Product Differentiation and Collusion in a Two-Sided Media Market*

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This paper investigates the incentives to differentiate products horizontally in a collusive two-sided media market. The findings show that externalities between advertisers and consumers participating in the media have important impacts on the sustainability of collusion and the optimal product choice, namely, that platforms differentiate their products to a greater degree for large externalities to prevent collusion from breaking down.

Keywords: Product differentiation, Collusion, Two-sided markets, Externalities

Ⅰ. Introduction

Empirical and anecdotal evidence shows collusive behavior by media firms. In the Italian newspaper market, subscription prices are collusive even after the regulatory regime switched to a more liberalized system.[1] In the United States in the 1890s, the Republic and the Globe-Democrat fixed advertising rates with the Post Dispatch in St. Louis. Additionally, the Chronicle and the Post Dispatch aligned to raise the subscription rate for country delivery. Also in the United States, San Diego's Sun and Tribune formed an agreement to raise subscription rates in the city.[2] Collusion in the media market can also involve newsgathering or coverage agreements. For example, in Honolulu, Hawaii, two stations simulcast their daily morning and evening news broadcasts. Additionally, NBC and FOX affiliates set up a local news service to create cooperative, general video news coverage.[3]

Given the media's important role in providing information, its collusive practices stimulate us to consider how media product is affected by collusion. This paper studies product choices made by collusive media platforms. Although the literature has investigated the effect of collusion on product differentiation, it has not incorporated the two-sided nature of media markets. On one side, the advertisers are interested in the number of consumers; hence consumers exert positive externalities on advertisement. On the other side, depending on the media format, advertisers can exert different externalities on consumers.[4] The interdependence between the consumer and the advertising market makes media platforms behave differently from those in traditional one-sided markets. This is particularly noticeable when platforms change prices on one side and market share changes on both sides due to externalities. This feedback effect, which does not arise in one-sided markets, affects platform profits and the incentives to deviate. We explicitly account for media's two-sidedness to investigate the incentives for collusive media platforms to differentiate products horizontally, focusing particularly on the impact of externalities.[5]

To analyze the above question, we use the Hotelling framework with two media platforms and two groups of agents—advertisers and co nsumers—uniformly distributed along the unit

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line. For two-sidedness, we restrict our attention to the case that externalities between advertisers and consumers exist only in one direction, namely, that advertisers prefer to place ads on a platform with more consumers than less but that consumers are indifferent to advertising levels. Both consumers and advertisers choose one media platform to join (i.e., both groups single-home). At the initial stage, media platforms decide on their product choice, which, once chosen, remains the same for the remainder of the game; after choosing the products, platforms charge prices repeatedly. To see the incentives behind product choice as simply as possible, we only consider a discrete case for product choice. We use the grim trigger strategy to investigate the optimal product choice under collusion.

Our paper contributes to the analysis of collusion in two-sided markets and reports on the following findings. Given the product choice of platforms, price collusion is harder to sustain for a higher externality. This is because the consumer share in the price deviation is larger than the share of 1/2 in punishment and collusion and because externalities enhance the deviation profit more than punishment and collusion profits. Furthermore, externalities affect product choice. If platforms choose products that are closer substitutes, the incentives to deviate in pricing grow due to the higher price elasticity, which induces platforms to lower collusive prices to sustain collusion. Because the magnitude of the price reduction for large externalities is higher than that of small ones, platforms differentiate more for larger externalities to save the deviation cost caused by closer substitutes.

There are several theoretical papers studying collusion in markets with product differentiation, but most focus on traditional one-sided markets.[6] Within the literature on two-sided markets, the papers closest to ours are Ruhmer (2011), Antonielli and Filistrucchi (2012).[7] Ruhmer (2011) investigates the impact of externalities on the sustainability of price collusion. The results show that collusion is harder to sustain when externalities increase, and further, that the high levels of asymmetry in externalities between market sides reduce the incentives to collude. Unlike in Ruhmer's (2011) paper with given product choice, our paper endogenizes the incentives to differentiate products under collusion and shows that larger externalities can cause more differentiation. Antonielli and Filistrucchi (2012) also discuss collusive choice of products, but they do not investigate how it is affected by externalities.

The remainder of the paper is organized as follows. The model is presented in section II. Section III discusses platforms' pricing strategies and analyzes the incentives to follow the agreed product choice. Section IV examines conditions to sustain unconstrained collusive prices. Section V is devoted to the analysis of the endogenous product choice for collusive platforms. Section VI presents our conclusions.

Ⅱ. Model

Consider a Hotelling model with two platforms, 1 and 2, each serving advertisers and consumers, denoted respectively by a and v. Media platforms have fixed and marginal costs in serving these groups, which are normalized to zero for simplicity. With each group's agents located uniformly on $[0,1]$, let $\theta_k \epsilon [0,1]$ $(k \epsilon \{a,v\})$ denote the location of an agent in group k. Consumers and advertisers single-home.[8] Assume that advertisers are concerned with consumer size in the specific platform but that consumers are indifferent to advertisers.[9] Let α be the externality parameter which measures the benefit an advertiser obtains from each consumer. In this paper, we assume that $0 \leq \alpha < 1$.[10] Platform i charges participation fees p_i and s_i for advertisers and consumers, respectively. If platform i attracts n_i consumers and m_i advertisements, each agent that homes

at platform i receives the following utility:

$$u_{i\theta_v}=H_v-s_i-x_{i\theta_v}^2 \text{ and } u_{i\theta_a}=H_a+\alpha n_i-p_i-x_{i\theta_a}^2, \quad (1)$$

