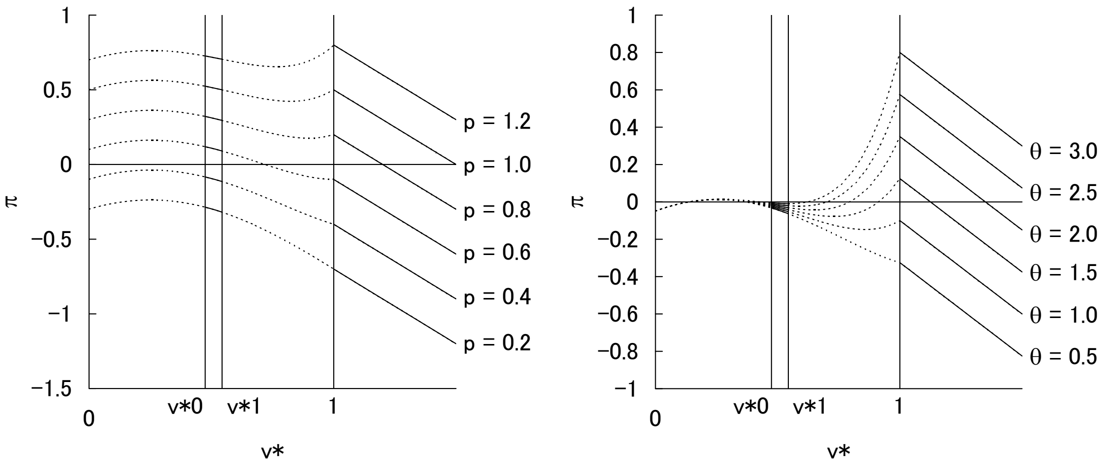

Figure VI: Behavior of Expected Profit for v^* in Optimistic Equilibrium.

4.2 Pessimistic Equilibrium

Case 1. $v^* \geq 1$. Substituting $s = 1$ into equation (5), we get equation (6). The relation between w and v^* for $v^* \geq 1$ is given by $w = v^*$ in equation (4). Note however that an equilibrium exists only for $v^* \geq w(v^{*M})$. Substituting $w = v^*$ into equation (6) gives

$$\pi = (1 + \theta)np - nv^*. \tag{10}$$

The above equation is identical to equation (7) that is for the optimistic equilibrium, but different in the ranges of v^* in which an equilibrium exists.

Case 2. $v^* < 1$. Substituting $s = v^*$ into equation (5), we obtain equation (8). The relation between w and v^* for $v^* < 1$ is given by $w = (v^*)^{2-n} - (1/2)(v^*)^{1-n} + 1/2$ in equation (4). But we have to note that an equilibrium exists only for $v^{*0} \leq v^* < v^{*M}$. Substituting this equation into equation (8) yields

$$\pi = n\left[\theta p(v^*)^n - (v^*)^2 + \frac{1}{2}v^* + p - \frac{1}{2}\right]. \tag{11}$$

The above equation is identical to equation (9), but different in the possible ranges of v^*.

Figure VII plots examples of π as the function of v^* for $v^* \geq 1$ and $v^* < 1$ (given the other parameters $n = 5, \theta = 0.5, p = 0.45$). Note here that an equilibrium exists only for $v^{*0} \leq v^* < v^{*M}$ and $v^* \geq w(v^{*M})$. The figure implies that the profit-maximizing value of v^* is in $v^{*0} \leq v^* < v^{*M}$ or equal to $w(v^{*M})$. It is not easy to find the general solution for the profit-maximizing value of v^*. But in our simulation results shown in Figure VII and those in the following section, π is maximized only at $v^* = v^{*0}$ or $v^* = w(v^{*M})$.

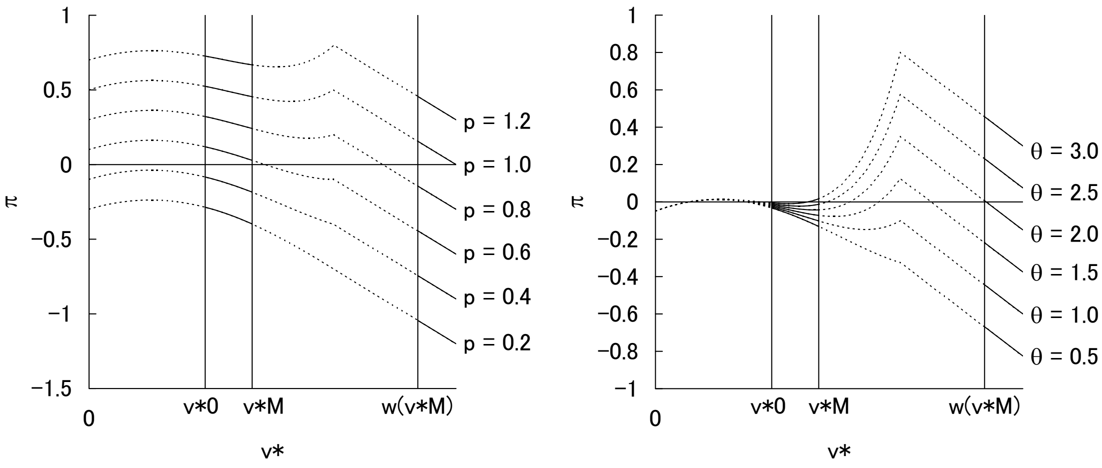

Figure VII: Behavior of Expected Profit for v^* in Pessimistic Equilibrium.

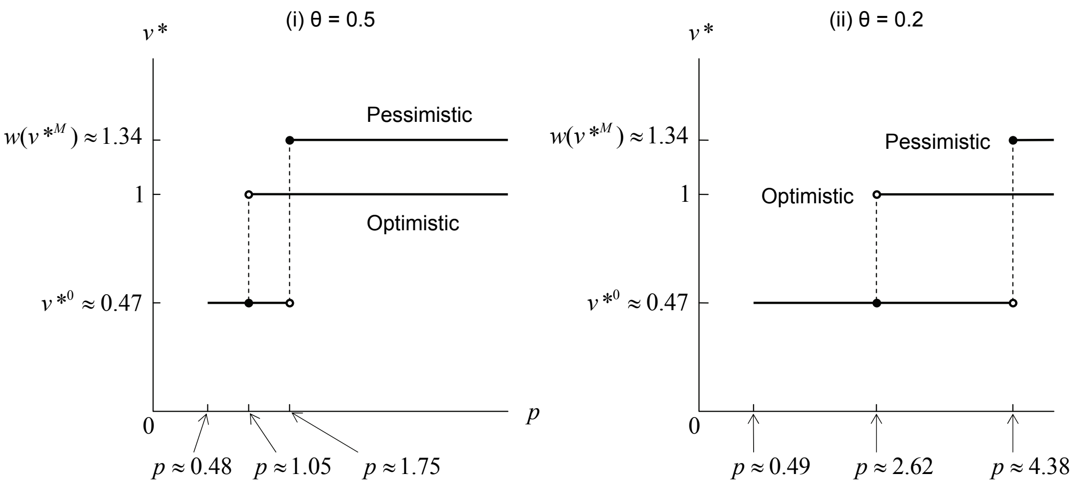

Figure VIII: Behavior of v^* for p (Compared between Different Values of θ).