where $x_{i\theta_k}$ denotes the distance between platform i and agent θ_k. Let d_1 and $1-d_2$ (where $d_1 \leq 1-d_2$) represent the respective product choices for platforms 1 and 2, then $x_{1\theta_k}=|\theta_k-d_1|$ and $x_{2\theta_k}=|1-d_2-\theta_k|$. In the following, we only consider the case $d_1, d_2 \epsilon \{0, 1/4\}$ to focus on the incentives to differentiate from the fully collusive choice of products.[11] We can extend our analysis to a continuous product choice in greater detail with considerable elaboration. Advertisers and consumers obtain respective intrinsic value H_a and H_v from joining a media platform,[12] which is sufficiently large enough to fully cover both market sides. For analytical simplicity, we assume that $H_a=H_v=H$. From the Hotelling specification, the number of each group that joins platform i $(i \neq j \epsilon \{1, 2\})$ is given by

$$\begin{aligned} n_i &= \frac{d_i-d_j+1}{2}-\frac{s_i-s_j}{2(1-d_i-d_j)} \\ m_i &= \frac{d_i-d_j+1}{2}-\frac{p_i-p_j+\alpha(n_j-n_i)}{2(1-d_i-d_j)}. \end{aligned} \quad (2)$$

The game is played as follows. Media platforms simultaneously decide on product choice d_i at the beginning of time, after which the product choice is fixed forever. In each of the following periods, the platforms will choose prices simultaneously.[13] Each agent then decides the platform in which to participate. This is a game of perfect monitoring: all the stage-game actions are revealed before the beginning of the next stage.

Ⅲ. Preliminary Analysis on Pricing Strategies and Product Choice Behavior

We consider a symmetric subgame perfect equilibrium in which (i) platforms' joint equilibrium profits are highest, (ii) platforms use grim trigger strategies to support collusion in pricing, and (iii) platforms use the stage-Nash equilibrium prices off the equilibrium path. At a symmetric equilibrium, each platform offers product $d_1=d_2=d \epsilon \{0, 1/4\}$ and the same price pair when deviations do not occur.

In our model platforms have two deviation possibilities: (i) to select a product that is different from the agreed choice or (ii) to deviate from the collusive prices but honor the agreed product choice.

In the following discussion, we use the below notations to denote the associated profits generated in each case: $\pi_i^N(d_i, d_j)$ is the stage-game Nash equilibrium profit of platform i, for the product choice (d_i, d_j) by platform i and j, respectively. Similarly, given the product choice, $\pi_i^C(d_i, d_j)$ represents the profit of platform i when both platforms adopt collusive prices, and $\pi_i^D(d_i, d_j)$ is the profit of platform i when platform i deviates from collusive prices. In the above notations, $d_i=d_j=d$ except in the case where deviation on product choice occurs.

Once deviation on product choice occurs, platforms will use the stage-Nash equilibrium prices in the following periods. In our model product choice is not decided repeatedly, so we use the following condition to sustain the agreed product choice: the average collusive payoff must exceed the average payoff in the subgame after any deviation at the initial stage. Thus, the condition to deter such deviation for platform i reduces to that

$$\pi_i^C(d, d) \geq \pi_i^N(d_i^D, d),$$

where $d_i^D \neq d$.

Given a symmetric product choice we use grim trigger strategies to sustain price collusion. The incentive condition for the sustainability of collusion in pricing can be formalized as follows:

$$\begin{aligned} &\pi_i^D(d, d)-\pi_i^C(d, d) \\ &\quad \leq \frac{\delta}{1-\delta}[\pi_i^C(d, d)-\pi_i^N(d, d)] \\ &\quad \Leftrightarrow \delta \geq \hat{\delta}=\frac{\pi_i^D(d, d)-\pi_i^C(d, d)}{\pi_i^D(d, d)-\pi_i^N(d, d)}. \end{aligned} \quad (3)$$

In the following, we will analyze platforms' pricing strategies and the incentives to follow the agreed product choice.

1. Off-Path Behavior

Whether product choice deviation or price deviation occurs, platforms will play the one-shot Nash equilibrium. We use superscript N to represent the one-shot Nash equilibrium variables. Because two revenue sources exist, the one-shot Nash profit for platform i is

$$\pi_i^N = m_i^N p_i^N + n_i^N s_i^N.$$

At the pricing stage, platform i maximizes the above profit function with respect to p_i^N and s_i^N. Solving the equilibrium prices from the first-order conditions for best response and plugging them, we have the following profit for platform i:[14]

$$\pi_i^N(d_i, d_j) = K\left\{\frac{(d_i-d_j)[2(d_i-d_j+6)K^2-3\alpha^2+\alpha(d_i-d_j)K]}{2(9K^2-2\alpha^2)}+1\right\}-\frac{\alpha}{2}, \tag{4}$$

where $K = 1-d_i-d_j$.

(1) Behavior and Profits following Price Deviation

Given a symmetric product choice, the punishment prices and profits following price deviation are as follows:

$$p_i^N = p_j^N = 1-2d$$
$$s_i^N = s_j^N = 1-2d-\alpha$$
$$\pi_i^N(d, d) = \pi_j^N(d, d) = 1-2d-\frac{\alpha}{2}.$$

The externality parameter α has a negative effect on the consumer price and the punishment profit. When advertisers value consumers more highly (higher α), platforms can obtain greater revenues from advertisers by increasing the number of consumers. Thus, consumer prices are lowered, but competition does not increase equilibrium consumer shares and causes lower profits. In addition, punishment prices and profits are higher at $d = 0$ than at $d = 1/4$. Due to symmetric product choice and price competition, when platforms move from $d = 0$ to $d = 1/4$, the equilibrium share is still 1/2 on each side, whereas prices fall significantly in the face of intense price competition caused by closer substitutes.

(2) Behavior and Profits following Product Choice Deviation

When platform i deviates on product choice, we can use equation (4) to express the profit generated in each stage game, denoted by $\pi_i^N(d_i^D, d)$, by substituting deviation location d_i^D and collusive location d for d_i and d_j, respectively.

2. Incentives to Follow the Agreed Product Choice

Suppose that platform 1 deviates to $d_1^D = 1/4$ from $(d_1, d_2) = (0, 0)$. To show that the per-period deviation profit, $\pi_1^N(1/4, 0)$, is smaller than the per-period collusive profit, $\pi_1^C(0, 0)$, we first note that $\pi_1^C(0, 0)$ is larger than $\pi_1^N(0, 0)$, i.e., the collusive profit must be larger than the non-collusive, stage-Nash profit. Therefore, it suffices to show that $\pi_1^N(1/4, 0) < \pi_1^N(0, 0)$ for this deviation to be unprofitable. By calculation, we have

$$\pi_1^N\left(\frac{1}{4}, 0\right)-\pi_1^N(0, 0) = \frac{18\alpha+224\alpha^2-621}{64(81-32\alpha^2)} < 0.$$

Since α must satisfy $0 \leq \alpha < 1$ for the second-order condition $(2(1-d_i-d_j) > \alpha)$ to hold with each profile of product choice, the sign of the above expression is negative.

To grasp the intuition behind the calculation, we check the changes in profit caused by product choice deviation by platform 1, which is roughly approximated by[15]

$$\frac{\Delta\pi_1^N}{\Delta d_1} \approx \left(\underbrace{\frac{\Delta n_1}{\Delta d_1}}_{(a)} + \underbrace{\frac{\Delta n_1}{\Delta s_2}\frac{\Delta s_2}{\Delta d_1}}_{(b)}\right)s_1(d, d)$$
$$+\left(\underbrace{\frac{\Delta m_1}{\Delta d_1}}_{(a)} + \underbrace{\frac{\Delta m_1}{\Delta p_2}\frac{\Delta p_2}{\Delta d_1}}_{(b)} + \underbrace{\frac{\Delta m_1}{\Delta n_1}\frac{\Delta n_1}{\Delta s_2}\frac{\Delta s_2}{\Delta d_1}}_{(c)}\right)p_1(d,d).$$

Note that this is the analogue of the envelope theorem; each fractional term in the right-hand side corresponds to the associated partial derivative. For instance, term $\Delta n_1/\Delta d_1$ denotes how n_1 changes as d_1 changes when other variables are held constant. Similarly, we can interpret other terms on the right-hand side.

When there are no externalities $(\alpha = 0)$, the external effect of the number of consumers on the benefit of advertisers, labelled by (c),

vanishes. The terms labeled by (a) represent the so-called demand effect, which captures the direct effect of product choice change on customer base, whereas the terms labeled by (b) represent the so-called competition effect, which captures the effect through the price competition following the product choice change. In the case of no externalities, consumer and advertiser side can be treated as two independent markets, and therefore we can apply the analysis of location choice in the standard Hotelling model with quadratic transportation costs (see D'Aspremont et al., 1979); by moving forward to platform 2, platform 1 expands its market share (i.e., demand effect) but endures low price (i.e., competition effect) on each side. It is well-known that the competition effect dominates in such a model, and therefore it's unprofitable to deviate toward the competitor. With the presence of externalities, term (c) is in effect, implying that the change in the consumer side affects the advertiser side. In this case, advertiser's utility is increasing with consumer size, so platform 1 has incentives to expand its size by moving forward to platform 2. However, this effect is limited, and indeed does not overturn the effects that are present without externalities; in order for the second-order condition to hold, we need the constraint that externality is small compared to the degree of differentiation between platforms, i.e., $2(1-d_i-d_j)>\alpha$.

Suppose that platform 1 deviates to $d_1^D=0$ from $(d_1, d_2)=(1/4, 1/4)$. In this case, $\pi_1^C(1/4, 1/4)$ also outweighs $\pi_1^N(0, 0)$, and therefore it suffices to show that $\pi_1^N(0, 1/4) < \pi_1^N(0, 0)$ for the deviation being unprofitable, which is quite straightforward as follows: note that the two products characterized by $(d_1, d_2) = (0, 1/4)$ are closer substitutes than those characterized by $(0, 0)$. Hence price competition for $(0, 1/4)$ is harsher than for $(0, 0)$. Additionally, platform 1 has a location disadvantage for $(d_1, d_2)=(0, 1/4)$ compared to the case of $(0, 0)$, so the equilibrium share of consumer and advertiser generated for $(0, 1/4)$ for platform 1 is smaller than that for $(0, 0)$. Thus, platform 1's Nash-equilibrium profit for $(d_1, d_2)=(0, 1/4)$ is lower than that for $(0, 0)$.

Therefore, from the above discussions we can conclude that deviation on product choice never occurs.

3. Collusive Price

Given product choice, platforms collude on prices subject to the incentive constraint to follow collusive pricing. Depending on whether the constraint is binding, collusive prices are called unconstrained collusive prices or constrained collusive prices. In this section, we analyze only the former case and use superscript C to denote the relevant variables.