5 Response to Negative Demand Shocks

5.1 Threshold for Workers' Turnover

We now turn to how negative demand shocks affect the value of v^* at which π is maximized (throughout this section, the notation v^* means the value of v^* at which π is maximized). In this model, the number of units that the firm produces is fixed to either $(1+\theta)n$ or n, so we think of decreased p as being a decline in the firm's revenue due to decreased demand. (The parameter p can be thought of as representing product market conditions relative to labor market conditions, since the distribution of v does not depend on p in our model.)

Figure VIII, which is obtained by numerical simulation, compares the behavior of v^* for decreased p between different values of θ (given the parameter $n = 5$). The figure (and

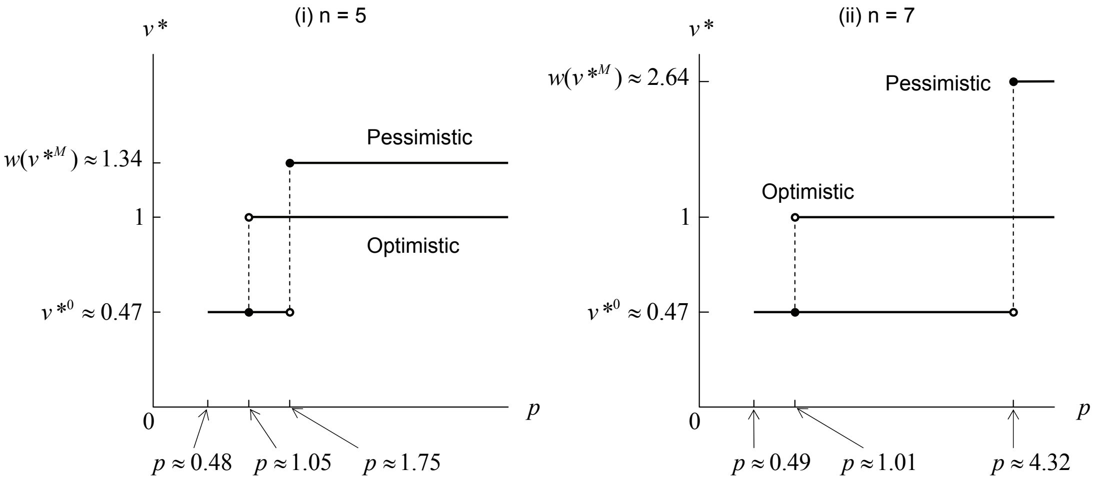

Figure IX: Behavior of v^* for p (Compared between Different Values of n).

the following ones) plots only for the range in which the firm expects non-negative profits (in the case of this figure, the expected profit is non-negative when p is greater than about 0.48 for $\theta = 0.5$ and about 0.49 for $\theta = 0.2$).

In the figure, the threshold v^* exhibits the following features. First, in the optimistic equilibrium, the threshold v^* takes the value of either 1 (a value greater than but sufficiently close to 1) or 0. In the pessimistic equilibrium, it takes the value of either $w(v^{*M})$ or v^{*0}.[22] Second, the value of p at which v^* falls to v^{*0} is lower in the optimistic equilibrium than in the pessimistic equilibrium. Third, the lower the value of θ (the lower the additional revenue from skill specificity), the higher the value of p at which v^* falls.

Figure IX compares the behavior of v^* between different values of n (given the parameter $\theta = 0.5$). From that figure, we can see the following. Firstly, in the optimistic equilibrium, the value of p at which v^* falls to v^{*0} does not differ greatly depending on n. By contrast, in the pessimistic equilibrium, the value of p at which v^* falls is much higher as n is larger. Secondly, in the pessimistic equilibrium, the larger the value of n, the higher the value of v^* at which no worker quits the firm. But in the optimistic equilibrium, it is constant at 1 (at a value greater than but close to 1) across different values of n.

5.2 Wage for Workers with Firm-specific Skills

Having obtained the value of v^* at which π is maximized, finding the profit-maximizing value of w (in this section, the notation w means the profit-maximizing value of w) is straight forward.

Figure X compares the behavior of w for decreased p between different values of θ (given the parameter $n = 5$). The features shown in the figure are as follows.

First, in the optimistic equilibrium, w takes the value of either 1 (a value greater than

[22]In the pessimistic equilibrium, $v^* = w(v^{*M})$ is the minimum value that satisfies $v^* \geq 1$, since there is no equilibrium with $v^{*M} \leq v^* < w(v^{*M})$.

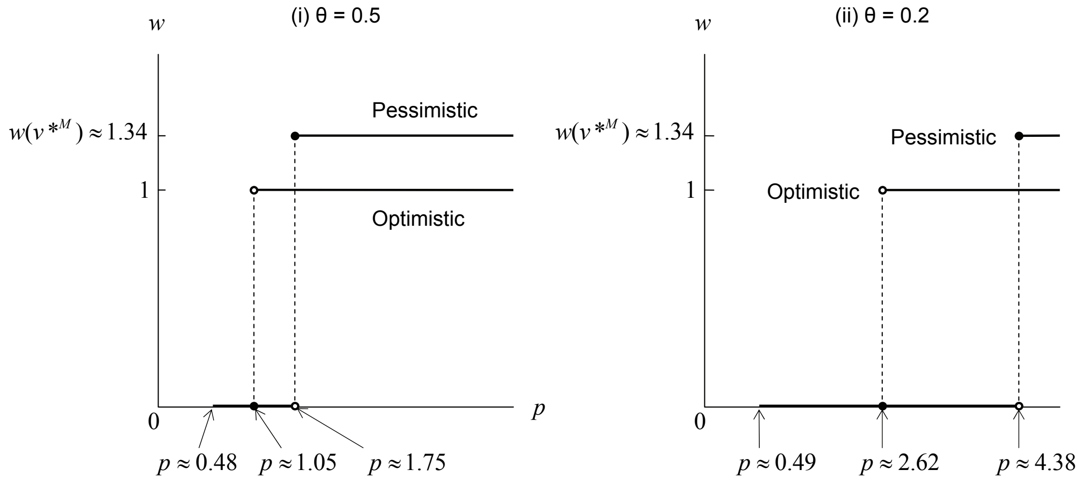

Figure X: Behavior of Internal Wage for p (Compared between Different Values of θ).

but sufficiently close to 1) or 0. This means that the firm sets w at the highest wage in the external labor market or at the minimum wage. In the pessimistic equilibrium, w takes the value of either $w(v^{*M})$ or 0.[23] Note, however, that setting $w = 0$ does not mean paying $w = 0$. If a firm sets $w = 0$, some workers will almost certainly quit, and thereby it hires all n workers at the wage of 1/2 (the wage that the firm expects to pay if $w = 0$ is given by $[1 - (v^{*0})^n]/2$).[24] Setting $w = 0$ therefore substantially means that the firm terminates its long-term contracts with all existing workers.

Second, the value of p at which w falls to 0 is lower in the optimistic equilibrium than the pessimistic equilibrium.

Third, the lower the value of θ (the lower the additional revenue from skill specificity), the higher the value of p at which w falls.

Finally, in the pessimistic equilibrium, w can be set at higher than 1, which means that the wage in the firm can be greater than the highest wage in the external labor market. This results from coordination failures in worker' decisions on their turnover. If coordination failures occur, setting w at 1 (at a value greater than but close to 1) is insufficient to reduce the probability of labor turnover to zero, as discussed in the previous section. Therefore, the firm can be willing to pay $w > 1$ in order to reduce costly labor turnover.

Figure XI compares the behavior of w between different values of n (given the parameter $\theta = 0.5$). The figure shows following. Firstly, in the optimistic equilibrium, the value of p at which w falls to 0 does not differ greatly by the value of n. By contrast, in the pessimistic equilibrium, it is much higher as n is larger. Secondly, in the pessimistic

[23]The reason why the firm pays $w > 1$ in the pessimistic equilibrium is that paying $w = 1$ is insufficient to keep $v^* = 1$. Given $w = 1$, the value of v^* is equal to v^{*1} in the pessimistic equilibrium.

[24]If the firm sets $w = 0$, it hires all workers at $w = 0$ with the probability of $[\Pr(v \le v^{*0})]^n = (v^{*0})^n$ and at the wage of 1/2 with $\{1 - [\Pr(v \le v^{*0})]^n\} = [1 - (v^{*0})^n]$. The wage that the firm expect to pay if $w = 0$ is thus $[1 - (v^{*0})^n](1/2)$.

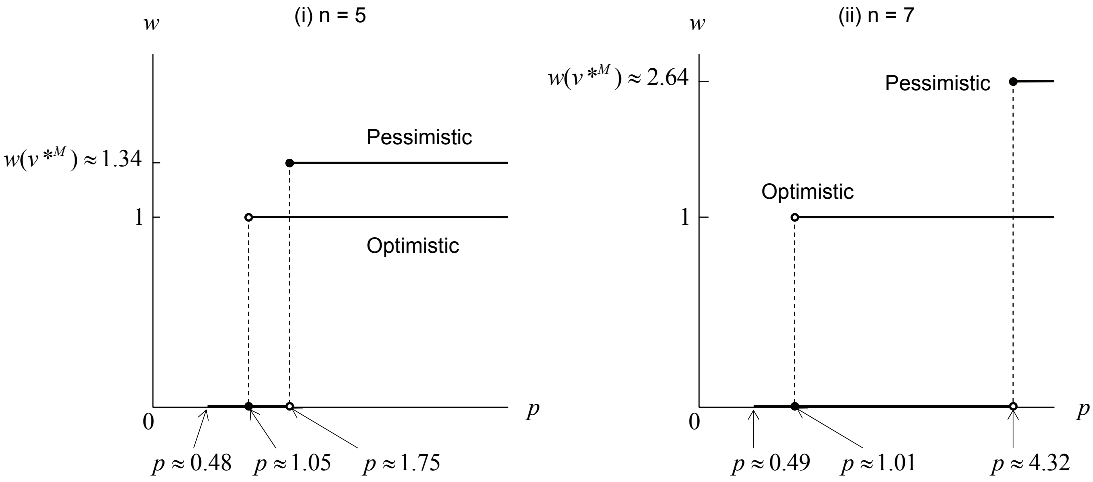

Figure XI: Behavior of Internal Wage for p (Compared between Different Values of n).

equilibrium, as n is larger, the value of w at which no one quits is higher and thereby can be far above the highest wage in the external market. This is consistent with the fact that larger firms pay higher wages.

5.3 Proportion of Skilled Jobs

Given v^*, the probability that skilled jobs are sustained in a typical firm is equal to $s^n = [\Pr(v \leq v^*)]^n$, since it pays the wage for skilled workers if and only if all n existing workers stay in the firm. Suppose now that an industry consists of an infinite number of firms, where the realizations of v are independent across workers in all firms. Then, by the law of large numbers, we can think of s^n as the proportion of firms with skilled jobs in the industry or the proportion of skilled workers.[25]

Figure XII compares the behavior of s^n for decreased p between different values of θ (given the parameter $n = 5$). In the figure, the proportion of skilled jobs exhibits the following features. First, the proportion of skilled jobs takes the value of either 1 or $(v^{*0})^n$. Second, the value of p that leads to the destruction of skilled jobs is lower in the optimistic equilibrium than the pessimistic equilibrium. Third, the lower the value of θ (the lower the additional revenue from skill specificity), the higher the value of p that leads to the job destruction.

Figure XIII compares the behavior of s^n between different values of n (given the parameter $\theta = 0.5$). From the figure, we can see the following. Firstly, in the optimistic equilibrium, the value of p that leads to the destruction of skilled jobs does not differ greatly by n. By contrast, in the pessimistic equilibrium, it is much higher as n increases.

[25] Some may think that an industry consisting of identical firms has only two extreme states: whether skilled jobs are sustained in all firms or not. Indeed, all firms in the industry announce the same value of w, and thereby workers in all firms have the same value of v^*. But, given $v^* < 1$, it is uncertain whether v exceeds v^* (and thus whether each worker quits), since v is the random variable. Therefore, skilled jobs are sustained in some firms and not in the others, even if all firms are identical.

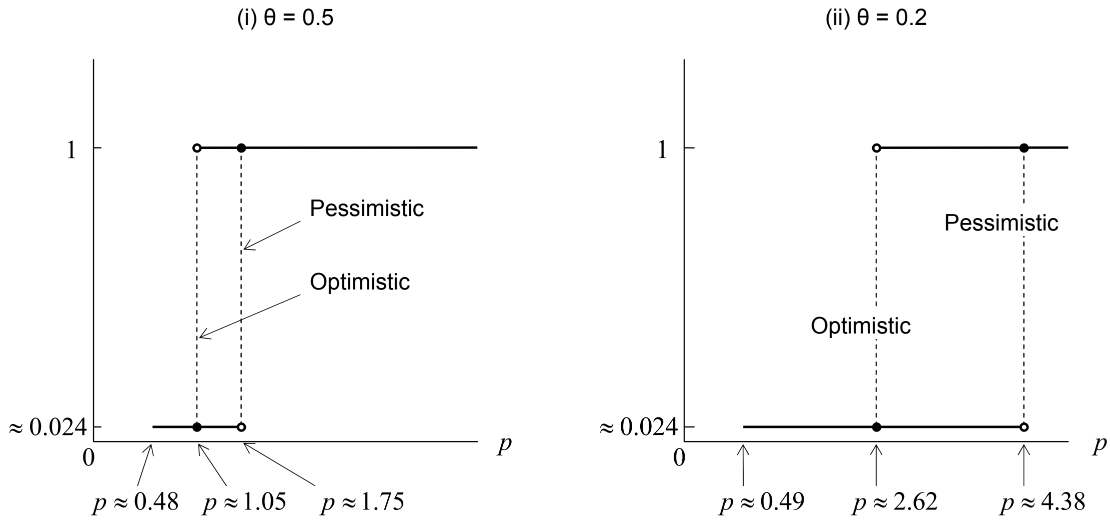

Figure XII: Proportion of Skilled Jobs for p (Compared between Different Values of θ).