Because of the discrete choice of location, the agents in the center of the Hotelling line $(\theta_k=1/2)$ pay the highest transportation costs despite of the collusive location. From the indifference conditions of the most remote agents, we can find the unconstrained collusive prices on both sides:

$$\begin{aligned} p_i^C &= p_j^C = V+\frac{\alpha}{2}-\left(\frac{1}{2}-d\right)^2 \\ s_i^C &= s_j^C = V-\left(\frac{1}{2}-d\right)^2. \end{aligned} \tag{5}$$

Due to the symmetric product choice and equal prices, each platform will split both market sides equally and obtain the following unconstrained collusive profit:

$$\pi_i^C(d, d) = \pi_j^C(d, d) = V-\frac{1}{4}(1-2d)^2+\frac{\alpha}{4}. \tag{6}$$

Collusion internalizes the externalities, inducing no competition for consumers and advertisers. Therefore, platforms have the market power to charge advertisers higher prices for larger α. With equal market share on both sides, the collusive profits also have an increasing relationship with α. In addition, prices and profits achieve their maximum at $d=1/4$, which minimizes transportation costs on each side.

4. Price Deviation

We consider the case in which platform i deviates on collusive prices and use superscript D to denote the relevant variables. Depending on the size of α, the agreed product choice and the collusive prices, it is optimal for the deviating platform to steal either a fraction of the market or the entire market from the competitor. In the price reaction functions below, the first equations in (7) and (8) correspond to the cases in which platform i has a partial share on advertiser and consumer side, respectively, while the second equations are the corresponding reaction functions of the full share cases. Similar to the work of Armstrong (2006) and Ruhmer (2011), the price charged to one side in expressions (7) and (8) can be expressed as the sum of the standard Hotelling price without externalities (term(A)) and the adjustment factors (terms (B) and (C)), which measure the external benefit or loss to the platform. Consider the first equation in (7): term (B) represents consumers' impact on advertising fee. As shown in equation (2), a unit of price advantage on the consumer side increases the number of consumers in the deviating platform by $1/[2(1-2d)]$. Thus, if the amount of undercut is $(s_j^C - s_i^D)$, consumer demand increases by $(s_j^C - s_i^D)/[2(1-2d)]$, which, in turn, raises advertisers' utility by $\alpha(s_j^C - s_i^D)/[2(1-2d)]$. Hence, platform i can charge a higher advertising fee if it has the advantage on the consumer side. In the first equation of (8), term (C) measures the external loss to platform i stemming from attracting an extra consumer, where $\alpha/[2(1-2d)]$ is the extra advertisers platform i attracts when it has an extra consumer and p_i^D is the profit earned from an extra advertiser. In our model, we allow advertisers to care about consumers but consumers to be indifferent to advertisers, so term (B) only appears in (7), while term (C) only appears in (8).

$$p_i^D = \begin{cases} \underbrace{\dfrac{(1-2d)+p_j^C}{2}}_{(A)} + \underbrace{\dfrac{\alpha(s_j^C - s_i^D)}{2(1-2d)}}_{(B)} \\ \quad if\ p_j^C < 3(1-2d) - \dfrac{\alpha(2\alpha + s_j^C)}{2(1-2d)} + \dfrac{\alpha}{2} \\ \underbrace{p_j^C - (1-2d)}_{(A)} + \underbrace{\dfrac{\alpha(s_j^C - s_i^D)}{(1-2d)}}_{(B)} \\ \quad if\ p_j^C \geq 3(1-2d) - \dfrac{\alpha(2\alpha + s_j^C)}{2(1-2d)} + \dfrac{\alpha}{2} \end{cases} \tag{7}$$

$$s_i^D = \begin{cases} \underbrace{\dfrac{(1-2d)+s_j^C}{2}}_{(A)} - \underbrace{\dfrac{\alpha p_i^D}{2(1-2d)}}_{(C)} \\ \quad if\ s_j^C < 3(1-2d) - \dfrac{\alpha(\alpha + p_j^C)}{2(1-2d)} - \dfrac{\alpha}{2} \\ \underbrace{s_j^C - (1-2d)}_{(A)} \\ \quad if\ s_j^C \geq 3(1-2d) - \dfrac{\alpha(\alpha + p_j^C)}{2(1-2d)} - \dfrac{\alpha}{2} \end{cases} \tag{8}$$

We have three share regimes in price deviation for platform i: (i) to have a partial share on each side, i.e., $m_i^D,\ n_i^D < 1$; (ii) to have a partial share on one side but a full share on the other side, i.e., $m_i^D < 1,\ n_i^D = 1$ and $m_i^D = 1,\ n_i^D < 1$;[16] and (iii) to have a full share on both sides, i.e., $m_i^D = n_i^D = 1$.

From above, we can get the following optimal deviation prices.

$$p_i^D = \begin{cases} (1-2d)\left[\dfrac{1-2d+p_j^C}{2(1-2d)+\alpha} + \dfrac{\alpha(s_j^C + p_j^C)}{4(1-2d)^2 - \alpha^2}\right] \\ \quad if\ m_i^D,\ n_i^D < 1 \\ \dfrac{1}{2}[p_j^C + \alpha + (1-2d)] \\ \quad if\ m_i^D < 1,\ n_i^D = 1 \\ p_j^C + \alpha - (1-2d) \\ \quad if\ m_i^D = n_i^D = 1 \end{cases} \tag{9}$$

$$s_i^D = \begin{cases} s_j^C + (1-2d)\dfrac{(1-2d)(2-4d-\alpha-2s_j^C) - p_j^C\alpha}{4(1-2d)^2 - \alpha^2} \\ \quad if\ m_i^D,\ n_i^D < 1 \\ s_j^C - (1-2d) \\ \quad if\ m_i^D < 1,\ n_i^D = 1 \\ s_j^C - (1-2d) \\ \quad if\ m_i^D = n_i^D = 1 \end{cases} \tag{10}$$

The market shares can be derived from the above prices, and the corresponding deviation profits are given by

$$\pi_i^D(d,d)=\begin{cases}\dfrac{L}{2[4(1-2d)^2-\alpha^2]} & \text{if } m_i^D,\ n_i^D<1\\ \dfrac{p_j^C}{4}+s_j^C+\dfrac{(\alpha+p_j^C)^2}{8(1-2d)}+\dfrac{\alpha}{4}-\dfrac{7(1-2d)}{8} & \text{if } m_i^D<1,\ n_i^D=1\\ p_j^C+s_j^C+\alpha-2(1-2d) & \text{if } m_i^D=n_i^D=1,\end{cases} \tag{11}$$

where $L=2(1-2d)^3+(1-2d)^2(2p_j^C+2s_j^C-\alpha)+(1-2d)[s_j^C\alpha-p_j^C\alpha+(p_j^C)^2+(s_j^C)^2]+(p_j^C-\alpha)s_j^C\alpha$.