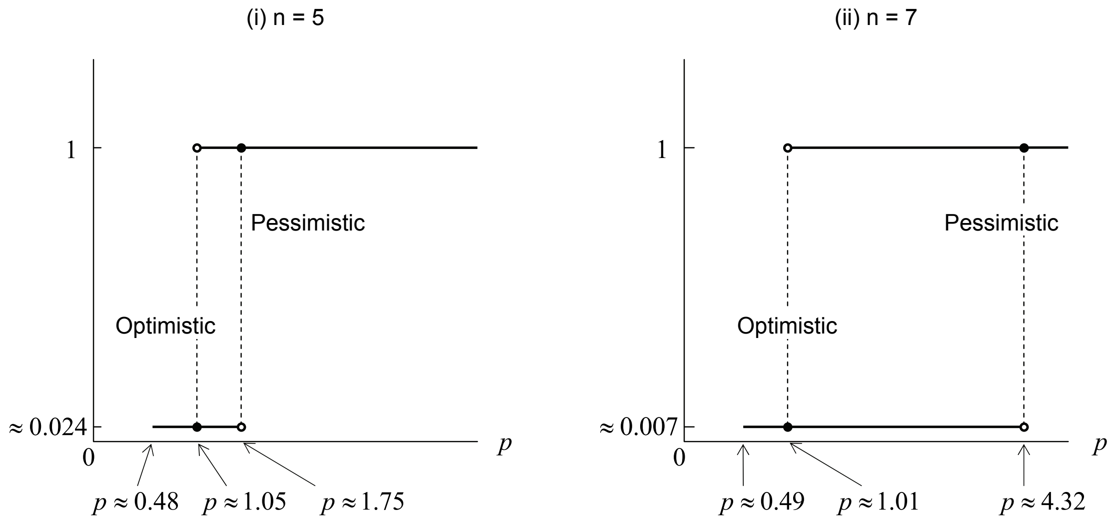

Figure XIII: Proportion of Skilled Jobs for p (Compared between Different Values of n).

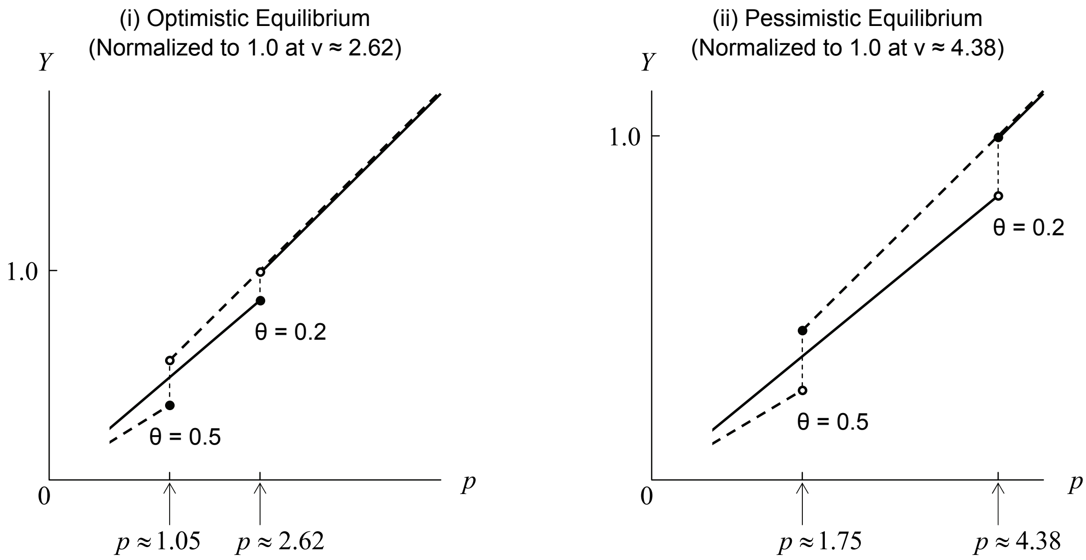

Figure XIV: Behavior of Expected Output for p (Compared between Different Values of θ).

Secondly, as n increases, the proportion of skilled jobs approaches 0 when p falls below some critical level.

5.4 Total Output of an Industry

We continue considering an industry consisting of an infinite number of firms, and we let the total number of the firms be normalized to 1. This assumption allows us to think of the expected output (or revenue) of a typical firm as the total output of the industry. Given v^*, a typical firm produces $(1+\theta)np$ with the probability of $s^n = \min[1, (v^*)^n]$ and np with $(1-s^n)$. Hence, the expected output of the firm and thus the total output Y is given by[26]

$$Y = np + \min[1, (v^*)^n]\theta np.$$

Figure XIV compares the relative declines in the total output Y between different values of θ (given the parameter $n = 5$). For the optimistic equilibrium, the output Y is normalized to 1 at $p \approx 2.62$, which is the minimum value of p that satisfies $s^n = 1$ in both cases $\theta = 0.5$ and $\theta = 0.2$. For the pessimistic equilibrium, Y is normalized to 1.0 at $p \approx 4.38$ for the same reason. The features exhibited in the figure are as follows. First, the value of p at which Y drops discretely is lower in the optimistic equilibrium than the pessimistic equilibrium. Second, the lower the value of θ (the lower the additional revenue from skill specificity), the higher the value of p at which Y drops discretely. Third, the higher the value of θ, the greater the percentage change in Y at which it drops

[26]The approximation of the total output can be written as

$$Y = s^n(1+\theta)np + (1-s^n)np.$$

Rewriting and substituting $s^n = \min[1, (v^*)^n]$, we get the equation in the text.

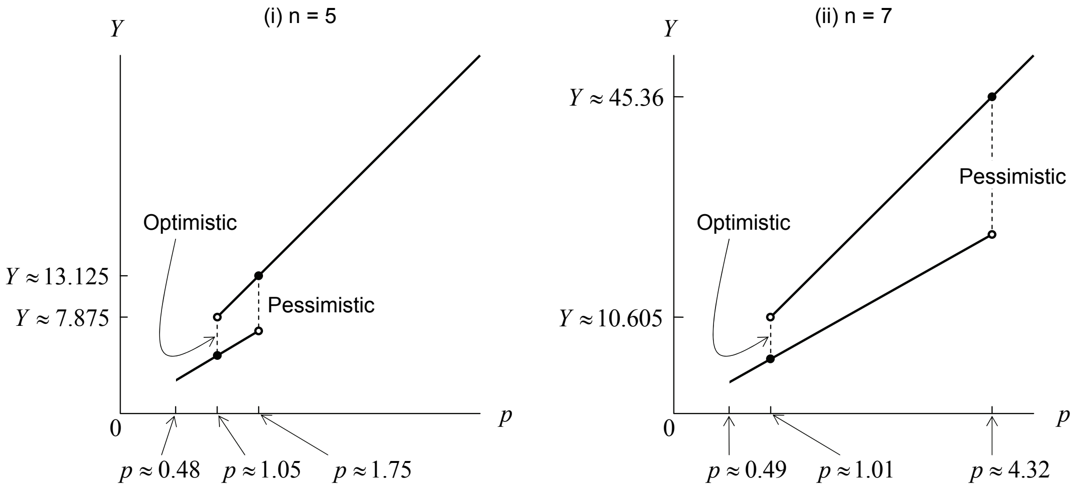

Figure XV: Behavior of Expected Output for p (Compared between Different Values of n).

discretely. Finally, take the case of the optimistic equilibrium as an example. For $p = 2.0$, the normalized Y is greater in the high-θ case than in the low-θ case. This means that when p falls to 2.0 from about 2.62, the percentage decrease in Y is smaller in the high-θ case. Thus, facing a slight drop in p (when p falls to between about 1.05 and 2.62), the percentage decrease in Y is smaller in the high-θ case than in the low-θ case. By contrast, facing a large drop in p (when p falls below about 1.05), the percentage decrease in Y is smaller in the low-θ case.