Substituting collusive prices in (5) into the above expressions, we have

$$\pi_i^D(d,d)=\begin{cases}\dfrac{M}{32[4(1-2d)^2-\alpha^2]} & \text{if } m_i^D,\ n_i^D<1\\ \dfrac{10V+3\alpha}{8}+\dfrac{(2V+3\alpha)^2}{32(1-2d)}+\dfrac{1-2d}{128}R & \text{if } m_i^D<1,\ n_i^D=1\\ 2V+\dfrac{3\alpha}{2}-\dfrac{(1-2d)(5-2d)}{2} & \text{if } m_i^D=n_i^D=1,\end{cases} \tag{12}$$

where $M=2(1-2d)^5+(1-2d)^4(\alpha-16)-4(1-2d)^3(4V+\alpha-8)-2(1-2d)^2(4V\alpha-32V-\alpha^2)+4(1-2d)(4V\alpha-\alpha^2+8V^2)+8V\alpha(2V-\alpha)$ and $R=(1-2d)^2-4(2V+3\alpha+38-20d)$.

It is easy to show that profits for price deviation are increasing with α. Suppose that platform i charges the same deviating price for consumers as before, regardless of the change in externalities, which still induces the same number of consumers.[17] Although the above situation is not optimal, platform i already obtains a higher profit for larger α. This is because advertisers gain a higher utility from the increase of α, which allows platform i not only to charge higher advertising fees but also to expand its market share of advertisers.[18] Because it is always able to mimic the pricing strategy for consumers based on the previous case without any change in externalities, platform i can obtain much higher profits under optimal conditions. Therefore, price deviation is more profitable when α increases. We also find that deviation profits are higher when the products are closer substitutes (larger d). This is because platforms can further expand their demand with a slight reduction in prices.

Ⅳ. Discount Factor Restriction to Sustain Unconstrained Collusive Prices

In this section, we will examine conditions under which unconstrained collusive pricing is sustainable.

By plugging the relevant profit functions, we can derive the minimum discount factor $\hat{\delta}$ required to sustain the unconstrained collusive prices. By doing the comparative statics for the minimum discount factors with respect to the externality parameter α and the product choice d, we have the following lemmas.

Lemma 1 *Given the product choices of platforms, it is more difficult to sustain the unconstrained collusive prices as the externality parameter increases.*

Lemma 2 *Given the intensity of externalities, it is more difficult to sustain the unconstrained collusive prices as products become closer substitutes.*

We consider the impact of the externality parameter α. We know that regardless of an increase in consumers, even for the same share, platforms can enjoy greater revenues by extracting more from advertisers due to their higher degree of reaction to the increase of α. Here, by lowering consumer prices, the deviating platform can enjoy more consumers, larger than the equilibrium share of 1/2 in the punishment and collusive phases. This implies that α has a greater effect on $\pi_i^D(d,d)$ than $\pi_i^N(d,d)$ and $\pi_i^C(d,d)$. Thus, platforms have high incentives to deviate for large α. The intuition for Lemma 2 is the following. By using

the same price differential for consumers, the deviating platform can obtain a larger consumer share at $d=1/4$ than that at $d=0$ due to the higher price elasticity for closer substitutes. Additionally, the fact that collusive prices at $d=1/4$ are higher than those at $d=0$ gives the deviating platform a greater advantage in trading off price reduction for consumer market expansion. Because the consumer size in deviation is much higher than that in competition or collusion, advertisers enjoy more and thus the deviating platform enjoys higher revenues from closer substitutes. Therefore, unconstrained collusive prices are harder to sustain as products become closer substitutes. Based on the case in which regime of $m_i^D=n_i^D=1$ occurs for both product choices, we draw Figure 1 to illustrate the results from Lemmas 1 and 2. Suppose that there are two values for $\alpha:\alpha_1$ and α_2 with $\alpha_1>\alpha_2$. On the δ-axis, the first two points denote $\hat{\delta}|_{d=1/4}$ for α_1 and α_2, respectively; the last two points denote $\hat{\delta}|_{d=0}$ for α_1 and α_2, respectively. As it can be seen for $d\epsilon\{0,1/4\}$, $\hat{\delta}$ required for α_1 is higher than that for α_2. In addition, for $\alpha_i\,(i=1,2)$, $\hat{\delta}$ required for $d=1/4$ is higher than that for $d=0$.

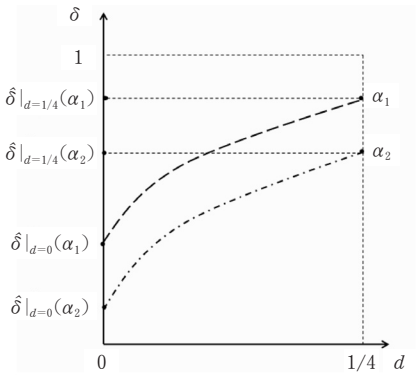

Figure 1: The Relationship Among d, α and δ

V. Product Differentiation for Collusive Platforms

In this section, we will consider the optimal product choice when collusion is sustainable. To this end, we need to find out the maximum collusive profit for a given product choice. Thus, we consider the following maximization problem for platform i:

$$\max_{s,p}\ \bar{\pi}_i^C(d,d)=\frac{s+p}{2}$$
$$s.t.\ \delta\geq\frac{\bar{\pi}_i^D(d,d)-\bar{\pi}_i^C(d,d)}{\bar{\pi}_i^D(d,d)-\bar{\pi}_i^N(d,d)}.$$

If the value of the discount factor is not lower than $\hat{\delta}$, unconstrained collusive prices are sustainable. If not, other prices are required to make collusion successful. Because the cases in which incentive constraints are nonbinding are similar to those in Section III-3, here we only analyze the binding constraint case. Let p and s denote the constrained collusive prices for advertisers and consumers, respectively, and let the top bar denote the binding constraint cases for profits. The one-shot profit from punishment is independent of collusive prices so that $\bar{\pi}_i^N(d,d)=\pi_i^N(d,d)=1-2d-\alpha/2$ as before. The profits for price deviation are similar to those in (11), which only requires replacing s_j^C and p_j^C by s and p, respectively. After performing the calculation, we have

$$\bar{\pi}_i^C(d,d)=\frac{s+p}{2}=\begin{cases}[2(1-2d)-\alpha]\dfrac{3\delta+1}{2(1-\delta)} & if\ \delta<1/3\\[2ex] [2(1-2d)-\alpha]\dfrac{2-3\delta}{2(1-2\delta)} & if\ 1/3\leq\delta<1/2.\end{cases}\tag{13}$$

In the above expressions, the first line corresponds to the constrained profit obtained when the deviation to have a partial share on each side is deterred. By contrast, the second line refers to the profit obtained when the deviation to capture the whole share on each side is deterred.[19] We only have constrained profit for $\delta<1/2$. This is because for large δ, the incentive constraint is nonbinding and thus the unconstrained collusive profits are obtained.

To find out the optimal product choice, we need to compare collusive profit in different locations and then choose the one which offers

the larger profit. In particular, we discuss how the optimal product choice depends on the level of discount factor, or the level of externality. By the formulas of the associated profits in (6) and (13), we can find the region of the parameter for which either close substitute ($d = 1/4$) or remote substitute ($d = 0$) is optimal. However, this argument involves tedious cases due to full/partial share consideration in deviation (by (13)). To simplify the argument without sacrificing the essence, we let $V \geq 13/4$,[20] so that the deviating platform's shares satisfy $m_i^D = n_i^D = 1$ in the unconstrained collusion regardless of product choice. With this restriction, the threshold of δ between constrained and unconstrained collusion is indeed between 1/3 and 1/2, and therefore the collusive profits are derived as the following manner; the constrained collusion with m_i^D, $n_i^D < 1$, for $\delta < 1/3$, the constrained collusion with $m_i^D = n_i^D = 1$, for $1/3 \leq \delta < \hat{\delta} \leq 1/2$, and the unconstrained collusion for $\delta > \hat{\delta}$. If V is smaller, the form of collusive profits need modification, which implies that the threshold value of δ for the following result must differ, but the argument is essentially the same.

Let $d^*(\delta, \alpha)$ denote the optimal product choice for collusive platforms as a function of the discount factor δ and the externality parameter α.

By analyzing the impact of δ on the optimal product choice, we have the following results for given α.[21]

Proposition 1 Suppose $V \geq 13/4$. *Then platforms will choose $d^*(\delta, \alpha)$ using the following method:*

Define $\tilde{\delta} = (4V - 5 + 5\alpha)/[8(V + \alpha - 1)]$.

If $\delta \geq \tilde{\delta}$, *platforms choose* $d^*(\delta, \alpha) = 1/4$. *Otherwise,* $d^*(\delta, \alpha) = 0$.

Note that $\tilde{\delta}$ is the threshold discount factor at which the colluding platforms are indifferent to locating between at $d = 1/4$ and $d = 0$.

The intuition of Proposition 1 is the following. When δ is high, platforms are not constrained by incentive constraints at both product choices, so they maximize collusive profits with respect to both product choices and prices. Because $d = 1/4$ minimizes transportation costs, platforms choose $d^*(\delta, \alpha) = 1/4$ and charge unconstrained collusive prices. When δ becomes sufficiently low, the constraint is binding for $d = 1/4$ but not for $d = 0$. In this case, platforms charge unconstrained collusive prices at $d = 0$ which are independent of δ, whereas at $d = 1/4$, they must reduce the constrained prices to sustain collusion as δ decreases. Because by our definition $\tilde{\delta}$ is the discount factor that makes the highest possible collusive profit equal at both product choices, as long as $\delta > \tilde{\delta}$, constrained profit at $d = 1/4$ is still higher than the unconstrained collusive profit at $d = 0$. In contrast, when $\delta < \tilde{\delta}$, the opposite case occurs. Finally, for very low discount factors, both constraints at $d = 1/4$ and $d = 0$ are binding, and thus constrained prices are charged. At $d = 1/4$, however, platforms cannot sustain the same maximum collusive prices as those at $d = 0$. This occurs because platforms have a higher incentive to deviate at $d = 1/4$. The reason is straightforward: by lowering consumer prices, platforms can obtain a larger consumer share in price deviation than in competition. Moreover, with the same price reduction, the consumer share in deviation at $d = 1/4$ increases much more than that at $d = 0$ due to the higher price elasticity for closer substitutes. This allows advertisers to obtain greater utility and platforms to enjoy higher revenues from price deviation at $d = 1/4$. Thus, due to the higher incentives to deviate, platforms must decrease collusive prices at $d = 1/4$ more than those at $d = 0$ to make collusion sustainable. Therefore, platforms increase differentiation to obtain higher collusive profits for a low enough δ (that is, $d^*(\delta, \alpha) = 0$).