Figure XV compares the behavior of Y (not normalized) for decreased p between different values of n (given the parameter $\theta = 0.5$). The figures show the following. Firstly, in the optimistic equilibrium, the value of p at which Y drops discretely does not differ greatly by n. By contrast, in the pessimistic equilibrium, the value of p at which Y drops is much higher as n is larger. Secondly, in the pessimistic equilibrium, the change in Y at which it drops discretely is greater as n is larger (however, the percentage change at which Y drops discretely is always equal to $-\theta/(1+\theta)$ regardless of the value of n).

6 Another Example: Continuous Production Function

Our basic model assumes that productivity falls to the lowest level if one or more existing workers quit. In this section, weakening this extreme assumption, we provide another model, in which productivity decreases continuously with the job separation rate. This weaker assumption produces the same results on destruction of skilled jobs as in the basic model. However, under a continuous production function, coordination failures cannot occur in workers' decisions on whether they stay or quit.

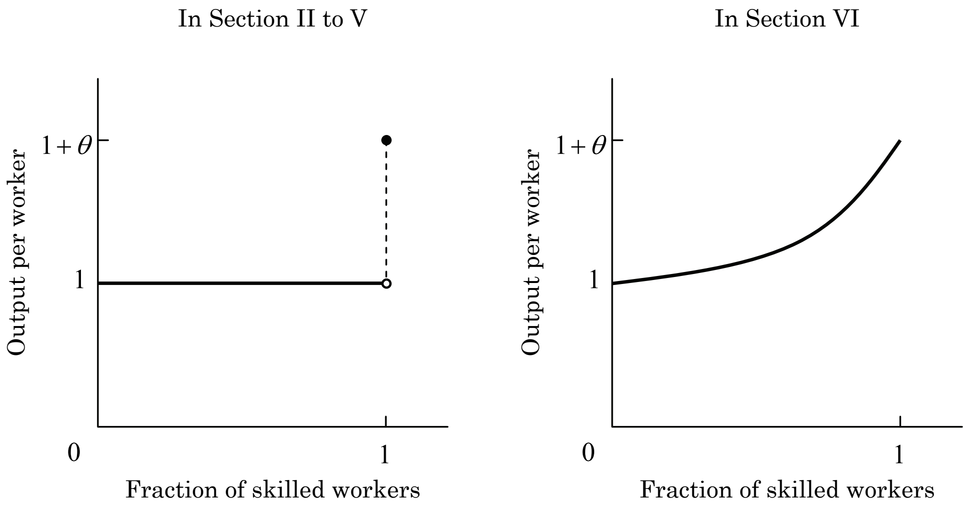

Figure XVI: Productivity and Fraction of Skilled Workers.

6.1 Production Technology and Labor Market

A Firm hires n workers at a wage w, and sells final goods at a price p, as well as in the basic model. But the output of the firm in this section is given by $[1+\phi(l)\theta]n$ units, where l denotes the fraction of skilled workers (the fraction of workers with firm-specific skills to all n workers), and $\phi(\cdot)$ is a twice continuously differentiable function, such that $\phi(0) = 0$, $\phi(1) = 1$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) \geq 0$. This means that skilled workers are independent or complementary to each other. Thus, productivity is initially equal to $(1 + \theta)$ for $l = 1$, decreasing continuously with decreasing l, and goes to 1 for $l = 0$ (see Figure XVI).

The external labor market is the same as in the basic model: each existing worker has an alternative earning opportunity v; the external wage v is distributed uniformly on $[0, 1]$; the realization of v is independent across all workers; each v is private information to each worker; if one or more workers quit, the firm has to hire replacement workers from the external labor market.

In the basic model, we assumed that, if one or more existing workers quit, the firm hires all n workers at the wage of $1/2$. This was because, if some workers quit, the output of the firm is constant at n units regardless of whether or not other workers have firm-specific skills. But in this section, even if someone quits, the workers who stay are still contributing to increased productivity, given a continuous production technology. Therefore, in this section, the firm always pays w to all n workers (including the replacement workers) regardless of whether someone quits. The wage w has to be greater than or equal to $\mathrm{E}(v) = 1/2$, or else the firm cannot hire replacement workers from the external market.

6.2 Firm's Profit Maximization

We first consider the expected profit of the firm. The firm sells $[1 + \phi(l)\theta]n$ units of final goods at the price p, so its revenue is thus $[1 + \phi(l)\theta]np$. On the other hand, it pays nw to workers. Hence, given w, its expected profit can be written as

$$\pi = [1 + \phi(l)\theta]np - nw. \tag{12}$$

The fraction l in equation (12) is dependent on w, since w affects workers' decisions on whether to stay or quit. The income that a typical worker can expect to receive if he stays is equal to w regardless of whether or not other workers stay. Therefore, each worker stays if and only if $v \leq w$. The probability that each worker stays is given by $\Pr(v \leq w) = \min(1, w)$, so the expected fraction of skilled worker is thus[27]

$$l = \min(1, w).$$

Since the expected fraction in the above equation may have different forms depending on whether w is greater than 1, we have to examine each case separately.

Case 1. $w > 1$. Substituting $l = 1$ into equation (12) gives

$$\pi = (1 + \theta)np - nw. \tag{13}$$

The expected profit π in equation (13) is a linear function of w with a slope of $-n$.

Case 2. $w \leq 1$. By substituting $l = w$ into equation (12), we get

$$\pi = \pi(w) \equiv [1 + \phi(w)\theta]np - nw. \tag{14}$$

The function $\pi(w)$ defined in equation (14) is non-concave, since $\pi''(w) \geq 0$ follows from $n, p > 0$ and $\phi''(\cdot) \geq 0$.[28]

Figure XVII shows the expected profit π as the function of w for both intervals. From that figure, it is clear that π is maximized at $w = 1$ if $\pi(1) \geq \pi(1/2)$ and $w = 1/2$ otherwise. Solving $\pi(1) \geq \pi(1/2)$ with respect to p gives[29]

$$p \geq \left\{2\theta\left[1 - \phi\left(\frac{1}{2}\right)\right]\right\}^{-1}. \tag{15}$$

[27] The expected number of the workers who stay is given by $n \cdot \min(1, w)$, so the expected fraction of skilled worker is thus $[n \cdot \min(1, w)]/n = \min(1, w)$.