Figure 2 illustrates the optimal product choice in the $\alpha - \delta$ plane. The solid curves, from top to bottom, represent $\hat{\delta}|_{d=1/4}$, $\tilde{\delta}$ and $\hat{\delta}|_{d=0}$ as

functions of α, respectively. From Proposition 1 we have $d^* = 1/4$ for $\delta \geq \tilde{\delta}(\overline{\alpha})$ for a fixed $\overline{\alpha}$. As it can be seen from Figure 2, the collusive platforms choose $d^*(\delta, \alpha) = 1/4$ in the dark area; by contrast, they choose $d^*(\delta, \alpha) = 0$ in the other area.

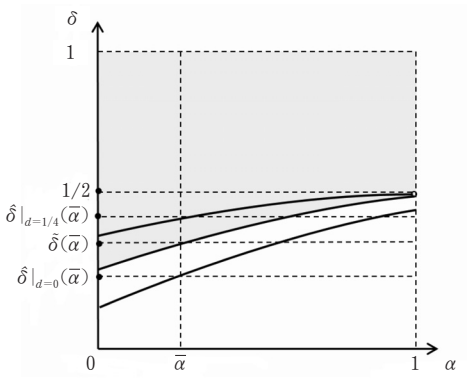

Figure 2: Optimal Product Choice for Given α or δ

In Figure 2, by fixing δ at some level, we can also show how α's change affects the optimal product choice. The findings are summarized in the following proposition.

Proposition 2 *Suppose $V \geq 13/4$. Then platforms will choose $d^*(\delta, \alpha)$ using the following method:*

Define $\tilde{\alpha} = [4V(1-2\delta)+8\delta-5]/(8\delta-5)$.

(i) For $\tilde{\alpha} \geq 1$, platforms choose $d^*(\delta, \alpha) = 1/4$ regardless of the size of α;

(ii) For $0 < \tilde{\alpha} < 1$, platforms choose $d^*(\delta, \alpha) = 1/4$ if $\alpha < \tilde{\alpha}$. Otherwise, $d^*(\delta, \alpha) = 0$;

(iii) For $\tilde{\alpha} \leq 0$, platforms choose $d^*(\delta, \alpha) = 0$ regardless of the size of α.

Note that $\tilde{\alpha}$ is the threshold externality parameter at which the colluding platforms are indifferent to locating between at $d = 1/4$ and at $d = 0$.

The intuition is as follows. As α increases, for each product choice, the deviation incentive gets larger and thus unconstrained collusion gets harder to sustain. To see the intuition, let us consider a particular δ between $\hat{\delta}|_{d=1/4}$ for $\alpha = 0$ and 1/2. If α is small, the deviation incentive is so low that unconstrained collusive prices can be sustainable at both choices. In this case, platforms choose $d^*(\delta, \alpha) = 1/4$. For medium α, since the deviation incentive caused by the increase of α affects first for $d = 1/4$ (by Lemmas 1 and 2), the constraint is binding only for $d = 1/4$ but not for $d = 0$. In this case, platforms compare the constrained collusive profit for $d = 1/4$ with the unconstrained collusive profit for $d = 0$: they choose $d^*(\delta, \alpha) = 1/4$ for $\alpha < \tilde{\alpha}$ but otherwise $d^*(\delta, \alpha) = 0$ for $\alpha \geq \tilde{\alpha}$. When deviation incentive is high enough to make unconstrained collusive prices unstainable at $d = 1/4$, colluding platforms must lower prices. For α such that $\alpha < \tilde{\alpha}$, the deviation incentive is not large. This implies that the price reduction is small, so that the constrained profit at $d = 1/4$ is larger than the unconstrained profit at $d = 0$. In contrast, for a large externality parameter such that $\alpha \geq \tilde{\alpha}$, colluding platforms must lower prices significantly at $d = 1/4$ to the level for which they receive a smaller collusive profit than the unconstrained collusive profit at $d = 0$. Therefore, platforms choose $d^*(\delta, \alpha) = 1/4$ if $\alpha < \tilde{\alpha}$ but $d^*(\delta, \alpha) = 0$ if $\alpha \geq \tilde{\alpha}$. Finally, for large α, the deviation incentive is so large that the constraints can be binding at both product choices. In this case, constrained collusive prices are charged. By using the logic analogous to that of Proposition 1, we have $d^*(\delta, \alpha) = 0$.

Ⅵ. Discussion and Conclusions

This paper investigates the impacts of externalities on collusive behavior in the media market. We use the single-homing model of two-sided markets with once and for all product choice and a pricing game that is repeated infinitely. Additionally, we assume grim trigger strategies to support collusion in pricing. Our findings show that externalities affect platforms' incentives to deviate and thus the optimal product choice: the larger the

externalities, the higher the degree of product differentiation for collusive platforms.

We now conclude by discussing the robustness of our findings in a more general setting. In the analysis, we allow consumers to be indifferent to advertising levels. This does not seem to be a very restrictive assumption. Ads in some media formats such as newspapers and magazines are not especially intrusive so that the disutility from ads is likely to be less pronounced and, in some cases, even neglected. If consumers and advertisers are concerned about each other, the analysis will be similar to the scenario in which the value of externalities is fixed on one side but changes on the other. Compared to the case without externalities on the consumer side, the feedback loop changes: the increased number of advertisers, caused by more consumers by undercutting the competitor's price, will reduce the number of consumers if they, for instance, dislike ads. In this case the incentives to collude are altered without changing the qualitative results. We also assume that the intrinsic values are identical and the transportation cost parameters are equal to 1 on both market sides. The modification of different intrinsic values directly affects only unconstrained collusion and does not change substantially any of the results. Transportation cost parameters can be interpreted as a kind of market power to charge prices such that their different values would alter the findings quantitatively.