[28] Taking the derivative of equation (14) with respect to w yields $\pi'(w) = \phi'(w)\theta np - n$. Taking the derivative again, we get $\pi''(w) = \phi''(w)\theta np$. Hence, we can see that $\pi''(w) \geq 0$, since $n, p > 0$ and $\phi''(\cdot) \geq 0$.

[29] From equation (14), $\pi(1) \geq \pi(1/2)$ can be rewritten as $(1 + \theta)np - n \geq [1 + \phi(1/2)\theta]np - n/2$. Solving with respect to p, we get equation (15).

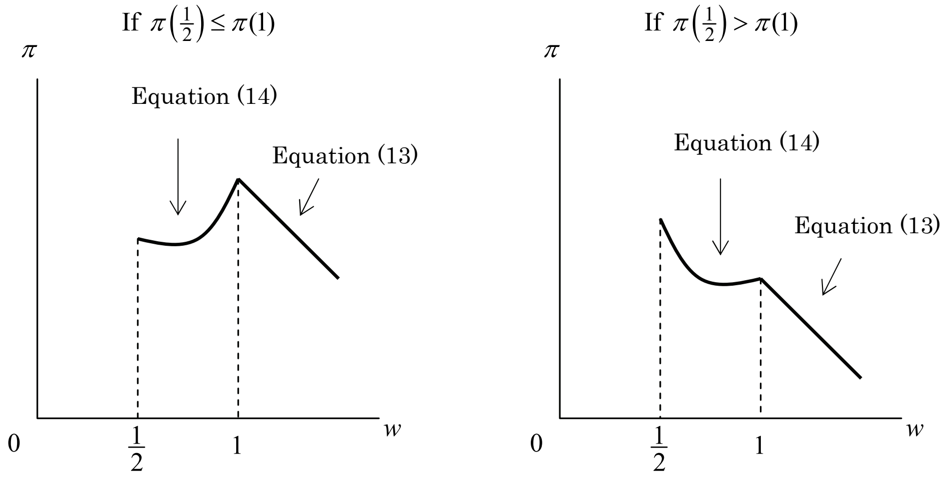

Figure XVII: Profit-maximizing Internal Wage.

Therefore, the profit-maximizing wage can be written as

$$w = \begin{cases} 1, & \text{if} \quad p \geq \left\{2\theta\left[1 - \phi\left(\frac{1}{2}\right)\right]\right\}^{-1} \\ \frac{1}{2}, & \text{otherwise.} \end{cases} \tag{16}$$

In addition, the non-negative profit condition for $\phi''(\cdot) > 0$ is given by

$$p \geq \begin{cases} (1+\theta)^{-1}, & \text{if} \quad \theta \geq \left[1 - 2\phi\left(\frac{1}{2}\right)\right]^{-1} \\ \left[2 + 2\phi\left(\frac{1}{2}\right)\theta\right]^{-1}, & \text{otherwise,} \end{cases}$$

and for $\phi''(\cdot) = 0$ is by

$$p \geq \left[2 + 2\phi\left(\frac{1}{2}\right)\theta\right]^{-1},$$

(see Appendix for details).

6.3 Response to Negative Demand Shocks

In the previous section, we examined the behavior of v^* for decreased p. But in this section, w itself acts as v^*, since the income that a typical worker expects to receive if he stays is always equal to w (regardless of the decisions of other workers).

Moreover, w can be thought of as the proportion of skilled jobs in an industry. If we suppose that an industry consists of an infinite number of workers, the probability that a typical worker remains in the current firm approximates the proportion of skilled jobs in the industry. The probability that a worker remains is given by $l = \min(1, w)$. Substituting equation (16) gives

$$l = \begin{cases} \min(1,1) = 1, & \text{if} \quad p \geq \left\{2\theta\left[1 - \phi\left(\frac{1}{2}\right)\right]\right\}^{-1} \\ \min\left(1, \frac{1}{2}\right) = \frac{1}{2}, & \text{otherwise.} \end{cases}$$

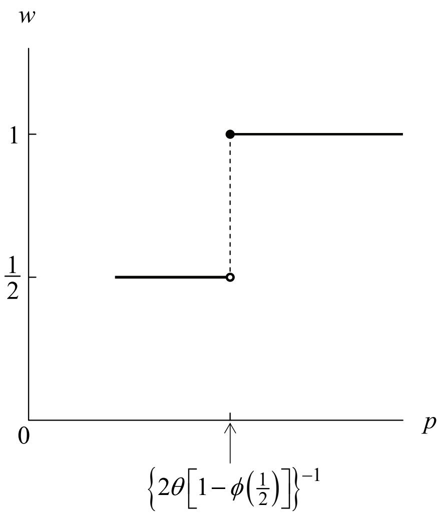

Figure XVIII: Behavior of Internal Wage w for p.

The above equation is identical to equation (16) itself, which gives the profit-maximizing value of w. Hence, we can think of w as the proportion of skilled jobs.

Figure XVIII shows the behavior of w for decreased p. In the figure, w exhibits features similar to those in the basic model: w takes the value of either 1 or 1/2; the larger the value of θ (the larger the additional revenue from skill specificity), the lower the value of p at which w falls discretely.[30] However, different from the basic model, there is only one equilibrium, and w does not depend on n. This is because, in this section, the income that a worker can expect to receive if he stays is independent of the decisions of $(n-1)$ other workers.

6.4 Total Output of an Industry

To examine the total output in an industry, we first consider the expected output (or revenue) of a typical firm. A typical firm produces $[1+\phi(l)\theta]n$ final goods and sells at the price p. Therefore, the expected output of the firm is given by $Y = [1+\phi(l)\theta]np$. Substituting $l = \min(1, w)$ yields

$$Y = \{1 + \min[1, \phi(w)]\theta\}np.$$

Substituting equation (16), we get

$$Y = \begin{cases} (1+\theta)np, & \text{if} \quad p \geq \left\{2\theta\left[1-\phi\left(\frac{1}{2}\right)\right]\right\}^{-1} \\ \left[1+\phi\left(\frac{1}{2}\right)\theta\right]np, & \text{otherwise.} \end{cases} \tag{17}$$

[30]Taking the derivative of $2\theta[1-\phi(1/2)]$ with respect to θ, we get $2[1-\phi(1/2)]$, which is positive, since $0 < \phi(1/2) \leq 1/2$ follows from $\phi(0) = 0$, $\phi(1) = 1$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) \geq 0$. Thus, $\{2\theta[1-\phi(1/2)]\}^{-1}$ is decreasing with θ.

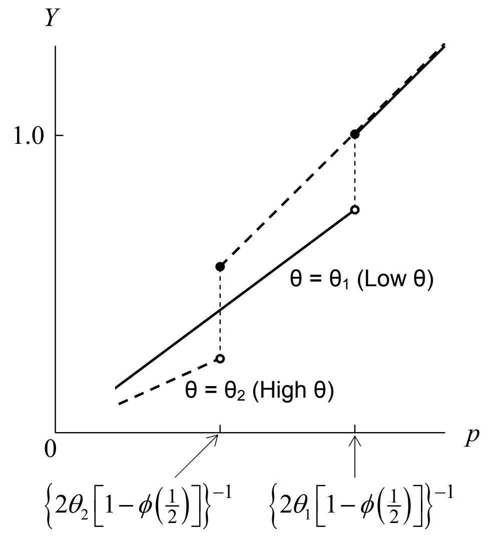

Figure XIX: Behavior of Total Output Y for p.

If we normalize the number of the firm in an industry to 1, Y can be thought of as the total output in the industry.

Figure XIX compares the relative decline in the total output Y between different values of θ (where $\theta_1 < \theta_2$). In the figure, the values of Y are normalized to 1.0 at $p = \{2\theta_1[1 - \phi(1/2)]\}^{-1}$, which is the minimum value of p that satisfies $l = 1$ in both cases $\theta = \theta_1$ and θ_2. The figure exhibits the same features as shown in the basic model: the higher θ (the higher the additional revenue from skill specificity), the greater the percentage change in Y at which it drops discretely;[31] facing a slight drop in p (when p falls to between $\{2\theta_2[1 - \phi(1/2)]\}^{-1}$ and $\{2\theta_1[1 - \phi(1/2)]\}^{-1}$), the percentage decrease in Y is smaller in the high-θ case; but facing a large drop in p (when p falls below $\{2\theta_2[1 - \phi(1/2)]\}^{-1}$), the percentage decrease in Y is smaller in the low-θ case.

7 Summary and Conclusions

In this paper, we have tried to examine how decreased demand for products leads to job destruction and thereby how total output of an industry decreases. Our two models, in which skilled workers are more or less complementary to each other, produce the following comparative statics results.

First, as shown in Figure XII and XVIII, once market conditions fall below some critical level, the number of skilled jobs can drop discretely, rather than decrease continuously. In

[31] From equation (17), the percentage change in Y at which it drops discretely is given by

$$\frac{(1+\theta)np - [1+\phi(1/2)\theta]np}{(1+\theta)np} = \frac{1-\phi(1/2)}{1/\theta+1},$$

which is increasing with θ, since $0 < \phi(1/2) \leq 1/2$.

addition, skilled jobs with higher specificity are sustained until market conditions worsen to a more serious level.

The second is shown in Figure XIV and XIX. Facing a large drop in demand, an industry with higher skill specificity has a greater relative decline in the value of output compared to that with lower specificity. But facing a slight drop in demand, higher specificity has a smaller relative decline. In short, skill specificity has an advantage in favorable market conditions, whereas it has a disadvantage in weak conditions.

Moreover, this paper has attempted to highlight the potential importance of coordination problems for the wage setting of profit-maximizing firms. With a Leontief technology and asymmetric information, coordination failure can occur in workers' turnover decisions, and thereby there can be multiple equilibria. In the pessimistic equilibrium where coordination failure arises, labor turnover can occur even if the wage paid by the current firm is greater than the highest wage in the external labor market (as described in Section III). This coordination problem makes firms willing to pay wages far above the external wages to reduce workers' turnover (as shown by Figure X and XI in Section V).

Now we turn to the question of which equilibrium is more likely to occur? Applying standard arguments, if workers can talk beforehand, it is possible to reach the optimistic equilibrium which is better for all of them. On the other hand, if each worker suspects that one or more other workers will quit irrationally, the pessimistic equilibrium is more likely to prevail. However, the optimistic equilibrium will sustain if the firm commits to paying workers who stay the wage equal to the highest wage in the external market regardless of whether other workers quit (Blanchard and Kremer [1997], p.1112). This implies that not only mutual trust among workers, but also the ability of firms to pay plays an important role in whether skilled jobs are sustained.

In some advanced countries, the polarization of the labor market has been observed especially in the 1990s or later (e.g., Autor, Levy and Murnane [2003], Goos and Manning [2003], Autor, Katz and Kearney [2006]). A standard explanation attributes the job polarization to computerization. Computer technology can replace human labor in middle-skilled jobs (routine cognitive tasks such as bookkeeping and clerical work). In contrast, it cannot replace human labor in high-skilled jobs (non-routine cognitive tasks such as problem solving and coordination activities) and low-skilled ones (non-routine manual tasks as performed by waiters and truck drivers).[32] Computerization therefore lowers the relative wages of middle-skilled jobs and leads to job polarization.[33]

Computerization not only saves labor but also simplifies tasks in middle-skilled jobs. This means that computerization decreases the importance of the long-term accumulation of experience and knowledge, and it has been one of the major causes of the rapid destruc-

[32]To be precise, computer technology is complementary to high and low-skilled jobs, whereas it is a substitute for human labor in middle-skilled jobs.

[33]According to Autor, Levy and Murnane (2003), computerization raises the wages of high-skilled jobs through complementarity. For middle-skilled jobs, it lowers the wages, since it substitutes for human labor. For low-skilled jobs, it has ambiguous effects on the wages because the impact of complementarity offsets additional labor supply from displaced workers from middle-skilled jobs.

tion of middle-skilled jobs. In our model, the decline in the importance of skilled labor can be represented by decreased θ, and this model shows that the number of skilled jobs decreases rapidly once θ falls below some critical level.

In this paper, the creation of new employment is beyond the range of our consideration, but in future work, it may be possible to include job creation in an analysis. Furthermore, it is also important to provide an empirical evidence to support our findings.

Appendix

Proof of Coordination Failures in Workers' Decisions

In Section III, the payoff of a worker (i.e., the expected income of a typical worker) depends on the strategies of other workers (i.e., other workers' decisions on whether they stay or quit). Therefore, workers' decisions on turnover can be thought of as one of response games. This subsection provides proof that this game involves coordination failures.

Cooper and John (1988) show that *strategic complementarities* and *spillovers* together can generate coordination failures. We must first formulate the payoff of a typical worker in order to show that the worker turnover game exhibits both strategic complementarity and positive spillovers.

In Section III, the threshold v^* is defined as follows: a typical worker stays if $v \leq v^*$ and quit otherwise. Throughout this subsection, v^* denotes the threshold that a typical worker takes, but we let $\bar{v}^*$ be the threshold that all other workers take. In addition, we consider here only the case that $v^*, \bar{v}^* \leq 1$.