In our research, we use the grim trigger strategy to analyze collusive behavior. Future research can investigate how our findings change when other punishment mechanisms, such as the stick and carrot strategy, are used to support collusion. Another area of research is studying the use of other pricing strategies (e.g., discriminatory pricing or no pricing for a group of agents) and investigating their impact on collusive product choice. For future research, we hope this paper has shed some light on the understanding of the fundamental interactions underlying collusion in two-sided markets.

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Notes

1) See Argentesi and Filistrucchi (2007) for details.
2) See Adams (1996, 2002) for details.
3) For further examples of collusion in the media, see Dewenter et al. (2011) and Ruhmer (2011).
4) Due to the strong intrusiveness on program content, commercials on television and radio are usually nuisances for consumers. In this case, advertising exerts negative externalities on consumers. For some other media formats, such as newspapers and magazines, ads do not interrupt consumers in the same way, so disutility of ads is likely negligible. In addition, consumers may prefer ads that provide information they are looking for. For instance, consumers prefer to read more fashion ads in Elle. In the latter two cases, advertising exerts zero or positive externalities on consumers.
5) Note that the analysis here can also apply to some other two-sided markets that share the same structure with the media.
6) Under the assumption of fixed and symmetric product choice in the Hotelling model, Chang (1991) analyzes the relationship between the degree of product differentiation and the ability to collude. The results show that collusion is more difficult to sustain the smaller the degree of product differentiation. Some papers such as Deneckere (1983), Majerus (1988), Wernerfelt (1989) and Ross (1992) also study such issues, but with a representative consumer model. Different from the above papers, Hackner (1995) instead endogenizes the incentives to differentiate products. He shows that firms will choose an intermediate degree of differentiation (i.e., the firms locate at 1/4 and 3/4, respectively) with a sufficiently high discount factor, and that the lower the discount factor, the more firms are forced to increase differentiation.
7) Relatedly, Dewenter et al. (2011) study the ef-

fects that collusion can have in newspaper markets and obtain the result that collusion may improve welfare. Because of the use of a representative consumer model, however, Dewenter et al. (2011) cannot deal with product choice as we have done in this paper.

8) The framework captures a common phenomenon in media markets. Consumers always choose one media platform at any given point in time. For some media such as newspapers, consumers can make choices of a newspaper that are persistent over time. In addition, due to contractual restrictions or limited budget, advertisers may select one media platform to host ads. The paper by Kaiser and Wright (2006) provides evidence that the model of two-sided single-homing fits German magazines best.

9) In our model, it is implicitly assumed that all media consumers watch the ads and purchase one unit of the corresponding product. Thus, each advertiser wants to reach each consumer exactly once, meaning that they only select the platform on which to advertise, not how many ads to place. Here, the number of advertisers is equivalent to the intensity of advertising.

10) The value restriction for α will be explained in footnote 14.

11) Product choice $d_1 = d_2 = 1/4$ jointly minimizes the total transportation cost.

12) On the consumer side, H_v is the benefit consumers obtain from consuming perfect media products without advertising and price. On the advertiser side, H_a can be stated as the indirect benefit of advertising, while the term αn_i constitutes the direct benefit of advertising. Here, even if no purchase occurs, i.e., $\alpha n_i = 0$, firms are still willing to advertise to promote their corporate philosophy and foster a positive company image.

13) Product choice is substantively inflexible compared to price policy. In practice, platforms must decide on their products in advance. Additionally, relocation requires factoring in sunk costs, reputation costs and/or transaction costs, which can be prohibitively high, rendering relocation unprofitable.

14) The second-order condition is satisfied by $2(1-d_i-d_j) > \alpha$. If externality is too large compared to the degree of differentiation between platforms, no interior solution exists. There could be equilibria where the deviating platform attracts all agents from at least one side. In this case, the deviation profit is not as high, but the main argument, discussed below, would still apply qualitatively. In this paper, to satisfy the second-order condition for any combination of product choice, we need to restrict parameter values for externality to be that $0 \leq \alpha < 1$.

15) This formula is the envelope condition in the case of continuous product choice, and the precision of approximation increases as the change in Δd_1 gets smaller. Since Δd_1 is 1/4 due to our restriction on product choice, the following intuition is only suggestive. Nonetheless, this formula captures the main factors of changes, and helps us understand the impacts of them.

16) Due to the model setup, the deviating platform has a higher incentive to obtain more consumers to increase revenues. Therefore, we only have $m_i^D < 1, n_i^D = 1$ for regime (ii).

17) From the expressions in (2), we have $n_i^D = 1/2 - (s_i^D - s_j^C)/[2(1-2d)]$. Because α has no effect on s_j^C, the number of consumers does not change if s_i^D is changed, which is similarly irrelevant to the change of externalities.

18) Note that the number of advertisers does not change when reaching to 1.

19) In our paper, the deviation to have a full share on only one side (i.e., the consumer side) does not arise for constrained collusion.

20) With expressions in (5) and the second conditional expressions in (7) and (8), we can get $V \geq (\alpha - 8\alpha^2 + 26)/[4(\alpha+2)] = g$. This condition guarantees that regime of $m_i^D = n_i^D = 1$ occurs for both product choices for unconstrained collusion. Because $g'_\alpha = -2(\alpha+3)(\alpha+1)/(\alpha+2)^2 < 0$ and $0 \leq \alpha < 1$, we can get that $V \geq g|_{\alpha=0} = 13/4$.

21) The formal proofs of the propositions are available from the author upon request.

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