We now suppose that a typical worker decides to stay in his current firm. If all $(n-1)$ other workers stay, a typical worker receives w. Otherwise, he gets $1/2$. The probability that all other workers stay is given by $[\Pr(v \leq \bar{v}^*)]^{n-1} = (\bar{v}^*)^{n-1}$. Thus, the income that a typical worker can expect to receive if he stays, denoted by $\bar{w}$, can be written as[34]

$$\bar{w} = \bar{w}(\bar{v}^*) \equiv \frac{1}{2} + (\bar{v}^*)^{n-1}\left(w - \frac{1}{2}\right). \tag{18}$$

If a typical worker stays, his expected income is given by $\bar{w}$ in the above equation. On the other hand, if he quits and takes up his external opportunity, he expects to receive $\mathrm{E}(v|v > v^*) = (v^* + 1)/2$.[35] The probability that he will decide to stay is given by $\Pr(v \leq v^*) = v^*$. Let $m = m(v^*, \bar{v}^*)$ denote the payoff of a typical worker, so we have

$$m(v^*, \bar{v}^*) = v^* \cdot \bar{w}(\bar{v}^*) + (1 - v^*)\frac{v^* + 1}{2}.$$

[34] The income expected if he stays is given by

$$\bar{w} = (\bar{v}^*)^{n-1}w + [1 + (\bar{v}^*)^{n-1}]\frac{1}{2}.$$

Rewriting, we get equation (18).

[35] $\mathrm{E}(v|v > v^*)$ is given by the simple average between v^* and 1, since v is distributed uniformly.

Substituting equation (18) gives[36]

$$m(v^*, \bar{v}^*) = \frac{1}{2}\{-(v^*)^2 + [1 + (\bar{v}^*)^{n-1}(2w-1)]v^* + 1\}. \tag{19}$$

Cooper and John argue that, if $m_{12}(v^*, \bar{v}^*) > 0$, the game exhibits strategic complementarity (where the subscript denotes a partial derivative in the usual manner). Taking the total derivative of equation (19), we get[37]

$$m_{12}(v^*, \bar{v}^*) = (n-1)\frac{1}{2}(2w-1)(\bar{v}^*)^{n-2}.$$

The above equation is always positive whenever $n > 1$ and $w > 1/2$, and thus proves that the game exhibits strategic complementarity for $w > 1/2$.

Moreover, Cooper and John define the following: if $m_2(v^*, \bar{v}^*) > 0$, the game exhibits positive spillovers. Taking the partial derivative of equation (19) with respect to $\bar{v}^*$ gives

$$m_2(v^*, \bar{v}^*) = (n-1)\frac{1}{2}v^*(2w-1)(\bar{v}^*)^{n-2},$$

which is always positive whenever $n > 1$ and $w > 1/2$. Hence, the game is proven to exhibit positive spillovers for $w > 1/2$.

Thus, this game involves coordination failures if $w > 1/2$, since it exhibits both strategic complementarity and positive spillovers.

Deriving a Non-negative Profit Condition in Section VI

This subsection derives the non-negative profit condition (NNPC) in Section VI. The firm sets w at 1 if equation (15) holds and 1/2 otherwise. Hence, all we have to do is to examine these two cases.

For the case that equation (15) holds, substituting $w = 1$ into equation (14), we get $\pi = (1+\theta)np - n$. Hence, NNPC is given by[38]

$$p \geq (1+\theta)^{-1}. \tag{20}$$

For the case equation (15) does not hold, substituting $w = 1/2$ into equation (14) yields

[36]Substituting equation (18) into $m(v^*, \bar{v}^*)$ gives

$$m(v^*, \bar{v}^*) = v^* \left[\frac{1}{2} + (\bar{v}^*)^{n-1}\left(w - \frac{1}{2}\right)\right] + (1-v^*)\frac{v^*+1}{2}.$$

Rewriting, we get equation (19).

[37]Taking the partial derivative of equation (19) with respect to v^* gives

$$m_1(v^*, \bar{v}^*) = \frac{1}{2}(2w-1)(\bar{v}^*)^{n-1} - v^* + \frac{1}{2}.$$

Again, taking the partial derivative with respect to $\bar{v}^*$, we get the equation in the text.

[38]Solving $(1+\theta)np - n \geq 0$ with respect to p gives equation (20).

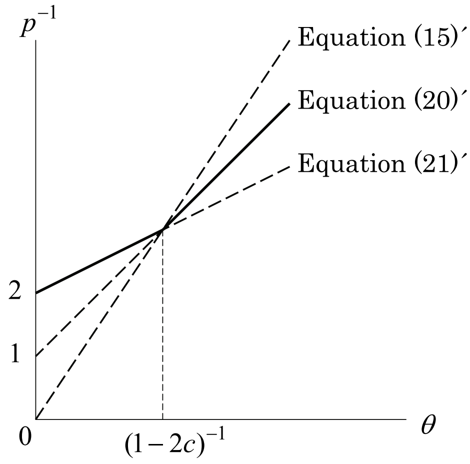

Figure A I: Non-negative Profit Condition for Section VI.

$\pi = [1+\phi(1/2)\theta]np - n/2$, so NNPC is thus[39]

$$p \geq \left[2 + 2\phi\left(\frac{1}{2}\right)\theta\right]^{-1}. \tag{21}$$

Therefore, NNPC can be written as

$$p \geq \begin{cases} (1+\theta)^{-1}, & \text{if} \quad p \geq \left\{2\theta\left[1-\phi\left(\frac{1}{2}\right)\right]\right\}^{-1} \\ \left[2+2\phi\left(\frac{1}{2}\right)\theta\right]^{-1}, & \text{otherwise.} \end{cases} \tag{22}$$

We denote $\phi(1/2)$ by c. The constant c satisfies $0 < c \leq 1/2$, since $\phi(0) = 0$, $\phi(1) = 1$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) \geq 0$. Taking the reciprocal of equation (15), (20), and (21), we get respectively

$$p^{-1} \leq 2(1-c)\theta, \tag{15$'$}$$

$$p^{-1} \leq 1+\theta, \tag{20$'$}$$

$$p^{-1} \leq 2+2c\theta. \tag{21$'$}$$

Thus, equation (22) can be rewritten as

$$p^{-1} \leq \begin{cases} 1+\theta, & \text{if} \quad p^{-1} \leq 2(1-c)\theta \\ 2+2c\theta, & \text{otherwise.} \end{cases}$$

Figure A I plots the right-hand sides of equation $(15)'$, $(20)'$, and $(21)'$ on the vertical axis against θ on the horizontal axis. Note however that, when $\phi''(\cdot) = 0$ (and thus $c = 1/2$), there is no intersection in the figure. In the figure, NNPC is given by equation $(20)'$ if p^{-1} is less than or equal to the right-hand side of equation $(15)'$ and by equation $(21)'$

[39] Solving $[1+\phi(1/2)\theta]np - n/2 \geq 0$ with respect to p yields equation (21).

otherwise. Hence, NNPC for $\phi''(\cdot) > 0$ can be written as

$$p^{-1} \leq \begin{cases} 1+\theta, & \text{if} \quad \theta \geq (1-2c)^{-1} \\ 2+2c\theta, & \text{otherwise.} \end{cases}$$

Rewritten, we get

$$p \geq \begin{cases} (1+\theta)^{-1}, & \text{if} \quad \theta \geq \left[1-2\phi\left(\frac{1}{2}\right)\right]^{-1} \\ \left[2+2\phi\left(\frac{1}{2}\right)\theta\right]^{-1}, & \text{otherwise.} \end{cases}$$

For $\phi''(\cdot) = 0$, NNPC is always given by equation $(21)'$, since there is no intersection in Figure A I. Hence, NNPC for $\phi''(\cdot) = 0$ is given by equation (21).

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